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## Structural Models for Fire Analysis

Modèles pour l'analyse des structures soumises au feu

Modelle zur Untersuchung von Tragwerken unter Feuer

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### SUMMARY

During the last decade a considerable amount of work has been performed in the Department of Bridges and Structural Engineering of the University of Liège in order to develop models for the analysis of the structural behaviour under fire conditions. An important part of the research has been devoted to concrete elements. Presently new developments are realized on composite structures using the same type of approach. The program has been modified and improved: it is now called CEFICOSS and has become a powerful numerical tool.

### RÉSUMÉ

Au cours de la dernière décennie, un travail considérable a été accompli dans le Service des Ponts et Charpentes de l'Université de Liège en vue de développer des modèles pour l'analyse du comportement au feu des structures. Une partie importante des recherches a été consacrée aux éléments en béton. A l'heure actuelle, des développements nouveaux sont réalisés pour l'étude des structures mixtes en utilisant le même type d'approche. Le programme a été modifié et amélioré: il porte maintenant le nom de CEFICOSS et est devenu un outil numérique très puissant.

### ZUSAMMENFASSUNG

In der Abteilung Hochbau der Universität Lüttich wurde während den letzten zehn Jahren ein erheblicher Aufwand zur Aufstellung von Modellen zur rechnerischen Untersuchung der Feuerbeständigkeit von Bauten betrieben. Der grösste Teil der Forschung wurde dem Betonbau gewidmet. Zur Zeit geht die Entwicklung auf dem Gebiet des Verbundbaues auf der Basis des gleichen Verfahrens weiter. Das Computer-Programm wurde geändert und verfeinert und läuft jetzt als leistungsfähiges numerisches Verfahren unter dem Namen CEFICOSS.



## 1. INTRODUCTION

The standard fire resistance test according to ISO 834 has been used quite intensively for the evaluation of the fire endurance of structural elements. In many countries it is still the only legal way to classify structural elements, regarding their fire resistance.

Due to several shortcomings of the standard test, important progress has been made in the development of models for the prediction of thermal and mechanical response. This is particularly true in the case of steel structures, for which simple methods have been elaborated [8]. Unfortunately, this type of method is not applicable to all types of elements, and furthermore it has several limitations.

To improve the prediction of fire behaviour it is useful to elaborate powerful numerical tools, i.e. computer models able to simulate the real behaviour of structures in a fire environment. This last decade models have been presented in order to study the thermal and mechanical response of structures. Most of this work has been performed at the University of Berkeley [2] [3], Braunschweig [9] [10] and Lund [11] [11] [12].

The Department of Bridges and Structural Engineering of the University of Liège has also developed various methods, devoted mainly to the mechanical response of concrete elements, and presently of composite elements. This article presents a brief summary of the research performed at the University of Liège in this field during the last decade and of the new developments planned in the near future.

## 2. MATERIAL BEHAVIOUR

Except for prestressing steels, few work has been devoted at the University of Liège to the elaboration of material behaviour models, i.e. thermal, physical and mechanical properties of steel and concrete at high temperatures. The models adopted can be found in [5] and [7]; they do not differ very much from those adopted in the CEB/FIP and ECCS Recommendations [4] [8]. Up to now simplified models have been used for the study of creep of steel and concrete.

## 3. THERMAL ANALYSIS

### 3.1 Environmental conditions

No study has been made at the University of Liège on the modeling of the environmental conditions created by a fire. In almost all comparisons that have been made the environmental temperature was represented by the standard gas temperature time curve defined in ISO 834.

Of course there is no difficulty to introduce other types of relations in the models developed.

### 3.2 Heat exchange

The determination of the heat flow  $\Phi_e$  transmitted to the element is a problem quite involved. This can be written as follows :

$$\Phi_e = h (\theta_e - \theta_s) + \sigma_o \epsilon_{es} (T_e^4 - T_s^4) \quad (1)$$

$\theta, T$  : temperature and absolute temperature  
 $h$  : coefficient of convection  
 $\sigma_0$  : Stefan-Boltzmann constant  
 $\epsilon_{es}$  : resultant radiation emissivity  
subscript e refers to environment, s to exposed surface.

The coefficient of convection and resultant emissivity are temperature dependent and are influenced by many parameters.

In order to ensure good agreement between theoretical and experimental results, many numerical tests have been realized with different values for  $h$  and  $\epsilon_{es}$ ; comparisons have been made with experimental results from various laboratories.

From all these computations it has been concluded that, for practical applications, constant values can be adopted for  $h$  and  $\epsilon_{es}$  [5]. The values proposed are very close to those adopted now in the ECCS Recommendations [8].

### 3.3 Heat propagation

#### 3.3.1 Steel elements

For steel elements convenient relations can be obtained by writing simplified heat balance equations in which it is assumed that the heat supplied is instantaneously uniformly distributed throughout the whole volume of the steel profile. The methods proposed now in the ECCS Recommendations [8] are based on these assumptions.

Applications of these methods have been made at the University of Liège and they appeared successful for protected and unprotected elements.

The most interesting contribution concerns the analysis of the longitudinal distribution of temperature in steel elements. To this end a numerical model based on the finite differences method has been proposed [5].

The distribution of temperature has been calculated in several cases including nodes where heated and non heated elements join together. It has been observed that the temperature gradient takes place on a very short length for unprotected elements, but this length increases for protected elements. In this case, the failure mechanism can be influenced, and therefore the fire endurance, since plastic hinges can form outside the nodes.

#### 3.3.2 Concrete elements

In the case of concrete elements it is no longer possible to use simplified relations, and the Fourier equation of heat conduction giving the temperature distribution under non steady state conditions must be used :

$$\text{div} (\lambda \text{ grad } \theta) + Q = c\rho \frac{\partial \theta}{\partial t} \quad (2)$$

$\lambda$  : thermal conductivity  
 $c$  : specific heat  
 $\rho$  : density  
 $Q$  : internal heat sources  
 $t$  : time

As the thermal properties of the materials are temperature dependent numerical methods, such as the finite difference and the finite element methods must be used. Both models have been applied at the University of Liège.



In the finite element method the set of non linear equations can be written as follows :

$$K(\theta) \cdot \theta + C(\theta) \cdot \dot{\theta} = g(\theta) \quad (3)$$

K : heat conductivity matrix

C : heat capacity matrix

$\theta$  : vector of nodal temperatures

g : vector of nodal heat flows.

In the method used here, the set of equations is written at a particular time inside the time increment; the solution is then obtained by an incremental procedure based on the method of the tangential conductivity, which is very close to the well-known Newton-Raphson procedure for the solution of non linear systems.

In the finite difference procedure an explicit scheme of equations is obtained by expressing the heat balance between adjacent elements in the discretized domain (figure 1).

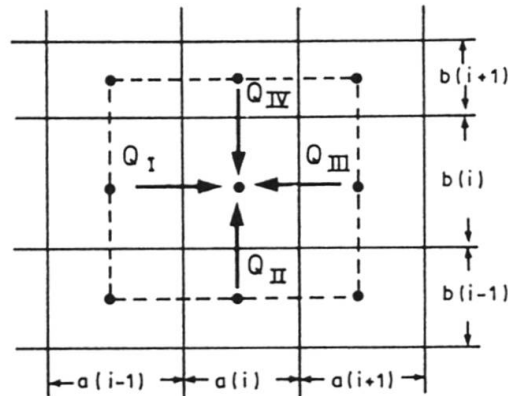


Fig. 1 Heat balance between adjacent elements

Both models have given good agreement between experimental and numerical results. The evaporation of free water can be taken into account inside a small domain, but up to now, no suitable approach has been proposed for the moisture and vapour diffusion through cracks and microcracks of concrete.

### 3.3.3 Composite elements

In the case of composite elements the same type of approach has to be used. However the computation time increases substantially due to the fact that, in composite structures, the percentage of steel becomes significant.

In the finite difference model, which is now incorporated in the programme for structural analysis, the procedure is not unconditionally stable. The criterion relating the time step to the mesh width can be written as follows for the mesh i :

$$\Delta t \leq \frac{c_i \rho_i}{4\lambda_i} (M_i)^2 \quad (4)$$

$M_i$  : minimum dimension of mesh i

$c_i, \rho_i, \lambda_i$  : values of  $c, \rho, \lambda$  in mesh  $i$

The criterion is very severe for steel meshes. Therefore, in order to reduce the computation time, several improvements have been introduced in the programme (7).

#### 4. STRUCTURAL ANALYSIS

##### 4.1 Steel structures

Some developments have been realized for the analysis of steel elements. The method of limit analysis using the concept of plastic hinge has been used [5]. This type of approach is now adopted everywhere, in particular in the ECCS Recommendations [8].

Based on this procedure a computer programme has been elaborated, in which the successive formation of plastic hinges is calculated automatically. This programme has been used to analyze the failure mechanism of various complex steel structures.

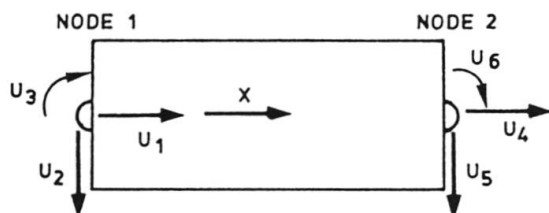
##### 4.2 Concrete structures

More work has been devoted to the development of models for the analysis of concrete elements.

For concrete structures simple methods related to the critical temperature in the bearing reinforcements can be used. Unfortunately this type of approach is acceptable only for simply supported beams and slabs. For continuous elements or columns it is no longer applicable, and more elaborated models have to be used. Therefore a numerical model for the analysis of concrete structures in bending has been developed.

In this model the structure is first analyzed under static loading, i.e. load applied incrementally and constant (ambient) temperature. It is then analyzed under fire conditions, i.e. time and, therefore, external temperature applied incrementally, constant load.

For the thermal analysis the finite difference approach mentioned in § 3.3.2 has been incorporated in the general structural programme. The structural analysis is based on the finite element method. The structure is divided in beam elements with 2 nodes and 3 degrees of freedom at each node (figure 2).



The midplane axial displacement is linear, whereas the lateral displacement is a cubic function of  $x$ . The Navier-Bernoulli hypothesis is respected and the shear energy is not taken into account.

Fig. 2 Beam element

The equilibrium conditions, based on the principle of virtual displacements, can be written :

$$\{F_e\} = (K) \cdot \{u\} \quad (5)$$

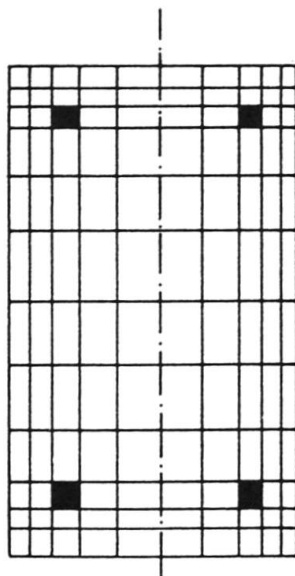


$\{F_e\}$  : vector of nodal forces applied to the structure

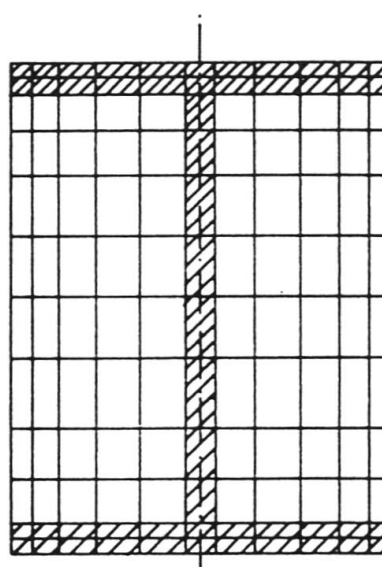
$(K)$  : structure stiffness matrix depending upon the material properties of the elements

$\{u\}$  : vector of nodal displacements.

In this approach, the cross section is divided into subslices forming a rectangular mesh, which is chosen in order to be the same as in the thermal analysis (figure 3.a).



a) reinforced concrete element



b) composite element

Fig. 3 Discretization of the cross section

Therefore the temperatures, strains and stresses can vary from one subslice to another. The integrals on the cross section appearing in the stiffness matrix, the internal bending moments and axial forces are computed in a numerical way [5] [7].

Stress-strain relations in the materials are non-linear and moreover they vary with temperature. Therefore an iterative approach is essential and the stiffness matrix has to be actualized at each step of the loading before fire occurs, and at each time increment during the development of the fire.

In the problem to be solved the materials are subjected to initial strains due to temperature changes ( $\epsilon_\theta$ ) and to creep effects ( $\epsilon_{cr}$ ); in this model, creep effects are not taken into account in an explicit way. Thus the stresses will be caused by the difference between the total strains ( $\epsilon_T$ ) derived from the nodal displacements and the initial strains :

$$\sigma = \sigma(\epsilon_\sigma) = \sigma(\epsilon_T - \epsilon_\theta - \epsilon_{cr}) \quad (6)$$

If the internal nodal forces  $\{F_i\}$  are calculated by integrating the internal stresses and compared with the applied nodal loads  $\{F_e\}$ , it can be observed that equilibrium is not reached.

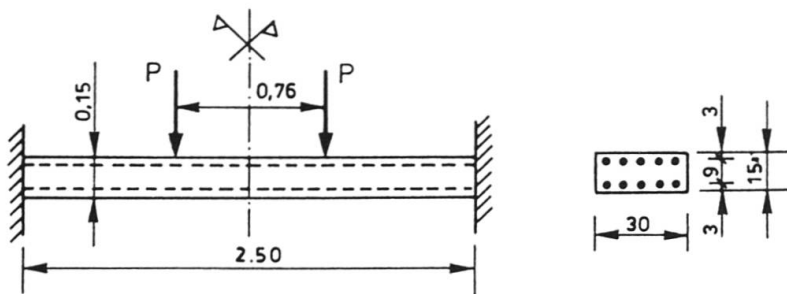
Thus, at every stage, the difference between the internal forces and the applied loads is determined at all nodes of the structure. These unbalanced residual forces are then redistributed throughout the structure to restore equilibrium. This, combined with the actualization of the stiffness matrix, gives rise to a Newton-Raphson process. Successive iterations take the form :

$$\{\Delta F_e\}_i^{(r)} = (K)_i^{(r)} \cdot \{\Delta u\}_i^{(r)} \quad (7)$$

$(K)_i^{(r)}$  : structure stiffness matrix updated at the beginning of the  $r^{th}$  iteration in the  $i^{th}$  increment taking into account the changes in material and geometrical properties  
 $\{\Delta F_e\}_i^{(r)}$  : unbalanced residual nodal forces.

Many comparisons between experimental and numerical results have been treated [5] [7]. To demonstrate the capabilities of the procedure described, results concerning a slab of rectangular cross section with end restraints are presented. This slab was tested at the University of Lund in Sweden [1].

The test conditions are described in figure 4.

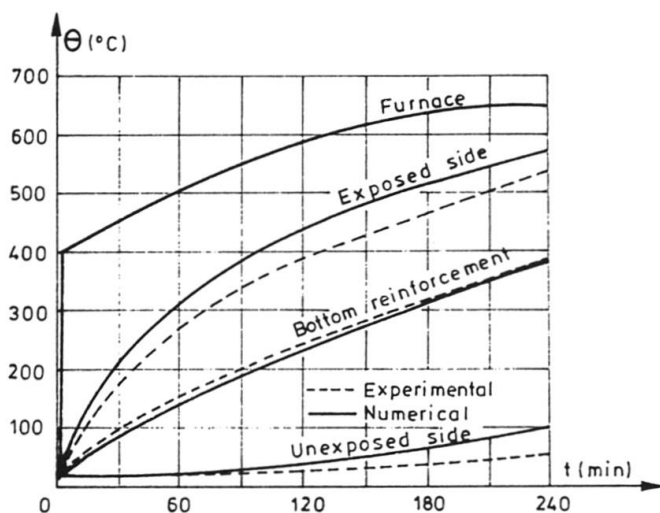


Only rotational restraint was imposed at both ends; the slab was free to expand axially.

Fig. 4 Reinforced concrete slab - rotational restraint at both ends

The increase of the environmental temperature was much less severe than that of the ISO standard curve. The temperature-time relation represented the simulation of a real fire with defined opening factor and fire load density.

Figure 5 also shows the temperature increase at various characteristic points of the cross-section : exposed side, bottom reinforcement and unexposed side. The agreement between numerical and experimental results is quite satisfactory.



In continuous beams and slabs the deflection versus time diagram does not give enough information to analyse the structural behaviour during the fire development. Due to rotational restraint at both ends, the most important factor is the evolution of the bending moment at the supports, which gives the redistribution of internal forces in the structure.

Fig. 5 Temperature increase at various points of the cross-section

Figure 6 shows the comparison between experimental and numerical results; the agreement is rather good. The differences observed are probably due to the modeling of material properties, maybe the influence of thermal creep of concrete.

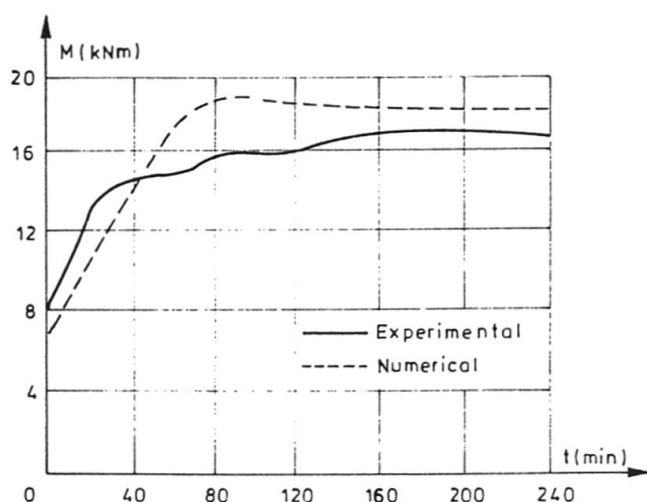


Fig. 6 Variation of the bending moment at the supports

The same type of example has also been used to study the influence of some constructive detailing characteristics. To this end several numerical simulations have been realized on the same type of slab, but using the ISO standard temperature time curve and several loading conditions. In particular the influence of the top and bottom reinforcement at the fixed ends has been studied. Some of these numerical tests are presented in figure 7.

| Numerical test                       | Detailing at the restrained end | Fire Resistance |
|--------------------------------------|---------------------------------|-----------------|
| 1                                    |                                 | $R_{f1}$        |
| 2                                    |                                 | $R_{f2}$        |
| 3                                    |                                 | $R_{f3}$        |
| Results : $R_{f1} > R_{f2} > R_{f3}$ |                                 |                 |

In the first case the simulation has been made with the same top and bottom reinforcement at the fixed end; in the second case the bottom reinforcement has been removed; in the third case, there is no bottom reinforcement, but the top reinforcement has been doubled.

Comparing the first two cases, the numerical results show that the fire endurance tends to decrease if the bottom reinforcement is removed. However this result depends significantly on the amount of ductility admitted for the concrete in compression at elevated temperatures.

Experimental investigations do not always corroborate this observation [4]. This tends to show that the ductility of concrete in compression at elevated temperatures is rather high.

Fig. 7 Numerical tests with various detailing characteristics

Comparing the third case with the second one, the numerical results show that it is not favourable to increase the amount of upper reinforcement at the support. This is due to the fact that increasing this reinforcement corresponds to decreasing the ductility of the section at the fixed end, which is unfavourable as soon as a plastic hinge has formed.

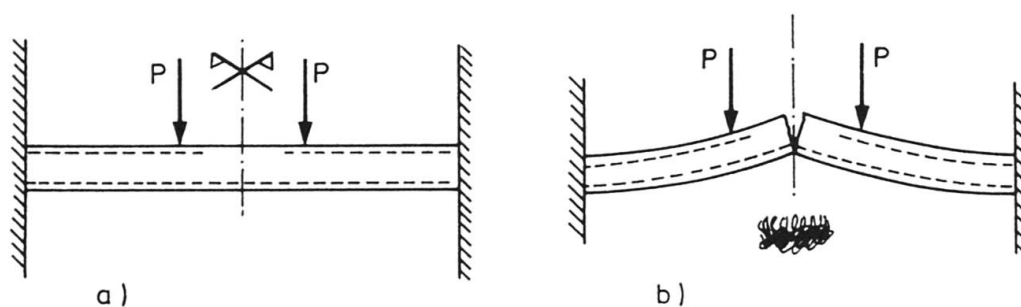
A lot of numerical simulations have been performed at the University of Liège and some of the conclusions obtained are presented here. Several of these conclusions are in good agreement with the design rules included in the FIP/CEB Recommendations. Most of them have been introduced in a book published recently in Belgium [6].

- The criterion of the maximum admissible deflection ( $f \leq l/30$ ) adopted in several countries as a failure criterion, for example in the Belgian standards, is not applicable to all types of concrete elements, mainly to simply supported slabs. In this case it does not represent correctly the true collapse of the element.

A criterion limiting the rate of deflection, such as for example the Ryan and Robertson criterion, seems to be much more appropriate.

- For elements in bending such as beams and slabs, the thermal creep of steel and concrete does not seem to have a significant influence on the failure mechanism and on the fire resistance of the element.
- A very simple method has been proposed for the determination of the fire resistance of simply supported beams and slabs with rectangular cross section.  
It is based on the consideration of the lever arm of the internal forces at failure; the value adopted has been checked by numerical experimentation.
- In hyperstatic beams and slabs the failure mechanism is much influenced by the amount of top and bottom reinforcement on the supports and along the spans.

The same type of slab as described previously (cf. figure 4) has been tested numerically, but with no upper reinforcement in the middle of the span (figure 8.a).



**Fig 8.** Reinforced concrete slab restrained at both ends  
No upper reinforcement near the middle section

After a relatively short time of fire exposure, a crack going through the slab is observed and the system behaves as indicated in figure 8.b. This is due to the change in the bending moment diagram : the bending moment at the supports increases until it reaches the ultimate moment. In the middle of the span the positive bending moment becomes negative after some time, which explains the formation of the crack.

However the system behaves very well; the situation described in figure 8.b can remain for a long time. This has been corroborated by experimental results obtained at the University of Ghent on continuous slabs.



Though this type of detailing seems convenient for continuous slabs, its practical realization is not easy; furthermore, if a point load is expected in the region where there is no upper reinforcement, punching failure should be studied carefully.

#### 4.3 Composite structures

During the last few years new types of composite beams and columns have been developed, and much attention has been paid to their behaviour under fire conditions. In order to improve the knowledge in this domain an E.C.S.C. research has been realized in collaboration with ARBED (Luxemburg). This research contains an experimental and a theoretical part. The tests have been executed in the laboratories of the Universities of Ghent and Braunschweig, while the theoretical part has been realized at the University of Liège [7].

The numerical model developed which is now called CEFICOSS is based on the same type of approach as the one described for concrete elements (figure 3.b), but it has been modified and improved in order to simulate the behaviour of composite elements, not only in bending, but also in bending and compression taking into account second order effects. Therefore it is suitable for the study of beams, columns and entire structures. Most of the improvements introduced in the programme are described in [7].

Though some improvements could still be made concerning material behaviour models, the agreement between numerical and experimental results has proved to be quite good.

To illustrate this the example of a composite T beam tested recently at the University of Ghent is briefly presented.

The loading and way of heat exposure are shown in figure 9. The beam is loaded and heated symmetrically. The thermal program is applied according to ISO 834. The dimensions of the cross section and the reinforcement arrangement are indicated in figure 9.

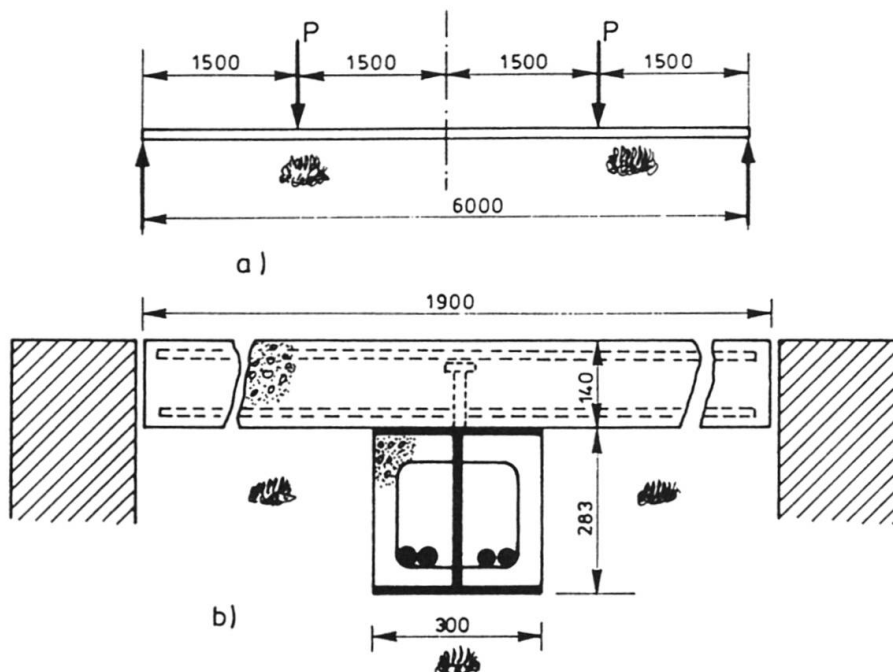


Fig. 9 Composite beam  
Loading and heating systems

The deflection versus time curve at mid point is represented in figure 10, showing the good agreement between experimental and numerical results.

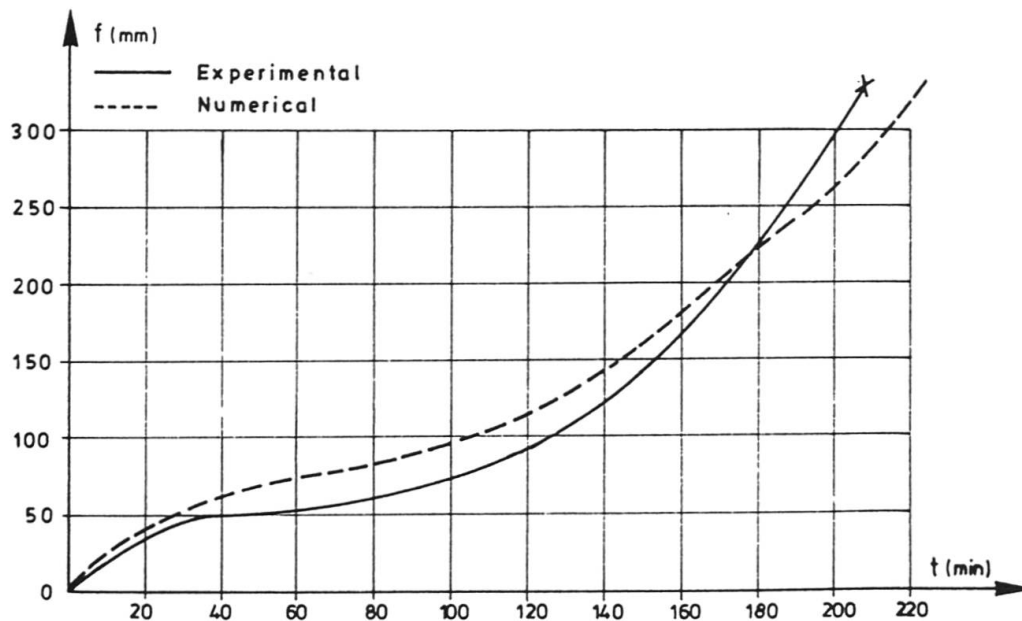


Fig. 10 Deflection versus time curve at mid-point

## 5. CONCLUSIONS

During the last decade a considerable amount of work has been performed in the Department of Bridges and Structural Engineering of the University of Liège concerning the analysis of the structural behaviour under fire conditions. A general view of the work made in this field is presented here.

This research concerns material behaviour models, thermal and mechanical analysis of steel, concrete and composite structures. Most of the work has been devoted to the elaboration of a model for the structural analysis of concrete elements. This model is based on the finite element method using beam elements with subdivision of the cross section in a rectangular mesh. The structure submitted to increasing temperatures is analyzed step-by-step using the Newton-Raphson procedure.

Numerical results have been compared with test results, and in most cases there is a good agreement. Numerical simulations have also been used in order to examine the influence of some structural detailings and of the amount and position of reinforcement in hyperstatic beams and slabs mainly. Some interesting practical conclusions have been presented and briefly discussed.

Presently new developments are realized on composite structures using the same type of approach but with several modifications and improvements. As can be seen in this paper comparisons with test results have already been realized. Though some improvements could still be made, there is no doubt that the model has now become a powerful numerical tool for the simulation of fire resistance and fire behaviour tests.



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