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Interaction of Load Speed and Mass on the Dynamic Response of Damped Beams

Comportement aux vibrations de poutres sous l'effet de charges dynamiques

Schwingungsverhalten von Balken unter fahrenden Lasten

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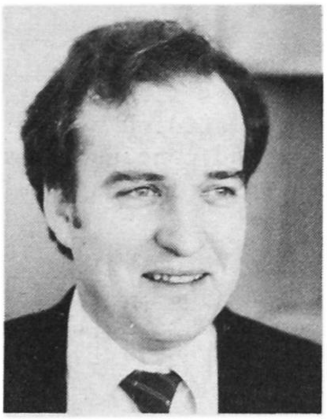

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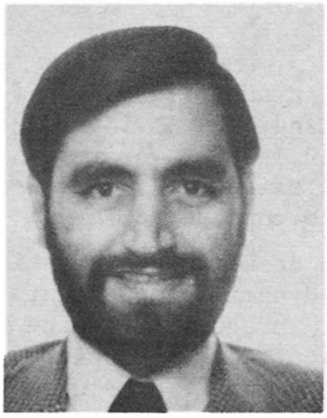

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SUMMARY

The interaction of load speed and mass and bridge damping, on the dynamic response of bridge type structures is presented in this paper. A comparison of closed form solutions obtained with the moving load idealised as a force or mass over a wide range of practical speed values has been made and impact factors for a wide range of damping values and load speeds are presented. An approximate linear relationship between impact factor and percentage critical damping has been shown to exist.

RÉSUMÉ

L'article traite de la relation entre la vitesse et la masse des véhicules d'une part et la capacité d'amortissement d'une structure de pont d'autre part. La comparaison de solutions, de type fermé, obtenues dans le cas d'une charge mobile, idéalisée par une force ou une masse, a été réalisée pour un large spectre de vitesses; les facteurs d'impact ont été déterminés en fonction de nombreuses valeurs d'amortissement et de vitesses. Une relation linéaire approximative semble exister entre le facteur d'impact et un pourcentage de la valeur critique d'amortissement.

ZUSAMMENFASSUNG

Der Zusammenhang zwischen Geschwindigkeit und Masse von Fahrzeugen und der Dämpfung von Brückentragwerken wird dargestellt. Ein Vergleich geschlossener, auf gewisser Idealisierung beruhender Lösungen werden für einen weiten Geschwindigkeitsbereich angestellt und entsprechende Stosszuschläge in Abhängigkeit von Dämpfung und Geschwindigkeit hergeleitet. Es wird gezeigt, dass näherungsweise ein linearer Zusammenhang zwischen Stosszuschlag und Bruchteilen der kritischen Dämpfung existiert.

1. INTRODUCTION

The historical development of the study of the effect of moving loads on bridge type structures is well documented [2,3,5,6,7]. However, despite considerable investigative efforts the effects of the interaction of load speed, damping and load mass on the dynamic response of such structures is not fully understood and hence assessment of appropriate impact factors for design is necessarily somewhat subjective.

This paper presents an analytical investigation into the dynamic response of beams subject to moving loads. The load is idealised as either a point force or a point lumped mass and comparison of the dynamic responses of the structure is presented for a wide range of speed and damping conditions. The interaction of the speed of the moving load and the damping associated with the beam is also evaluated and impact factors for a wide range of damping values and load speeds are presented.

2. IDEALISATIONS

2.1 Moving Load

The differences in dynamic response of a number of simple structures with the moving load idealised as a moving force or a moving mass has been studied by various authors [12,19,20]. Their results however, lead to no broad conclusions and they cannot be effectively compared because different particular load speeds and structures were considered.

Attempts have also been made to quantify the effects of various other physical parameters on the dynamic response of structures subject to moving loads. The effects of surface irregularities and ramps has been considered [4,8,15,16,17], as well as the effects of axle spacing [16] and braking loads [4]. Idealising the load system on a spring mass has also been considered [8,10,11,17,18].

The dynamic response of actual highway bridges will be influenced by all these factors. This parametric study however, is concerned with the interaction of load, load speed and damping of the structure over a broad range of values and the effects of these other factors, which are primarily site dependent, have not been included. Where possible, however, direct comparisons with case studies considered by other authors have been included.

Throughout this study the loading is considered to be a single constant velocity moving load idealised as either a moving force or a moving mass. The effect of a series of loads may be readily obtained by simple superposition provided the structure remains within the elastic range.

2.2 Bridge Structures

Walker [16] considered the effect of transverse stiffness on the response of bridge structures subject to moving loads and suggested that beam and slab bridges and simple slab bridges tend to behave like simple beams. Smith [11] demonstrated that the dynamic response of a bridge, obtained using the finite strip method, was slightly greater than the response obtained idealising the bridge as a simple beam, whilst results from Yoshida and Weaver [19] for an idealisation incorporating either a simple span beam or an orthotropic plate structure showed little difference. Gupta and Traill-Nash [4] showed that the dynamic response obtained using a simple beam idealisation is greater than the dynamic response obtained using plate theory. Moreover, it was also shown that the inclusion of torsional freedom in the simple beam idealisation is not justified. Previously Tan and Shore [13] had come to similar conclusions.

As the simple beam idealisation appears to produce conservative results when compared with almost all other idealisations it will be adopted for the purpose of the following parametric study.

3. CONSTANT VELOCITY MOVING LOAD ON A SIMPLY SUPPORTED UNDAMPED BEAM

Consider a simply supported beam with uniform cross-sectional area and mass per unit length traversed by a constant velocity moving load as shown in Fig. (1). Hooke's Law, Navier's hypothesis, Saint-Venant's principle and the theory of small deformations are assumed to apply. It is also assumed that the beam is initially at rest, before the load enters the span.

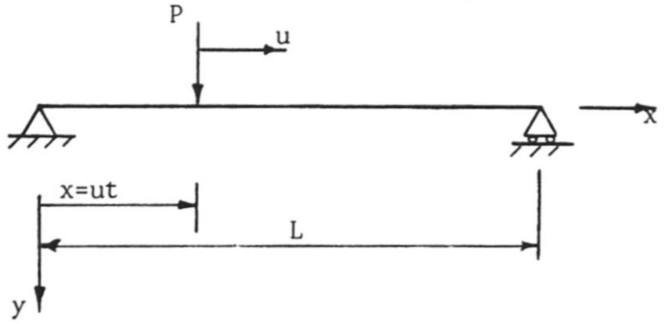


Fig. 1 Moving load on a simply supported beam

The load velocity or speed may be conveniently represented, non-dimensionally, in the form of a "speed" parameter α where

$\alpha = \frac{1}{2}$ (fundamental period of vibration of the beam/time taken for load to cross beam)

$$\alpha = \frac{1}{2}\,\frac{2\pi/\omega_1}{L/u} = \frac{\pi u}{L\omega_1}$$

where u = velocity of the load
L = span of beam
ω_1 = fundamental circular frequency of beam.

3.1 Constant Moving Force

If it is assumed that the mass of the moving load is small compared with the mass of the beam, ie only the gravitational effect of the load is considered, and any variations in the dynamic characteristics of the beam due to the participation of the load mass are ignored the beam motion may be described by the Bernoulli-Euler equation [1].

$$EI\,\frac{\partial^4 y(x,t)}{\partial x^4} + \rho\,\frac{\partial^2 y(x,t)}{\partial t^2} = \delta(x-ut)P \qquad (1)$$

where EI = flexural rigidity of the beam
ρ = mass per unit length of beam
P = magnitude of the load
ut = distance load has moved from origin after time t (see Fig. (1))
and $\delta(x-ut)$ is the Dirac delta function defined to be zero everywhere except at $x = ut$.
ie $\delta(x-ut) = 0$, $x \neq ut$ and additionally $\int_o^L \delta(x-ut)dx = 1$.

Boundary conditions

$$y(o,t) = y(L,t) = 0$$

$$\frac{\partial^2 y(o,t)}{\partial x^2} = \frac{\partial^2 y(L,t)}{\partial x^2} = 0$$

and initial conditions

$$y(x,o) = \frac{\partial y(x,o)}{\partial t} = 0$$

are assumed to apply.

The solution of equation (1) may be obtained using Laplace-Carson transforms [7,1]

$$y(x,t) = \frac{2P}{\rho L \omega_1^2} \sum_{j=1}^{\infty} \frac{1}{j^2(j^2-\alpha^2)} [\sin j\,\omega t - \frac{\alpha}{j} \sin \omega_j t] \sin \frac{j\pi x}{L} \tag{2}$$

where ω_j = j^{th} natural frequency of the beam, and $\omega = \frac{\pi u}{L} = \alpha\omega_1$

3.2 Constant Moving Mass

If both the gravitational and inertia effects of the moving mass are to be included the equation of motion of the beam is [1]:

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \rho \frac{\partial^2 y(x,t)}{\partial t^2} = \delta(x-ut)[P - m \frac{\partial^2 y(x,t)}{\partial t^2}] \tag{3}$$

Assuming the same boundary conditions as in section 3.1, equation (3) may be solved using Fourier finite sine and cosine series transforms to obtain the transformed equation of the problem. The transformed equation is:

$$\ddot{y}(j,t) + \omega_j^2\, y(j,t) + \frac{1}{\rho L} \sum_{i=1}^{n} m_i [\ddot{y}(j,t) + 2 \sum_{k=1}^{\infty} y(k,t) \sin \frac{k\pi ut}{L} \sin \frac{j\pi ut}{L}]$$

$$= \frac{g}{\rho} \sum_{i=1}^{n} m_i \sin \frac{j\pi ut}{L} \tag{4}$$

A conservative solution is obtained by considering only the linear inertia term of equation (4) [Ref. 6]. Thus the solution becomes:

$$y(x,t) = \frac{2P}{L} \sum_{j=1}^{\infty} \frac{1}{\omega_j^2 \left(1 - \frac{(j\,\omega \sqrt{1+R})}{\omega_j}\right)} \left[\sin j\omega t - \frac{j\sqrt{1+r}}{\omega_j} \sin\left(\frac{\omega_j t}{\sqrt{1+R}}\right) x \sin \frac{j\pi x}{L} \right] \tag{5}$$

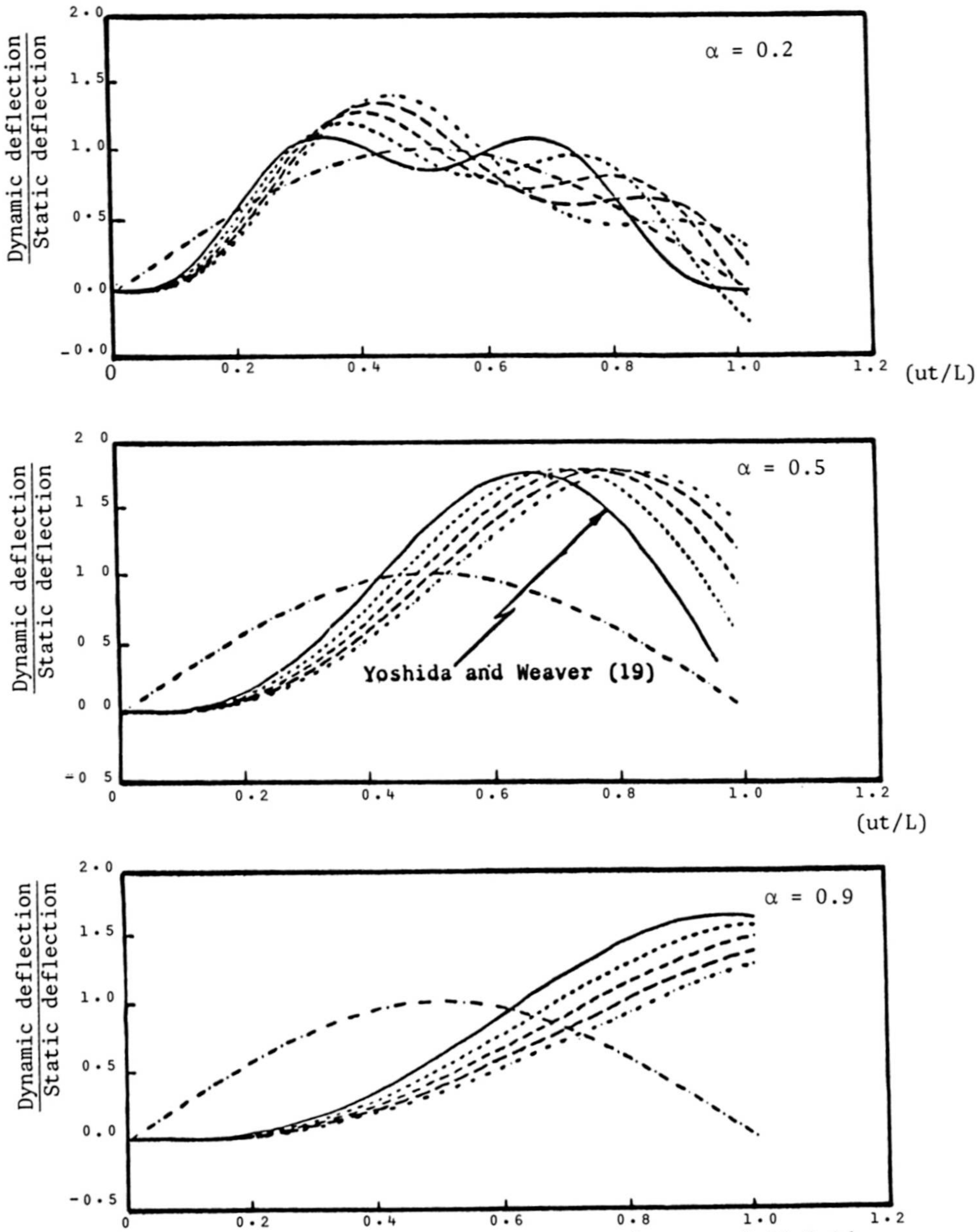
where mass ratio $R = \frac{M}{\rho L}$

4. COMPARISON BETWEEN MOVING FORCE AND MOVING MASS SOLUTIONS

It has been shown [1] that the critical section of a uniform simply supported beam which is traversed by a single constant velocity point load is the section at, or very close to mid-span. Hence the various comparisons that follow are made at the mid-span section.

Equations (2) and (5) may be solved numerically for varying values of the mass ratio R and various values of the load velocity parameter α. For each value of α, the results may be presented in non-dimensional form as a plot of the ratio of mid-point dynamic to mid-point static deflection versus the dimensionless parameter ut/L, where u is the constant speed of crossing, L is the span and t is the time elapsed since the load entered the span. These plots or dynamic influence lines at mid-span are presented in Figure (2). Each graph is for a particular value of speed parameter α and five ratios (0,0.25, 0.5, 0.75,1.0) of the mass of the load to the mass of the beam - the cases where the mass ratio is zero correspond to the moving load being considered as a force. Figure (3) shows the difference between the maximum force response and maximum mass response versus speed parameter for a series of mass ratios.

The limited results obtained by Yoshida and Weaver [19] using a finite element approach and considering the load as a moving force are shown in Figure(2) for $\alpha = 0.5$. Good agreement is observed.



—— Load mass = 0.0 (force)
------ Load mass = 0.25 Bridge mass
— — — Load mass = 0.5 Bridge mass
— — Load mass = 0.75 Bridge mass
-·-·-·- Load mass = 1.0 Bridge mass
-·—·- Static deflection

Fig 2 Comparison between dynamic influence lines for mid-span deflection of a moving force and a moving mass with various mass ratios. (Mass ratio = mass of load/mass of bridge) Speed parameter α = 0.2, 0.5, 0.9 . β = 0

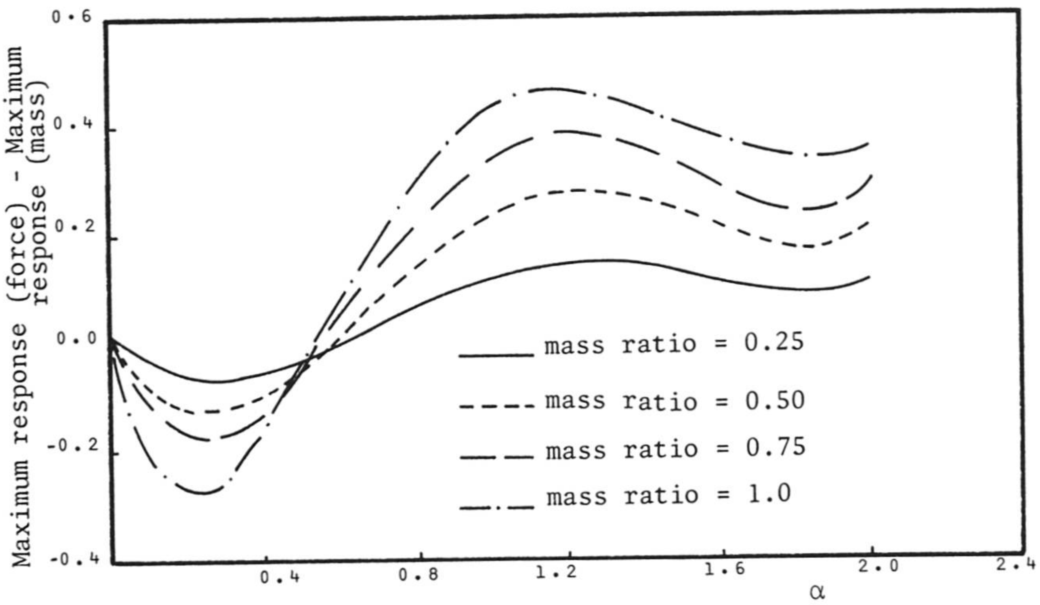


Fig. 3 Relationship between moving force, moving mass and speed parameter α

The figures indicate that the differences in deflection found using either a moving force or moving mass idealisation vary significantly with the speed parameter and the mass ratio (see Figure 3)). At speed parameter values of α less than approximately 0.5, the dynamic influence lines for deflection of the mid-span due to the moving force idealisation is somewhat less than the equivalent deflection obtained from the moving mass idealisation. The difference is greatest for high mass ratios and values of α in the region of 0.2. For all other values of α the moving force idealisation provides a conservative solution.

In most realistic structures the mass ratio is usually very low [9] and hence the difference between the idealisations is minimised. Moreover, damping effects (considered in the following sections) would lead to further reduction in the difference between the two idealisations and therefore, it may be concluded that modelling the moving load as a force will give conservative beam responses for the cases of speed parameter greater than 0.5 and sufficiently accurate responses for cases where α is less than 0.5.

5. MOVING FORCE IDEALISATION INCLUDING DAMPING

For the simply supported beam shown in Fig. (1) if the beam damping is assumed to be proportional to the velocity of vibration (viscous damping) the equation of motion may be written as:

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + \rho \frac{\partial^2 y(x,t)}{\partial x^2} + 2\rho\omega_b \frac{\partial y(x,t)}{\partial t} = \delta(x-ut)P \qquad (6)$$

where ω_b is the circular frequency of beam damping.

Equation (6) may be solved, incorporating the boundary conditions for the simply supported beam, by employing Fourier finite since integral transformations [1,2], viz:

$$y(x,t) = \frac{2P}{\rho L\omega^2} \sum_{j=1}^{\infty} A_j[j^2(j^2-\alpha^2)\sin j\omega t - 2j\alpha\beta \cos j\omega t - e^{-\omega_b t}(B_j \sin \omega_j' t + 2j\alpha\beta \cos \omega_j' t)]\sin \frac{j\pi x}{L} \qquad (7)$$

where $A_j = 1/[j^2(j^2(j^2 - \alpha^2)^2 + 4\alpha^2\beta^2)]$

$B_j = j\alpha[j^2(j^2 - \alpha^2) - 2\beta^2]/(j-\beta^4)^{\frac{1}{2}}$

and j = mode number

ω = circular frequency $= \dfrac{\pi u}{L}$

ω_j = circular frequency at jth mode of vibration $= \dfrac{j^2\pi^2}{L^2}\sqrt{\dfrac{EI}{\rho}}$

β = damping parameter $= \dfrac{\omega_b}{\omega_1}$

$\omega_j' = \sqrt{\omega_j^2 - \omega_b^2} \qquad (\omega_j > \omega_b)$

$\omega_j' = \sqrt{\omega_b^2 - \omega_j^2} \qquad (\omega_b > \omega_j)$

Equation (7) may be solved numerically for varying values of load velocity parameter α and damping parameter β. Figures (4) and (5) show a typical plot of mid-point dynamic/static deflection versus ut/L for α = 0.5 and various values of damping parameter β. It may be seen from Figure 5 (and Ref[1]) that the effects of damping are most appreciable after the moving load has left the span.

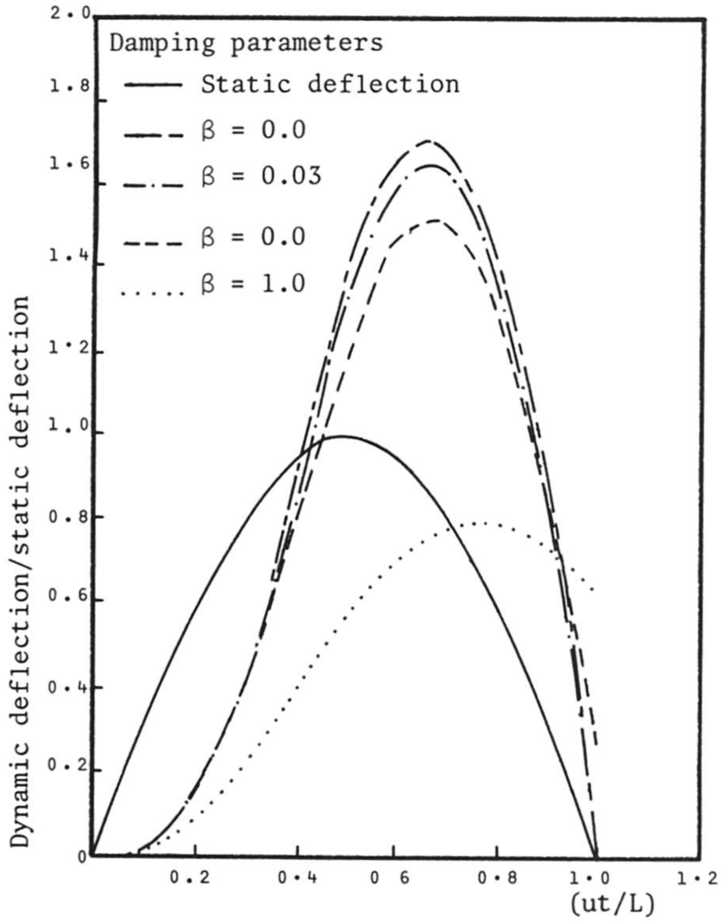


Fig. 4. Effect of damping on the mid-span response of a simply-supported beam.
Speed parameter α = 0.5

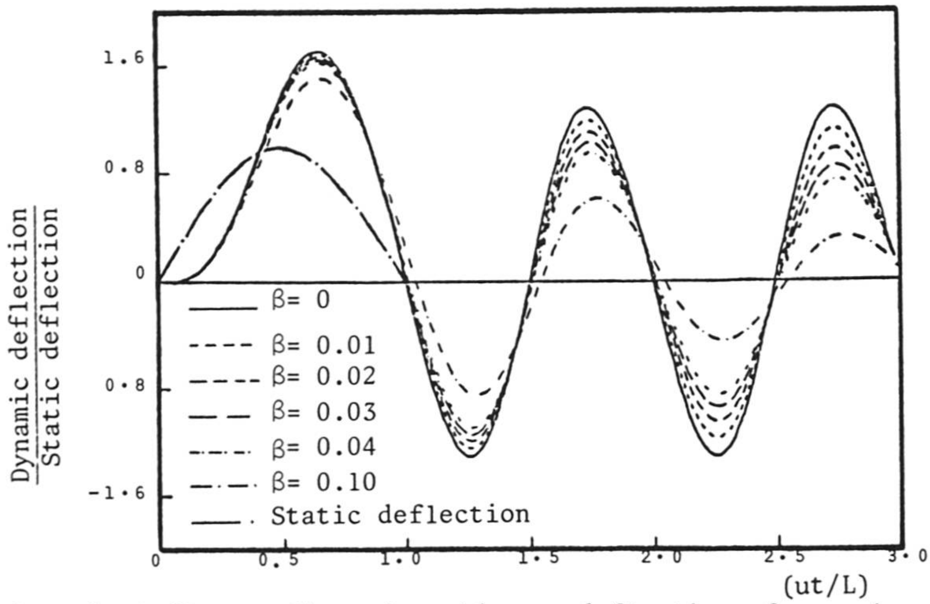


Fig. 5 Dynamic influence lines for mid-span deflection, for various values of damping.
Speed parameter α = 0.5.

6. MAXIMUM RESPONSES - IMPACT FACTORS

For practical purposes the maximum responses of a beam, with varying damping ratios, subject to loads with varying speed parameter are of most interest. The impact factor for the beam, may be considered as the ratio of the maximum mid-span dynamic deflection to maximum mid-span static deflection. Equation (7) may be solved for various values of α and β and the maximum responses selected.

Figure (6) shows an enlarged plot of the impact factor versus speed parameter for the case of zero damping in the range $0 < \alpha < 0.3$. The local maxima of 1.172 occurs when $\alpha = 0.15$. Figure (7) shows the same plot, but extended to cover cases up to $\alpha \leq 2$. The variation in the maximum response at low values of α arises

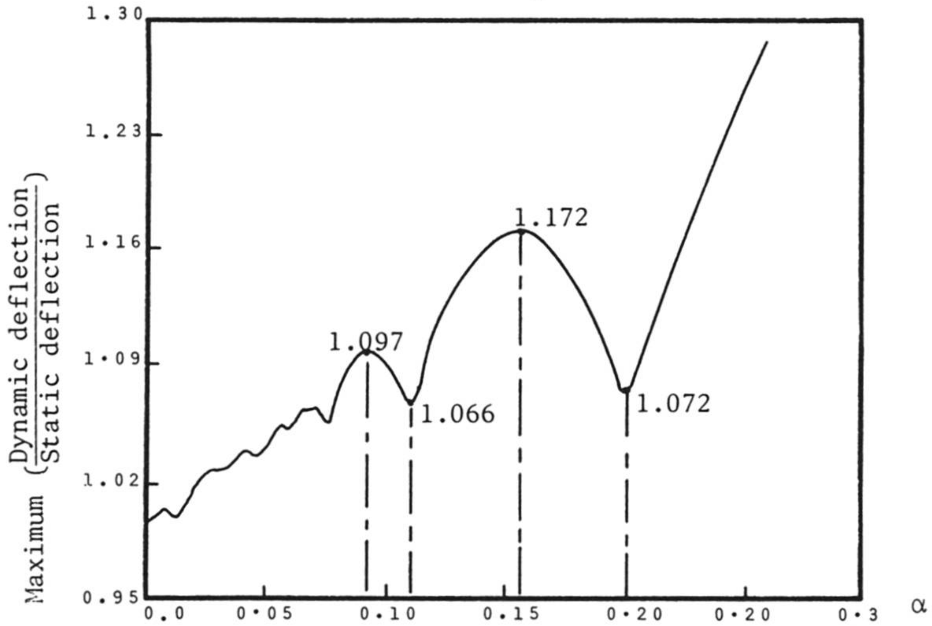


Fig. 6. Maximum dynamic influence line values for mid-span deflection versus speed parameter α
(Closed form solution), $\beta = 0$

because at low speeds the beam tends to vibrate with small amplitudes about the statically deflected shape.

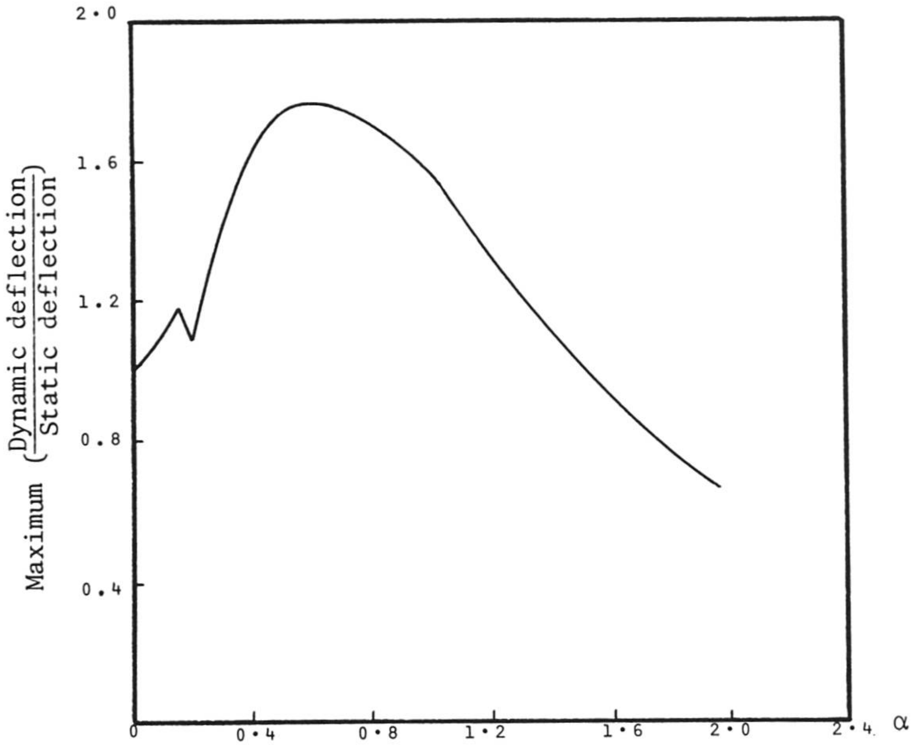


Fig. 7 Maximum dynamic influence line values for mid-span deflection versus speed parameter α.
(Closed form solution), β = 0

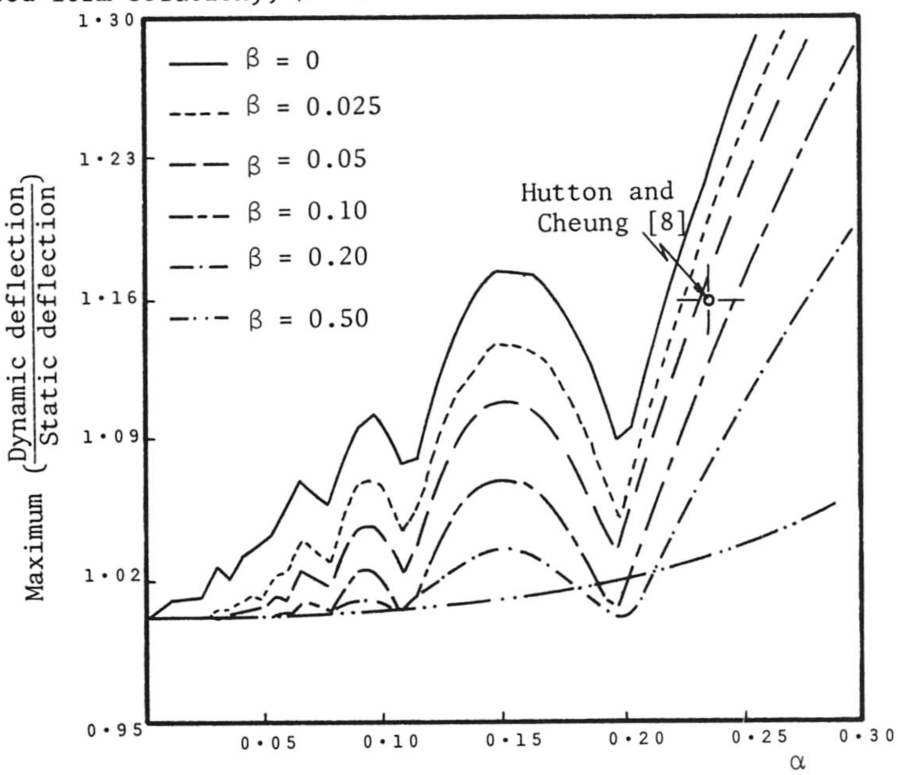


Fig. 8 Maximum dynamic influence line values for mid-span deflection versus speed parameter α with various damping values.
(Closed form solution)

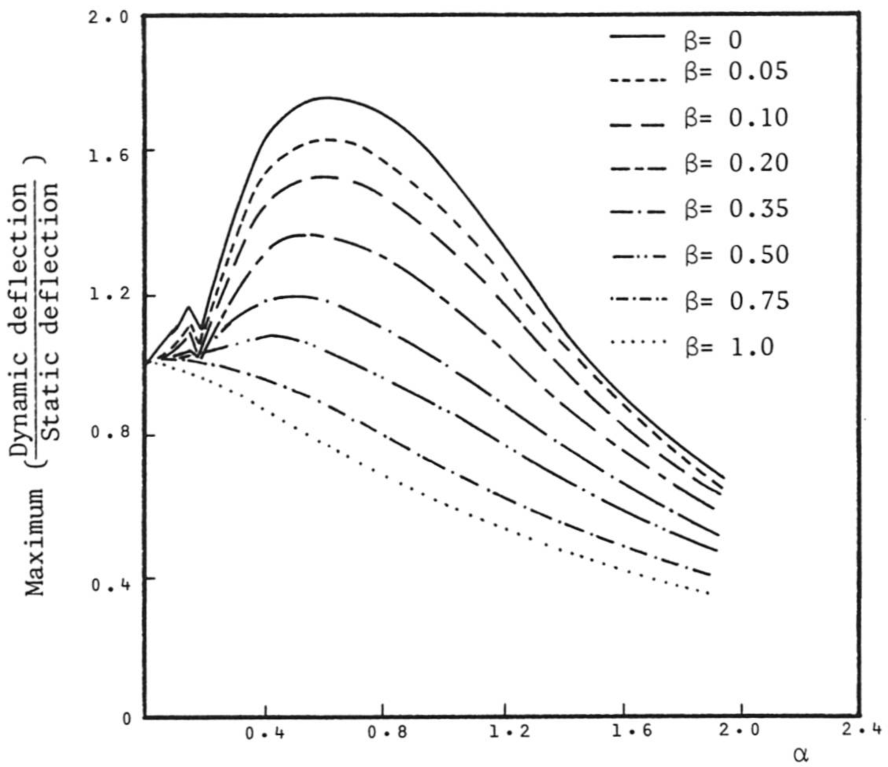


Fig. 9 Maximum dynamic influence line values for mid-span deflection versus speed parameter α with various damping values. (Closed form solution)

Figures (8) and (9) show corresponding plots of impact factor versus speed parameter with varying values of damping parameter β. As the damping of the structure is increased the dynamic response at a particular load speed α decreases, and hence the impact factor decreases.

Hutton and Cheng [8] used the finite strip method to consider the effect of a realistic two axle sprung vehicle on an actual simply supported concrete slab deck with viscous damping ratio of 0.1%. Results are presented for the case of α = 0.236 with ratios of ϕ_V, the lowest natural frequency of the vehicle on its tyres to the lowest natural frequency of the bridge deck, varying from 0.5 to 2.0. The ratio of the maximum dynamic to static deflection obtained for the case corresponding to the analyses presented above (ie $\phi_V = 1$) is superimposed on Figure (8) and a reasonable correlation may be observed.

The ratio of the maximum dynamic to static deflection may be considered as the impact factor for the structure provided the structure remains in the linear elastic range of behaviour. Figure (10) shows that, at mid-span the relationship between impact factor and damping is not quite linear over the wide range of damping values considered but, with the range of damping values applicable to practical structures (β < 5%) linearity is maintained and impact factor and damping may be related by the equation:

$$\text{Impact factor} = 1.71 - 5.52\beta$$

where 1.71 is the maximum impact factor for the undamped case.

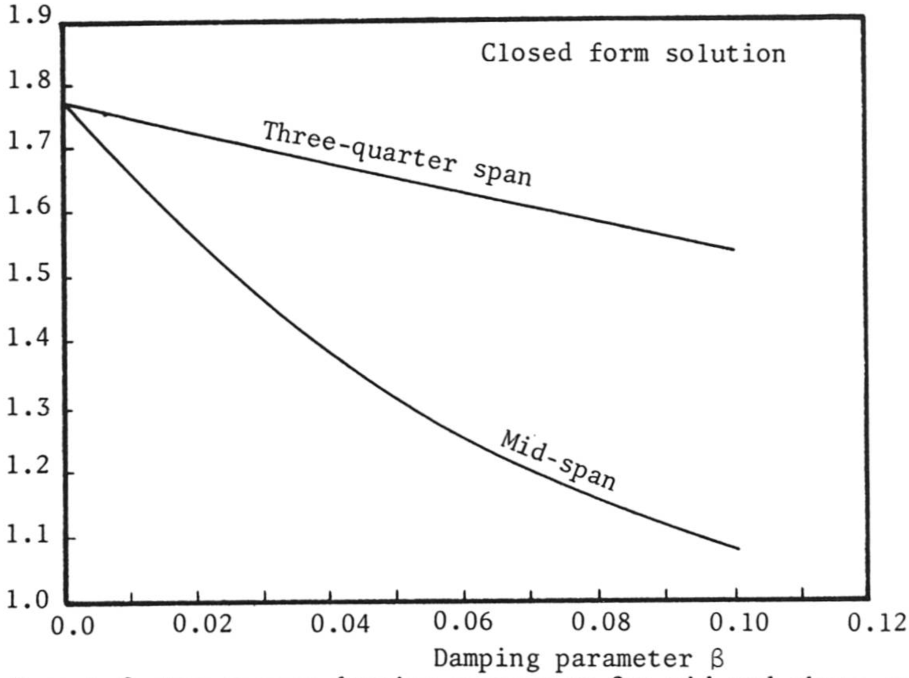


Fig. 10 Impact factor versus damping parameter for mid and three-quarter span of a simply supported beam. (Closed form solution)

7. CONCLUSIONS

The modelling of simply supported bridge type structures as simple beams when they are subjected to moving loads has been shown by other authors to produce conservative results. Comparison of closed form solutions obtained with the moving load idealised as a force or mass over a wide range of practical speed and damping ratios indicates that the moving force idealisation produces conservative results for values of speed parameter, α, greater than 0.5. For values of α less than 0.5, the results produced using either load idealisation are similar and for practical cases, where the mass ratio is usually low and damping exists, it may be concluded the idealising the moving load as a force over this range of α is satisfactory.

The effect of damping of the structure in reducing the dynamic response is appreciable while the moving load is still on the beam and it becomes more significant in the subsequent beam free vibration after the load has passed off the beam.

An approximate linear relationship between impact factor and percentage of critical damping of the structure has been shown to exist. Documentation concerning the computer programs used in this study may be found in Ref. [1] along with further supporting detailed analytical results.

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