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New Definition of Slenderness of Reinforced Concrete Columns

Nouvelle définition de l'élancement de colonnes en béton armé

Neue Definition der Schlankheit bei Stahlbetonstützen

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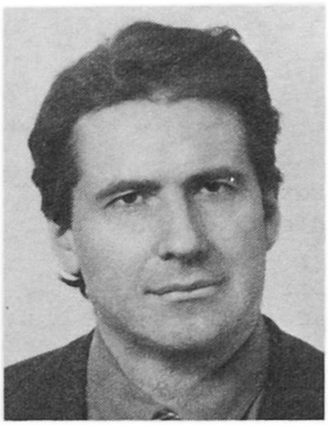

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SUMMARY

The definition of column slenderness according to which the field of slender frames is defined in Codes of Practice, including the CEB-FIP Model Code, does not take into account the influence of axial load, reinforcement and concrete strength, despite the fact that these factors have a major influence on the non-linear behaviour of the structure. In this paper, a modified definition of slenderness is proposed which takes these parametres into consideration and which can apply to both simple and multistorey sway frames. This definition is justified by parametric numerical tests as well as by theoretical considerations.

RÉSUMÉ

Dans les codes, et en particulier le code modèle CEB-FIP, la définition de l'élancement des structures élancées ne tient pas compte des influences de l'effort normal, de l'armature et de la qualité du béton. Ces facteurs ont une influence considérable sur le comportement non linéaire de la structure. Une définition modifiée de l'élancement est proposée, tenant compte de ces paramètres, et elle est valable pour les cadres simples et multiples. Cette définition est basée sur des essais numériques paramétriques et sur des considérations théoriques.

ZUSAMMENFASSUNG

In den Normen, und besonders in der CEB/FIP-Modellvorschrift, wird bei der Definition der Schlankheit der Einfluss der Normalkraft, der Bewehrung und der Betonqualität nicht berücksichtigt. Diese Faktoren haben einen bedeutenden Einfluss auf das nichtlineare Verhalten der Struktur. Eine modifizierte Definition der Schlankheit, welche diese Parameter berücksichtigt, wird vorgeschlagen. Sie ist gültig für einfache und mehrfache Rahmen. Die Definition stützt sich auf Versuche und theoretische Überlegungen ab.

1. INTRODUCTION

The CEB Model Code |1| (as well as any other code of practice) defines the field of "slender"frames: if a frame belongs to this field, second order effects cannot be disregarded, and must be calculated using accurate or simplified procedures of non-linear analysis; the procedures can be long and difficult to apply, expecially in the case of sway frames, where a nonlinear analysis of the frame as a whole needs be performed, followed by a verification of each column, according to approximate nonlinear procedures (such as the "model column" method). These procedures are really justified only when second order moments are a sizable part of the total design moments in critical sections. Therefore to define the field of slender frames means to delimit the field in which every conceivable R.C. frame shows a sizable nonlinear behaviour.
To measure the "degree of nonlinearity" in structural behaviour a suitable parameter is needed. Up to now the "Eulerian" slenderness of columns was adopted, defined by the following expression:

$$\lambda = l_o/i$$

where:
l_o = effective buckling length
i = minimum radius of gyration of the gross section of concrete only.
However the nonlinear behaviour of a structure does not depend only on the geometrical parameters syntetized by λ.
The axial load and material strength are also important factors.
However the slenderness as previously defined could still be considered a valid way to measure the nonlinear behaviour of a structure made of an homogeneous material; it is not suitable therefore for a non-homogeneous material such as Reinforced Concrete, which is subject to cracking and viscous behaviour, in which an essential role is played by reinforcing steel.
This considered, a parameter including at least the most important variables of structural behaviour of Reinforced Concrete would be useful to define in a more correct way the cases where second order effects must be considered. This is however no easy task, given the complexity of the phenomenon and the number of variables involved. This number is already considerable if, for the sake of simplicity, isolated columns are studied.
If however, in a more realistic way, complete frames are considered, these variables and the complexity of the structural behaviour increase considerably. Besides,redistributions of moments take place among different sections in the frame. For this reasons the more comprehensive definitions of slenderness which were up to now formulated |3||4|, are only in part based on theoretical considerations.
Their validity is based also on the result of parametric analyses, which were performed using nonlinear computer programs.
These programs permit to obtain a realistic picture of the evolutive behaviour of R.C. plane frames subject to a given load history, taking into account cracking of concrete, second order effects, plastic behaviour of concrete and, in case, initial imperfections and creep.
Purpose of this work is to illustrate a possible formulation of a modified definition of column slenderness which can better define the field where second order effects need be considered in structural analysis of R.C. plane frames.

2. DEFINITION OF "SLENDER" FRAMES ACCORDING TO CEB MODEL CODE

As already stated, the CEB Model Code defines the field of slender frames using the Eulerian Slenderness ; in particular the following limits are given:
a) for $\lambda < 25$ no second order effects need be considered;
b) for $\lambda \leq 140$ approximate methods of calculation of second order effects can be used, such as the "model column" method.

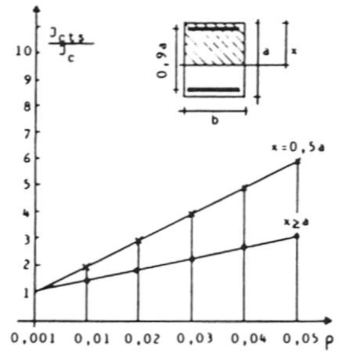

Fig.1 - Influence of stiffening effect of steel in cracked and uncracked elements

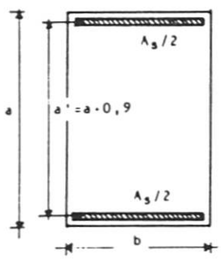

Fig.2 - Simmetrically reinforced concrete section

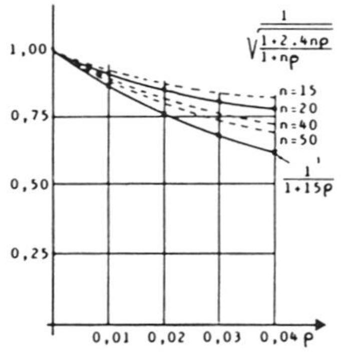

Fig.3 - Correction factor of λ for influence of reinforcement

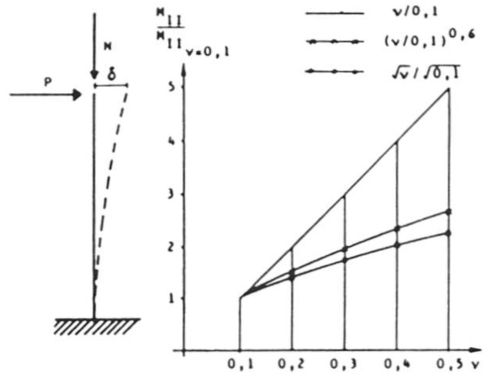

Fig.4 - Variation of M_{II} in function of ν

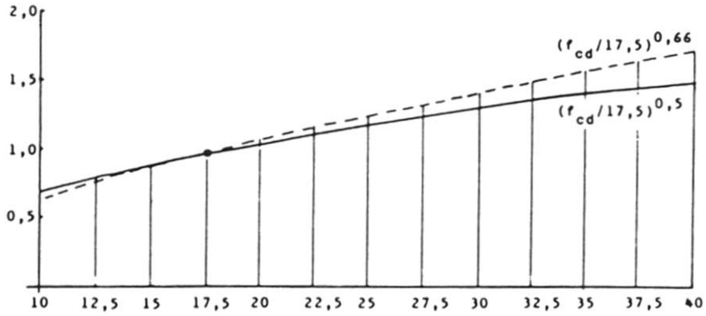

Fig.5 - Influence of concrete resistance on second order moments

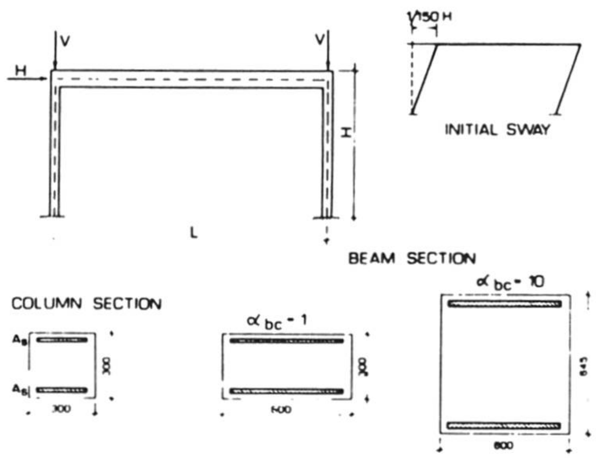

Fig. 6 - Typical frame

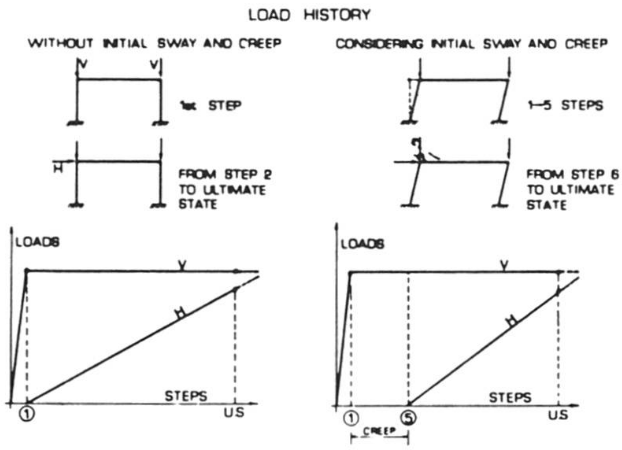

Fig. 7 - Load history (from Ref. 4)

Such methods, which consider each column as a separate element, cannot be used alone in the case of sway frames; in the latter case, as already stated, a non-linear analysis of the frame considered as a whole, must be made:

c) for $140 < \lambda < 200$ an "accurate" nonlinear analysis must be performed in any case, while a slenderness > 200 is not recommended.

The experience, which has up to now been accumulated by the authors, shows however that in many cases frames whose columns have a slenderness between 25 and 40 often show very limited second order effects.

This is the case of the sway frames of Example 14 of the "Trial and Comparisons Calculations" performed to check the applicability and validity of the Model Code |5|. This is an example of sway frame, having a slenderness of 35 for which a time consuming nonlinear analysis is required; however second order effects are negligible, as the common sense would suggest for a structure whose columns are 50 X 50 cm. square over a storey height of 3.50 m.

A more accurate lower limit of slenderness needs therefore be defined.

Let us also notice, on this subject, that the ACI Code 318/77 |2| sets a lower limit of slenderness (also using the "Eulerian" expression of λ) which has a value of 22, that is even more conservative that the CEB value.

To establish a more reasonable lower limit of slenderness is however much more important in the case of CEB as,according to this Code, easy to use approximate methods of analysis (such as the "Moment Magnifier" method of ACI) are not permitted in the case of sway frames.

3. FACTORS AFFECTING THE NONLINEAR BEHAVIOUR OF SLENDER R.C. FRAMES

As already said, the nonlinear behaviour of R.C. frames is a quite complicated phenomenon, involving the influence not only of the parameters defining the column under consideration but also those concerning the other members of the structure, the distribution of loads, the load history and so on.

If we try to identify the most relevant among the parameters and we confine ourselves to the column under consideration, in addition to the slenderness , these factors appear to be the most important:

a) the axial load on the column;

b) the reinforcement ratio;

c) the concrete strength.

A more satisfactory definition of "equivalent" slenderness for R.C. columns should include at least these primary variables. This has been done by Menegotto-Via |3|, who include in their expression axial load and reinforcement ratio, and by the authors who added also the influence of concrete strength, as will be explained in the following paragraph.

4. FORMULATIONS OF "EQUIVALENT" SLENDERNESS

Once the most relevant variables which influence the nonlinear behaviour of columns have been identified, the following criteria can be followed:

- an approximate relationship is determined between second order moment M_{II} (or magnifier factor $\mu = (M_I + M_{II})/M_I$) and each of these variables;
- the same kind of relationship is adopted for a modified definition of slenderness λ^*;
- in this way, instead of obatining a family of curves $\mu = f(\lambda)$ corresponding to different values of the relevant variables (see Ref. 4), a very "compact" group of points can be represented on the μ / λ^* plane, so that an unique curve $\mu = f(\lambda^*)$ can be extrapolated.

This curve represents in a sintetic way the nonlinear behaviour of the type of frame under consideration and therefore permits to identify the field of slender frames for the type, determining appropriate values of the modified slenderness λ^*.

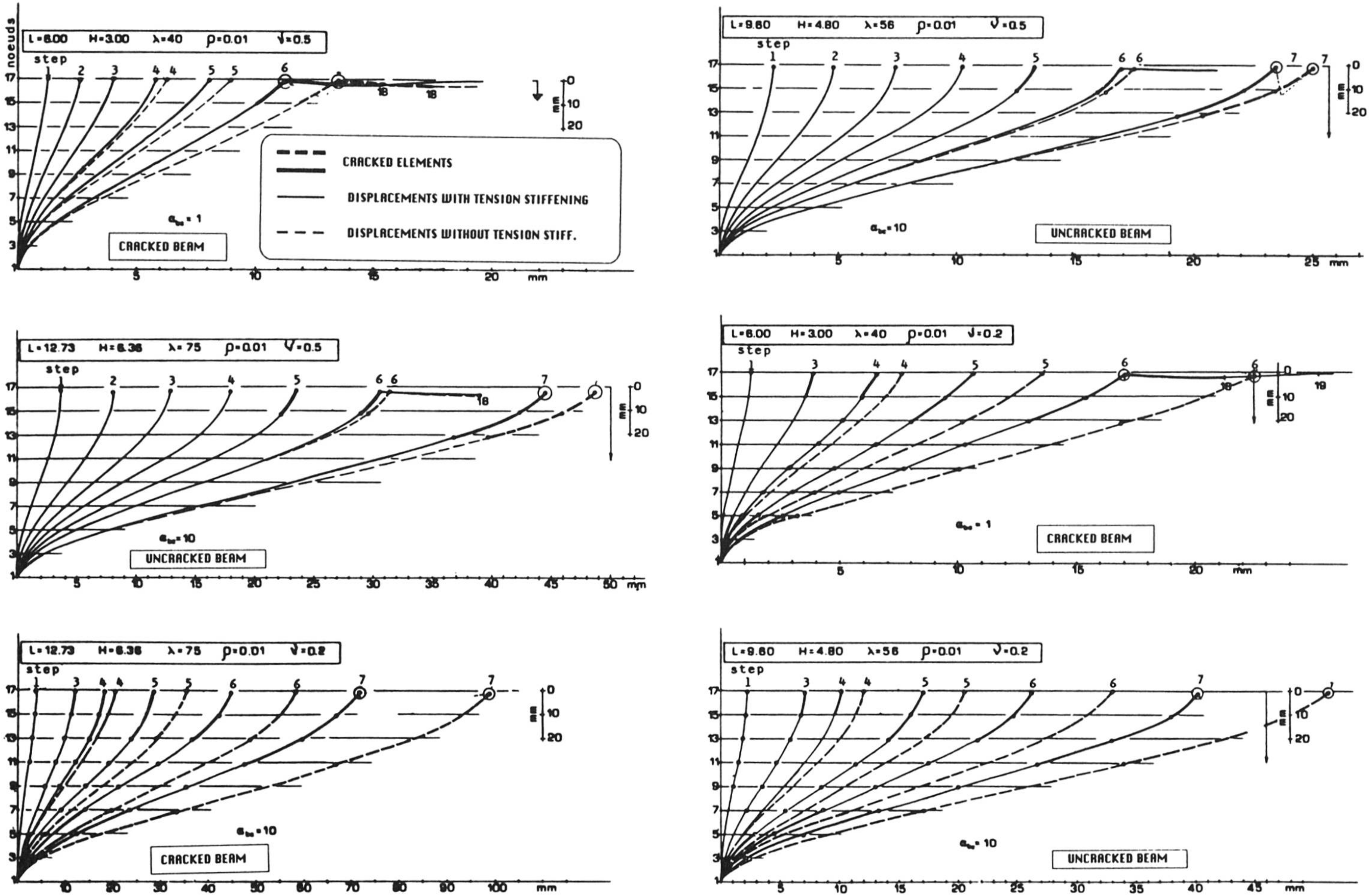

Fig.8/13 - Displacements of left-hand column

Two formulations of the equivalent "slenderness" have been up to now proposed. Menegotto and Via have suggested the following expression:

$$\lambda^{**} = \lambda^{s+c}\sqrt{\nu}$$

where is

$$\lambda^{s+c} = \frac{l_o}{\sqrt{\dfrac{J_c + 20\ J_s}{A_c + 20\ A_s}}}$$

λ^{s+c} is the slenderness of the column with reference to an homogeneized R.C. section (with n coefficient = 20);

ν is the reduced axial load on column.

This expression has been verified by a series of numerical tests performed on isolated columns (not considering the effects of creep and initial imperfections). The authors have suggested instead the following expression:

$$\lambda^{*} = \lambda \frac{\nu^{0.6}}{1 + 15\rho} \left(\frac{f_{cd}}{17.5}\right)^{0.5}$$

where is

λ = "Eulerian" slenderness;

ρ = geometrical reinforcing ratio.

This expression has been verified by a series of parametric numerical tests which were performed on portal R.C. sway frames.
These tests are described in detail in |4|. This description is summarized in paragraph 6. In appendix II a more detailed description is given.

5. THEORETICAL CONSIDERATIONS ON "EQUIVALENT" SLENDERNESS

As already said a satisfactory definition of "equivalent"slenderness should include at least the influence of the following basic parameters:

- steel reinforcement;
- reduced axial load ν;
- concrete strength f_{cd}.

Let us see in some detail how these factors were accounted for in the considered expression of "equivalent" slenderness.

Steel reinforcement

In the expression by Menegotto-Via |3|, reinforcement was considered by introducing the expression:

$$\lambda^{s+c} = \frac{l_o}{\sqrt{\dfrac{J_c + 20\ J_s}{A_c + 20\ A_s}}} \qquad (1)$$

This expression represents the slenderness of an "equivalent" reinforced concrete uncracked element, where an homogeneization factor n = 20 is assumed.
The adopted value of n is considerably higher than the value resulting from the ratio of elastic modules of steel and concrete.
This increase should take into account the increasing stiffening effect of reinforcing steel due to the following factors:

- creep of concrete;
- cracking of the column.

The latter effect is very strong as in shown in the plot of Fig. 1 were the ratios

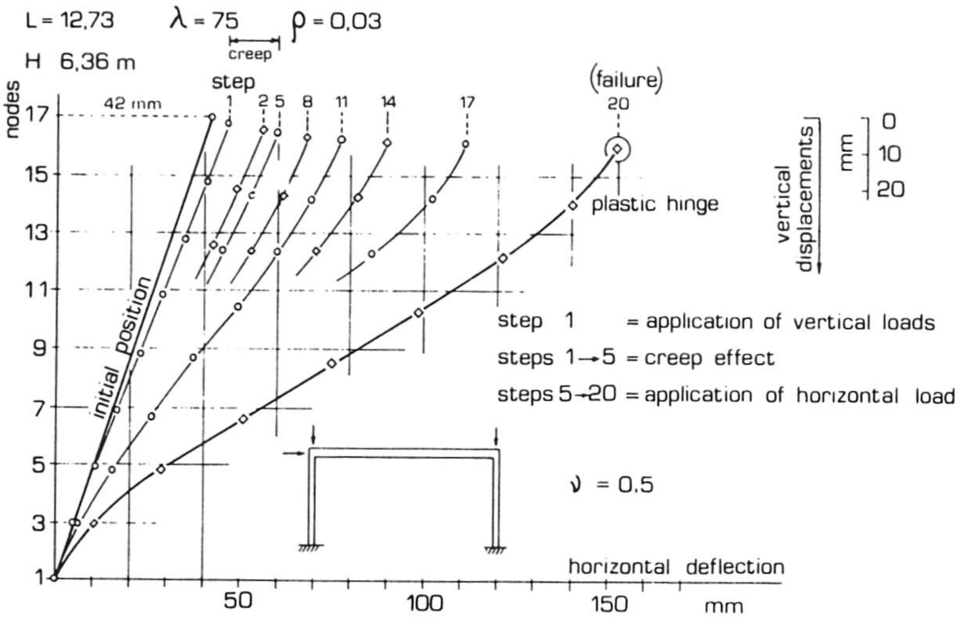

Fig. 14 - Displacements of left-hand column and beam (with creep and initial imperfections)

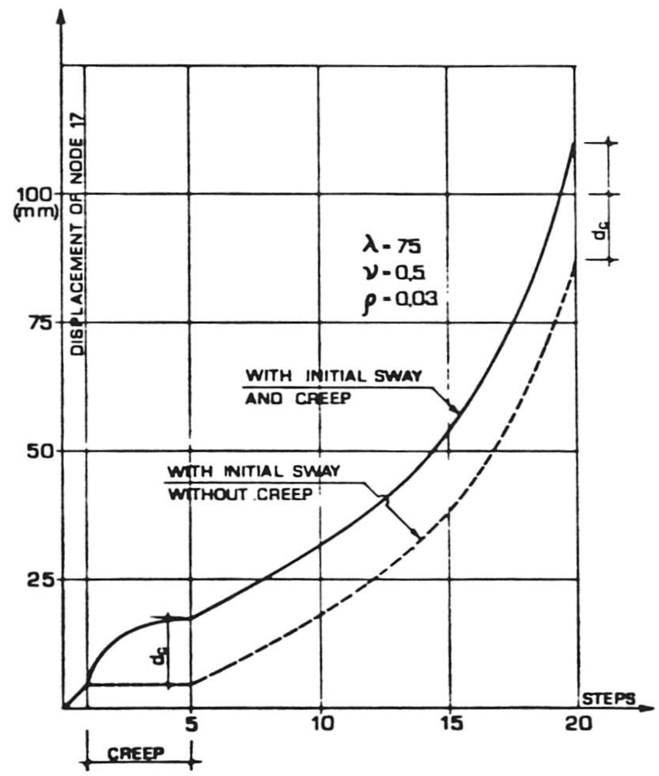

Fig. 15 - Displacements of upper joint of left-hand column for the two load histories

of moment of inertia J_{c+s} corresponding to a negligible reinforcement (ρ=0.001), are plotted for different values of ρ, for an uncracked ($x \geq a$) and cracked (x=0.5) rectangular doubly symmetrically reinforced section.
The adopted value n=20 is based on considerations deduced by criteria adopted in the ACI Code for the definition of second order effects |11|. In this Code the moment of inertia of the column is computed according to the following expression:

$$J = \frac{J_c/5 + (E_s/E_c)\ J}{1 + R_m}$$

where R_m (ratio of permanent load to total load) takes into account the influence of creep. A factor n = 5 (E_s/E_c) = 20 as adopted by Menegotto-Via seems unsufficient in many cases to take into account both cracking and creep, not only because the ratio E_s/E_c can be as high as 10 for f_{ck}= 20 Mpa but also because the influence of creep on the stiffening effect of steel cannot be considered using the ACI formula for this particular problem.
It is now useful for our purpose to transform the expression (1) so that it be expressed in function of reinforcement ratio ρ, and slenderness λ. This can be easily done only in the case which is relevant here, that is the case of a simmetrically reinforced rectangular section as represented in Fig.2. With obvious passages expression (1) is transformed as follows:

$$\lambda^{s+c} = \frac{l_o}{\sqrt{J_c/A_c}} \frac{1}{\sqrt{\dfrac{1 + n(J_s/J_c)}{1 + n(A_s/A_c)}}} = \lambda \frac{1}{\sqrt{\dfrac{1 + n(J_s/J_c)}{1 + n\rho}}} \qquad (2)$$

being for the rectangular section

$$J_c = \frac{ba^3}{12} \qquad J_s = A_s \frac{a'^2}{2}$$

$$\rho = \frac{2A_s}{ab} \qquad A_s = \frac{\rho ab}{2} \qquad a' = 0.9\ a$$

$$J_s = \frac{\rho ab}{2} \frac{(0.9\ a)^2}{2} = \frac{\rho ba^3\ 0.8}{4} = \frac{\rho ba^3}{5}$$

$$J_s/J_c = \frac{\rho ba^3/5}{ba^3/12} = 2.4\ \rho$$

expression (2) becomes

$$\lambda^{s+c} = \lambda \frac{1}{\sqrt{\dfrac{1 + n\ 2.4\rho}{1 + n\rho}}} \qquad (3)$$

Expression (3) is plotted on Fig.3 in function of ρ for different values of n (for λ = 1).
The authors have proposed, to consider the influence of ρ , a much simpler expression

$$\frac{1}{1 + 15\ \rho} \qquad (4)$$

which was also plotted on Fig. 3, where it can be seen that adopting expression (4) means to give to the stiffening effect of reinforcing steel a much greater importance than using the previous expression; this seems rather reasonable, given the influence of creep and the presence of cracked sections in the element at ultimate limit state.

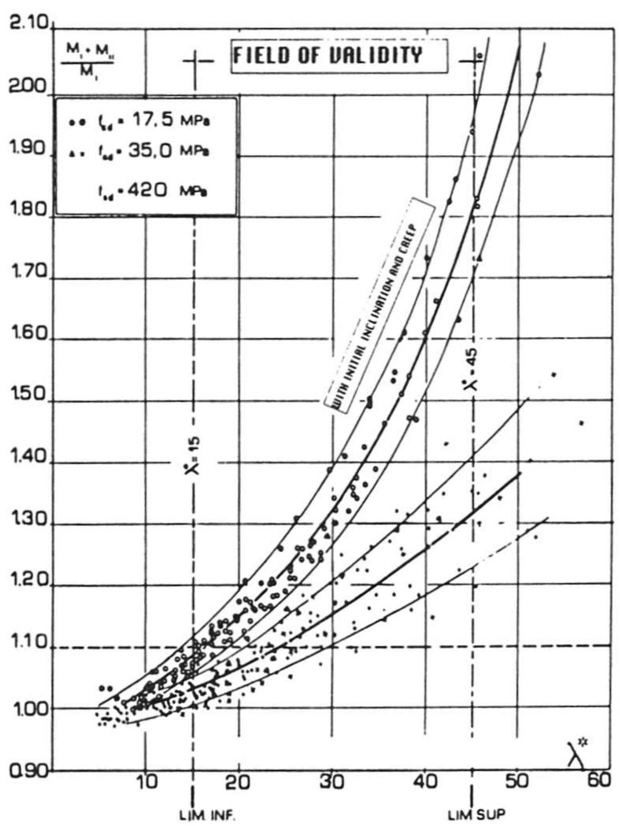

Fig. 16 - Results of basic research plotted against λ^*

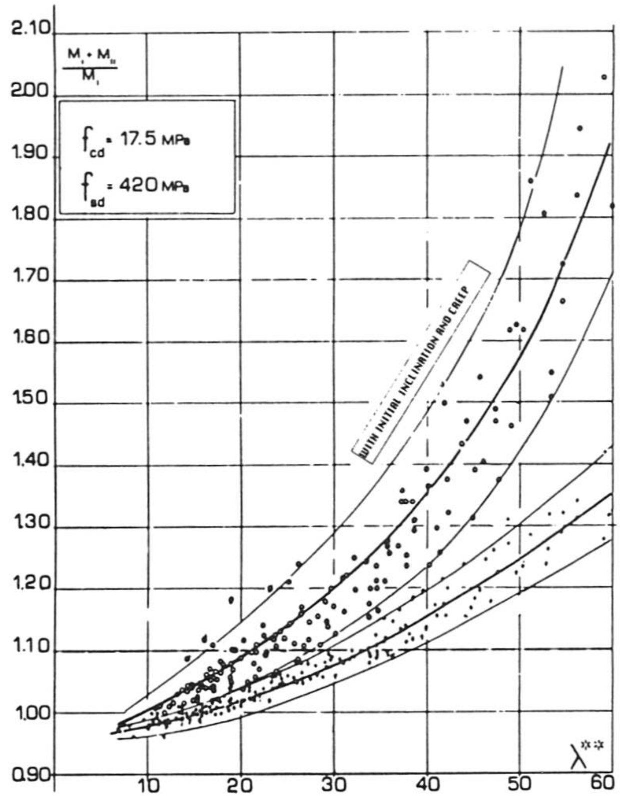

Fig.17 - Results of basic research plotted against λ^{**}

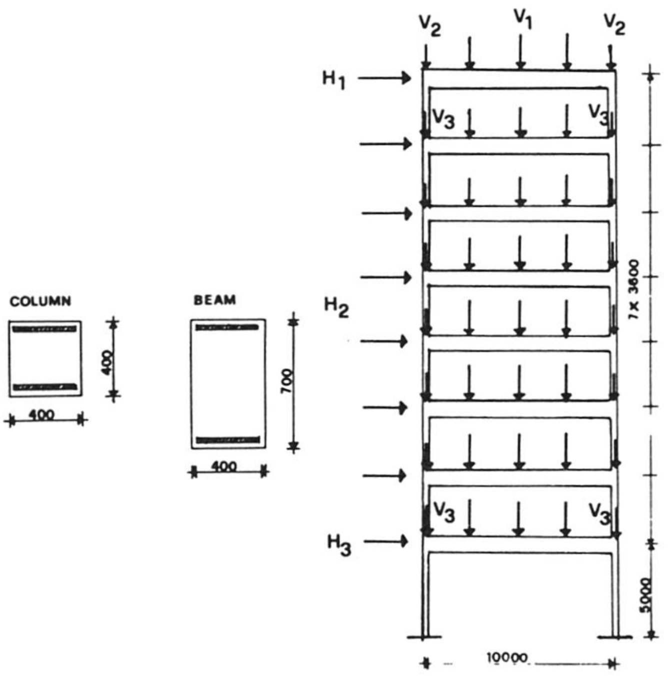

Fig.18 - Multistorey frame; example1

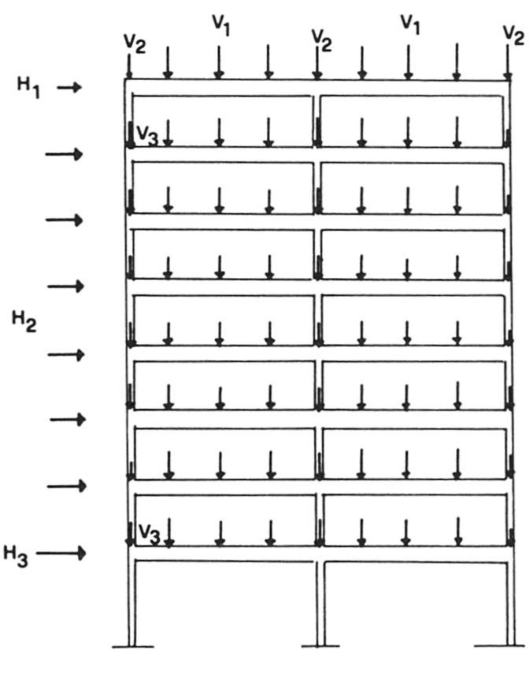

Fig. 19 - Two bay frame - Example 3

Reduced axial load

In the expression by Menegotto-Via | 3 |, the influence of axial load was considered by introducing the expression

$$\sqrt{\nu}$$

This expression is obtained by computing the second order moment according to the so called "secant" formula |8||3| which is based on the assumption that the first order moment be constant along the column axis. This is not the case in most situations, where the first order moment varies linearly reaching the maximum values at the extremities. If a "model column" scheme is considered as in Fig.4/a, and the lateral displacement δ is computed according to the elastic theory, considering the stiffness constant along the column, it can be demostrated that the relationship between M_{II} and ν is approximately linear (see appendix 1). In the expression which was adopted for λ^* however the formulation

$$\nu^{0.6}$$

was adopted, which seems to fit satisfactorily with the results of the parametric study.
In Fig. 4/b the laws of variation of M_{II} in function of ν are represented for the considered formulations.

Concrete strength

Concrete strength influences nonlinear behaviour in two opposite ways: an higher value of f_{cd} corresponds to higher tensile strength and therefore to reduced cracking, it corresponds also to higher elastic module of concrete and therefore to an increase in column stiffness. These two factors produce a decrease in second order effects. Or the other hand an increase of f_{cd} (for given value of ν) corresponds to higher values of axial load N at ultimate limit state and therefore to higher second order effects.
In fact the elastic module E_c can be expressed in function of f_{ck}.
The following expression could be adopted

$E_{cm} = 4809 \sqrt{f_{ck}}$	ACI Code	|Mpa|
$E_{cm} = 5700 \sqrt{f_{ck}}$	Italian Code	|Mpa|
$E_{cm} = 9.5\ (f_{ck}+8)^{1/3}$	CEB Model Code	|Gpa|

If the ACI (or Italian Code) expression is adopted, M_{II} can be expressed as

$$M_{II} = k \frac{f_{cd}}{E_{cm}} = k' \frac{f_{cd}}{\sqrt{f_{cd}}} = k' f_{cd}^{0.5}$$

In fact, at equal value , M_{II} is proportional to f_{cd} and inversely proportional to E_{cm}. If an f_{cd} = 17.5 Mpa is conventionally assumed as a reference value (This convention influences the numerical value of λ^*, but not the validity of the expression) the following expression to be introduced in the formulation of λ^* is obtained

$$(f_{cd}/17.5)^{0.5}$$

This expression is plotted on Fig.5, in function of the values of f_{cd} which are practically interesting; in this field (from 10 to 40 Mpa) the equivalent slenderness is increased by a factor of 2.
If the expression of E_{cm} given by CEB Model Code is adopted, the following expression is obtained (approximately)

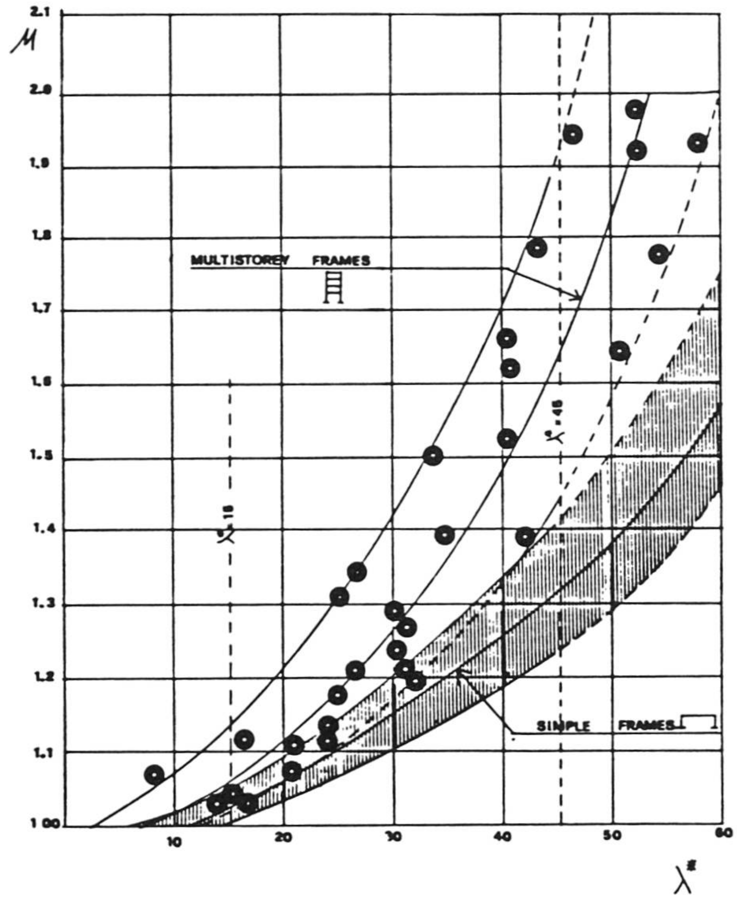

Fig.20 - Moment magnification factors in function of λ* (Cauvin-Macchi expression).

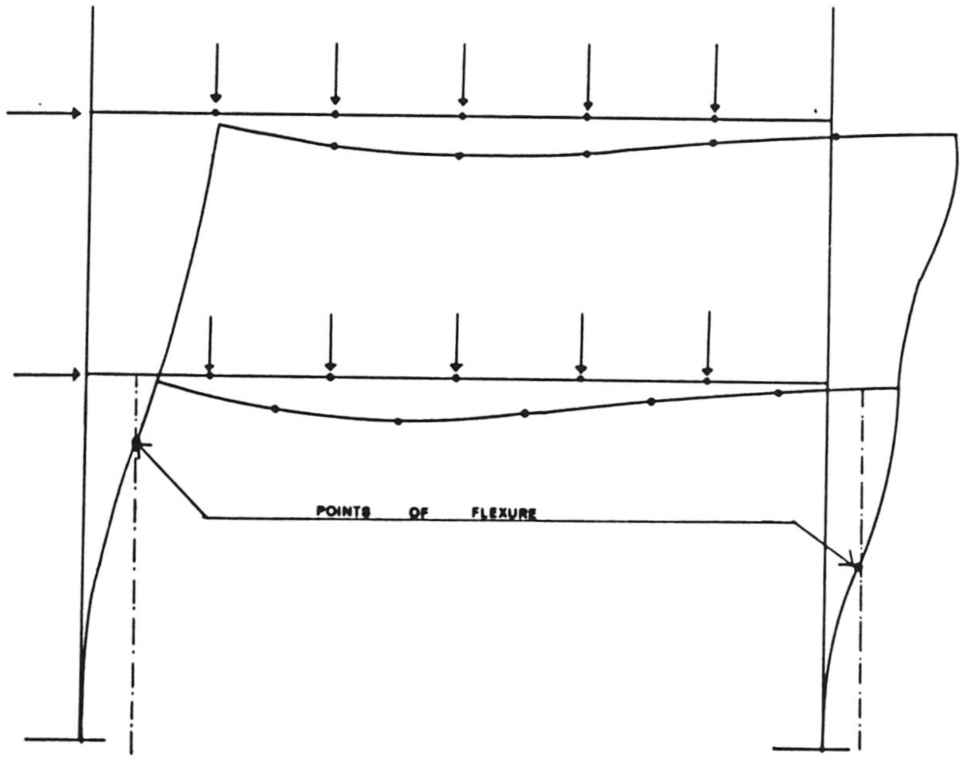

Fig.21 - Deformed shape of Example frame at Ultimate Limit State

$$M_{II} = k \frac{f_{cd}}{(f_{cd} + 8/1.5)^{1/3}} \quad k \frac{f_{cd}}{(f_{cd})^{1/3}} = k\, f_{cd}^{0.66}$$

and therefore the following term should be introduced in *

$$(f_{cd}/17.5)^{0.66}$$

This expression is also plotted on Fig.5; as can be seen from this figure the variation of $f_{cd}/17.5$ is not markedly different from the one previously obtained.

6. RESULTS OF PARAMETRIC NUMERICAL TESTS ON SIMPLE SWAY FRAMES

About 420 nonlinear analyses were performed on one storey, one span portal frames. Details of these tests are given in Ref.|4|.
The variable which were considered are the following:
a) column "Eulerian" slenderness (from 25 to over 100);
b) concrete quality (f_{cd} varies from 17.5 to 35 Mpa);
c) column reinforcement (the geometrical reinforcement ratio was varied from $\rho = 0.01$ to $\rho = 0.04$);
d) axial load on columns (the reduced axial load ν was varied from 0.1 to 0.5).
The analyses were performed considering two different load histories:
- in the first load history vertical concentrated loads were applied in the first load step and then a horizontal force was proportionally applied until collapse. of the frame (Fig.7);
- in the second load history vertical loads (considered as permanent loads) were applied to a frame having a non intentional inclination of 1/150 H. Then the creep due to these forces was simulated; at last the horizontal force was proportionally applied until collapse.
As a consequence, two sets of results were obtained: in the first creep was not influent, in the second the combined effect of creep and non intentional inclination was considered.
As may be expected much greater values of moment magnification factors were obtained in the second case.
The influence of non intentional inclination is not, correctly speaking, a non linear effect; it is however reasonable, for practial purposes, to include it in the value of M_{II}. In fact, the CEB Model Code prescribes the computation of this effect for slender structures only; therefore non intentional inclination "contributes", so to speak, to the definition of the field of slender frames, which is the purpose of this work; that is, when non intentional inclination has sizable influence on the results, the frame must be considered slender.
In Fig.s 8 to 13 displacements of joints of the left hand column during the application of loads were plotted for the first load history and for some significant cases; dashed lines indicate the displacements which were obtained without taking into account the so called "Tension Stiffening" effect, according to the procedure described in |7|.
Tension stiffening has a considerable influence on displacements, as can be seen from the diagrams, but a much more limited one on action effects, at least in the case which were considered.
In Fig.14 these displacements were plotted for the second load history. In all these diagrams the gradual diffusion of cracked zones is also indicated.
In Fig. 15 the "displacement history" of the upper joint of the left hand column is plotted for the two load histories.
Being the main purpose of this research to check the validity of the expressions of "equivalent" stiffness, the moment magnification factors at ultimate limit

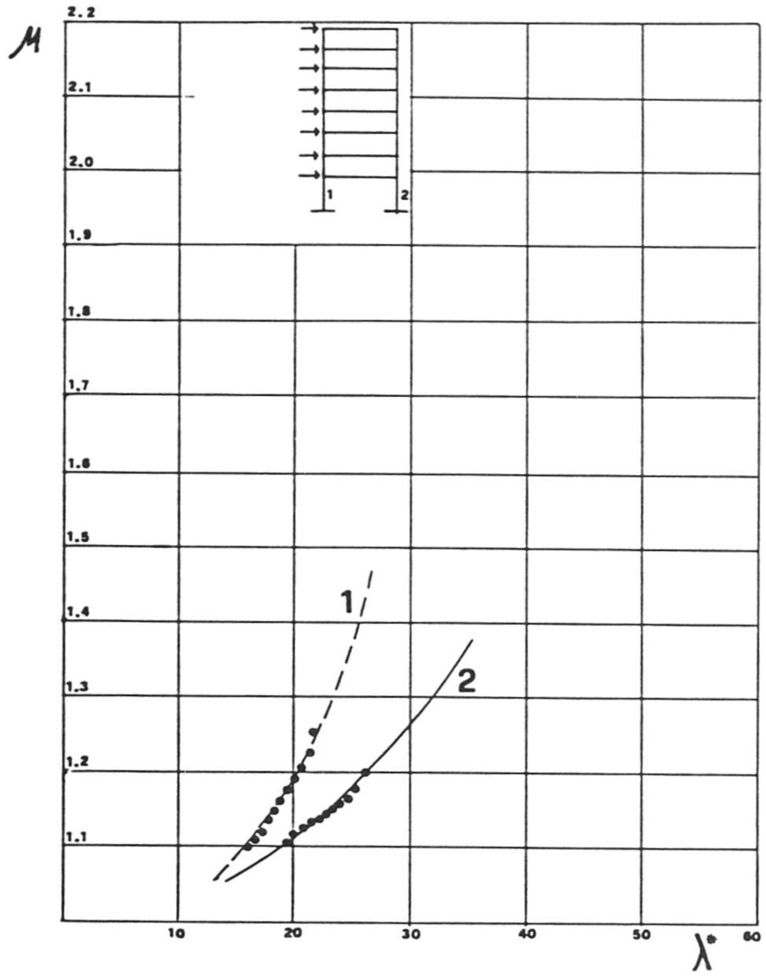

Fig. 22 - μ in ground floor columns of frame N.1 with increasing loads

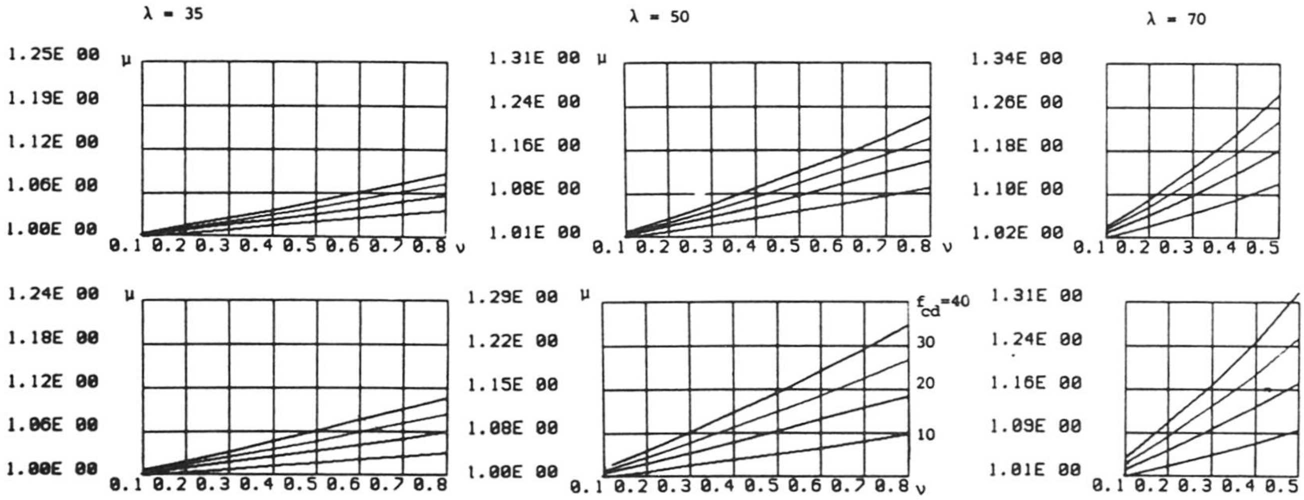

Fig. 23 - Values of μ (Methods 1 and 2 of appendix 1)

state (with reference to the base section of left hand column) of all the considered cases, for both load histories, were plotted against the values λ^* (Cauvin-Macchi) and λ^{**} (Menegotto-Via) of the equivalent slenderness. See Fig.s 16 and 17.
The ultimate limit state which is considered is the one corresponding to the formation of the first "plastic hinge" in the column.
The moment magnification factor is defined as $(M_I+M_{II})/M_I$, M_I and M_{II} being first and second order moments.
The moment magnification factors were also plotted against the corresponding values of the "Eulerian" slenderness. The diagrams are reported in Appendix II.

7. RESULTS OF NUMERICAL TESTS ON MULTISTOREY FRAMES

Results based on simple portal frames are by no means general and need to be checked against more realistic cases. This was done in a limited way by studing four multistorey frames as described in Fig.s 18 and 19.
Examples 1 and 2 are one bay multistorey frames which are identical excepting the fact that in example 2 ground floor columns are hinged at the base, while the same columns of example 1, are built-in the foundation (Fig.18).
Examples 3 and 4 are multistorey two bays frames which differ from one-another in the same way.
Design of reinforcement was performed according to a linear elastic analysis (for example 1 an approximate non linear analysis, using the so called F-Δ method was performed and the reinforcement modified accordingly).
The influence of creep and non intentional inclination were not considered.
Loads were applied proportionally until collapse of the structure.
Slenderness of columns at each floor was computed according to the well known expression given by CEB Model Code |1|

$$\lambda = \sqrt{\frac{12KA}{h}}$$

where is
k relative horizontal displacement between floors for a unit load applied at the top being assumed E=1
A total area of column sections at considered floor
h floor height.
It is interesting to notice that this method for the determination of λ (as well as the criterium to compute the buckling length in a column belonging to a frame, according to the ACI Code) considers loads as applied on joints of the frame only. The influence of loads distributed along the beams is disregarded. As a consequence the value of λ are the same at each floor.
For each column of the four frames, where plasticization of the most stressed section is reached, before the collapse of the frame under consideration, the moment magnification factor has been represented in function of the "equivalent" slenderness λ^*. The μ factor corresponds to the ultimate state in the column. The results are represented on Fig.20, where also the corresponding results of the parametric study previously performed on simple portal frames are represented. In both cases the influence of creep and non intentional inclination were disregarded. A curve $\mu=f(\lambda^*)$ can be extrapolated which does not correspond to the one obtained from simple frames, but gives higher values of μ.
A suitable definition of "equivalent slenderness λ^* should lead to the extrapolation of a $\mu=f(\lambda^*)$ curve, which is unique for a broad class of plane frame layouts and loading conditions.
Once this curve had been determined two aims could be achieved:
- The frontier between slender and non slender frames could be defined by a suitable value of λ^*;
- The moment magnification factor μ could be derived directly from this curve, thus avoiding complicated nonlinear analyses of the frame under

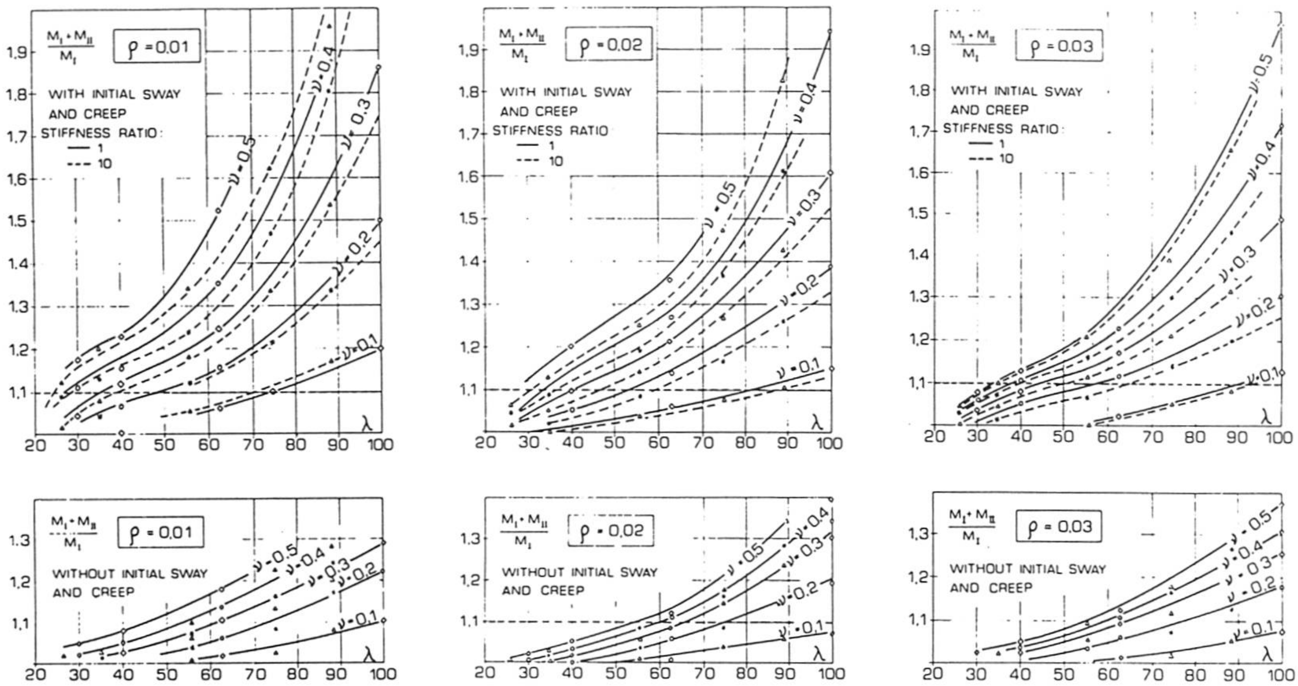

FIG. 24. Moment magnification factors in function of λ and ν for given value of ρ.

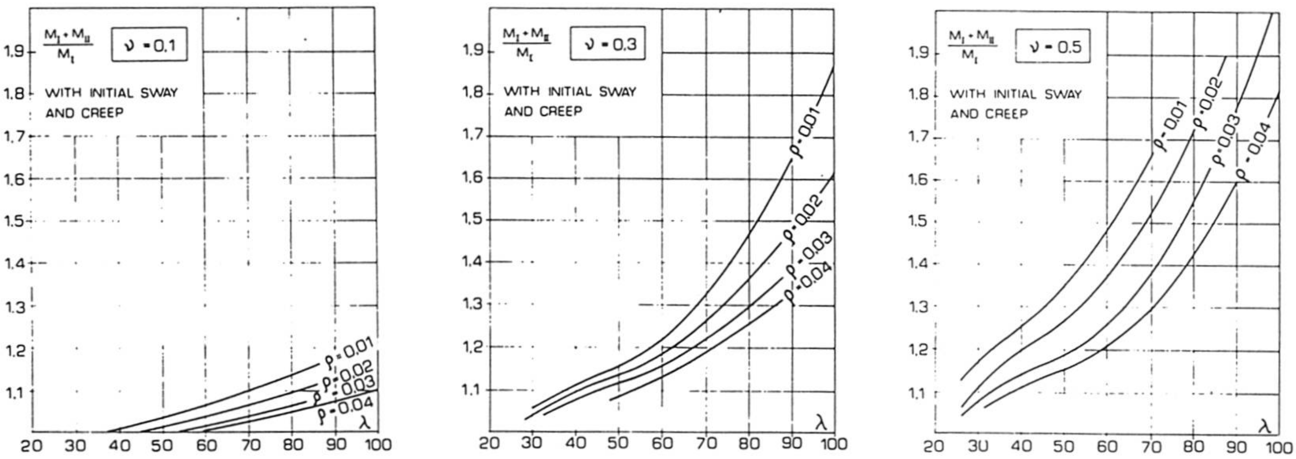

FIG. 25 - Moment magnfication factors in function of λ and ν for given values of ρ.

consideration.
If the values of μ obtained for multistorey frames 1, 2, 3 and 4 are represented on the μ/λ^* plane, it easy to see that they do not fit well in the band of points from which the curve concerning simple frames was derived.
A second curve can be drawn which for high values of $\lambda^*(\lambda^*>25)$ gives higher values of μ.
However, the lower limit of $\mu^*=15$ (corresponding to a magnification factor of less than 1.10) can still be accepted, as the two curves do not differ much around this value.
Therefore the definition of the frontier between slender and non slender frames, which was defined for simple, one bay, one storey frames can be accepted also for multistorey frames.
To understand the nonlinear, behaviour of frames 1, 2, 3 and 4 with reference to the parametric studies concerning simple frames, the basic difference between the two sets of frames must be examined.
In multistorey frames, sizable load uniformely applied along the beams were considered, while only loads concentrated on joints were applied on simple frames. While, according to CEB rules |1|, the "Eulerian" slenderness at each floor was computed taking into account the geometric properties of the frame only and therefore the same value of λ was found for each column of the same floor, the load distribution along the beams can influence the effective length of the columns and therefore their slenderness. As an example let us consider the deformed shape of ground floor of frame number 1 at ultimate limit state (Fig.21); the presence of a considerable vertical load distributed along the beam produces a displacement toward the top of the left-hand point of flexure,thus increasing the effective length of the same column. As a consequence, in the real behaviour of the frame, the slenderness of the columns belonging to a given floor is not the same, greater being the slenderness of the left-hand side column (if of course horizontal loads are applied from left to right). This can also be derived from the diagram of Fig.22, where values of magnification factors μ are plotted for the two ground floor columns of frame N.1, in function of λ^* at different load levels and not only at Ultimate Limit State as in the preceding diagrams. It is easy to see that the values for the left-hand column are considerably higher. This phenomenon is relevant whenever the distributed load is high, while the beam /column stiffness ratio is relatively low. This is the case in many practical situations.

8. CONCLUSIONS

An "equivalent" slenderness expression

$$\lambda^* = \frac{1}{1 + 15\rho} \nu^{0.6} (f_{cd}/17.5)^{0.5}$$

was proposed and justified with the purpose of individuating a parameter capable of giving an approximate measure of the "degree" of non linearity of structural behaviour.
To perform this task, the expression must contain the variables which are mostly relevant to nonlinear behaviour and give a very compact representation of moment magnification factors.
This result is with acceptable approximation attained with both proposed formulations (Fig.s 23 and 24).
It must be noticed that the expression proposed by the authors seems more effective when creep and non intentional inclination are considered in nonlinear analysis (upper curve of Fig.23) while good results are obtained with the expression by Menegotto-Via, only in the cases where these factors are disregarded.

This fact may be explained by what was previously said about the stiffening effect of reinforcement: the Menegotto-Via expression tends to under-estimate this effect, which is, on the other hand, emphasized by creep behaviour. Both formulations however permit to individuate clearly a lower limit of slenderness, below which second order effects can be disregarded. In the case of λ^* formulation, if the assumption is made that second order moments of less than 10% of first order moments can be disregarded, a lower limit of $\lambda^*=15$ can be assumed, as already stated in Ref. |4|.
This limit seems to remain valid when more complex frames are considered as results from the parameteric study illustrated in paragraph 7.
The same results show also however that the $\mu=f(\lambda^*)$ diagrams cannot be used for the direct determination of second order effects, at least as long as the procedure to determine the effective length of columns does not take into account the distribution of vertical loads as explained in paragraph 7.
The proposed formulation modified in some detail will be introduced in the new Italian Code for the definition of the field of slender frames (See Appendix III)

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APPENDIX 1 - Theoretical relationship between M_{II} and ν

The second order moment M_{II} in a "model" column as represented in Fig.4/A can be derived from the beam column elastic theory (see Timoshenko and Gere |8|) or by assuming a deformed shape of the column, such that the displacement on top be a function of curvature at the built in end of the column. Although the expressions which are obtained in both cases are complex, it can be shown that, for moderate slenderness and axial load ($\lambda<70, \nu<0.5$) and in the field of normally used concrete design resistances ($10 Mpa < f_{cd} < 30 Mpa$) M_{II} can be considered approximately as a linear function of ν; therefore ν should theoretically be introduced in the expression of λ^* affected by an exponent $i \simeq 1$ (however a lower value $i=0.6$ was introduced, to better fit the results of "accurate" nonlinear analyses).

- First method

This method was used to derive the "moment magnifier" procedure suggested by the ACI Code |2| to compute M_{II} in slender columns.
The top deflection of the column can be expressed as

$$\delta = \frac{P\,l^3}{24EJ}\;\chi(u)$$

where

$$u = \frac{l_o}{2}\sqrt{\frac{N}{EJ}}$$

and

$$\chi(u) = \frac{3(\tan u - u)}{u^3}$$

an approximate expression of $\chi(u)$ can be obtained by developing tan u as a series

$$\tan u \cong u + \frac{u^3}{3} + \frac{2u^5}{15} + \ldots$$

$$\chi(u) = 1 + \frac{2u^2}{5} = 1 + \frac{1}{10}\,l_o^2\,\frac{N}{EJ} + \ldots$$

The second order moment is therefore given by

$$M_{II} = N\,\delta = ab\,f_{cd}\,\nu\,\frac{Pl_o^3}{24EJ}\left(1 + \frac{1}{10}\,\frac{l_o^2\,ab\,f_{cd}\,\nu}{EJ}\right) \tag{1}$$

M_{II} is represented by a polynomial expression where the quadratic term is small if the parameters influencing the slenderness (l_o and J) are sufficiently small. If the third term of the series representing tan u is disregarded, a linear expression is obtained for $M_{II} = f(\nu)$. $\chi(u)$ could be also expressed approximately by the following relationship (adopted by the ACI Code, see Ref.s |2| and |8|)

$$\chi(u) \cong \frac{1}{1 - N/N_{crit}} = \frac{1}{1 - \dfrac{\nu ab\,f_{cd}}{(\pi^2 EJ)/l^2}}$$

M_{II} is therefore expressed by

$$M_{II} = \frac{Pl^3}{24EJ}\,\frac{ab\,f_{cd}\,\nu}{1 - \dfrac{ab\,f_{cd}\,\nu}{(\pi^2 EJ)/l^2}} = \frac{k\nu}{1 - k'\nu} \tag{2}$$

If the column slenderness is not too high (and therefore the design load/critical load ratio is sufficiently small) again a linear expression is obtained for $M_{II}=f(\nu)$.

- Second method

The second order moment could be also computed using the basic criteria adopted by CEB Model Code |1|, in the so called "Model Column" method.
M_{II} can be expressed as

$$M_{II} = N\, l_o^2/10 \cdot 1/r$$

where 1/r is the curvature at the column built in base

$$1/r = \frac{Pl_o}{2EJ} + \frac{M_{II}}{EJ} = \frac{Pl_o + 2\, M_{II}}{2\, EJ}$$

$$M_{II} = \frac{Nl_o^2\, (Pl_o + 2\, M_{II})}{20\, EJ}$$

$$M_{II} = \frac{N_o\, l_o^3\, P}{20\, EJ - 2\, N\, l_o^2} = \frac{\nu l_o^3\, P}{\frac{20\, EJ}{baf_{cd}} - 2\, \nu l_o^2} = \frac{k\nu}{k' - k''\nu} =$$

$$= \frac{k/k'\, \nu}{1 - k''/k'\, \nu} = \frac{k'''\, \nu}{1 - k^{iv}\nu} \qquad (3)$$

$$k^{iv} = k''/k' = \frac{1.2\, f_{cd}\, l_o^2}{E\, a^2} \quad \text{(for rectangular section)}$$

the k^{iv} constant is small in most cases and therefore expression (3) can be considered a linear function of ν.
The values of the moment magnifier $\mu=(M_I+M_{II})/M_I$ have been plotted using expression (2) (first method)and expression (3) (second method) for ν variing from ν=0.1 to ν=0.8, for three values of λ (35,50,70) and 4 values of f_{cd} (10,20,30,40 Mpa).
These values are reported on Fig. 23 It is interesting to note that similar values of μ are obtained with the two methods.

APPENDIX II - DETAILS ON NUMERICAL TESTS

1. Computer program

The program used for this research, named SICA NL, can execute a non linear analysis of generalised concrete plane frames taking into account both material and geometrical non linearities.
The "displacement" approach was chosen and a step-by-step procedure is used to track the evolutive behaviour of the structure up to collapse.
In brief the operations that the program can perform can be summarized as follows:
a) Structural and load data are read.
b) The load vector {F} is subdivided in a given number of steps Δ_i{F} of constant or variable intensity. Subdivision of loads in steps can be performed in a number of ways according to necessity; in particular some loads may be increased up to a given level (permanent loads) and then left

constant while others can be increased up to collapse of the structure. It is thus possible to consider non proportional loads.

c) Solution, at every step, of an equilibrium system of the kind

$$\Delta_i\{F\} = |\overline{K}_{i-1}|\Delta_i\{u\} \qquad (1)$$

where $\Delta_i\{u\}$ is the displacement vector corresponding to $\Delta_i\{F\}$ and $|\overline{K}_{i-1}|$ is the structure stiffness matrix, relative to step i-1, which is corrected at every step to take into account material and geometrical non-linearities.

d) Computation, at every step, of progressive displacements

$$\{u\} = \sum_{1}^{n}{}_i \Delta_i\{u\}$$

and of the corresponding internal forces.

Operations described in c) and d) are repeated until the predefined number of steps has been exhausted. The accumulated errors implicit in this kind of linearization are eliminated using an iterative procedure

Material non-linearities have been taken into account using the following procedure:

a) Beams and columns are automatically divided in a given number of elements introducing additional joints.
b) The plastic effects are concentrated in those section where the ultimate moment is reached, in the sense that in those sections a plastic, limited rotation hinge is introduced. The ultimate moment is computed in every section in function of axial load at every step, using the constitutive laws of materials prescribed by CEB (or ACI).
c) Cracking of the member is considered by substituting the stiffness of the cracked section in those elements where the ultimate tensile stress of concrete has been reached.
d) The "tension stiffening effect", that is the influence of tensile resistance of concrete between cracks, has been simulated with the method described in |4|.

2. SIMULATION OF CREEP

Creep deformations which are produced by permanent (and semi-permanent) loads can be computed step-by-step by the program, so that evolution of the phenomenon can be followed.
The computation procedure can be summarized as follows:
a) Permanent (and semipermanent) load is applied step-by-step.
b) When the maximum value of permanent load is reached, the structure is analysed with that load for a given number of steps which correspond to time intervals.

At every time step the load is kept constant, while a moment-curvature diagram is used, which is obtained from the original diagram, by increasing, at equal moment, the curvature and the corresponding concrete deformation ε_c) by a quantity $k\phi(t)$.
$\Phi(t)$ is the creep coefficient, which, can be expressed by

$$\Phi(t) = \beta_1 \beta_2(t)$$

where β_1 was assumed equal to 2.07.

$\beta_2(t)$ expresses the dependance of $\phi(t)$ upon time.
The following expression has been assumed for $\beta_2(t)$

$$\beta_2(t) = \frac{\log t - \log t_i}{\log t_f - \log t_i}$$

where is

t_i = initial time
t_f = final time
t = current time.

The new moment-curvature expression is therefore given by

$$k' = k(1 + \phi(t)) = \frac{M}{E_{cr} J} \tag{3}$$

where E_{cr} is the equivalent elastic modulus of concrete relative to permanent loads (and therefore considering creep).
If creep is not considered the moment-curvature expression is given by

$$k = \frac{M}{E_c J} \tag{4}$$

where E_c is the elastic modulus of concrete relative to short duration loads (and therefore not considering creep).
From (3) and (4) we obtain

$$E_{cr} = \frac{E}{1 + \phi(t)} \tag{5}$$

Therefore, from a practical point of view, this method can be adopted by substituting at every step the equivalent elastic modulus given by (5). The procedure of application of loads is illustrated by fig. 7.
In the first steps permanent loads are applied.
In the steps from 2 to 5 the load is kept constant, the modulus of concrete is varied according to (5), while, in the calculation of member stiffnesses, according to 2nd order theory, the deformations relative to the previous steps are considered.
From step 5 onward the short duration loads are applied, while the E_c value of elastic modulus is restored.

3. ONE SPAN SWAY FRAMES CONSIDERED IN THIS STUDY

A total of about 420 non linear analyses were performed, using the computer program described in the previous paragraphs, on five series of one span one story reinforced concrete sway frames, to investigate the influence of some relevant variables on the non linear behaviour of this kind of structures, and determine useful rules of design
The variables which were considered are as follows:
1) Column slenderness
2) Concrete quality
3) Column reinforcement
4) Initial sway due to imperfections
5) Axial load on columns.

The analyses were performed first disregarding the influence of initial sway and creep and subsequently considering it.
For each series of frames two different values of ratios between beam and column stiffenesses were considered

$$\alpha_{bc} = \frac{J_b h_c}{J_c l_b} = \begin{cases} 1 \\ 10 \end{cases}$$

In this expression J_b and J_c are the moments of inertia of the uncracked setion of concrete of beam and column respectively, while l_b and h_c are their lengths.
The vertical load V was applied directly on columns and five values were considered corresponding to a reduced load

$$\nu = \frac{V}{b\ d\ f_{cd}}$$

equal to 0.1, 0.2, 0.3, 0.4, 0.5.
The reinforcement in columns was designed according to a linear elastic analysis. Two critical sections were considered (at bottom and on top of column) and the reinforcement designed for each of these section was extended for 1/2 the height of the column.
The same reinforcement has been assumed for each series.
Four values of ρ were considered.
A symmetrical reinforcement was adopted throughout.
For $\rho = 0.02$ analyses were also performed with an improved quality concrete.

The design value for concrete compressive strength was obtained from the characteristic value f_{ck} dividing it by a safety coefficient γ_m equal to 1.5 according to CEB Model Code |1|. Such a value (f_{cd}) is introduced only in the calculation of the strength of the critical section, while the initial modulus of elasticity E_c is computed for the characteristic strength f_{ck}.
Two basic load histories were considered for each case; as previously explained.

4. DEFINITION OF SWAY-FRAME ACCORDING TO CEB

A first check of the obtained results concerns the CEB definition of "sway frame" In table 1 the frames of the studied family are considered, and the corresponding values of the expression

$$h_{tot}\sqrt{F_V/E_{cm}\ J}$$

are calculated, F_V being the total vertical load on the frame (see |1|).
In the table, the shaded area shows the frames which are not considered sway-frames according to CEB rule. The larger area separated by dots includes the frames for which the second order effects increase by less than 10% the first order moments in the critical section (according to the performed calculations), with a reinforcement ratio $\rho = 0.01$ in the columns.
If the reinforcement is increased to $\rho = 0.04$, the area is further increased the new border is indicated by crosses.
Considering for instance the second column of the table, (h_c = 3 m and therefore $\lambda = 35 \div 40$) it means that only for an axial load $\nu = 0.1$ the frame is considered "non sway" by CEB, but with $\nu = 0.2$ the second order effects are less than 10% with a reinforcement $\rho = 1\%$; and it is still less than 10% even with $\nu = 0.4$ if the reinforcement is increased to $\rho = 4\%$.
It can be concluded that the CEB rule is conservative, but that the limits could be considerably raised taking into account the reinforcement ratio in the measure of the conventional slenderness.

λ	26÷29	35÷40	56÷63	75	85÷100	127
ν \ h_c	2.18	3.00	4.80	6.36	7.58	10.82
0.1	0.19	0.26	0.42	0.55	0.66	0.94
0.2	0.27	0.37	0.59	0.78	0.93	1.33
0.3	0.33	0.45	0.72	0.96	1.14	1.63
0.4	0.38	0.52	0.84	1.11	1.32	1.88
0.5	0.42	0.58	0.93	1.24	1.47	2.11

Table 1 - Values of $h_{tot} \sqrt{F_V / E_{cm} J}$

APPENDIX III - PLOTS OF MOMENT MAGNIFICATION FACTORS IN FUNCTION OF "EULERIAN" SLENDERNESS

The most sinificant results of the analyses were organized and plotted in the following manner:

a) The critical section at the base of the left-hand side column was considered

b) In that section the magnification factor

$$\frac{M_I + M_{II}}{M_I}$$

was computed, where $(M_I + M_{II})$ is the moment resulting from a non linear analysis with program SICA NL, while M_I is the moment resulting from a conventional linear elastic analysis.

c) The value of the magnification factor were plotted against the slenderness λ of the column. This slenderness was computed in a conventional way, that is considering the uncracked concrete section only, and using the CEB expression already mentioned (paragraph 2).

The moment magnification factors are represented in fig.24 for the reinforcement ratios ρ = 0.01,0.02, 0.03

On each of these diagrams the interpolated curves corresponding to the considered values of ν are represented; for each value of ν two curves are plotted correpon-ding to the considered values of beam to column stiffness ratios.

In the upper set of curves the combined effects of initial sway due to imperfec-tions and creep are considered.

In the lower set such effects are disregarded.

In fig.25 the same results are plotted for three values of ν , to emphasize the influence of this factor on the results.

The given diagrams emphasize the considerable influence of ν and ρ on second order effects.

APPENDIX IV - PROVISIONS OF THE FUTURE ITALIAN CODE CONCERNING THE DELIMITATION OF SLENDER FRAMES

According to this code columns are considered slender when

$$\lambda = \frac{lo}{i} \leq \frac{15(1 + 15\rho)}{\sqrt{a \nu}}$$

being $a = f_{cd}/17,5$.
The limit of $\lambda^* = 15$ has therefore been accepted as the frontier between slender and non slender columns while the espression for λ^* has been slighth modified

$$\lambda^* = \frac{\lambda}{1 + 15\rho} \sqrt{a\nu}$$

as ν is affected by an exponent of 0.5 instead of 0.6