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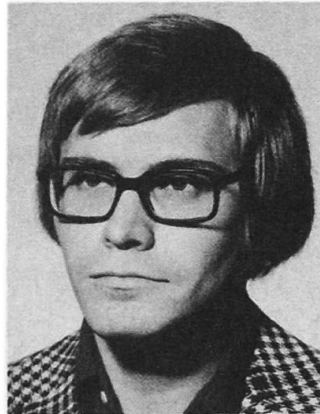
## A Stochastic Model of the Concrete Strength

Modèle stochastique de la résistance du béton

Ein stochastisches Modell für die Betonfestigkeit

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### SUMMARY

A new stochastic model to represent the main characteristics of the failure strength of concrete is presented. The approach can be seen as an extension of fracture mechanics within a stochastic framework. Both time and the volume dependence, which is typical for a brittle material like concrete, are included. Several examples of the applicability of the model, including fatigue of plain concrete, are presented. The results are primarily of interests in limit state designs.

### RÉSUMÉ

L'article présente un nouveau modèle stochastique décrivant les principales caractéristiques de résistance à la rupture du béton. Cette approche peut être vue comme une extension de l'étude mécanique des fractures dans un cadre stochastique. Ce modèle rend compte de la dépendance du temps et du volume qui est typique pour un matériau cassant comme le béton. Plusieurs exemples d'applications sont présentés, y compris le cas de fatigue du béton non armé. Les résultats sont principalement d'intérêt pour l'étude de structures à l'état limite.

### ZUSAMMENFASSUNG

Ein neues stochastisches Modell für die Darstellung der wichtigsten Festigkeitseigenschaften des Betons wird vorgestellt. Der Ansatz kann als stochastische Erweiterung der Bruchmechanik angesehen werden. Die für sprödes Material wie Beton typische Zeit- und Volumen-Abhängigkeit wird erfasst. Mehrere Beispiele für die Anwendbarkeit des Modells einschliesslich der Ermüdungsfestigkeit von Beton werden gezeigt. Die Ergebnisse sind vor allem für die Bemessung auf Grenzzustände von Interesse.



## 1. INTRODUCTION

The strength characteristics of materials have traditionally been formulated in accordance with the theory of elasticity or the theory of plasticity. With such an approach it is possible to make use of a well developed theory in the calculation of stresses and strains in a body. But this often leads to a poor representation of the strength properties of materials. This is especially noticeable in modern limit state designs where the notion of a better use of the strength of a material have been introduced.

It has been obvious for a long time that the strength of materials need to be better characterized. One approach in a better understanding of the strength characteristics is fracture mechanics. The original theory [5] has been improved in many respects dependent on the application. For concrete several approaches are available based on nonlinear assumptions [6] or other special assumptions [1, 8]. All such models have increased the understanding of the behaviour of concrete. But fracture mechanics is difficult to apply on structures in general. It is primarily a research method when it is applied to concrete.

A modern approach to represent the strength of materials is based on statistical principles. This method can be used in a traditional way to reflect the variability found in the failure strength of materials or as the basis for a stochastic strength theory. A stochastic strength theory can, to a certain extent, be seen as a generalization of fracture mechanics, but it also adds new features which makes this approach of interest in practical applications.

## 2. STRENGTH CHARACTERISTICS OF CONCRETE

### 2.1 Some material qualities

Concrete materials and all cement based composites are basically discontinuous, anisotropic, heterogeneous, multiphase composite systems. The main components, the matrix and the aggregate, may have different characteristics as well as the proportions between them can vary dependent on desired qualities.

The properties and strength of any heterogeneous system depend on the physical and chemical characteristics of its constituents, and the interaction between them. In the concrete system, randomly distributed aggregate inclusions of various sizes and surface texture are embedded in a continuous visco-elastic matrix. The properties of the aggregate are normally well defined and can often be represented independent of time. The characteristics of the matrix varies with stress and time and is also subject to environmental influences. The interaction between the two would therefore not only vary with stress and time but also with mineralogical composition of the materials in contact. The aggregate - matrix contact zone is a very significant phase in the concrete system and is often the weakest link in a heterogeneous system.

### 2.2 The strength of concrete

It is characteristic of concrete materials that they have a high compressive strength and a low tensile strength. A high compressive strength is primarily due to chemical bonding forces within the crystal structure. The tensile strength is primarily dependent on physical bonding and is of a lower order of magnitude.

The strength characteristics of a specific concrete will depend on the characteristics of the constituents, the matrix and the aggregate, and the bond interphase between them. There are many factors which inter-relate the strength of the composite. A significant amount of research has been devoted to explaining these factors. It is also possible to explain the influence of a large number of factors on the strength characteristics of concrete.

From a practical point of view tensile stresses play an important role. In normal concrete with random distribution of aggregates moisture movements creates biaxial and triaxial stress conditions in the matrix and large tensile stress concentrations occur around aggregate inclusions. Many experiments have shown that the breakdown invariably starts around the aggregate - matrix interphase. This is because the tensile bond strength of the aggregate - matrix interphase is much lower than the tensile strength of the matrix itself.

The implications of the stress concentrations can be illustrated with the creep characteristics. In Fig 1 the stress strain curve for concrete in uniaxial compression is shown. If a sustained stress lower than the failure stress is applied, concrete will exhibit creep. If the stress level is below some critical value, tensile stresses around aggregate inclusions will not be high enough for cracks to propagate. The creep deformations can in this case be seen as a consolidation. If the stress level is high enough cracks will propagate and will finally result in a brittle failure. The higher the stress level the shorter the duration of this delayed brittle failure.

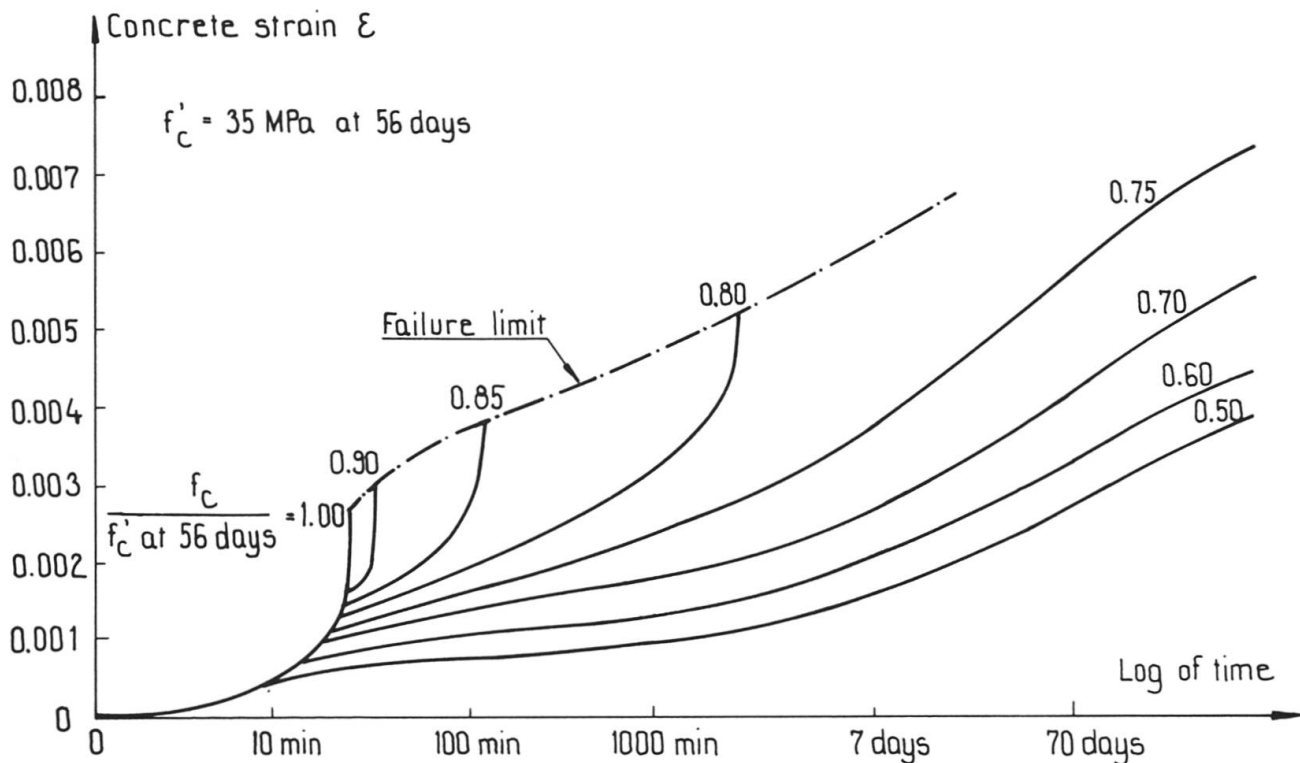


Fig 1 Creep characteristics as a function of time and applied stress. From [10].

The brittle behaviour of concrete, which depends on microcracks or flaws, will also be dependent on the volume under stress. The larger the volume the greater the probability of larger flaws being present. The consequence of this is that the failure load will decrease for increasing volumes under stress.

The dependence of the strength of concrete on the volume under stress and the duration is important in connection with limit state evaluations. The assumption of a constant concrete strength based on standardized testing of cubes or cylinders is hardly justified.



### 3. A STOCHASTIC MODEL OF CONCRETE STRENGTH

The statistical theory of brittle failure was introduced to explain the dependence of the failure load on the volume under stress [14]. Concrete loaded to failure does also exhibit this volume dependence, which will be exemplified later. In normal applications concrete will not be loaded to failure. But the stress level might be high enough for cracks to propagate. In this case the duration of the sustained stress has to be considered. Concrete, which will behave potentially brittle, can be modelled by an extension of the original theory [12].

#### 3.1 The volume dependence

In a brittle material the strength will depend on flaws in the material. The size of these flaws are assumed to follow some statistical distribution  $F(r)$  giving the probability of finding a flaw whose failure load is less than  $r$ . If there are  $n$  flaws in a unit volume the strength of a larger volume can be represented with the statistical distribution of the minimum value

$$F_V(R) = 1 - \{1 - F_X(R)\}^{nV} \quad (1)$$

If  $nV$  is large eq 1 can be written in its asymptotic form

$$F_V(R) = 1 - \exp\{-cnV R^k\} \quad (2)$$

which is the Weibull distribution. For concrete a brittle behaviour can only be expected above a certain stress level where cracks may propagate. If this stress level is called  $r_0$  eq 2 will have the form

$$F_V(R) = 1 - \exp\{-cnV (R - r_0)^k\} \quad (3)$$

In many applications it is an advantage to express eq 3 in a way which relates any volume  $V$  to a standardized volume  $V_0$ . This is the case for concrete in compression where  $V_0$  is the volume of a standard test cube or cylinder. Equation 3 can now be written

$$F_V(R) = 1 - \exp\left\{-\frac{V}{V_0} \left(\frac{R - r_0}{m}\right)^k\right\} \quad (4)$$

which is just a change of variables. The variables  $m$ ,  $r_0$  and  $k$  are determined from experimental information.

#### 3.2 The time dependence

The time dependence of a brittle failure of concrete has to be considered if the stress level is high enough for cracks to propagate. This time dependence is most likely to be dependent on the volume under stress. The time dependence is thus conditional on the volume under stress. This can be expressed as

$$F_T(R|V) = 1 - \left\{1 - F_Y(R|V)\right\}^{nT} \quad (5)$$

It is possible to assume different types of dependences between the volume under stress and the duration to reflect material characteristics like a memory. For concrete subject to sustained high stresses the time dependence can be seen as a delayed brittle behaviour. Such an assumption will lead to an expression similar to eq 3. In a normalized form this can be written

$$F_T(R|V) = 1 - \exp\left\{-\frac{T}{T_0} V^P \left(\frac{R - r_0}{m}\right)^h\right\} \quad (6)$$

where  $T_0$  is a reference duration and  $V^P$  some reduced volume. It is here assumed that  $m$  and  $r_0$  are the same as in eq 4. Only the parameter  $h$  need to be estimated from experimental information.

### 3.3 Extensions to the model

The stochastic model described is the simplest possible. More realistic models can be introduced based on additional assumptions.

The flaws are often different in the interior of a body compared to the surface. On the surface cracks might be present as a result of an uneven temperature distribution during the curing or as a result of temperature or humidity variations afterwards. This can be taken into account by expressing eq 1 as a function of both volume and surface area. It is possible to introduce a memory assumption in the derivation of eq 6 to consider a certain self-healing if the stresses are not sustained.

All extensions to the proposed model will probably improve the description of strength characteristics of concrete. But at the same time the model will be more complicated and require more background information. This is normally not desirable from a practical point of view.

## 4. EVALUATION OF THE MODEL

The stochastic model presented in the previous chapter relates the brittle failure load of concrete to the volume under stress and a delayed brittle failure in addition to the duration of the stress. In applications the expressions given need to be evaluated and expressed in a more practical way.

### 4.1 The volume dependence

The brittle failure load as it is expressed in eq 4 will give the probability distribution of a certain volume  $V$  under stress. Often it is more practical to express this in terms of the mean value and the variance. The mean is given by

$$E [R] = r_0 + \int_{r_0}^{\infty} \exp \left\{ - \frac{g(R)}{V_0} \right\} dR \quad (7)$$

and

$$g(R) = \int_{R_s > r_0} \left\{ \frac{R_s(x, y, z) - r_0}{m} \right\}^k dV \quad (8)$$

where  $s(x, y, z)$  is a dimension free, normalized function which describes the stress distribution in a body, and  $R$  some characteristic stress, normally the stress at failure. The variance can be expressed in a similar manner.

An explicit evaluation of the mean and variance can only be done for simple stress distributions. In the case of a uniform stress distribution the mean value and the variance are

$$E [R] = r_0 + m (V_0/V)^{1/k} \Gamma (1 + 1/k) \quad (9)$$

$$V [R] = m^2 (V_0/V)^{2/k} \left\{ \Gamma (1 + 2/k) - \Gamma^2 (1 + 1/k) \right\} \quad (10)$$

where  $\Gamma(\cdot)$  is the gamma function. For most other stress distributions it is not possible to derive expressions as simple as eqs 9 and 10. In such cases the volume under stress has to be evaluated numerically. If the original volume is divided into sub-volumes, each large enough to be considered independent of the strength of others, then eqs 9 and 10 can be used as an approximation for each volume. With such an approach it is possible to use a finite element evaluation of stresses in a body.



For practical purposes it is often better to write eqs 9 and 10 as

$$R = R_0 \left\{ a + b \left( V_0/V \right)^{1/k} \right\} \quad (11)$$

$$C O V = \frac{b \left( \frac{V_0}{V} \right)^{1/k}}{a + b \left( \frac{V_0}{V} \right)^{1/k}} \sqrt{\frac{\Gamma(1 + 1/k)}{\Gamma^2(1 + 1/k)} - 1} \quad (12)$$

Now the mean strength  $R$  for a volume  $V$  is related to the mean strength  $R_0$  of some standardized volume  $V_0$ . This relation is visualized in Fig 2 where it can be seen that the mean strength decreases with increasing volumes under stress. The variability, which is represented by a Weibull distribution decreases with increasing volumes.

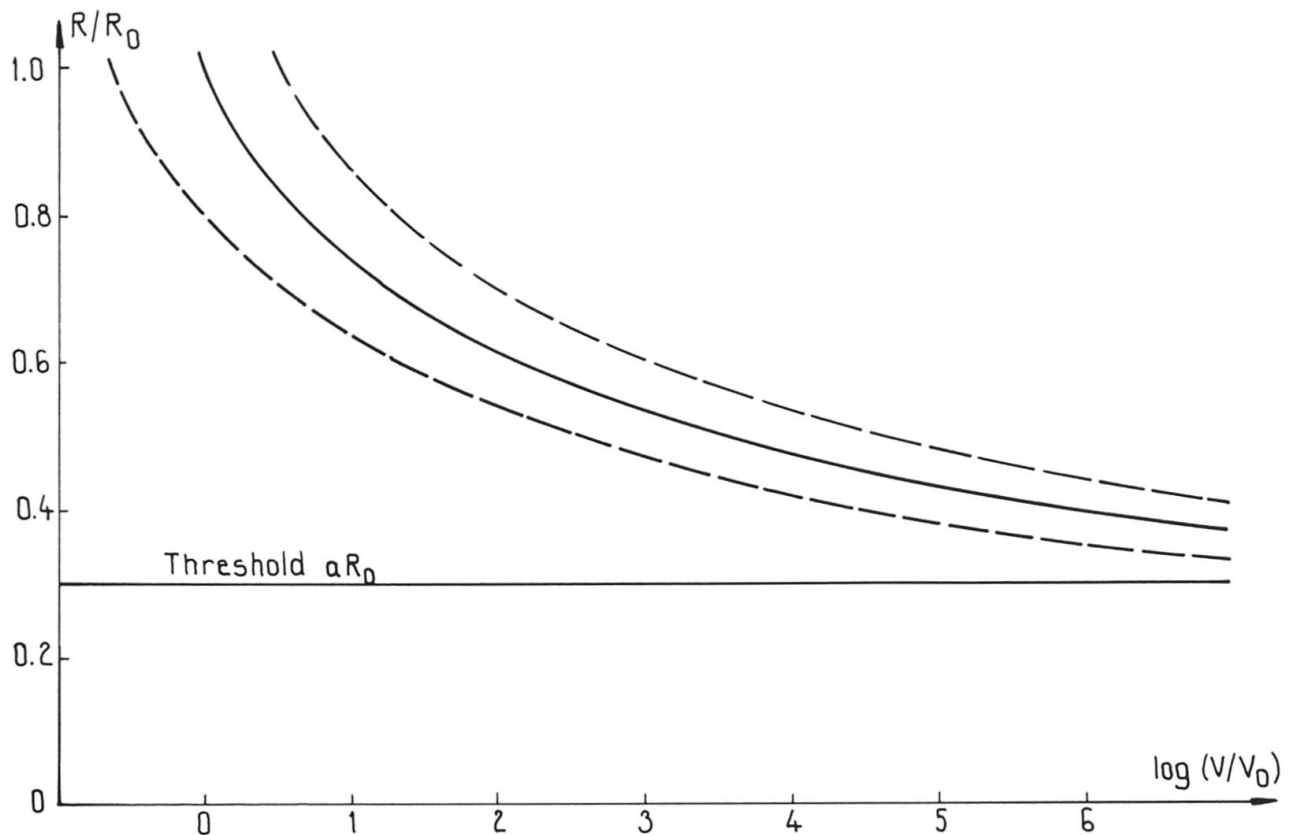


Fig 2 The volume dependence of the brittle failure load. The solid line and the broken lines correspond to the mean behaviour and the spread around the mean value respectively.

#### 4.2 The time dependence

The delayed brittle failure load as it is expressed in eq 6 can be evaluated in a similar way as the volume dependence. Without going into detail, the mean value and the coefficient of variation are

$$R = R_0 \left\{ a + b \left( V_0/V \right)^{1/k} \left( T_0/T \right)^{1/h} \right\} \quad (13)$$



$$C_0 V = \frac{b \left(\frac{V_0}{V}\right)^{1/k} \left(\frac{T_0}{T}\right)^{1/h}}{a + b \left(\frac{V_0}{V}\right)^{1/k} \left(\frac{T_0}{T}\right)^{1/h}} \sqrt{\frac{\Gamma \left(1 + \frac{2}{h}\right)}{\Gamma^2 \left(1 + \frac{1}{h}\right)} - 1} \quad (14)$$

where, in a similar way as for eqs 11 and 12,  $R$  is the mean strength when the sustained duration is  $T$  with reference to the strength  $R_0$  for the duration  $T_0$ . The dependence of the mean strength on the volume under stress and the duration is visualized in Fig 3. The time dependence should be seen as conditional on the volume dependence.

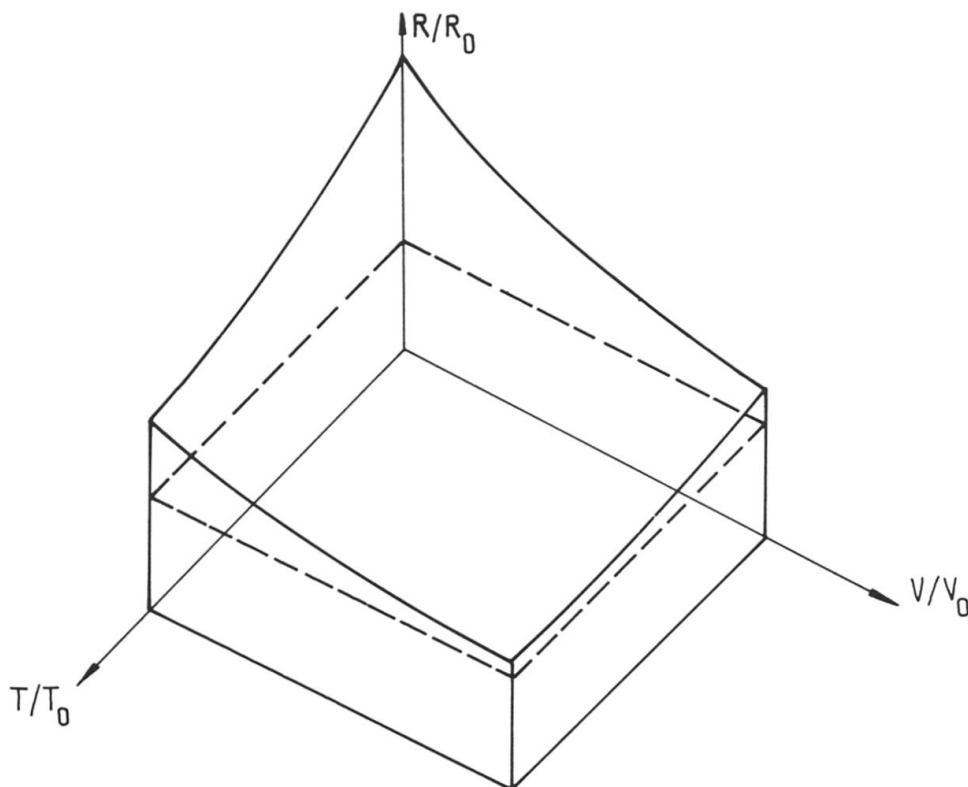


Fig 3 The brittle strength as a function of volume under stress and the duration. The dotted line corresponds to the threshold stress.

The time dependence can also be used for durations shorter than  $T_0$ . This will give a higher mean failure load and a larger spread based on eqs 13 and 14. This will be applicable for impact loads.

#### 4.3 Parameter values

In an application of the theory of brittle behaviour or delayed brittle behaviour information is needed which makes it possible to estimate the three parameters in the model. Such information is not directly available for general use. There is, however, certain information available which can be used for a verification of the model.





A brittle behaviour can only be expected when the stress level is above a certain threshold value where cracks will propagate. This threshold value, which is represented with  $r_0$  in the original formulation or a  $R_0$ , will depend on the qualities of the concrete, especially the bonding between the matrix and the aggregate. If the proportional limit is used as measure of a  $R_0$  the threshold seem to be in the range of 30 - 40 % of the failure load for cubes tested in compression [7]. For small samples tested in tension the proportional limit seems to be in the range of 60 - 70 % of the failure load [7].

The parameter  $k$  which determines the volume dependence can be estimated from eq 12 based on a knowledge of the mean value and the variance of the concrete strength. Such information is available in a large scale from standardized testings of cubes or cylinders for the compressive strength. Based on such tests the value of  $k$  will vary from 2 for low strength concrete to approximately 12 for high strength concrete. In real structures the mean strength is normally lower and the spread higher compared to the values obtained in standardized tests. This is because the compaction and curing is not as good as for test specimens. If this is taken into account the value of  $k$  will be reduced. For concrete subject to tension no general information is available to estimate the volume dependence.

The failure strength will be represented by a Weibull distribution. In Fig 4 normalized plots of the Weibull distribution are shown for some different values of  $k$ . For  $k = 3.4$  a Weibull distribution will have a similar shape as a Gaussian distribution. A normal distribution will thus give a good representation of concrete with low strength.

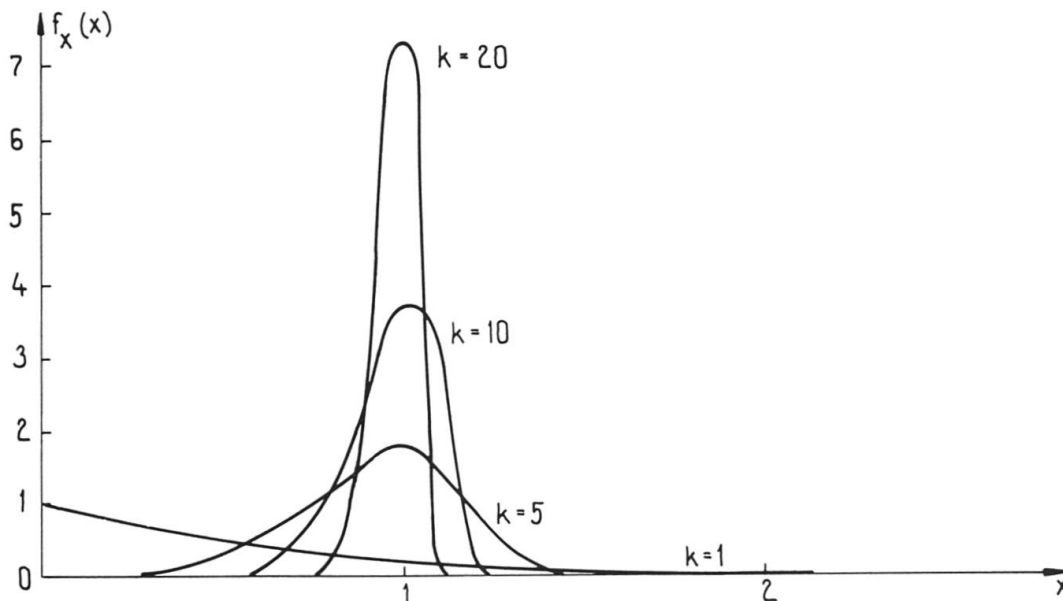


Fig 4 Weibull density functions

The parameter  $h$  which determines the time dependent behaviour can be estimated from eqs 13 and 14. Unfortunately very few experiments have been performed of the type shown in Fig 1 which could be used in the evaluation of the time dependence. Normally such experiments have to last for several years. Otherwise it will be difficult to estimate  $h$  because of the high variability encountered in the beginning of the creep process. Information available [10, 13] indicate a range of  $k$  between 15 - 30 for sustained loads under stationary environmental conditions.

When a concrete body is subject to temperature and humidity variations the creep rate will be significantly higher compared to stationary conditions. The time parameter  $h$  is thus reduced when outdoor environmental conditions have to be considered. Variable loads and especially dynamic loads will also increase the rate of crack propagation which will decrease the  $h$  parameter. The stress range seems to be of special importance in determining the reduction.

## 5. APPLICATIONS

In an evaluation of the brittle failure load the stress distribution need to be expressed in terms of compressive or tensile stresses. This can be done based on Mohr's hypothesis

$$\sigma_{1,2} = \frac{1}{2} (\sigma_x + \sigma_y) \pm \sqrt{\frac{1}{4} (\sigma_x - \sigma_y)^2 + \tau^2} \quad (15)$$

where an evaluation based on  $\sigma_1$  and  $\sigma_2$  will relate to a compressive failure or a tensile failure respectively. Often the stress distribution can be simplified to normal stresses in one dimension only.

### 5.1 Compressive failure loads

Even though it has been a well - known fact that concrete exhibits a volume dependence very few experiments seem to have been performed in a large scale to demonstrate this. One of the few larger experiments was performed in the thirties in connection with the construction of the Boulder dam [2]. Cylinders with a height to diameter ratio of two, ranging in height size from 100 mm to 1800 mm were tested in compression. The 28 days strength of a standard cylinder was 26 MPa with a water cement ratio of 0.5. This slow growth of the compressive strength is typical of concrete used at that time.

The result of the tests is presented in Table 1 with reference to the strength of a standard test cylinder,  $d \times h = 150 \text{ mm} \times 300 \text{ mm}$ . A few tests on standardized cylinders showed a coefficient of variation of 0.08.

Cylinder size		Rel. strength
d, mm	h, mm	%
50	100	109
75	150	106
150	300	100
200	400	97
300	600	91
450	900	87
600	1200	84
900	1800	82

Table 1 The relative failure strength of cylinders as a function of volume. From [2].

Based on the information available the volume dependence shown in Fig 5 can be obtained. Of normal practical interest is the result for volumes larger than a standard cylinder which is shown as a straight line in Fig 5. The limiting value  $aR$  is somewhat high but this might reflect a faster growth of the tensile strength than for the compressive strength which would result in an early obtainable threshold value. With a water cement ratio of 0.5 the final strength should reach 40 - 50 MPa and  $a \approx 0.3$ .

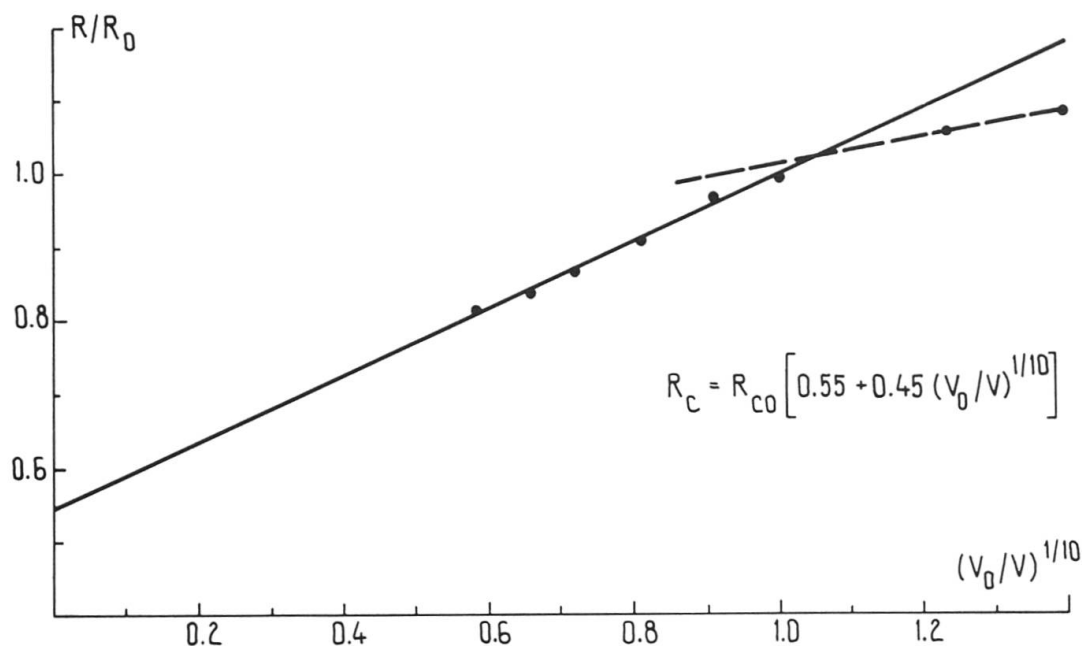


Fig 5 The volume dependence of the brittle fracture load in uniaxial compression.

## 5.2 Tensile failure loads

The tensile strength of concrete is low which makes it difficult to perform direct tensile testing. Instead alternative methods like the splitting test and the flexural test are used. Here the volume dependence of concrete tested in flexure is presented.

The basic equations for the brittle failure load, eqs 11 and 12, are here simplified by setting  $a = 0$ . This makes it possible to derive explicit expressions for a beam in flexure. For the two point load situation shown in Fig 6 the mean failure load and the coefficient of variation are

$$R_t = R_{t0} \left\{ V_0/V \cdot \left( 1 + \frac{e}{L} k \right) \right\}^{1/k} \quad (16)$$

$$C_0 V = \sqrt{\frac{\Gamma (1 + 2/k)}{\Gamma^2 (1 + 1/k)}} - 1 \quad (17)$$

based on the theory of elasticity for the volume under tension. Even if such a stress distribution does not reflect reality eqs 16 and 17 are reasonable approximations.

A laboratory experiment [15] where beams of different sizes have been tested with one point load and two point loads is presented in Table 2. Based on eq 16 the relation  $R_1/R_2$ , where  $R_1$  is the failure load with one point and  $R_2$  is the failure load for two point loads with  $e = L/3$ , should be constant and equal to

$$R_{t1}/R_{t2} = (1 + k/3)^k \quad (18)$$

This is not the case. The reason to this is that if no major flaw is present in the small volume under peak stress for the case of one point load, concrete will exhibit a certain ductile behaviour. In two, or maybe four cases, the relation  $R_1/R_2$  takes a low value of around 1.16 which reflects an instantaneous brittle

behaviour. This information can be used to determine the  $k$  value from eq 17. This gives a  $k$  value of around 10. In Fig 6 the result of Table 3 is shown together with the mean failure load. In this case eq 9 is used because there is no generally accepted flexural test method which can be used to obtain a reference volume. It can be seen in Fig 6 that a threshold value is rather high which reflects the assumption that  $a = 0.6 - 0.7$  for concrete in tension. Besides, flexural testing with small beams or with one point load is not advisable.

Beam dimensions			Rupture loads		
d, mm	b, mm	L, mm	$R_1$ , kN	$R_2$ , kN	$R_1/R_2$
75	75	225	4.2	3.3	1.27
100	100	300	3.9	3.0	1.30
150	150	450	3.0	2.6	1.15
200	200	600	2.8	2.3	1.21
75	75	338	3.7	3.2	1.15
100	100	450	3.8	3.1	1.22
150	150	675	2.9	2.5	1.16
75	75	450	3.9	3.3	1.18
100	100	600	3.6	2.9	1.24
75	75	675	4.0	3.1	1.29

Table 2 The flexural strength for one point load,  $R_1$ , and two point loads,  $R_2$ . From [15].

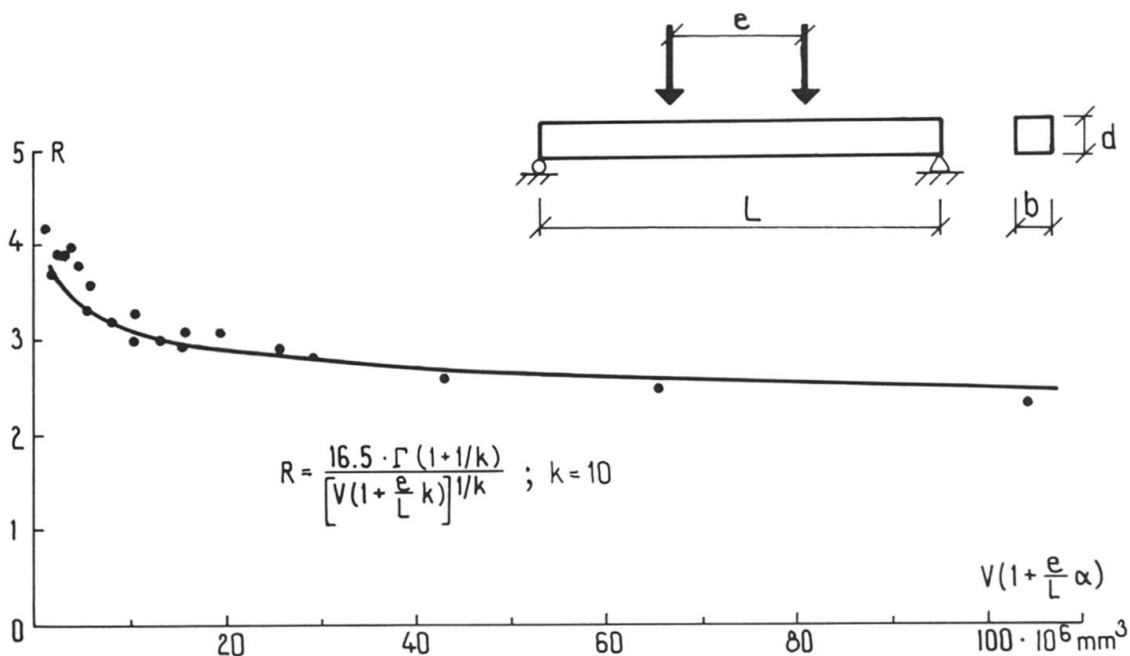


Fig 6 The flexural failure load as a function of the volume under tensile strength.



### 5.3 Time dependent compression failure loads

The time dependent creep characteristics of concrete have been studied extensively. But very few investigations have considered creep to failure for sustained stresses. This requires investigations which will last for years.

One study which is presented in [13] has included delayed brittle failures for a duration of 30 years. Compared to the brittle failure load obtained in standardized testing the failure load after a certain duration is given in Table 3. No additional information is given. If it is assumed that the threshold level is  $0.3 R_0$  where  $R_0$  is the strength at the duration 2 minutes. the result shown in Fig 7 will be obtained.

Duration of loading T				Percentage of failure load after duration T
min	hours	days	years	
2				100
10				95
30				92
60	1			90
	4	0.17		88
		100		78
		365	1	77
			3	73
			30	69

Table 3 The reduction of the failure load after sustained loading. From [13].

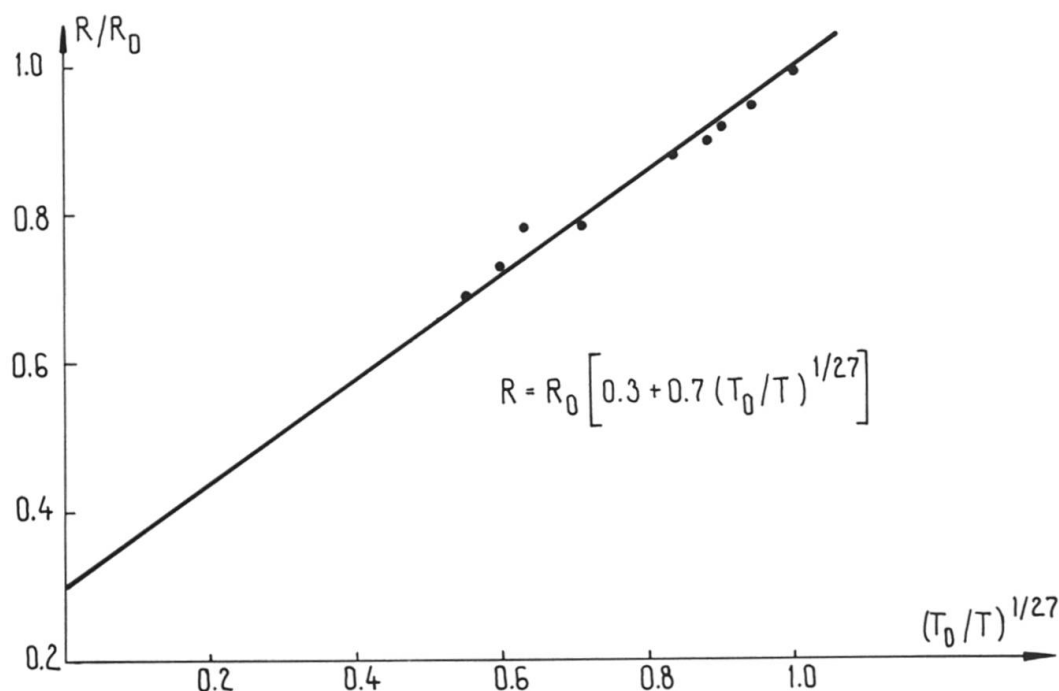


Fig 7 The time dependence of the brittle failure load in uniaxial compression.

Another result [10] shows a similar trend. But in this case the sustained loading has lasted for less than a year.

#### 5.4 Volume and time dependent compression failure loads

Any investigations of the combined influence of volume and durations on the reduction of the failure load for sustained loads do not seem to exist. But if the damage accumulation is simply a function of the time spent above the threshold level, fatigue experiments of plain concrete can be used.

In the case of fatigue of concrete the time dependence is normally expressed as the number of cycles to failure,  $N$ , instead of the duration. But during each cycle a certain creep will take place dependent on the stress level and the time spent on a high stress level. The number of cycles can thus be seen as a measure of the time spent above the threshold  $\sigma_0$  in eq 13. In a simplified way the mean fatigue strength of concrete can be represented with

$$R = R_0 \left\{ a + b (V_0/V)^{1/k} (N_0/N)^{1/n} \right\} \quad (19)$$

and the coefficient of variation in a similar way as eq 14.

In a fatigue experiment [9] three different stress gradients were obtained by applying a compressive load excentrically in two cases. The stress gradients desired are shown in Fig 8. These stress gradients do probably not reflect reality but they are used here in the evaluation of the volume under stress. The compressive strength of the test specimens were around 41 MPa. The result of the test, where failures are given as a function of maximum stress applied compared to the static strength and cycles to failure, is presented in Fig 9.

A direct evaluation of the fatigue experiment based on eq 12 cannot be done because of a lack of information concerning the variability of the static compressive strength. An indirect estimation of  $k$  based on the difference in mean fatigue strength after  $10^5$  cycles will give the mean fatigue curves shown in Fig 9.

Similar results can be obtained for other fatigue experiments [12] of plain concrete. When evaluating the fatigue life of concrete both the volume under stress and the time spent above the threshold have to be considered. Fatigue curves for practical use can be obtained from a knowledge that the spread can be represented by a Weibull distribution.

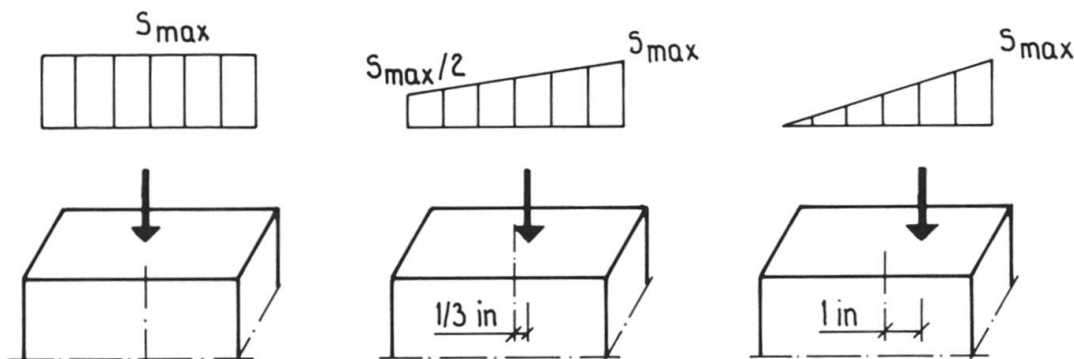


Fig 8 Stress gradients used in a fatigue experiment [9].

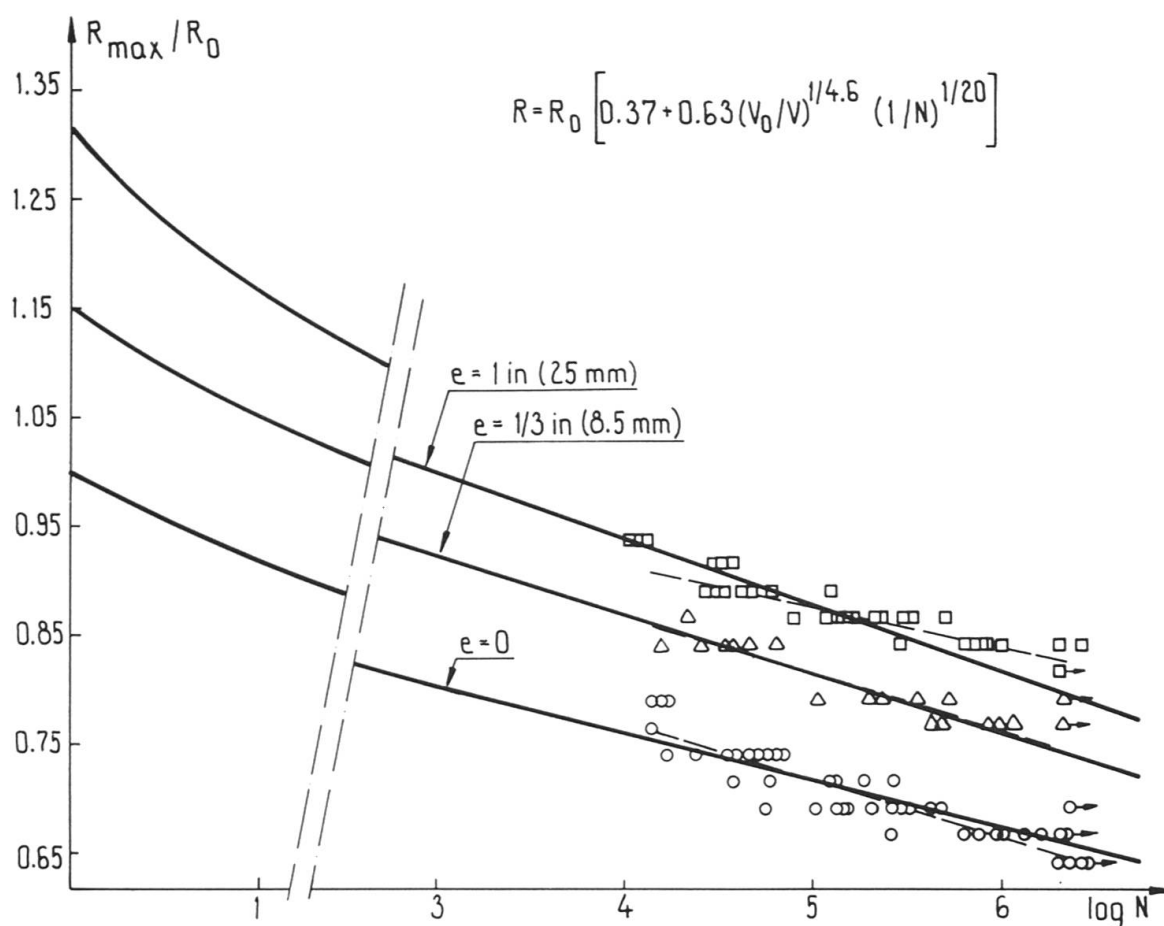


Fig 9 Fatigue curves corresponding to the stress gradient shown. Dashed lines originally proposed curves. From [9].

### 5.5 Volume and time dependent shear failure loads

It is recognized in most codes that the shear capacity of a beam depends on the location of the load in relation to the depth of the beam. Besides there is also a maximum allowable shear capacity when the load is close to the support. This is visualized in Fig 10.

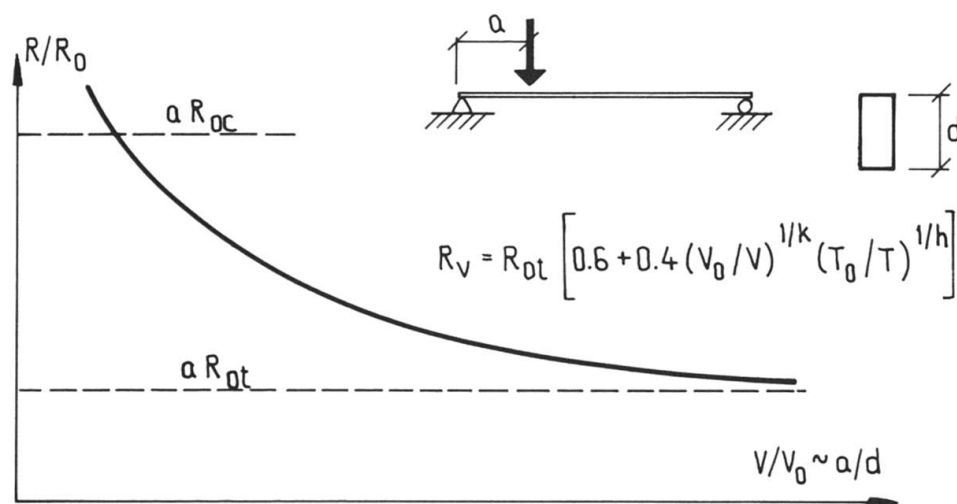


Fig 10 The failure shear strength as a function of the location of a load.



The shear strength of a beam is normally assumed to be related to the tensile strength of concrete. This is because failures some distance away from the support will appear as diagonal tension failures. The curve shown in Fig 10 can be obtained from an evaluation of the volume under tension. The stress distribution is given by eq 15 where it is enough to consider only the shear stress. Because of the volume dependence the shear capacity is also time dependent. This can be a very important aspect which is not considered in codes which allow an increase of the shear capacity close to supports.

The upper limit of the shear strength when a load is close to a support seems to be related to the threshold value for concrete in compression. This does not reflect a volume dependence, merely that cracks will propagate directly because of a complex stress distribution.

## 6. CONCLUSIONS

In this paper a new model of concrete strength has been proposed. The model includes the volume and time dependence of the brittle strength within a stochastic framework. A stochastic approach has mainly two advantages, which will be discussed briefly below.

Stochastic models are valuable not only because they are capable to represent uncertainty, but also because they are effective instruments of decision. The model presented does not reflect reality in detail. It is an attempt to include the main influences on the brittle strength of concrete. As such it cannot describe special features which sometimes are of interest.

The model clearly demonstrates that the failure strength of concrete is dependent on the volume under stress and the duration of a sustained stress. This requires a new approach in the evaluation of the safety of structures. Loads cannot be represented by some maximum value alone. It is necessary to include a measure of the duration of loads too [11]. In the future it is desirable to consider environmental influences in terms of their impact on the time dependence. The formal evaluation of the safety of structures also requires a new approach. The present model code [4] does not reflect the failure behaviour of concrete as it is demonstrated in this paper. The modifications needed are, however, not as drastic as it may seem.

This paper has mainly dealt with the failure strength of concrete. But the time dependent behaviour of concrete, which normally is associated with creep, can also result in cracks. Such cracks can reduce the durability of concrete structures. This is an area which is equally important as the one dealt with here.

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