

**Zeitschrift:** IABSE proceedings = Mémoires AIPC = IVBH Abhandlungen  
**Band:** 12 (1988)  
**Heft:** P-128: Back-analysis for ground parameters in tunneling  
  
**Artikel:** Back-analysis for ground parameters in tunneling  
**Autor:** Hisatake, Masayasu / Maeda, Yukio  
**DOI:** <https://doi.org/10.5169/seals-41130>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 22.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

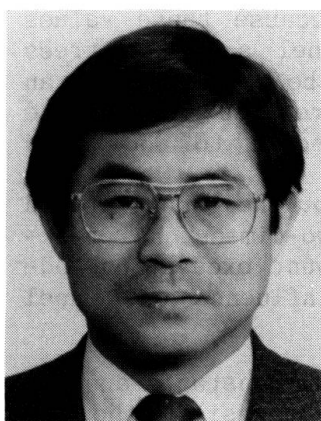
## Back-Analysis for Ground Parameters in Tunneling

Calcul inverse des indices de sols dans la construction de tunnels

Rückwärtsermittlung von Bodenkennwerten im Tunnelbau

### Masayasu HISATAKE

Assistant Professor  
Kinki University  
Osaka, Japan



Masayasu Hisatake, born 1948, is an assistant Professor of Civil Engineering at Kinki University. He obtained the degree of Dr. Engineering at Osaka University in 1983 and has been engaged in research on numerical analysis of tunneling.

### Yukio MAEDA

Professor of Civil Engineering  
Kinki University  
Osaka, Japan



Yukio Maeda was formerly a Professor of Civil Engineering at Osaka University, where he was awarded the title of Professor Emeritus in 1985. He has been a Professor of Structural Engineering at Kinki University since April 1985. He is the Chairman of the Technical Committee of IABSE.

### SUMMARY

In order to estimate initial stresses and mechanical properties of the ground around a tunnel, a three dimensional back-analysis method based on an optimization technique is proposed. By using field measurements of displacements caused by tunnel face excavations, equivalent values of the ground parameters are back-analyzed, in which non-homogeneity of the ground, stress relief condition and structural conditions such as cracks and joints are taken into consideration. Application of the method to two actual tunnels in extremely squeezing and time dependent grounds shows appropriateness of the method through comparison of analytical and field results.

### RÉSUMÉ

Une méthode tridimensionnelle, basée sur une technique d'optimisation, permet de déterminer les contraintes initiales et les propriétés mécaniques du sol à proximité d'un tunnel. Les mesures de déplacement effectuées lors de l'excavation du tunnel permettent le calcul inverse des valeurs équivalentes des paramètres de base; ces valeurs tiennent compte de la non-homogénéité du sol, de la relaxation du sol et des conditions structurales telles que fissures et joints. La méthode a été appliquée dans deux cas concrets de tunnels: dans un sol très compressible et dans un sol de qualité variable dans le temps. La comparaison des résultats de calcul et de mesures démontre la valeur de cette méthode.

### ZUSAMMENFASSUNG

Zur Abschätzung der Initialspannungen und der mechanischen Bodeneigenschaften im Bereich eines Tunnels wird eine dreidimensionale, auf einer Optimierungsmethode basierende Rückwärtsbewegung vorgeschlagen. Unter Verwendung der Verformungsmessungen beim Tunnelausbruch werden die Bodenkennwerte unter Berücksichtigung der Inhomogenität, des Spannungsabbaus und der vorhandenen Risse und Klüfte ermittelt. Die Methode wurde bei zwei Tunnelbauprojekten in sehr kompressiblem Boden bzw. einem Baugrund mit zeitabhängigem Verhalten angewendet. Die Genauigkeit der Resultate wird durch den Vergleich mit den analytisch bestimmten Werten bestätigt.



## 1. INTRODUCTION

In planning of a tunnel, it is fundamentally important to estimate virgin stresses and geological conditions. The movement of the tunnel is, however, affected not only by these factors, but also by structural conditions such as cracks and joints which exist in rocks before the tunnel excavation. Therefore, the values of mechanical properties of the ground depend on the size of the tunnel. Also, the tunnel excavation may be interpreted as a kind of stress relief. This means that the mechanical characteristics of the rocks should be determined under the stress relief condition by taking the structural characteristics into account.

In a stage of tunnel planning, however, it is impossible to find the values of virgin stresses and the mechanical properties above mentioned along an entire tunnel line, because of long distance tunnel execution.

If displacements measured after the excavations of the tunnel face are able to be back-analyzed to estimate the values of the parameters three-dimensionally, these values may be recognized as the equivalent ones, because these values reflect the structural characteristics, size of the tunnel and the stress relief condition. In other words, this back-analysis can be recognized as an analysis for in-situ experiments of a real scale. The results of sequence analysis which uses the back-analyzed values may be expected to show good agreements with the field measurements.

Some back-analysis methods for a tunnel lining[1]-[3] and for the ground around a tunnel[4]-[6] have been proposed. As shortcoming of two-dimensional back-analysis for the ground is that it can not take into account excavation-and-lining-construction sequence, even though it gives big influence on tunnel movement.

In this paper, in order to evaluate the equivalent mechanical constants and the initial stresses of the ground, a three-dimensional back-analysis method is proposed and is applied to practice at tunnel sites, in which a simplex and a finite element methods are employed. The difficulty of the three-dimensional back-analysis is mainly due to computation time, costs and input data. In order to overcome the difficulty, a simplified method is presented here.

In the first part, a back-analysis method for an elastic and time dependent tunnel is developed, and the demonstrative examples are shown.

In the second part, two practical applications of the method are shown to determine an optimum shape of a tunnel in an extremely squeezing ground and to estimate displacements of a time dependent tunnel. In the both applications, the analytical results are compared with field records, and the applicability of this method is confirmed.

## 2. BACK-ANALYSIS BY OPTIMIZATION METHOD

### 2.1 Elastic analysis

#### 2.1.1 Formulation

In a practical tunnel construction, shotcreting, steel supporting or rock bolting is carried out at the newly appeared tunnel inside after a tunnel face is excavated, and this construction sequence is repeated. Since the construction sequence affects tunnel movement very much, so it is extremely important in the back-analysis to take this sequence into account. In other words, the three-dimensional analysis is essential.

The general concept of the back-analysis coupled with an optimization method is that the analytical results would coincide with the measured ones, if the equivalent input values could be given to the analysis. The equivalent input values are back-analyzed by making the following objective function minimum:

$$J = \sum_{i=1}^{n1} (u_i - u_i^*)^2 + \sum_{i=1}^{n2} (p_i - p_i^*)^2 + \sum_{i=1}^{n3} (q_i - q_i^*)^2 \longrightarrow \text{Minimum} \quad (1)$$

Constrains:  $E > 0$ ,  $0 < \nu < 0.5$ ,  $K > 0$ ,  
 where  $u$ : displacement,  $p$ : stress,  $q$ : strain,  $\nu$ : Poisson's ratio,  
 $E$ : Young's modulus,  $K$ : lateral pressure ratio

$n1, n2$  and  $n3$  are the number of measurement, and "\*" means the measured values.  
 $u$ ,  $p$  and  $q$  are the functions on unknowns such as virgin stresses and mechanical properties.

In an elastic back-analysis,  $E$ ,  $\nu$ , and  $K$  are analyzed by the optimization method under consideration of the construction sequence. It is assumed in this analysis that the vertical and horizontal initial stresses  $\sigma_v, \sigma_h$  of the ground are defined as  $\sigma_v = \gamma h$  and  $\sigma_h = K \sigma_v$ , where  $\gamma$  and  $\gamma h$  are, respectively, unit weights of the ground and overburden. Nodal forces in FEM which are released at the excavation of the tunnel face are calculated from  $\sigma_v$  and  $\sigma_h$ . If displacements measured in the tunnel are symmetrical to the vertical plane through a tunnel axis, this assumption may be applied to this case. The measurement of stresses and strains at the tunnel inside is much more difficult than that of the displacements, so the displacement measurement is usually conducted.

All unknowns, of course, may be searched simultaneously by the optimization method. In this case, however, the computation time increases very much, particularly in the three-dimensional analysis. If the number of unknowns can be decreased, the computation time decreases exceedingly. So, the following method is proposed here:

In ordinary cases, an approximate range of the  $K$  value can be estimated from the measured displacements before doing back-analysis. So, it is possible to calculate the minimum value of  $J$  ( $=J_{min}$ ) by the optimization method for respective given  $K$  value, and the relation between  $J_{min}$  and  $K$  is drawn as shown in Fig.1. The optimum value of  $K$  ( $=K_{opt}$ ) which gives the minimum value of  $J_{min}$  is easily estimated from Fig.1, and the optimum values of  $E$  ( $=E_{opt}$ ) and  $\nu_{opt}$  are also calculated by giving  $K_{opt}$  into Eq.(1).

### 2.1.2 Numerical example

The validity of the method is checked by numerical experiments in the followings:

Firstly, in order to calculate displacements and stresses in the linings caused by one excavation of the tunnel face, three-dimensional finite element analysis is carried out. Fig.2 and Table 2 show tunnel geometry employed and analytical conditions respectively. This analysis is called as SA(Sequence Analysis) from now on. Secondly, the back-analysis is carried out by giving the

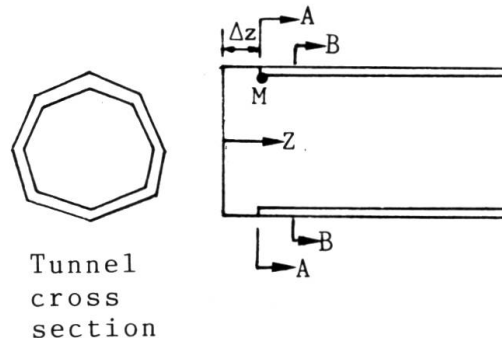
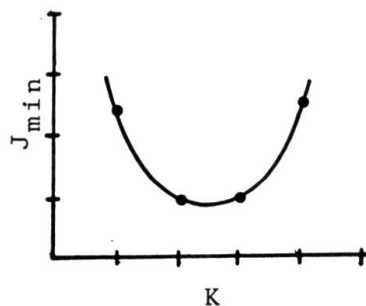


Fig.1 Relationship between  $J_{min}$  and  $K$  Fig.2 Excavation ( $\Delta z$ ) of tunnel face



Table 1 Analytical conditions

Maximum excavation width	10 m
Excavation length along tunnel axis	2.5 m
Thickness of lining	0.2 m
E of the ground	5000 tf/m <sup>2</sup>
Poisson's ratio of the ground	0.3
Lateral pressure ratio	0.5
Unit weight of the ground	2 tf/m <sup>2</sup>
Overburden	100 m
E of lining	2,000,000 tf/m <sup>2</sup>
Poisson's ratio of lining	0.15

displacements obtained by SA. In this back-analysis, four values of  $J_{min}$  are calculated by putting four arbitrary values of  $K$  into Eq.(1) and the convergency characteristics of  $J$  and the relationship between  $J_{min}$  and  $K$  are shown in Fig.3.

From Fig.3  $K_{opt}$  is easily estimated as 0.5, so  $E_{opt}$  and  $\nu_{opt}$  are also calculated by applying  $K=0.5$  to Eq.(1). Fig.4 shows the convergency characteristics of  $J$ ,  $E$  and  $\nu$  in the case of  $K=0.5$ .

In the practical application of this method, it is difficult to avoid error in determining  $K_{opt}$ . Accordingly, the stability of the results obtained by this method is checked as follows:

Consider such a case that  $K_{opt}$  is determined from Fig.3 as 0.45 which has 10% error, and that the optimum values of  $E$  and  $\nu$  are analyzed by putting  $K_{opt}=0.45$

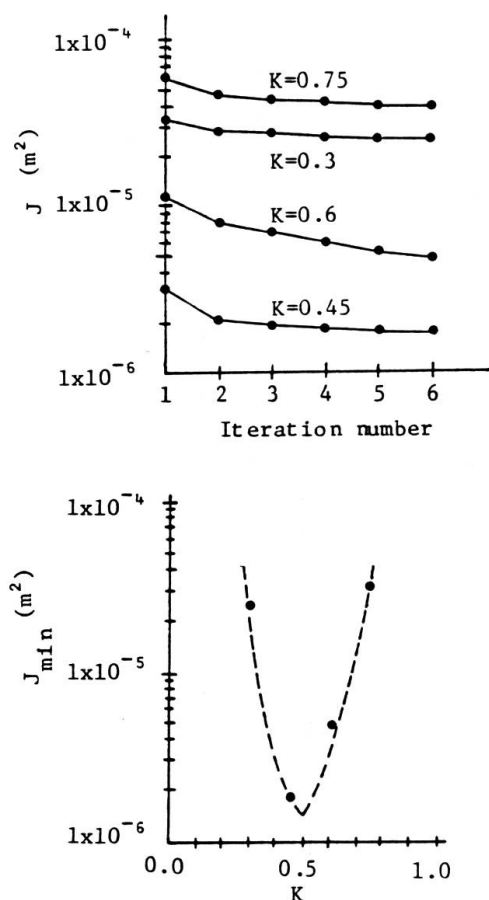


Fig.3 Convergency characteristics of  $J$  and relationship between  $J_{min}$  and  $K$

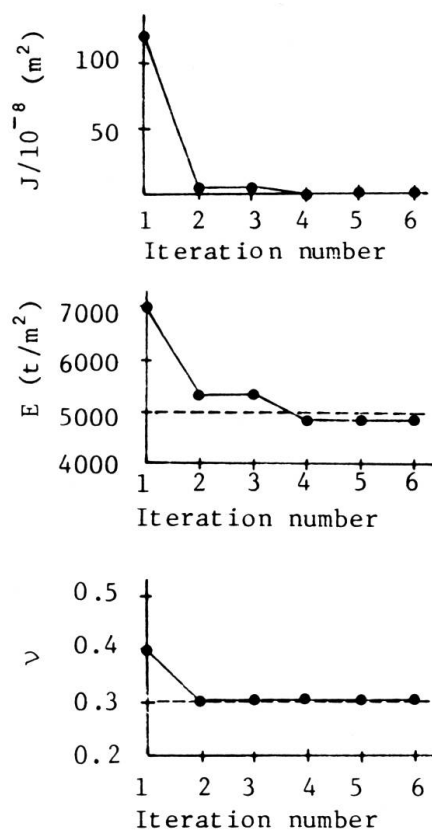
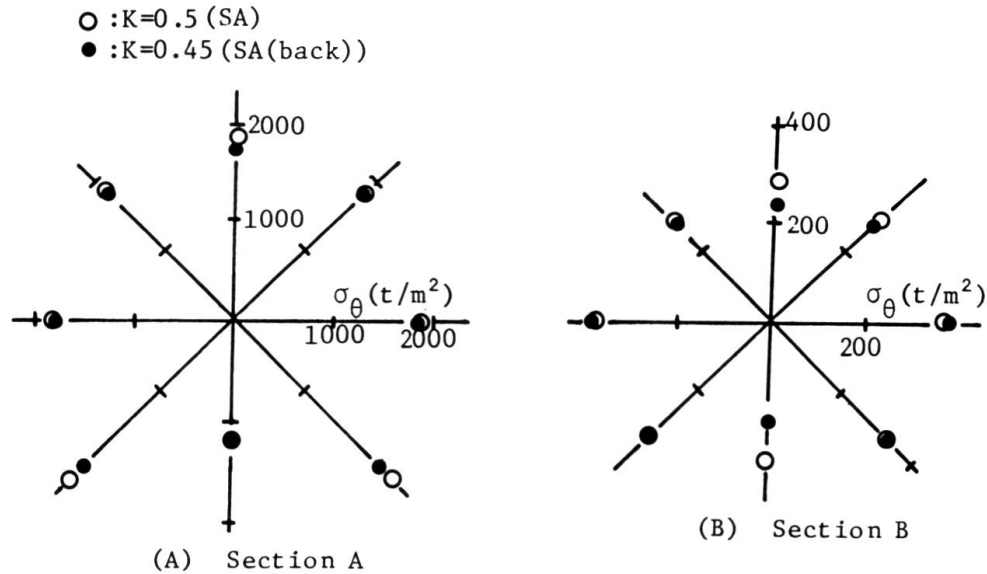


Fig.4 Convergency characteristics of  $J$ ,  $E$  and  $\nu$  in case of  $K=0.5$



**Fig.5** Comparison of circumferential stresses  $\sigma_\theta$  of the linings for  $K=0.5$ (real) and  $k=0.45$ (10% error)

into Eq.(1). Fig.5 shows the comparison of circumferential normal stress  $\sigma_\theta$  on the inner boundary of the linings in the sections A and B in Fig.2, the both of which are calculated by SA with real input data and by SA(back) with input data obtained by back analysis(having 10% error in  $K_{opt}$ ). From the results in Fig.5, it is understood that the lining stresses obtained by SA(back) are stably analyzed, even though the estimated optimum K-value contains error. The reason of this is that the values of  $E_{opt}$  and  $\nu_{opt}$  are determined in Eq.(1) by adjusting the error in  $K_{opt}$ .

## 2.2 Viscoelastic analysis

In three dimensional back-analysis for a time dependent tunnel, it takes exceeding computation time and costs and which makes no use of this kind of back analysis to practical applications. Even though viscoelastic back analysis is applied, converged values of parameters may not be obtained in some cases because of existence of a lot of unknown parameters in analysis. Therefore some simplicity of the back analysis should be developed. Considering the above facts, a simplified back analysis is recognized as still more useful even if it contains some error. From the above engineering view point, the following time dependent back-analysis is developed.

Fig.6 shows the time dependent settlements(= $u_v(t)$ ) measured on the ground surface directly above the tunnel crown due to a shallow tunnel excavation in soft rock[7], which may be expressed by a logarithmic function as

$$u_v(t) = c_1 + c_2 \log(1+t), \quad (t:\text{days}), \quad (2)$$

where  $c_1$  and  $c_2$  are constants.

On the other hand, the two-dimensional viscoelastic analysis for this problem gives

$$u_v(t) = H(h/r, \nu) r^2 \gamma \Phi(t), \quad (3)$$

where  $H$  is an influence function with the parameters  $\nu$  and  $h/r$ ( $h$ : tunnel depth,  $r$ : tunnel radius). By equating the above two equations, the creep function  $\Phi(t)$ , is obtained.

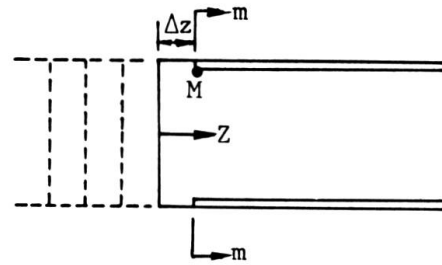
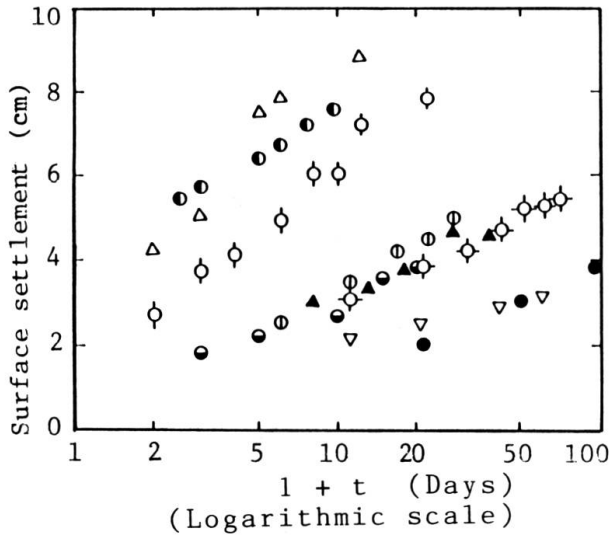


Fig.6 Time dependent settlements due to shallow tunnel excavations

Fig.7 Excavation of tunnel face

$$\Phi(t) = 1/E_1 + \log(1+t)/E_2, \quad (4)$$

where,  $E_1 = H(h/r, \nu)r^2 \nu / c_1$  and  $E_2 = H(h/r, \nu)r^2 \nu / c_2$ .

Eq.(4) is adopted in this back-analysis. The values of  $E_1$  and  $E_2$  are unknowns and will be determined by two operations of back-analysis in the followings:

Let's consider the displacement  $u_m(t)$  at the measuring point M in Fig.7. If the tunnel face is excavated by  $\Delta z$  as shown in Fig.7,  $u_m$  can be expressed by  $\Delta u$  as

$$u_m(t_1) = \Delta u(z=\Delta z, \Phi(t=t_1)), \quad (5)$$

where  $\Delta u$  means the displacement caused by one excavation of the tunnel face, "t" is the time measured after the face has been excavated and z is the distance measured from the tunnel face to the entrance direction. If one more excavation of  $\Delta z$  is made at the time  $t=t_1$  after the linings have been constructed,  $u_m$  becomes as follows at  $t=t_2=2t_1$

$$u_m(t_2) = \Delta u(\Delta z, \Phi(t_2)) + \Delta u(2\Delta z, \Phi(t_1)). \quad (6)$$

By the way, the time dependent displacement of the viscoelastic body can be analyzed by using the correspondence principle[8].

$$[u(t)] = L^{-1}[u^*(s)], \quad (7)$$

where \*,  $L^{-1}$  and s are the Laplace transform, the Laplace inversion and the transform parameter, respectively. If the Schapery's direct method[9] is applied to Eq.(7),  $[u(t)]$  is approximately calculated by the following equation:

$$[u(t)] = L^{-1}[u^*(s)] \div [su^*(s)]_{s=0.5/t} \\ = [s[Km(s)\Phi^*(s)]]^{-1}[f/s]_{s=0.5/t} = [Km(s)\Phi^*(s)]^{-1}[f]_{s=0.5/t} \quad (8)$$

where  $[Km]$  and  $[f]$  are, respectively, stiffness matrix and nodal forces which are released at the excavation of the tunnel face, and it is assumed that it does not change with time. Then Eq.(8) can be reformed as



$$[u(t)] = [K_m(1/E^s, 1/E^t, 1/E(t))]^{-1}[f], \quad (9)$$

where  $1/E(t) = (s \Phi^*(s))_{s=0.5/t} = 1/E_1 + \ln(1+t)/E_2$ .

$E^s$  and  $E^t$  are the Young's moduli of shotcrete and rock bolt, respectively. In deriving Eq.(9), such a relation as  $\Phi(t) = s \Phi^*(s)_{s=0.5/t}$  is used.

It is seen from Eq.(9) that the displacements are directly connected with  $E_1$  and  $E_2$  in the real space. Therefore, if the measured displacement  $u_m(t_1)$  is applied to the back-analysis as input data, the optimum values of  $\nu$ ,  $K$  and the Young's modulus( $E(t_1)$ ) are easily obtained in the same manner as the elastic back-analysis. In this case, however, it is obvious from Eq.(9) that the estimated Young's modulus( $E(t_1)$ ) does not equal  $E$ , but has the following relationship with  $E_1(=E)$  and  $E_2$ :

$$\frac{1}{E(t_1)} = \frac{1}{E_1} + \frac{\ln(1+t_1)}{E_2}. \quad (10)$$

In other words,  $E(t_1)$  is the apparent Young's modulus at  $t=t_1$ .

In the second step,  $E(t_2)$  is related with  $E_1$  and  $E_2$  in the followings.  $\Delta u(2\Delta z, (t_1))$  in Eq.(6) can be calculated by the usual FEM, because  $\nu$ ,  $K$  and  $E(t_1)$  are already known, so  $\Delta u(\Delta z, \Phi(t_2))$  can be represented by the known values improving Eq.(6)

$$\begin{aligned} \Delta u(\Delta z, \Phi(t_2)) &= \bar{u}_m(t_2), \\ \text{where, } \bar{u}_m(t_2) &= u_m(t_2) - \Delta u(2\Delta z, \Phi(t_1)) \end{aligned} \quad (11)$$

As  $\Delta u(\Delta z, \Phi(t_2))$  is the displacement at  $t=t_2$  due to the first one excavation, the application of  $\bar{u}_m(t_2)$  to the back-analysis as input data will give the following expression to  $E(t_2)$ :

$$\frac{1}{E(t_2)} = \frac{1}{E_1} + \frac{\ln(1+t_2)}{E_2}, \quad (12)$$

in which  $E_1$  and  $E_2$  are obtained by solving Eqs.(10) and (12),

$$E_1 = \frac{E(t_1)E(t_2)\ln[(1+t_2)/(1+t_1)]}{E(t_1)\ln[(1+t_2)/(1+t_1)] + E(t_1) - E(t_2)}, \quad (13)$$

$$E_2 = \frac{E(t_1)E(t_2)\ln[(1+t_2)/(1+t_1)]}{E(t_1) - E(t_2)}$$

The tunnel movement in future can be estimated by performing three-dimensional sequence analysis of viscoelasticity, because all of the mechanical parameters concerning the time dependent ground have been determined.

### 3. APPLICATION TO PRACTICAL TUNNELS

In order to check the applicability of the method, it is applied to two actual tunnels excavated in a squeezing and a time dependent grounds.

#### 3.1 Determination of optimum tunnel shape

The optimum tunnel shape of the upper part of a bench excavation tunnel in an extremely squeezing ground is determined based on the back-analysis. An analytically determined shape is compared with an actual shape determined by construction records of the tunnel movement. The construction records are obtained by changing the tunnel shape eight times.



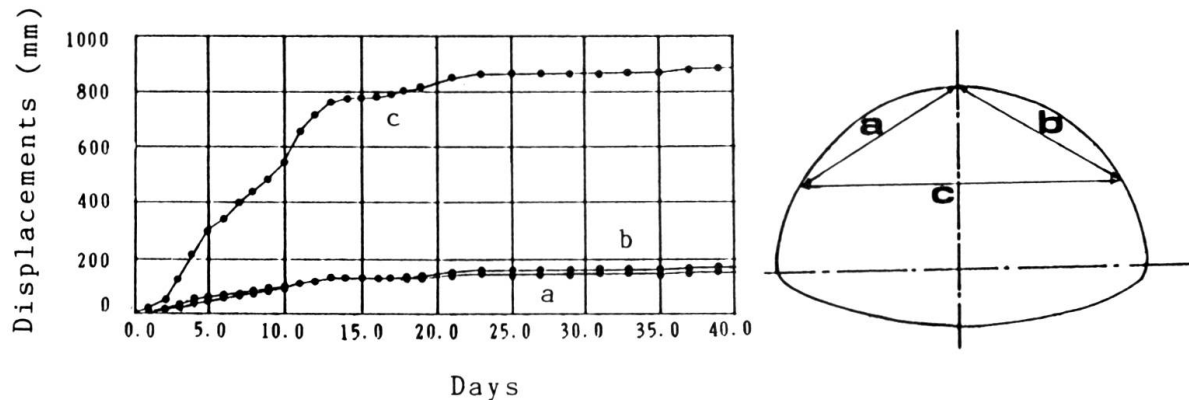


Fig.8 Convergency records measured

Fig.8 shows the convergency records measured at the site. Fig.9 shows a relationship between  $K$  and the minimum value of  $J_{min}$  calculated by the elastic back-analysis. From Fig.9 the optimum  $K$  value ( $=K_{opt}$ ) is obtained as 1.75. This value, which is greater than 1.0, is reasonable because horizontal displacements at the spring line are greater than settlements at the crown, even though the tunnel width is greater than the tunnel height. In the analysis, to decrease the number of unknowns, the Poisson's ratio of the ground is assumed as 0.3, because this value does not give much influence on estimation of the equivalent Young's modulus.

As the displacement convergency observed exceeds 0.8 meters and the damage of the shotcrete is recognized, one fifth of the Young's modulus of the shotcrete measured in a laboratory ( $E_c$ ) is used as the analytical value ( $E_{ca}$ ) in Fig.9. When one tenth of  $E_c$  is used in the analysis, the same value of  $K_{opt}$  ( $=1.75$ ) is also obtained from Fig.10. This means that the value of  $K_{opt}$  is not much influenced by the  $E_{ca}$ -value.

The back-analyzed value of  $E$  of the ground ( $=E_{opt}$ ) becomes  $60 \text{ tf/m}^2$  under the conditions of  $K_{opt}=1.75$  and  $E_{ca}=E_c/5$ , which is very small. But a qualitative check can be done for this value by calculating a plastic radius around the tunnel based on the following elasto-plastic theory presented by Kastner[10].

$$R = r[(1-\sin\phi)(P_{tan}\phi/C+1)]^b, \quad (14)$$

where  $R$  : radius of plastic region ,  $C$  : cohesion ,  
 $\phi$  : angle of internal friction,  $b = (1-\sin\phi)/(2\sin\phi)$ ,  
 $P$  : overburden pressure.

The value of  $R/r$  calculated by Eq.(14) is about 28, in which the values of  $C$  and  $\phi$  are determined by rock specimens in a laboratory. So the value of  $E_{opt}$  obtained by the back-analysis is recognized as a practical one.

Fig.11 shows several types of tunnel shape used by the sequence analysis, in which back-analyzed values of  $K_{opt}$  and  $E_{opt}$  are used. The optimum tunnel shape defined here is the shape in which the maximum shearing stress produced in the shotcrete becomes the minimum in any actual tunnel shape. As the shotcrete stresses increase with an increase in a distance between the tunnel face and the measuring tunnel cross sections, the effect of the tunnel face progress is taken into account in calculation of the shotcrete stresses.

As the tunnel shape changes from section 1 to section 4, the tangential stress at points ① and ③ increases, but decreases at point ②. In the four cases of calculation, the maximum shearing stress in case 3 becomes the minimum. Therefore, the section 3 is determined as the optimum tunnel shape for this

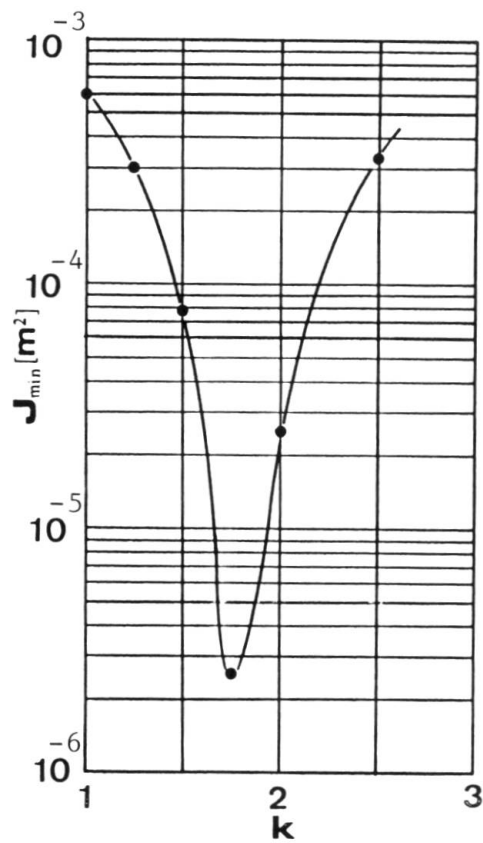


Fig.9 Relationship between K and  $J_{\min}$  ( $E_{ca}=E_c/5$ )

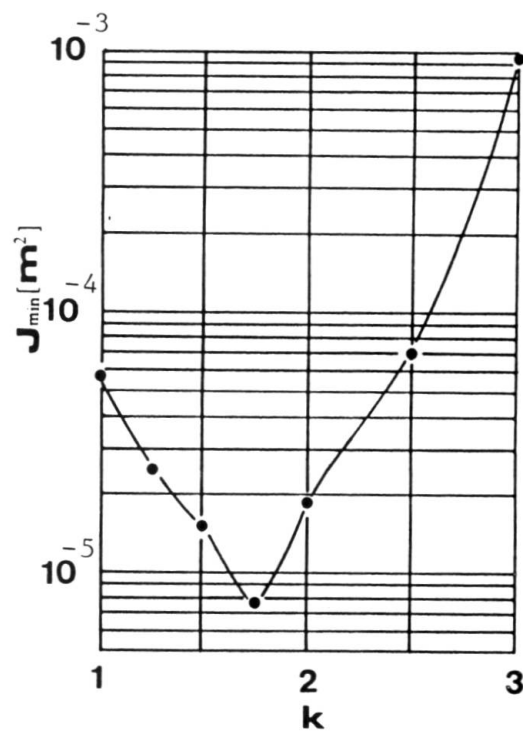


Fig.10 Relationship between K and  $J_{\min}$  ( $E_{ca}=E_c/10$ )

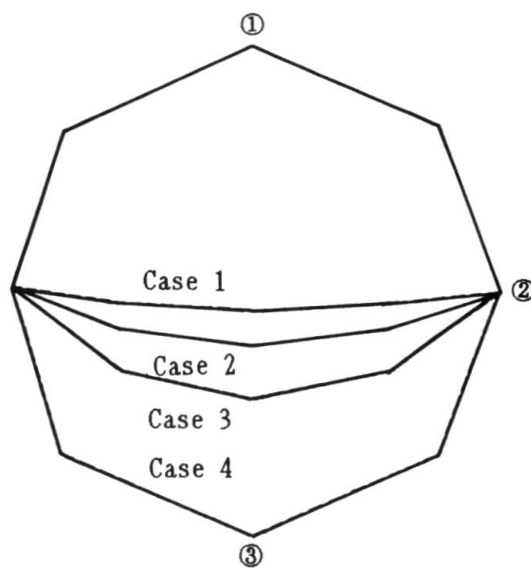


Fig.11 Tunnel shapes in analysis



site. The analytically obtained tunnel shape shows small difference in invert height (about 20 cm) with the shape sought out at the site from the actual records of tunnel movements.

### 3.2 Prediction of tunnel displacements

#### 3.2.1 Executive conditions

Fig.12 shows a tunnel cross section and displacement measuring points. Construction conditions of the tunnel are shown in Table 2. As the Young's modulus of the shotcrete ( $E^S$ ) increases with time after its execution,  $E^S$  is determined in the analysis by using test values as shown in Fig.13.

#### 3.1.2 Back-analysis

Fig.14 illustrates finite element meshes used, and the measured displacements at "e" and "f" shown in Fig.12 are indicated in Table 3.

Fig.15 illustrates analytical relationship between  $J_{min}$  and  $K$  obtained with measured displacements at  $t=1$  day, so the optimum value of  $K$  can be determined

Table 2 Executive conditions

Tunnel radius	5.1m
Number of rock bolts in a cross section	15
Rock bolt length	4m
E of rock bolt	$2.1 \times 10^7 \text{ t/m}^2$
Cross section area of rock bolt	$5.07 \times 10^{-4} \text{ m}^2$
E of steel support (H-200)	$2.1 \times 10^7 \text{ t/m}^2$
Cross section area of steel support	$63.53 \times 10^{-4} \text{ m}^2$
Thickness of shotcrete	20cm
E of shotcrete	Fig.13
Overburden	8.5m
Unit weight of the ground	$2.46 \text{ t/m}^3$
Excavation length at the tunnel face	1.25m

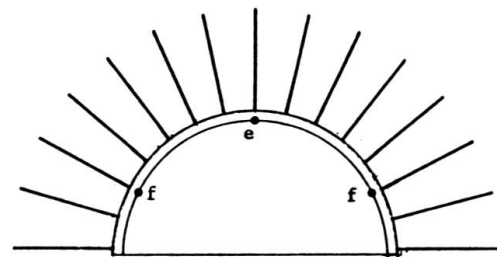


Fig. 12 Tunnel cross section and displacement measuring points

a	b	c	d	e
	$E_a^S = 1,451,000 \text{ t/m}^2$			
	$E_b^S = 1,827,000 \text{ t/m}^2$			
	$E_c^S = 2,052,000 \text{ t/m}^2$			
	$E_d^S = 2,189,000 \text{ t/m}^2$			
	$E_e^S = 2,359,000 \text{ t/m}^2$			

Fig. 13 Young's modulus of shotcrete

as  $K_{opt}=1.3$ . As the values of  $K$  and  $\nu$  are already fixed, the values of  $E(t_1)$  and  $E(t_2)$  are easily analyzed by following 2.2. Figs.16 and 17 illustrate the convergency characteristics of  $(J, E(t_1))$  and  $(J, E(t_2))$ , respectively, and from these figures  $E(t_1)$  and  $E(t_2)$  are determined by Eq.(13)

$$E(t_1=1\text{day})= 6768 \text{ tf/m}^2 \quad \text{and} \quad E(t_2=2\text{days})= 4517 \text{ tf/m}^2 \quad (15)$$

As all of the unknowns have been estimated, the sequence analysis of viscoelasticity can be done.

Fig.18 shows the comparison of measured and analytical displacements of time dependency at the measuring point "e" on the tunnel crown shown in Fig.12. From the results of Fig.18, it may be concluded that the analytical results are in comparatively good agreement with the measured ones and that the back-analysis proposed here may be applicable to practical tunnel problems.

Table 3 Measured displacements at "e" and "f" shown in Fig.12 (v:vertical, h:horizontal)

	$t_1 = 1 \text{ day}$	$t_2 = 2 \text{ days}$
$u_{e,v}$	1.0	1.58
$u_{f,h}$	1.5	2.38

(Unit: mm)

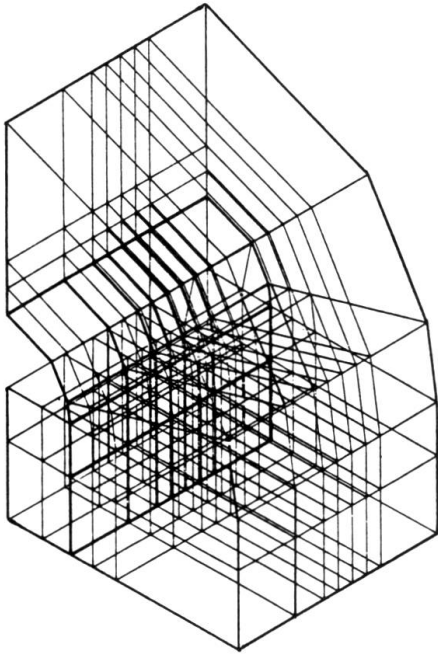


Fig.14 Finite element meshes

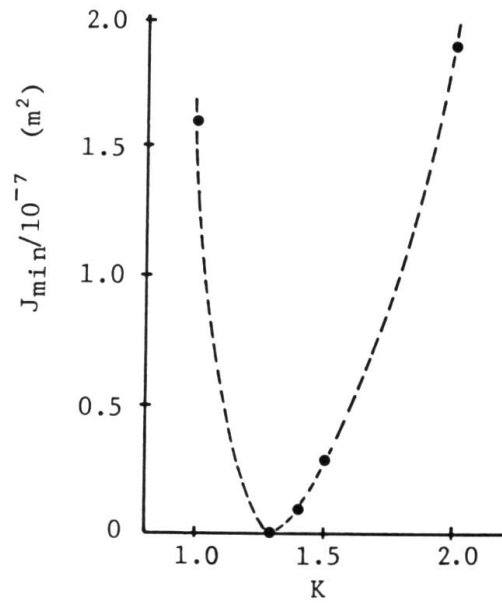


Fig.15 Relationship between K and Jmin

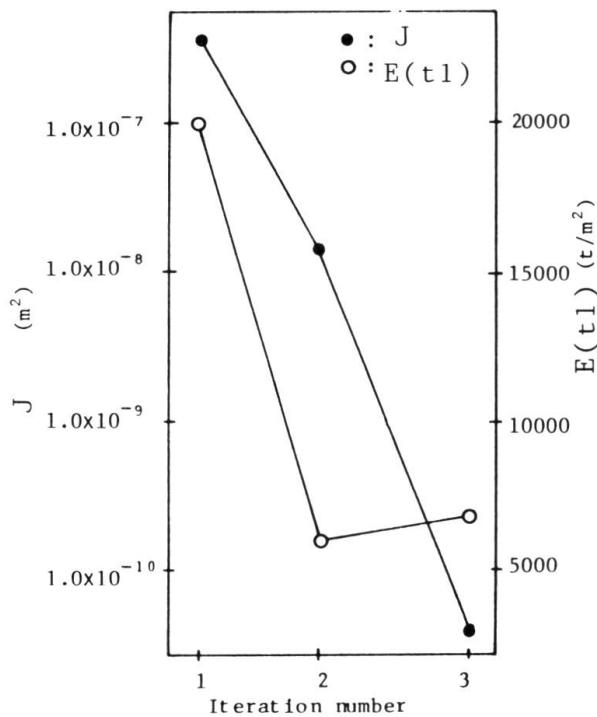


Fig. 16 Convergence characteristics of J and E(t1)

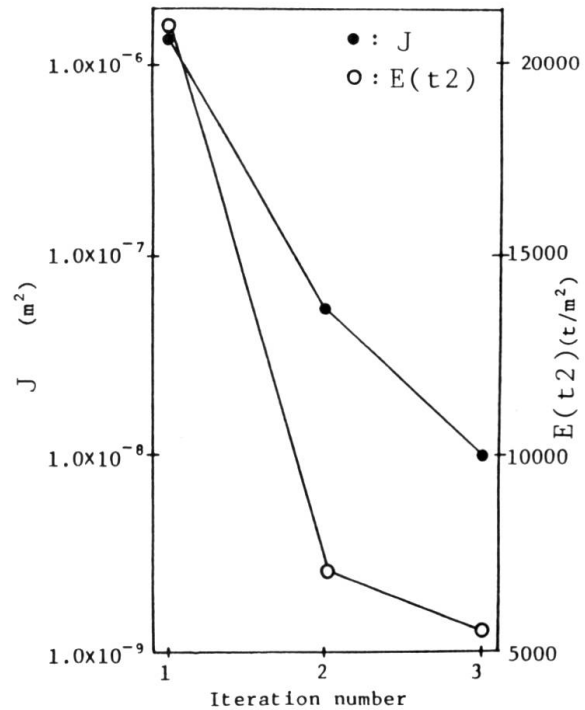


Fig.17 Convergence characteristics of J and E(t2)

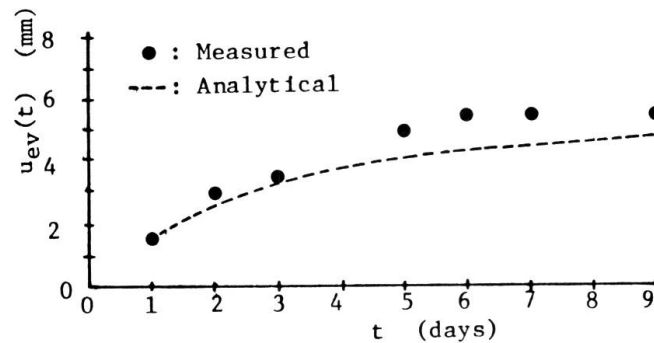


Fig. 18 Comparison of measured and analytical displacements at "e"

## CONCLUSIONS

The present study will give the following conclusions:

1. A three-dimensional back-analysis method based on an optimization technique has been developed, by which the Young's modulus, the Poisson's ratio and the lateral pressure ratio are estimated with a few displacements measured on the inside of the tunnel linings.
2. A back-analysis method with an idea of the apparent Young's modulus was presented for the time dependent ground.
3. In order to determine the optimum tunnel shape and to forecast the time dependent tunnel displacements, the proposed method was applied to practical tunnels and good applicability of the method was demonstrated by comparing the analytical results with the actual records.

## REFERENCES

- [1] Kovari, K., Amstad, Ch. and P. Fritz, Integrated Measuring Technique for Rock Pressure Determination, Proc. Int. Symp. on Field Measurements in Rock Mech., 1977.
- [2] Gioda, G. and L. Jurina, Identification of Earth Pressure on Tunnel Liners, Proc. 10th Int. Conf. Soil Mech. and Found. Engg., Vol.1, 1981.
- [3] Hisatake, M. and T. Ito, Back Analysis Method to Tunnel Linings, Proc. Int. Symp. Field Measurements in Geomech., 1983.
- [4] Sakurai, S. and K. Takeuchi, Back analysis of Displacement Measurements in Tunneling, Proc. Japan Socie. Civil Engrs, No.337, 1983.
- [5] Hisatake, M., Assessment of Tunnel Face Stability by Back Analysis, Proc. 2nd Int. Symp. Field Measurements Geomech., 1986.
- [6] Hisatake, M., Three Dimensional Back Analysis for Tunneling, Proc. Japan Socie. Civil Engrs., No.376, 1986.
- [7] Ito, T. and M. Hisatake, Ground Pressure Acting on Arbitrary Shaped Tunnel Lining in Visco-elastic Ground, Proc. Japan Socie. Civil Engrs., Vol.307, 1981.
- [8] Lee, E.H., Stress Analysis in Viscoelastic Bodies, Quart. Appl. Math., Vol. 13, No.2, 1955.
- [9] Schapery, Approximate Methods of Transform Inversion for Viscoelastic Stress Analysis, Proc. 4th U.S. National Cong. Appl. Mach., Vol.2, 1962.
- [10] Kastner, H., Statik des Tunnel-und Stollenbaues, Springer-Verlag, chapter 4, 1971.