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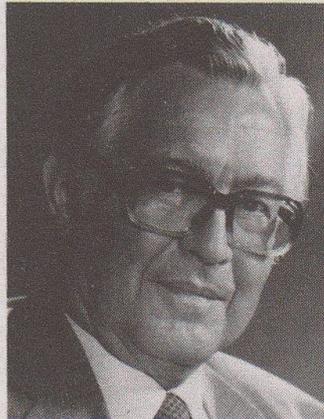
## Cracks and Crack Control at Concrete Structures

Fissures et contrôle des fissures dans les structures en béton

Risse und Risskontrolle bei Betonbauwerken

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### SUMMARY

Cracks are almost unavoidable in large concrete structures. Their causes and their meaning for the serviceability and durability of the structures are treated. Simple rules for the design and sizing of reinforcement or prestressing are given in order to keep the crack width in admissible limits.

### RÉSUMÉ

Il est pratiquement impossible d'éviter les fissures dans les grands ouvrages en béton. Leurs causes et leurs conséquences sur l'aptitude au service et la durabilité des structures sont traitées. De simples règles pour le projet et le dimensionnement de l'armature passive et de précontrainte sont proposées, afin de maintenir les fissures dans des limites acceptables.

### ZUSAMMENFASSUNG

Risse im Beton sind in grossen Bauwerken fast unvermeidlich. Ihre Ursachen und ihre Bedeutung für die Gebrauchsfähigkeit und Dauerhaftigkeit der Bauwerke werden behandelt. Einfache Regeln für die Bemessung der Bewehrung oder Vorspannung werden angegeben, um zulässige Grenzen der Rissbreiten einzuhalten.



## 1. FOREWORD

We wish to achieve concrete structures without cracks, because laymen as clients or users consider cracks as damage or as beginning deterioration, they make the engineer or contractor liable and demand repair. On the other side we assume in the design analysis that the tensile zone of the concrete member is cracked - what a contradiction! Prestressing of concrete structures was invented and applied in order to eliminate tensile stresses and hereby to prevent cracks. But soon we found cracks also in prestressed concrete structures. Why? Are these cracks harmful or harmless? More than 30 years of research and observations referring to the causes and consequences of cracks allow helpful answers.

## 2. CAUSES OF CRACKING

### 2.1 Tensile strength of concrete

The main cause of cracking is the very low and widely scattering tensile strength of concrete. A statistical evaluation of laboratory tests by H. Rüsçh [1] gave the following values for axial tension, related to the 28 day compression cube strength  $f_{c,W}$

$$\begin{aligned} 5 \% \text{ fractile } f_{c,t} &= 0,18 f_{c,W}^{2/3} \\ 95 \% \text{ fractile } f_{c,t} &= 0,36 f_{c,W}^{2/3} \quad \text{N/mm}^2 \end{aligned}$$

In structures the tensile strength may even be lower for reasons which are described in section 2.2.

The flexural tensile strength is slightly higher in beams with a depth between  $d = 15$  to  $30$  cm, however, it is better to neglect this in practical work.

Concrete members crack if the tensile strain  $\epsilon_{ct}$  exceeds  $0,01\%$  to  $0,012\%$ . This rupture strain is almost independent of the concrete strength.

The 5 % fractile of  $f_{ct}$  has to be assumed in design analysis in order to find those zones in the structure which may be affected by cracks. The 95 % fractile of  $f_{ct}$  must be considered for the calculation of the maxima of restraint forces and the necessary amount of reinforcement for the crack width limitation.

### 2.2 Causes of cracking during the hardening period of the concrete

In numerous cases it could be proven that the cracks occurred already during the first days after placing the concrete before any loads acted on the structure. They are caused by "Eigenstresses" (self equilibrating stresses) due to differential temperatures  $\Delta T$  (Fig. 1) which are higher than the slowly developing tensile strength  $f_{ct}$  of the concrete (Fig. 2). These  $T$  must mainly be traced to the heat of hydration which the cement produces during the hardening period and which so far was usually neglected (with the exception of massive structures like concrete dams, see for example [2]). Depending on the type and the quantity of cement, concrete members 20 to 30 cm thick can warm up by about  $20^\circ\text{C}$ , 1 m thick up to  $60^\circ\text{C}$  during the first two days. If the heated member cools down too quickly by cold air, mainly at night, then

the stresses  $\sigma_{ct}$  get easily higher than the still low tensile strength  $f_{ct}$  and the concrete must crack. Even if only micro-cracks form, they will reduce the final tensile strength of the hardened concrete. However, quite often wide cracks show up due to these effects, even when much reinforcement was placed, because the young concrete gives not sufficient bond strength for making rebars effective to limit the crack width.

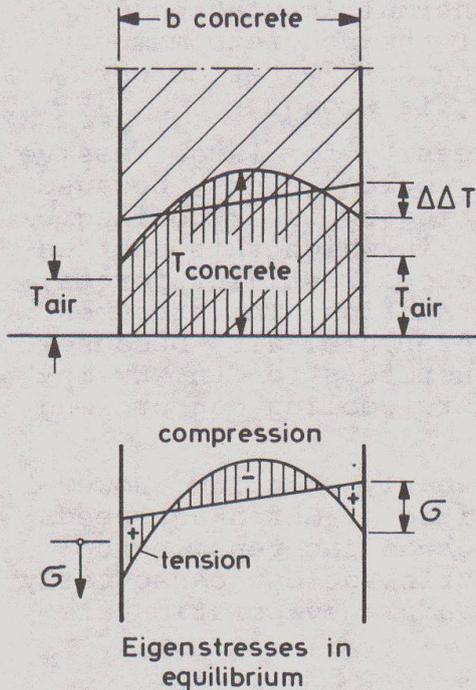


Fig. 1 Heat of hydration gives high temperature  $T$ . Cooling from outside causes "Eigenstresses"

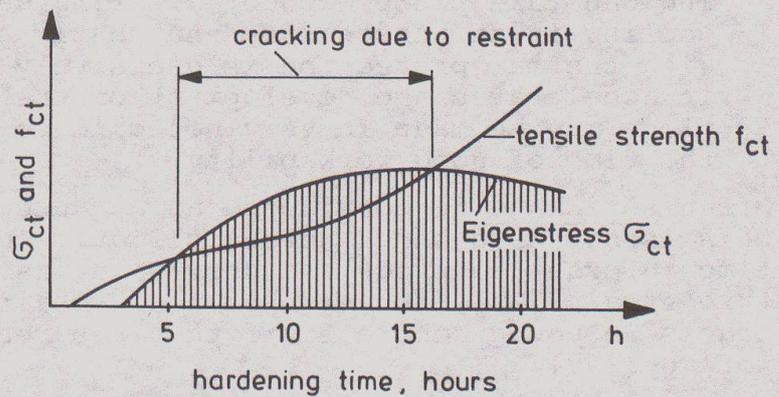


Fig. 2 Development of the tensile strength of concrete  $f_{ct}$  and of "Eigenstresses" due to  $\Delta T$  caused by early cooling

It is necessary to prevent such early cracks by keeping the  $\Delta T$  so low that the  $\sigma_{ct}$  remain smaller than the  $f_{ct}$  (Fig. 2). This can be reached by the following measures, single or in combination:

- Choice of a cement with low initial heat of hydration. Table 1 shows how different the heat development of German cements is, given in Joule per gram cement at  $20^\circ\text{C}$  initial temperature. The quantity of the cement per  $\text{m}^3$  of concrete should be kept as low as possible by good grading of the aggregates. The heat development can be slowed down by adding fly ash or using slag furnace cement.

Table 1: Heat of hydration of German cements in J/g

cement class	1 day	3 days	7 days	28 days
Z 25 Z 35 L	60 to 170	125 to 250	150 to 300	210 to 380
Z 35 F Z 45 L	125 to 210	210 to 340	275 to 380	300 to 420
Z 45 F Z 55	210 to 275	300 to 360	340 to 380	380 to 420



- Curing. First, evaporation of water must be prevented at all open surfaces of the concrete structure by spraying a vapour barrier or by covering the concrete with a dense membrane.
- Curing by thermal insulation. Too quick cooling of exterior zones must be prevented. The degree of thermal insulation depends on the climate and the thickness of the concrete member, but also upon the type of cement. Spraying cold water on warm young concrete, as it was done for years, is wrong.
- Cooling of young concrete. This is a necessity for large massive concrete structures like dams with construction joints, because the shortening of the concrete after joining by later cooling must be prevented. For normal structures, in which this shortening can take place without creating dangerous restraint forces, cooling is an unnecessary and costly aggravation. The treatment with thermal protection is decisively preferable, also because it accelerates the development of the concrete strength. Exemptions may be made in very hot climate where cooling can prolong the time of good workability.

Often shrinkage is considered as a cause of early cracking. However, this is not true under normal climatic conditions. Shrinkage needs time in order to produce a shortening as high as the tensile rupture strain. Only in very hot and dry air shrinkage can cause early cracks in young concrete, if the measures against evaporation are not applied.

### 2.3 Causes of cracks after the hardening of the concrete

The tensile stresses  $\sigma_{ct}$  due to dead loads DL and live loads LL, producing action forces  $M, N, V$  may first be mentioned. The necessary amount of reinforcement or prestressing must be calculated to satisfy ultimate limit state capacity and simultaneously to keep crack widths in admissible limits in the serviceability limit state. These tensile stresses due to service loads can fully or partially be suppressed by prestressing. The degree of prestressing  $\mathcal{K} = M_D / M_{DL+LL}$  can be chosen  $\mathcal{K} \leq 1,0$  ( $M_D$  = moment of decompression) along structural or economic criteria. Normally  $\mathcal{K} = 0,4$  to  $0,6$  lead to better serviceability than full prestressing if the reinforcement is designed, following the rules given in section 5.

Cracks can also occur by tensile stresses which are produced by restraining deformations caused by strains due to rising or falling temperatures or due to shrinkage and creep of the concrete. Imposed deformations like differential settlement between foundations can also cause cracks.

We speak of restraint forces - there is internal restraint causing "Eigenstresses" as shown in Fig. 1, and external restraint in hyperstatic (redundant) structures, as shown in Fig. 3.

Cracks due to these causes in prestressed concrete bridges have taught us that they were mainly due to temperature differences produced by sunshine and following cooling by rain or night. Extreme weather conditions must be considered as they may come every 20 to 50 years. The possible maxima of  $\Delta T$  depend much upon the local climate. The highest  $\Delta T$  were found in continental climate and in high mountains in zones of moderate or cold climate. In several countries measurements of  $\Delta T$  at bridges have been made - see [3,4,5].

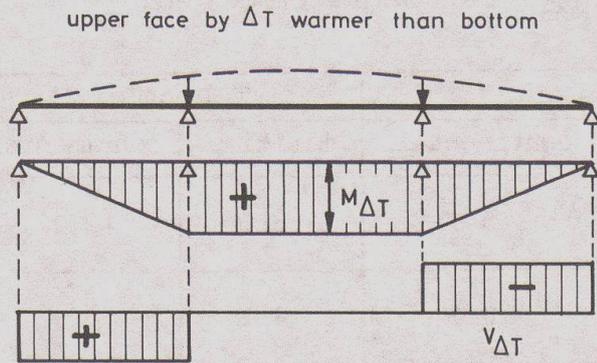


Fig. 3 Restraint action forces  $M_{\Delta T}$  and  $V_{\Delta T}$  at a continuous beam due to  $\Delta T$

Lately the Transportation Research Board of USA has published the Report 276 on "Thermal effects in concrete bridge superstructures" (September 1985).

These  $\Delta T$  have to be superimposed to the mean temperature changes  $T_m$  which must be assumed for calculating the max or min changes of the lengths of the structures. In central Europe these  $T_m$  are specified for concrete bridges with  $+20^\circ\text{C}$  and  $-30^\circ\text{C}$  from a mean of  $+10^\circ\text{C}$ .

The extreme temperature diagram can be subdivided into three parts (Fig. 4). The linear part of  $\Delta T$  causes restraint forces in hyper-

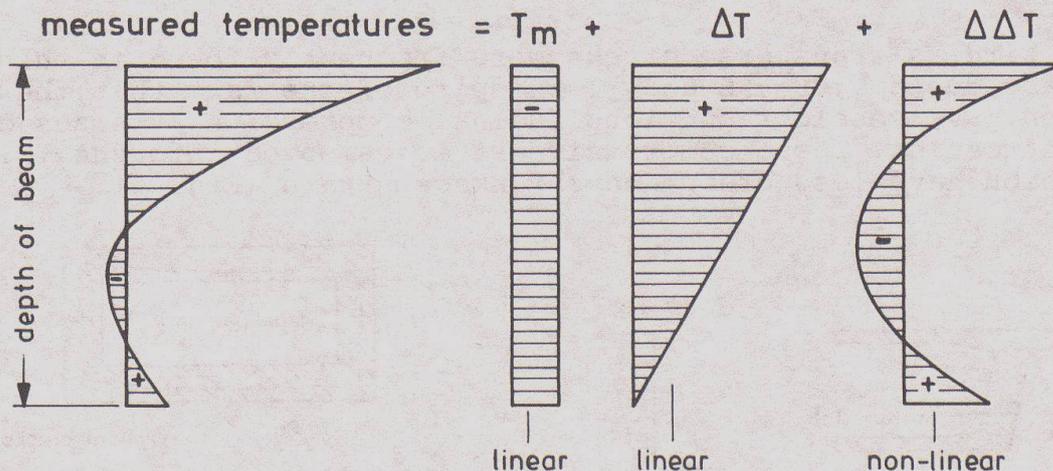


Fig. 4 Division of a temperature diagram into linear  $\Delta T$  and non-linear  $\Delta\Delta T$

static structures, e.g.  $M_{\Delta T}$  and  $V_{\Delta T}$  in a three span continuous beam as shown in Fig. 3. The non-linear part causes Eigenstresses, which are in equilibrium over the cross-section and produce no action forces, but exist also in statically determinate structures. These Eigenstresses due to  $\Delta\Delta T$  can simply be calculated:

$$\sigma_{c,T} = \Delta\Delta T \cdot \alpha_T \cdot E_c$$

$\alpha_T$  = thermal expansion factor,  $10^{-5}$  per 1 K for normal concrete.

Only cooling causes tensile stresses at the edge zones.



For bridges in Europe, the following  $\Delta T$  can be recommended:

Type of structure	box girder		T beams	
	maritime	continental	maritime	continental
upper face warmer than bottom	$\Delta T = 10 \text{ K}$	15 K	8 K	12 K
bottom edge warmer than upper face	$\Delta T = 5 \text{ K}$	8 K	4 K	6 K

Differential shrinkage  $\Delta S$  can in addition to  $\Delta T$  cause such stresses if the shortening of the concrete is restrained.  $\Delta S$  often lead to cracks if thin members are connected to thick members. Also differential creep  $\Delta Cr$  can cause cracks like those found in construction joints of some German bridges, built spanwise, if all tendons were coupled in the web. This was not the case when the incremental launching method was used with tendon couplers distributed over the whole cross section.

In box girders transverse cracks were frequently found in thin bottom slabs due to  $\Delta Cr$ ,  $\Delta S$  and  $\Delta T$  in spite of the fact that the calculation gave considerable longitudinal compressive stresses due to prestressing. These compressive stresses moved into the thick webs which have less creep and shrinkage strain (Fig. 5).

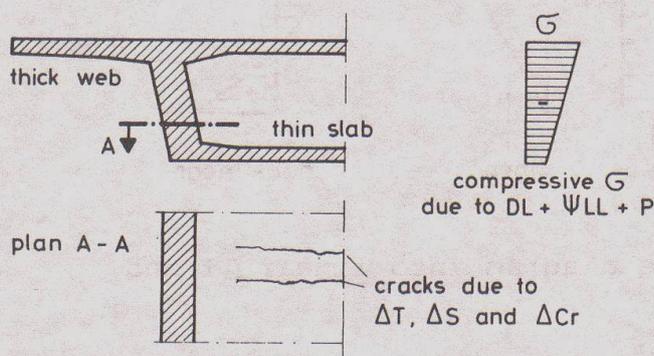


Fig. 5 Transverse cracks in thin bottom slab due to  $\Delta S$ ,  $\Delta Cr$ ,  $\Delta T$  in spite of high prestressing

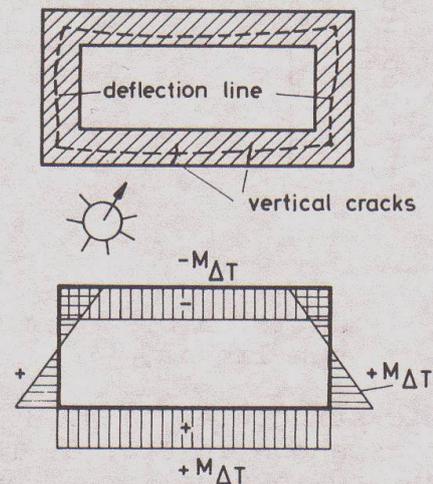


Fig. 6 Bridge pier,  $M$  due to sunshine by restrained deformations

Box sections are redundant frames and therefore they are affected by restraint moments if they are heated on one side, e.g. by sunshine. This leads to vertical cracks in bridge piers or tower shafts (Fig. 6).

Examples of temperature cracks at p.c. bridges are published in [6] with additional references.

### 3. DETERMINATION OF ZONES ENDANGERED BY CRACKS AND TREATMENT OF ACTION FORCES DUE TO RESTRAINT

Cracks occur in zones of the structures in which the principal tensile stresses  $\sigma_{ct}$  due to loads or due to restraint forces or due to the addition of  $\sigma_{ct}$  both in service condition exceed the tensile strength of the concrete  $f_{ct}$ . The  $\sigma_{ct}$  are normally calculated for the uncracked state I with  $\sigma_{ct}$  the linear theory of elasticity. The 5 % fractile of  $f_{ct}$  should be assumed as the limit strength.

The tension flange of beams under bending is crack-endangered over the length in which  $M_{load+restr} > M_{crack}$ , where this cracking moment is defined by  $\sigma_{ct} = f_{ct,5\%}$  in the edge fibre.

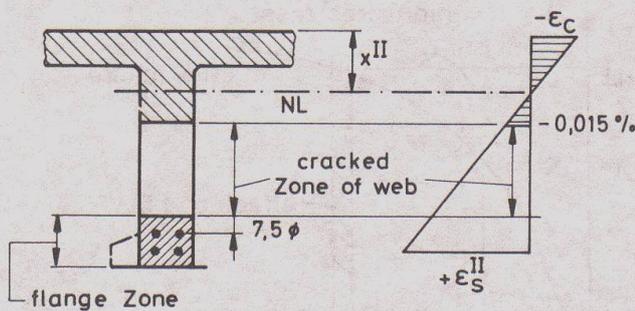


Fig. 7 Cracking zone in webs of beams under max  $M_{load}$  or  $M_{DL+LL+restraint}$

When the flange zone cracks, then the crack tends to continue into the web. The upper limit of the crack-endangered zone in the web has to be found by calculating the strain diagram for the cracked state II under max  $M$ . The limit is given by  $\epsilon = 0,015\%$  (Fig. 7).

The max possible action forces caused by restraint, preferably bending moments, have to be calculated with the maxima of the causing forces, like  $\Delta T$ , assuming that the 95 % fractile of the tensile strength  $f_{ct}$  has to be overcome in the tension flange.

This  $M_{restr}$  has to be added to the moments due to loads, at least for the frequent ones, and it hereby lengthens the zones in which  $\sigma_{ct} > f_{ct,5\%}$  occurs in the flange (Fig. 8).

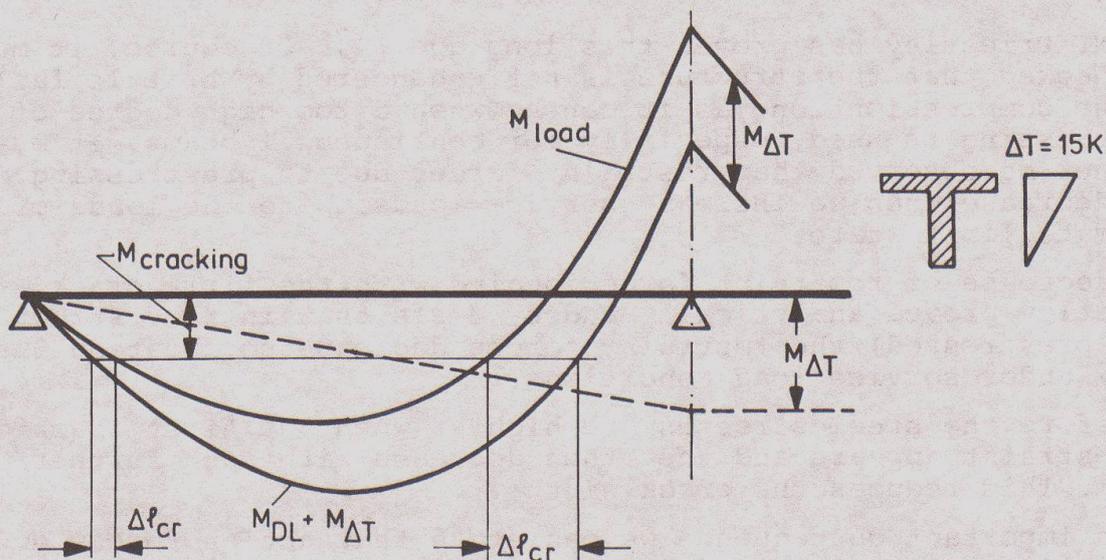


Fig. 8 Additional length  $\Delta l_{cr}$  of the crack endangered zone of the bottom flange of a continuous beam due to  $\Delta T$



Favourable live load moments, like negative moments, can of course not be superimposed onto positive moments due to restraint forces. The sectional forces due to restraint define only location and quantity of the reinforcement or of prestressing forces necessary to limit the crack width in the serviceability state. They do not decrease the ultimate carrying capacity because these  $M_{restr}$  are reduced and finally disappear by cracking and plastic deformation when we increase the loads with the required safety factor to reach the ultimate limit state, which defines the necessary quantity of steel ( $r_c + p_c$ ) for the carrying capacity (Fig. 9).

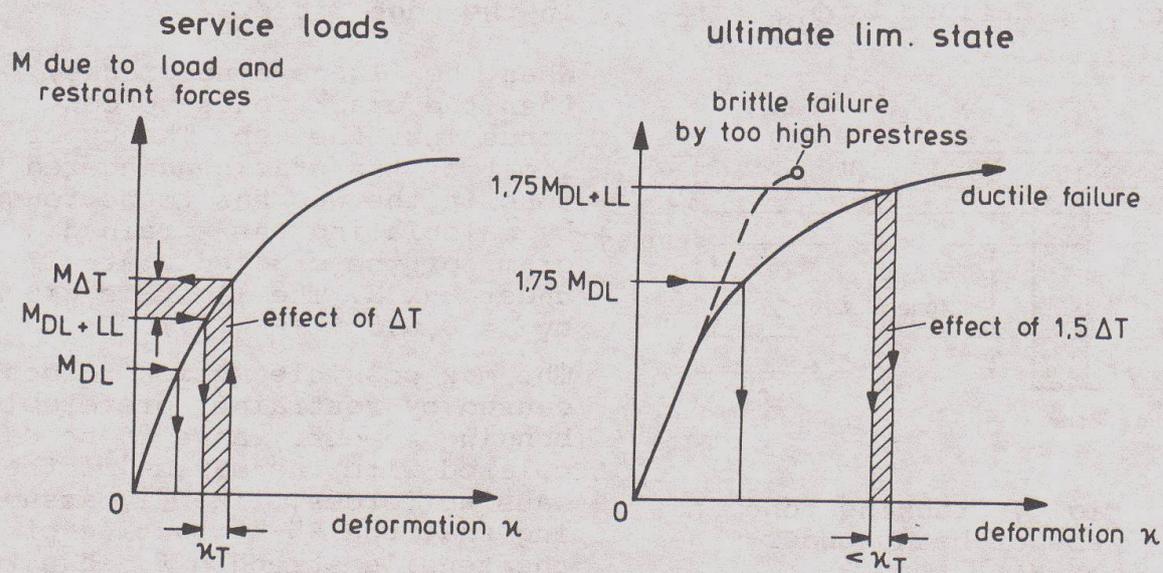


Fig. 9 Priestley's display how restraint forces  $M_{\Delta T}$  disappear in hyperstatic structures due to cracking and plastic deformation if loads are increased to the ultimate limit state, here full prestressing for load moments.

M.J.N. Priestley has proven this long ago [4]. Of course, it must be checked that the structure is not endangered by brittle failure of the compression zone as it can be when a too high degree of prestressing is used, especially for continuous T beams. It must further be observed that restraint forces due to prestressing do not decrease when we increase service loads up to the loads of the ultimate limit state.

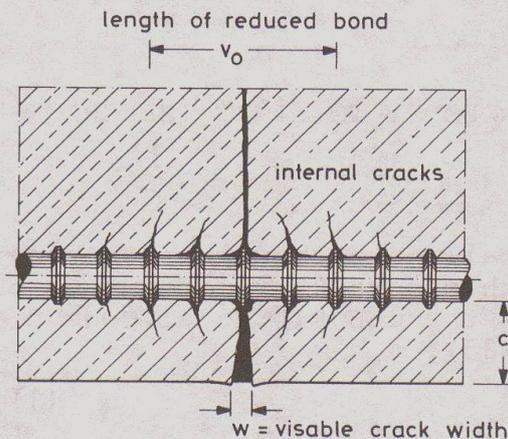
The decrease of restraint forces begins with the first crack. Priestley proved analytically and by tests that in r.c. structures (not prestressed) the restraint forces decrease to about 50 % already under service load conditions.

Therefore the steel stresses are highest when the first crack due to restraint appears and they then decrease with each further crack. This reduces the crack widths.

As an important consequence we can state that action forces due to restraint shall not be added to load forces for the ultimate limit state which defines the sizing of the steel  $A_s + A_p$  in tension flange members. For the serviceability limit state they must be added to define mainly  $A_s$  for crack width limitation.

#### 4. VALUATION OF CRACKS

Cracks are judged by the crack width  $w$  at the surface of the concrete (Fig. 10) which decreases towards deformed rebars. Long years of research [7] and [8] and experience showed that crack widths up to  $w = 0,4$  mm do not significantly harm the corrosion protection of the rebars, if the concrete cover is sufficiently thick and dense.



Polluted air, especially  $\text{CO}_2$  causing carbonation, and  $\text{SO}_2$  forming acids, or chlorides<sup>2</sup> from deicing salts, damage the concrete independent of cracks. Structures must be protected against such attacks, having cracks or no cracks.

Fig. 10 The crack width  $w$  at the surface serves as a scale

Cracks are harmful for the image of the engineers if they are easily visible, because laymen consider them a damage. Therefore at concrete faces which are often seen from a short distance, crack widths  $w > 0,2$  mm should be avoided just for appearance or image sake.

Different grades of environmental aggression and different sensibility of steel types against corrosion led to different requirements for the concrete cover. It makes sense to scale also the admissible limits of the crack width for different environmental conditions. Herefore the limit values should be defined with the 90 % fractile  $w_{90}$  in order to keep a sufficient margin for occasionally surpassing crack widths, which should prevent claims for repair liability be raised too quickly.

On the other side, a max  $w$  should be given and when this will be surpassed, then a damage must be admitted.

For the environmental criteria of CEB and Eurocode No. 2, we can define the following crack widths:

Table 2: Allowable crack widths

environment	$w_{90}$	max $w$	appearance
a low aggressivity	0,3 mm	0,5 mm	easily visible
b medium aggressivity	0,2 mm	0,4 mm	scarcely visible for the unarmed eye
c high aggressivity	0,1 mm	0,3 mm	



These values are valid for a normal concrete cover  $c = 30$  mm and hereby for bar diameters  $\phi < c/1,2 \leq 25$  mm. For a larger cover, the allowable crack width should increase with  $c/30$  ( $c$  in mm). For  $c > 60$  mm and bar  $\phi > 32$  mm an anchored skin reinforcement with thin bars inside the concrete cover must be recommended in order to prevent cracks to open too wide.

5. SIMPLE METHODS FOR SIZING REINFORCEMENT TO LIMIT THE CRACK WIDTH

5.1 Basic analysis

The sizing must be based on theoretically and experimentally derived formulae for calculating the width of cracks which can be displayed as follows (The author follows the CEB-FIP Model Code of 1978 and the CEB Manual of October 1983).

The mean crack width is  $w_m = s_{rm} \cdot \epsilon_m$  (1)

The strain  $\epsilon_m$  is found in the stress - strain diagram of an axially tensioned r.c. bar according to Fig. 11:

$\epsilon_m = \epsilon_s^{II} - \Delta\epsilon_s$  and here is  $\Delta\epsilon_s = \frac{1}{E_s} \frac{\sigma_s^2, 1.cr.}{\sigma_s^{II}}$  (see [9]) (2)

$\Delta\epsilon_s$  corresponds to the strain reduction by concrete in tension between cracks, the so-called tension stiffening.

$\epsilon_s^{II}$  and  $\Delta\epsilon_s$  include the considerable influence of the concrete strength and of the relative amount of reinforcement  $\rho_r = \frac{A_s}{A_c}$

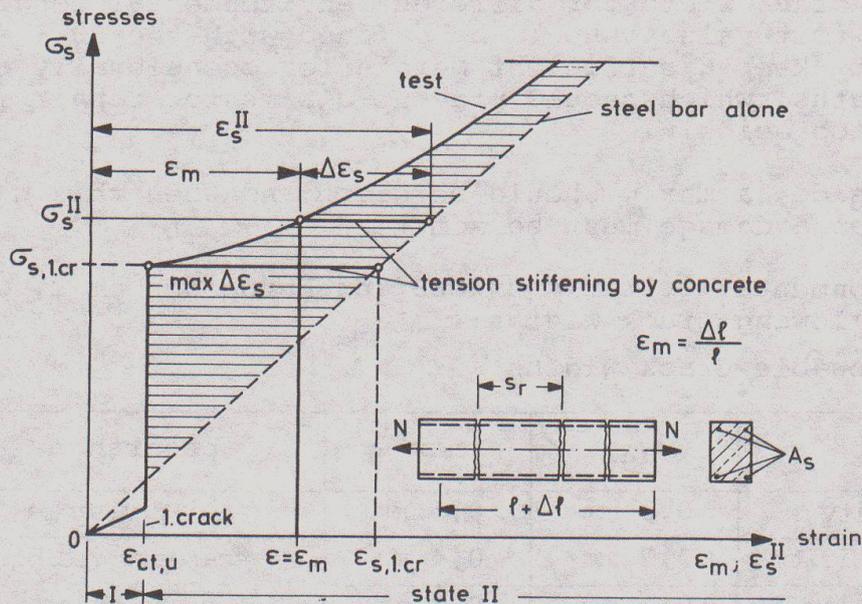


Fig. 11 Stress-strain diagram of a reinforced concrete bar under axial tension. Definition of the  $\epsilon$  values for crack width formulae

The mean spacing of cracks can be written

$$s_{cr,m} = 2 \left( c + \frac{s}{10} \right) + k_1 k_2 \frac{\sigma}{\sigma_r} \quad [\text{mm}] \quad (3)$$

herein is

$c$  = concrete cover in mm

$s$  = transverse bar spacing in mm

$k_1$  = 0,4 for normal ribbed bars, factor to consider the bond strength

$k_2$  = 0,125 for bending, factor to consider shape of  $\xi$  diagram

$k_2$  = 0,25 for centric tension

$k_2$  < 0,125 for bending + axial compression (M with  $-N_p$ )

$\sigma$  = diameter of rebar in mm

$\rho_r$  = degree of reinforcement  $A_s/A_{c,eff}$  related to the effective zone, see Fig. 13.

With these formulae, the mean width of cracks can be calculated.

The characteristic value  $w_{90} = k_4 w_m$  depends on the  $k_4$  factor

for the width of scatter which was found to be as low as  $k_4 = 1,3$  in tests with restraint forces because the steel stress decreases at cracking. Values of  $k_4$  up to 1,7 were found by evaluation of crack measurements at structures. The Eurocode gives  $k_4 = 1,3$  for restraint forces and  $k_4 = 1,7$  for load actions. This differentiation is too complicated for practical design. The author recommends to use generally  $k_4 = 1,5$ .

The effect of repeated loads can be considered by a reduction of  $\Delta \mathcal{E}_s$  in equation (2) with the factor  $k_5$

$$\Delta \mathcal{E}_{s,rep.} = k_5 \frac{\sigma_{s,1.cr.}^2}{\sigma_s^{II} E_s} \quad \text{with } k_5 = 0,4 \text{ to } 0,8$$

depending on the severeness of the dynamic loading (see [9]).

If the direction of the rebars is not rectangular to the crack, like in shear and torsion, then the crack width increases with  $k_\alpha$  which can be assumed to

$$k_\alpha = 1,0 \quad \text{for angles up to } \alpha = 15^\circ$$

$$k_\alpha = 2,0 \quad \text{for angles of } \alpha = 45^\circ$$

for intermediate angles,  $k_\alpha$  can be linearly interpolated.

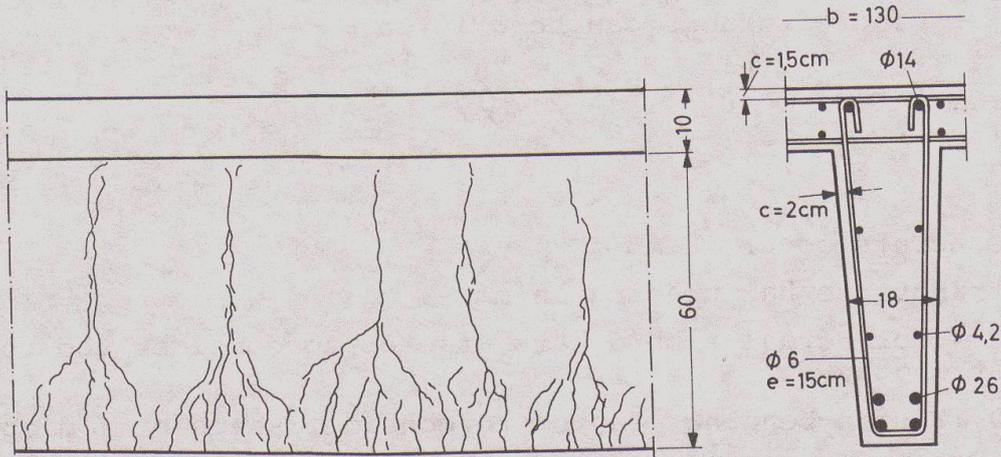


Fig. 12 Cracks of a T beam prove that the small crack spaces and the corresponding small crack widths obtained by the four bars  $\phi$  26 mm in the bottom flange are restricted to a small zone around the bars. Outside this zone, the web reinforcement was too weak to prevent wide cracks

Fig. 12 shows that the reinforcement limits the crack width only within a small zone around the bars which was defined in the CEB-FIP Model Code as the effective zone  $A_{C,eff}$  as shown in Fig. 13.

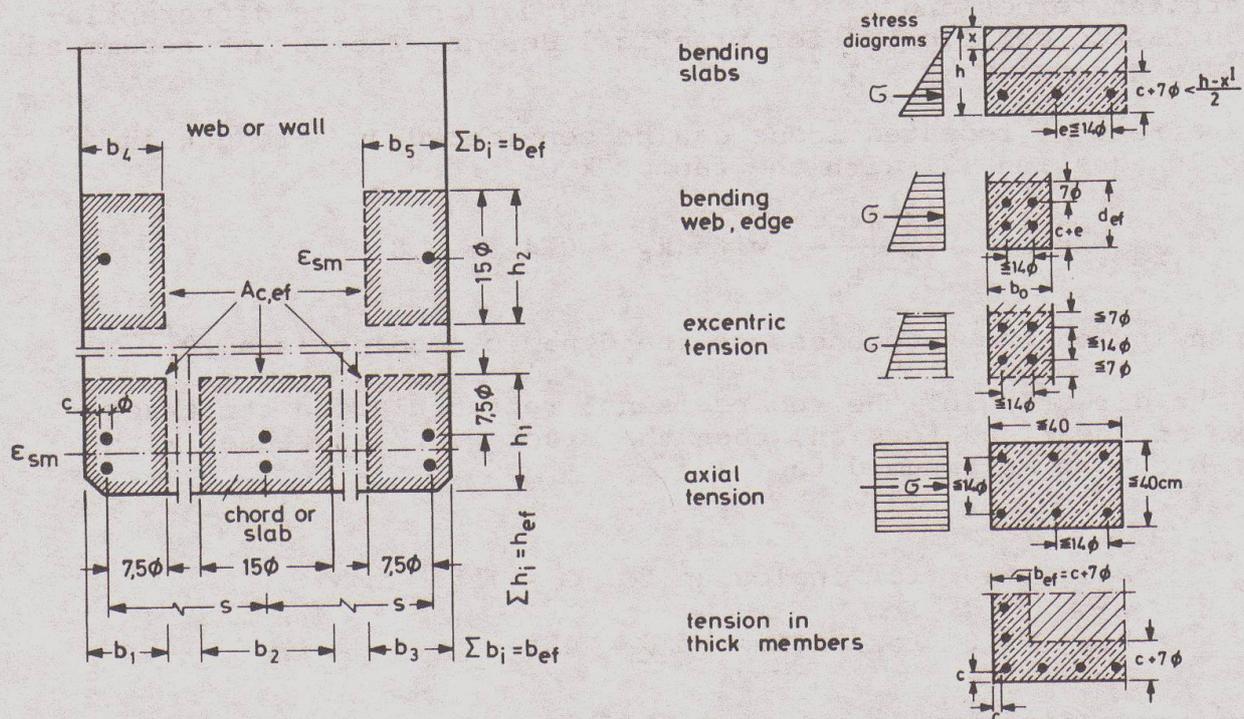


Fig. 13 Definition of the effective zone  $A_{C,eff}$  according to CEB

Definition of  $A_{C,eff}$  for zones with different stress diagrams

The degree of reinforcement must be related to the rather small effective area. Outside this area, wide cracks can form which are harmless for the carrying capacity but should be avoided by additional reinforcement, if appearance counts. Such wide cracks inside massive structures must also be avoided if the structure must be tight against water pressure.

For practical design work it was not intended to calculate crack widths for an assumed amount of rebars with these theoretically based formulae. As early as 1969 it was recommended to use simple charts for sizing the necessary reinforcement (see [10]) and such charts have been published in the CEB Manual of October 1983 in section 2.42. Their use will be explained in the following chapter.

### 5.2 Sizing reinforcement for crack control under axial tension

The  $\rho_r - \phi$  diagram in Fig. 14 allows to read the necessary amount of deformed bars  $A_s$  related to the effective concrete area  $A_{c,eff}$  with  $\rho_r = A_s/A_{c,eff}$  for a chosen bar diameter  $\phi$  and for a specified limit of crack width  $w_{90} = 1,5 w_m$ . The diagram is valid for axial tension due to loads or restraint forces under free elongation conditions.

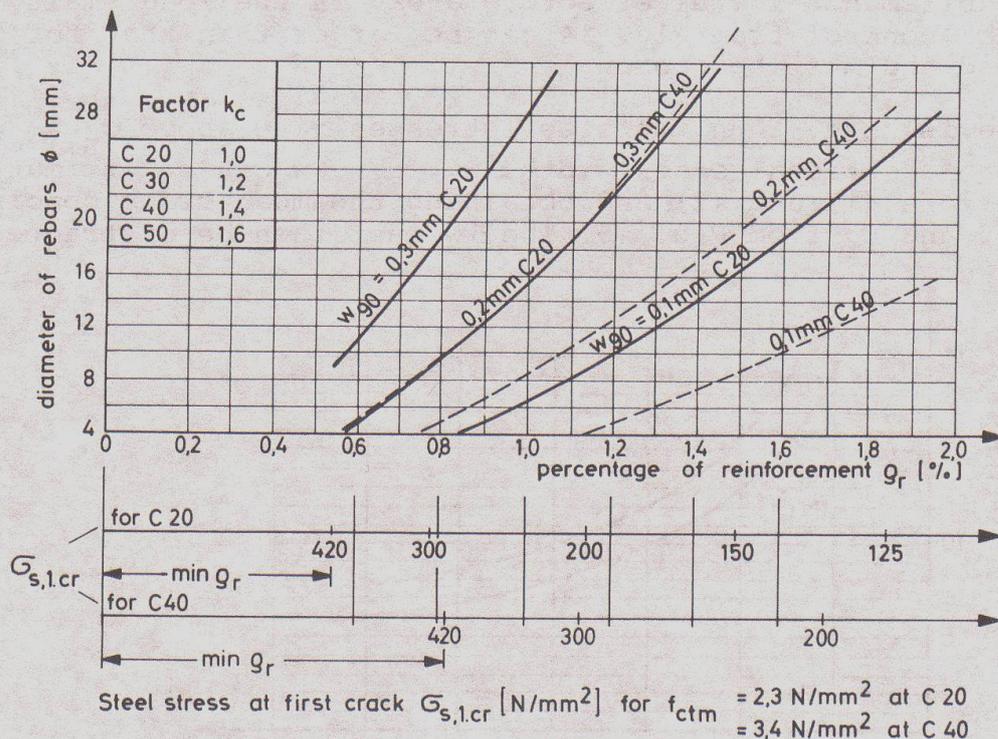


Fig. 14  $\rho_r - \phi$  diagram for axial tension, see text

The full lines refer to a characteristic cube strength of the concrete C 20, the dotted lines to C 40. For other strengths, the factor  $k_c$  has to be used. For crack control one should always choose the concrete class above the one specified for ultimate strength of the structure.

The bar  $\phi$  should be chosen for getting small bar spacings, see section 5.5.



Below this diagram, there are the steel stresses  $\sigma_{s,1.cr}$  given which exist at the first crack, they are

$$\sigma_{s,cr} = \frac{f_{ctm}}{\rho_r} = \frac{0,27 f_{ck}^{2/3}}{\rho_r}$$

This stress shall not exceed the yield strength of the steel and therefore a min  $\rho_r$  is noted, assuming a steel quality of St 420/500. For C 20 we get min  $\rho_r = 0,6 \%$ , for C 40 min  $\rho_r = 0,8 \%$ .

The steel stresses at cracking are in a wide range higher than allowable stresses in former times for service conditions. This is acceptable for restraint forces because they decrease by further cracks. For loads, however, such high stresses are prevented by the dimensioning for ultimate limit state with loads being multiplied with the safety factor, leading to

$$A_s = \frac{\gamma S}{f_{sy}}$$

Normally this  $A_s$  due to loads is sufficiently large to satisfy crack control requirements in the effective area. If the load is small, then  $\rho_r$  for crack control from Fig. 14 can be larger than that for carrying the load and must be chosen.

If the load is high, then the steel stresses rise above  $\sigma_{s,1.cr}$  and cause an additional crack width  $\Delta w$ . This  $\Delta w$  can be estimated, using equations (1) and (2) and obtaining the mean crack spacing for given  $\rho$  and  $\rho_r$  from Fig. 15. The  $\Delta w$  must then be deducted

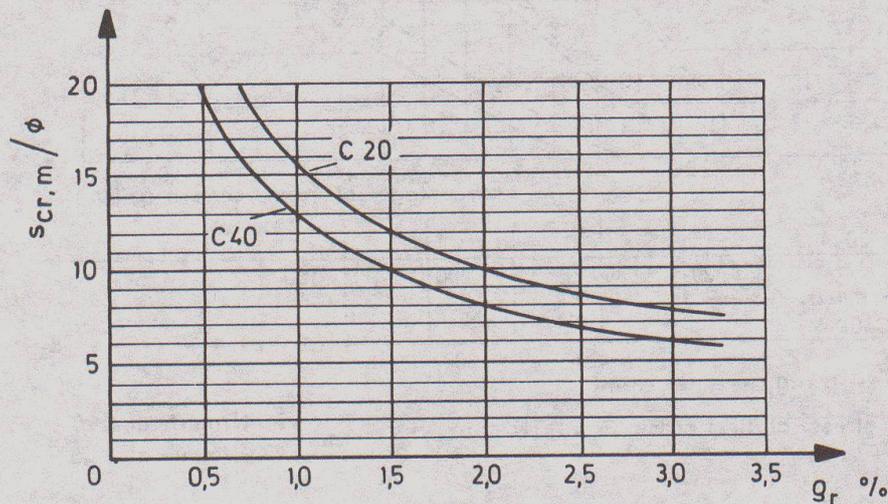


Fig. 15 Mean crack spaces  $s_{cr,m}$  at a r.c. bar under tension, related to bar  $\phi$  and  $\rho_r$

from the specified  $w_{90}$  in order to read the higher  $\rho_r$  from Fig. 14 along a line for  $w_{90} - \Delta w$ . Rough estimations are sufficient.

### 5.3 Sizing reinforcement for crack control for bending and bending with normal force due to prestressing

In a member stressed by bending or bending plus longitudinal compression, a much smaller quantity of reinforcement is sufficient for crack control than for axial tension. This is easily understood if we consider the jump of steel stress at cracking in Fig. 16 and Fig. 17.

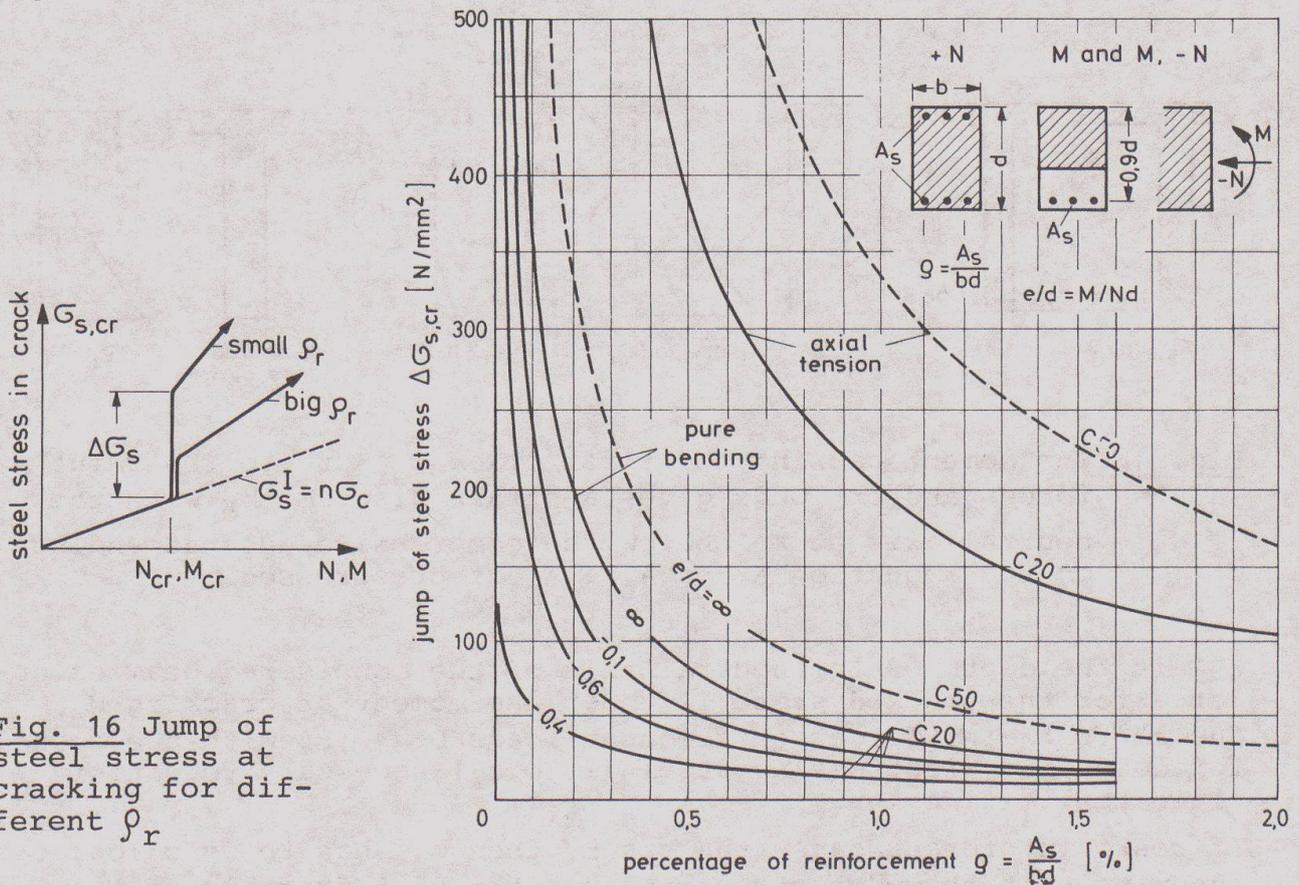


Fig. 16 Jump of steel stress at cracking for different  $\rho_r$

Fig. 17 Jump of steel stress  $\Delta G_{s,cr}$  at cracking of concrete in a r.c.bar under tension  $+N$  or under bending by  $M$  or bending with longitudinal compression  $-N$  with different eccentricity  $e/d$ , for  $f_{ct,m} = 0,19 f_{ck}^{2/3}$  [ $N/mm^2$ ].

This jump of steel stress depends on the concrete quality  $f_{ct}$ , the percentage of reinforcement  $\rho_r$  and the stress characteristic: tension or bending or bending with axial compression of varying eccentricity as for example by prestressing. It must be noted that in Fig. 17  $\rho_r$  is always related to  $A_c = bd$ .

The big difference of  $\Delta G_s$  between tension and bending is obvious. For p.c. structures it is important to see how small  $\Delta G_s$  is getting by the axial compression due to prestressing. The range  $e/d = 1,0$  corresponds to a moderate prestressing degree,  $e/d < -0,4$  corresponds to "limited" prestressing and  $e/d = -0,17$  would be full prestressing. A moderate prestressing ( $\lambda = 0,3 - 0,5$ ) leads already to low steel stresses at cracking in the service state and therefore small  $\rho_r$  are sufficient for crack control.



Also for bending and for  $M$  plus -  $N$  we can use the  $\rho_r - \delta$  diagram of Fig. 14 for finding the necessary  $\rho_r$  if we apply the correction factor

$$k_B = \frac{h - x^{II}}{h} \quad \text{as explained in Fig. 18}$$

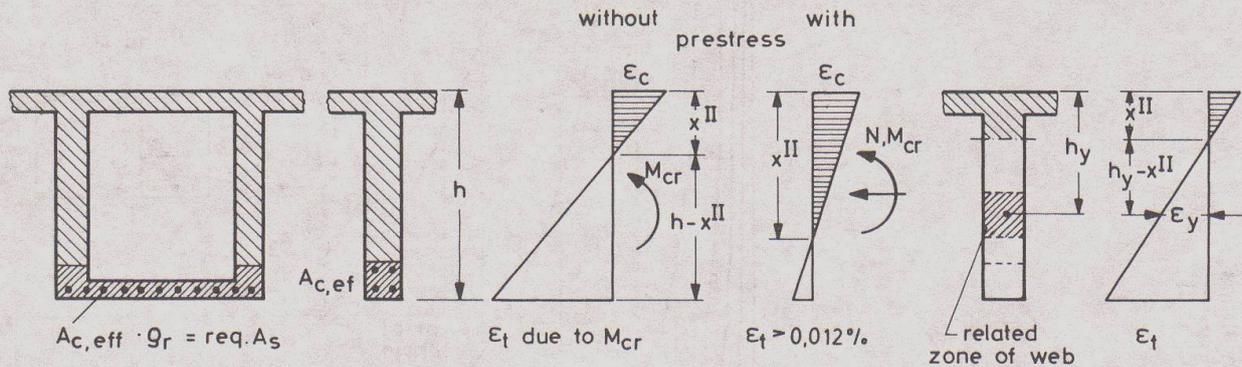


Fig. 18 The moment causing the first crack  $M_{cr}$  due to restraint forces and/or loads gives a strain diagram  $\epsilon_c$  with the neutral axis at  $x^{II}$  below the compressive edge depending on the amount of  $A_s$  or  $A_s + A_p$  if prestressed.

$x^{II}$  is the depth of the neutral axis of the considered beam calculated for the cracked state II under the moment at cracking  $M_{cr}$  due to restraint forces or frequent loads  $DL + LL$  with the reinforcement and prestressing steel (if p.c.) necessary to satisfy ultimate limit design.

If restraint forces cause the crack, then  $M_{cr}$  has to be calculated assuming that the edge stress reaches the 95% fractile of  $f_{ct}$ . For calculating  $x^{II}$  we assume as usual that the cross section remains plane (straight strain diagram!). In fact, this is not true if shear forces act simultaneously which reduce  $x^{II}$ , but so far there is no simple method to consider this correctly.

If partial prestressing is applied, then  $k_B$  can easily be as low as 0,2 or 0,3 leading to small  $\rho_r$  for satisfying crack control. Here again we have to be aware that this  $\rho_r$  read from the diagram Fig. 14 is related to cracking load. Should higher loads later cause stresses considerably above  $\sigma_{cr}$ , then a correction is necessary for

$$\Delta \sigma_s = \sigma_{loads} - \sigma_{cr} \quad \text{with} \quad \Delta w = s_{cr,m} \cdot \epsilon_m \approx s_{cr,m} \cdot 0,74 \epsilon_s$$

( $s_{cr,m}$  from Fig. 15).

In box girders with thin bottom slabs the strain  $\epsilon^{II}$  is restrained by the connection to the webs (Fig. 18). The slab is almost under axial tension - but not unrestrained, as assumed for Fig. 14. However, at such slabs we have to think also of restraint stresses due to differential shrinkage  $\Delta S$ , therefore a supplement to  $k_B \rho_r$  is recommended. This supplement can be roughly calculated

assuming  $\Delta S$  with  $\epsilon_s = 0,01 \%$  which increases the crack width by  $\Delta w_s = s_{cr,m} \cdot 0,6 \epsilon_s$  and  $k_{B,r}$  has to be read in Fig. 14 for  $w_{90} = \Delta w_s$ .

For sizing the reinforcement needed for crack control in the webs above the flange zone

$$k_B = \frac{h_y - x^{II}}{h_y} \quad \text{as shown in Fig. 18}$$

has to be used. The depth of the web should be subdivided into several portions.

For crack control in members stresses by shear or torsion, the formulae given in [9] should be used.

#### 5.4 Crack control without reinforcement

In massive concrete structures or in moderately prestressed structures which get tensile stresses due to  $\Delta\Delta T$  or  $\Delta T$  (see Fig. 1 and 4) it can occur that cracks remain fine hair cracks with widths below  $w_{90}$  even without reinforcement. This is so because the tensile strain  $\epsilon_{ct}$  is restraint by the adjoining zone under compression (Fig. 19).

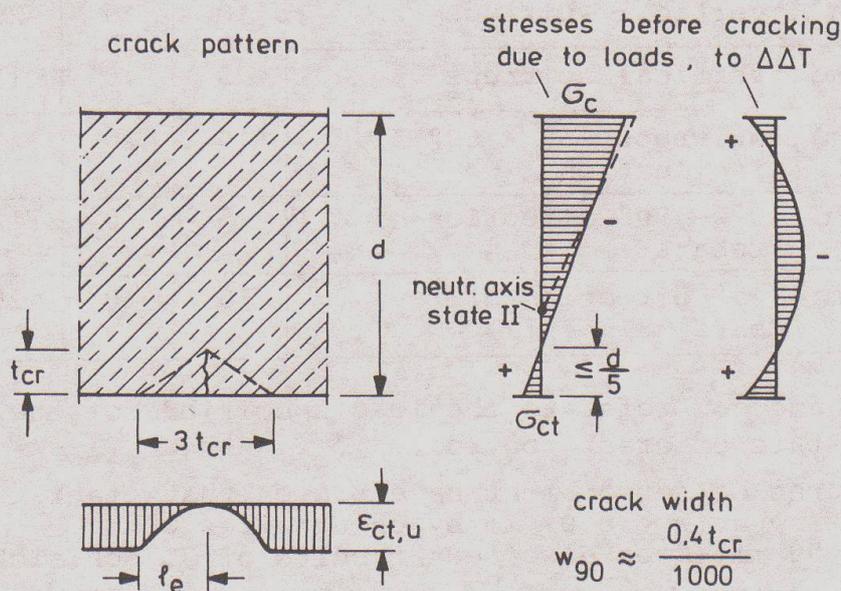


Fig. 19  
If  $\epsilon_{ct} < 0,015 \%$   
then crack width  
remains small with-  
out reinforcement

The width of such cracks depends upon the possible depth  $t_{cr}$  of the crack and can be calculated from the max tensile strain of concrete  $\epsilon_{ct,u} \leq 0,012 \%$  with  $k_4 = 1,6$

$$w_{90} = 1,6 \cdot 2 t_{cr} \cdot \epsilon_{ct,u} \approx 0,4 t_{cr} \cdot 10^{-3} \text{ [mm]}$$

In dry climate, shrinkage of the cracked zone should be considered with  $\Delta S \approx 0,01 \%$ , then we get

$$w_{90} = 1,6 \cdot 2 t_{cr} (\epsilon_{ct,u} + \epsilon_s) = 0,6 t_{cr} \cdot 10^{-3}$$



For  $w_{90} = 0,1$  mm the depth of the cracks can be as large as 25 cm, resp. 17 cm. For restraint bending (e.g. unreinforced but moderately prestressed slabs or beams) the depth should remain below  $t_{cr} \leq d/5$ .

### 5.5 Recommendations for spacing and diameters of rebars

The small effective area of  $7,5 \emptyset$  around the rebars requires small spaces between bars  $s \leq 15 \emptyset$ . The crack width is further almost linearly depending on the bar diameter. Therefore, optimal crack control is obtained by choosing small  $\emptyset$  and small spacing which lead also to the lowest steel quantities. The following table gives

Recommended upper limits of bar spacings measured rectangularly to the bars, in cm

Allowable crack width $w_{90}$ in mm	0,1	0,2	0,3
tension	10	15	20
tension by bending with $\sigma_s^{II} = 240 \text{ N/mm}^2$	10	15	20
tension by bending with $\sigma_s^{II} = 120 \text{ N/mm}^2$	15	20	30
shear with $\tau_o \approx 2 \text{ N/mm}^2$ , vertical stirrups	10	15	20
shear with $\tau_o \approx 3 \text{ N/mm}^2$ , vertical stirrups	5	10	15
shear with $\tau_o \approx 3 \text{ N/mm}^2$ , stirrups $45^\circ - 60^\circ$ inclination	10	20	25
torsion for $\tau_T > 2 \text{ N/mm}^2$ , $0^\circ - 90^\circ$ direction of rebars	5	8	12
torsion for $\tau_T > 2 \text{ N/mm}^2$ , $45^\circ$ direction of rebars	10	20	25

The stresses  $\sigma_s^{II}$ ,  $\tau_o$  and  $\tau_T$  refer to the load specified for the serviceability limit state of crack control.

Nervi's famous structures of ferrocement have proven that crack widths can be kept as low as  $w < 0,01$  mm by using wires with  $\emptyset = 2$  mm spaced 30 to 50 mm - see also test results of J. Schlaich in [11].

## 6. MINIMUM REINFORCEMENT

The minimum reinforcement has to fulfil two requirements:

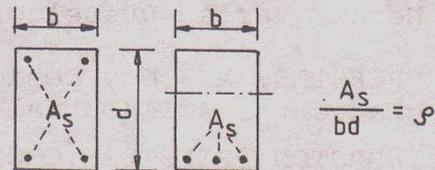
- To secure the load-carrying capacity which has to be calculated for  $\max \gamma (DL + LL)$ , ( $\gamma$  = safety factor, global or split according to codes) but without restraint forces. The rebars with  $A_s$  and prestressed steel with  $A_p$  must remain within the ultimate limits of strain  $\epsilon_s$  and  $\epsilon_p$ .

In order to prevent sudden failure at cracking, which can be caused by restraint forces, the minimum amount of reinforcement must be for pure tension:

$$\min \rho = \frac{f_{ct,95\%}}{f_{sy}} = \frac{0,36 f_{ck}^{2/3}}{f_{sy}} \quad [N/mm^2]$$

for bending:

$$\min \rho = 0,2 \frac{f_{ct,95\%}}{f_{sy}} \quad [N/mm^2]$$



in both cases  $\rho$  must be related to the full cross section  $A_C = b d$ . If cracking is primarily caused by restraint forces due to  $\Delta T$  or  $\Delta S$  or differential settlement, then the related area  $A_C$  can be limited to two or three times the  $A_{C,eff}$  according to Fig. 13.

This requirement leads to the following  $\min \rho$  in [%]:

concrete strength $f_{ck}$ N/mm <sup>2</sup>	20	30	40	50	related area
$\min \rho$ for tension %	0,75	0,93	1,10	1,26	$A_C = b d$
$\min \rho$ for bending %	0,15	0,18	0,22	0,25	$A_C = b d$

- b) For the serviceability limit state the  $\min \rho$  must secure to keep the crack widths below the required limit. The  $\min \rho_{cr}$ , therefore, depends on the allowed  $w_{90}$ , the concrete strength and the chosen bar  $\phi$  and must be related to the effective area  $A_{C,eff}$ . This  $\min \rho_{cr}$  must be built into all zones where the concrete tensile stresses, calculated for the uncracked state I due to loads or restraint forces become higher than the 5 % fractile of the tensile strength of concrete, this is where

$$\sigma_{ct}^I \geq 0,18 f_{ck}^{2/3} \quad [N/mm^2]$$

In these zones,  $\min \rho$  is found from fig. 14 together with the  $k_B$  factor according to Fig. 18 if bending or prestressing is involved.

In zones without this cracking danger, the min reinforcement can be chosen along constructional criteria.



## REFERENCES

1. RÜSCH H., Die Ableitung der charakteristischen Werte der Betonzugfestigkeit. Beton, Heft 2/1975.
2. WIDMANN R., Massenbetonprobleme beim Bau der Gewölbemauer Kölnbrein. Lectures of Betontag 1977. Deutscher Beton-Verein.
3. KEHLBECK F., Einfluß der Sonnenstrahlung bei Brückenbauwerken. Werner Verlag Düsseldorf, 1975.
4. PRIESTLEY M.J.N., Design of concrete bridges for temperature gradients. ACI Journal, 1978, pp. 209-217.  
THURSTON, PRIESTLEY, COOKE, Influence of cracking for thermal response of reinforced concrete bridges. Concrete International, August 1984.
5. DILGER W., GHALI A., Temperature stresses in composite box girder bridges. Journal of Structural Engineering, ASCE, June 1983, p. 1460.
6. LEONHARDT F., Rißschäden an Betonbrücken, Ursachen und Abhilfe. Beton- und Stahlbetonbau, Heft 2/1979.
7. SCHIESSL P., Admissible crack width in reinforced concrete structures. Preliminary Report Vol. 2 of IABSE-FIP-CEB-RILEM-IASS, Colloquium Liège, June 1975.
8. Concrete International, May 1985, p. 24, A.W. BEEBY's contribution.
9. LEONHARDT F., Vorlesungen über Massivbau, 4. Teil, Springer Verlag, 1977.
10. FALKNER H., Zur Frage der Rißbildung durch Eigen- und Zwangsspannungen infolge Temperatur in Stahlbetonbauteilen. Deutscher Ausschuß für Stahlbeton, Heft 208.
11. SCHLAICH J., DIETERLE H., Versuche mit Ferrozement. 15. Forschungskolloquium des DAFStB in Stuttgart, April 1984.