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# Durability – a Probabilistic Approach

## Durabilité – une approche probabiliste

## Dauerhaftigkeit – ein wahrscheinlichkeitstheoretischer Ansatz

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**SUMMARY**

The scatter in observed service lifes of building structures is very high. Designing an optimal structure in relation to durability is therefore not an easy task. This paper investigates whether reliability analysis can be used to solve the problem. Essentially the same techniques are used as those which have proven to be successful as a design tool for ultimate and serviceability limit states without deterioration effects. As an example a reinforced concrete slab for an outdoor gallery of an apartment building is analysed for the limit state of corrosion of the reinforcement.

**RÉSUMÉ**

Les durées de vie des constructions présentent une dispersion importante. Il n'est pas facile de concevoir une structure optimale du point de vue de la durabilité. L'article essaie de déterminer si une approche probabiliste pourrait être utile. Les mêmes techniques sont utilisées que celles qui ont fait leurs preuves comme outil de conception pour des états de rupture et de service, sans effets de détérioration. L'état limite de corrosion de l'armature d'une dalle de balcon est étudié à titre d'exemple.

**ZUSAMMENFASSUNG**

Die Streuung der beobachteten Lebensdauer von Bauwerken ist beachtlich. Das Entwerfen einer in bezug auf Dauerhaftigkeit optimalen Konstruktion ist daher keine einfache Aufgabe. In der vorliegenden Arbeit wird untersucht, ob dieses Problem mit Hilfe der Zuverlässigkeitsanalyse gelöst werden kann. Dabei wurden die gleichen Methoden benützt, die sich bei der Überprüfung von Grenzzuständen der Tragfähigkeit ohne Einbezug von Alterungsproblemen bewährt haben. Als Beispiel dient eine Stahlbetonplatte für eine Aussengalerie eines Hochhauses, die für den Grenzzustand auf Korrosion der Bewehrung untersucht wird.

## 1. INTRODUCTION

Every building structure has in principle a finite service life. Due to loadings, environmental conditions, usage and so on, the structure will in course of time deteriorate. Finally it has to be pulled down, renovated, replaced or repaired. The deterioration effects can be related to several aspects such as safety, serviceability, comfort or aesthetics.
For the design of a structure that must be new build or renovated, it is important to have a good insight in the various processes that lead to deterioration. In that way an impression can be gained of the durability. As these processes in general are stochastic, it is necessary to take into account that the service life will be scattered. In general this scatter will be even relatively high.

An example of this is given by Bekker [1]. From his study it follows, that the service lifes of houses in the Netherlands vary from about 45 to 125 years (see Fig. 1). The mean service life is approximately 84 years. The end of the service life was in these cases not always based on technical reasons. Political, economical and social reasons have also played a part. It is stated, however, that the technical service life will also have a high scatter.

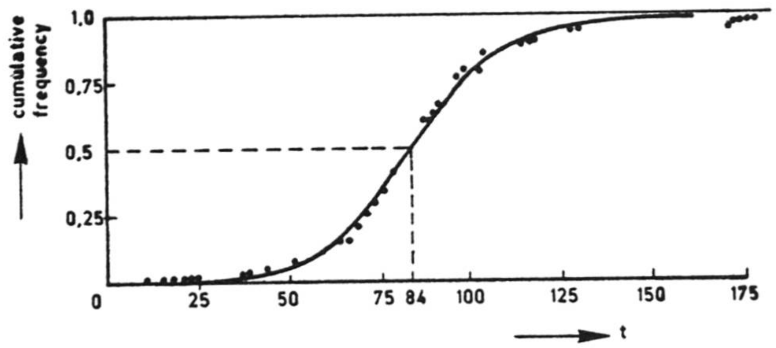



Fig. 1 Service life distribution of houses in the Netherlands [1]

In general, it is not possible to indicate exactly what the reasons are for the scatter. One reason will be the complex and hardly known set of deterioration mechanisms involved and the interactions between them. Another reason is the random nature of the environmental conditions and the structural properties. Even when conditions are apparently the same, the scatter in observed service lifes proves to be very high. In such circumstances it is not easy for a designer to relate the durability of the various building components to each other and to optimise the design in an economic manner.

On many aspects a great resemblance can be observed between durability on the one side and (time independent) safety and serviceability on the other. Among these aspects are:

- the stochastic environmental conditions;
- the stochastic structural properties;
- the mutual relations between building components;
- the necessity of an economical optimization of the design.

In the case of safety and serviceability problems, there is an increasing tendency to approach the design in a systematic and rational way from a probabilistic point of view. Such an approach is called a reliability analysis. A logical extension of this development therefore is to apply probabilistic

methods also in the cases of durability and service life problems. In several publications this has already been proposed [2,3]. The present publication demonstrates primarily, by means of an elaborated example, that this approach is feasible indeed and that the results can serve as a sound basis for design decisions.
Reliability analysis can also be interpreted as a kind of sensitivity analysis. The most relevant parameters in relation to the scatter problem will be distinguished, even when only little information is available. This can simplify the problem and serve as a guide-line to focus attention on the important aspects.
Finally it should be mentioned, that probabilistic methods can be considered as a valuable tool for code making activities. Using the concept of partial safety factors, results of probabilistic analysis can be translated into simple design requirements. It would be advantageous to have a code for durability, developed along the same line.

## 2 ECONOMIC OPTIMIZATION

In order to build a structure high investments have to be done. The interest and the redemption that are related to this investment belong to the operating costs. These consist moreover of:

- maintenance costs (including inspection and cleaning);
- repair costs in case of damages;
- energy costs;
- administration costs;
- working costs, like guards and security.

For a greater part these costs are determined by the design and execution of the building structure. For this reason these costs should be well considered during the design and the construction stage.

Between the direct building costs and the maintenance and repair costs exists a relation. A right investment can lead to a reduction of maintenance costs. In principle it is therefore possible to determine an optimal investment, leading to the lowest sum of investments, repair and maintenance costs.
In order to establish the optimum (in fact the minimum) costs, it is necessary to compare the direct building costs with the costs that will be encountered in the future. This can either be done by capitalizing the future costs or by comparing costs on a yearly basis.

In capitalizing the costs, all the future expenses are converted into an amount of money that -in a manner of speaking- has to be reserved in addition to the direct costs. This reservation can be lower than the amount of the future expenses, because in the course of time it will increase due to the bearing of interest. In a sound economy the interest will be higher than the inflation. Therefore the real rate of interest will in general be positive.
If the target service life of the building structure equals N years, then the total costs are equal to:

$$E\{C_{kap}\} = S + \sum_{i=1}^{N} \frac{V_i}{(1+r')^i} + \sum_{i=1}^{N} \frac{P\{F_i\} \cdot D_i}{(1+r')^i} \qquad \ldots.(1)$$

Where:

$E\{C_{kap}\}$ = expected capitalized costs;
$S$ = direct investment;
$V_i$ = maintenance costs in year i;
$r'$ = real rate of interest (nominal interest minus inflation);
$P\{F_i\}$ = probability of failure in year i;
$D_i$ = damage as a consequence of failure in year i.

The advantage of taking the real interest is that no allowance has to be made for increasing prices due to inflation. To simplify the problem, in relation (1) a number of working costs have not been considered. This simplification does not affect the result of this study.
Under normal circumstances it is not certain that damage will occur, so the risk or cost expectation has to be considered. Therefore, in relation (1) damage costs are multiplied with a probability of occurence.

Instead of capitalizing the expected maintenance and repair costs, it is also possible to make a cost-accounting on an annual basis. In that case, the investments have to be converted to an annual amount for redemption and interest. If the costs for maintenance and possible damage are the same for every year, the expected annual costs can be expressed as:

$$E\{x\} = \frac{S.r'.(1+r')^N}{(1+r')^N - 1} + V_i + P\{F_i\} \cdot D_i \qquad \text{....(2)}$$

Where:

$E\{x\}$ = expected annual costs;
$S$ = investment (direct costs);
$V_i$ = yearly maintenance costs;
$r'$ = interest;
$P\{F_i\}$ = probability of failure in one year;
$D_i$ = amount of damage in case of failure.

If in relation (2) r' represents the nominal interest, then this relation refers to a normal annuity (every year nominal the same amount). For durability problems it is often better to use the real interest, as this keeps step with the price level.

If the costs for maintenance and possible damage are not constant in every year, they have to be capitalized and added to the direct costs. As in general these costs will not be constant, it is more convenient to use relation (1) and to make the cost-accounting on a capitalized basis. It will be clear that the results of the optimization calculation are not dependent of the basis for the economical consideration.

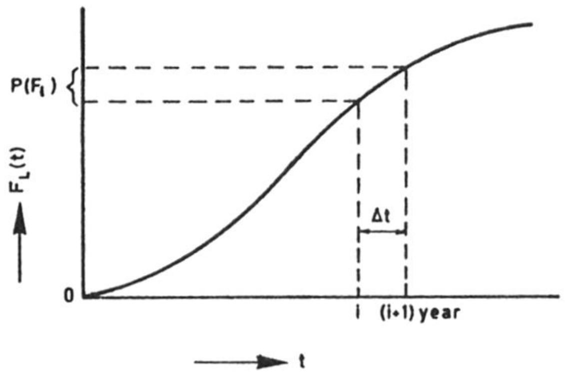



Fig. 2 Distribution of L(t)

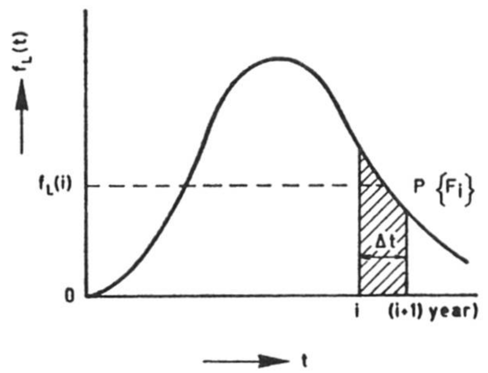



Fig. 3 Probability density of L(t)

From the relations (1) and (2) it follows, that for an economic optimization it is necessary to know the yearly probability of failure $P\{F_i\}$. Sometimes it is possible to asses a value for this probability on the basis of experience. This can be done if sufficient statistical information is available. For instance for fire and window pane failures this is in general the case.

Another possibility to establish the probability of failure is to deduce the distribution of the service life on the basis of probabilistic methods. The probability $P\{F_i\}$ follows (see Fig. 2) from the distribution $F_L(t)$ of the service life:

$$P\{F_i\} = P\{t - \Delta t < L < t\} = F_L(t) - F_L(t - \Delta t) \qquad ....(3)$$

Where t = i years and $\Delta t$ = 1 year. It is also possible to start (see Fig. 3) from the probability density function $f_L(t)$ according to:

$$P\{F_i\} = f_L(t) \cdot \Delta t \qquad ....(4)$$

The determination of $F_L(t)$ will be discussed in the next section.

## 3. RELIABILITY ANALYSIS

The first step in a reliability analysis is a thorough investigation of all relevant hazards and failure mechanisms. In a systematical way this can be done for example by means of a "Failure Mode and Effect Analysis" (FMEA). In a table or a checklist a survey is given of all relevant hazards (the causes of deterioration), the belonging failure mechanisms (the way the hazards affect the building structure) and the effects (damage). For reinforced concrete, an example is given in table 1, based on information from [4 - 7].

| hazard | mechanism | effect |
|---|---|---|
| 1. alternating load | fatigue | cracking/failure |
| 2. streaming water | erosion | surface defects |
| 3. turbulent water | cavitation | excavations |
| 4. driving, walking | wastage | impracticability |
| 5. freezing | expansion | cracking |
| 6. acids | neutralization | corrosion |
| 7. chloride etc. | depassivation | pit corrosion |
| 8. sulphates | crystal growth | desintegration |
| 9. de-icing salts | withdrawing heat | scaling |
| 10. freezing/de-icing | withdrawing heat | scaling |
| 11. soft water | neutralization | corrosion |
| 12. sugar,glycerine | forming of acids | corrosion |
| 13. micro-organisms | production of acids | corrosion |
| 14. acidifying gases | neutralization | corrosion |
| 14a. carbondioxyde | carbonation | corrosion |
| 15. corrosion | reducing bar diameter | failure |
| 16. corrosion | rusting | cracking of cover |
| 17. pollution | crystallization | pop-outs |
| 18. alkali aggregate | alkali-silica reaction | expansion |

Table 1 FMEA related to the durability of concrete

Next, a selection should be made in order to distinguish between important and less important hazards. Only the hazards that might result in effects with unacceptably high risks are taken into further consideration. If the risks are not directly known from experience, this means that mathematical models for failure mechanisms have to be developped, numerical values should be assigned to the various parameters (best estimates as well as measures of uncertainty) and economical consequences should be estimated.

The simplest mathematical model for a failure mechanism consists of one load parameter S and one resistance parameter R, both time independent. Failure occurs when the load exceeds the resistance:

$$\{F\} = \{\text{failure}\} = \{R < S\} \qquad \text{....(5)}$$

The probability of failure can be calculated from the convolution integral:

$$P\{F\} = \int_0^\infty F_R(s) \cdot f_S(s) \cdot ds \qquad \text{....(6)}$$

where $F_R(s)$ is the cumulative probability distribution function of the resistance R and $f_S(s)$ is the probability density function of the load S. The exact mathematical solution of (6) is in many cases too time consuming to be carried out. In the course of time approximation methods have been developed [3] to work out this equation.

In durability problems R and/or S are time dependent (see Fig. 4). Failure occurs if for at least one value of the time $\tau$ in the considered period (0 - t) the resistance R(t) is lower then the load S(t) at the same moment:

$$P\{\text{failure in } (0 - t)\} = P\{R(\tau) < S(\tau)\} \text{for at least one } \tau \text{ in } (0,t) \qquad \text{....(7)}$$

Since the event $\{\text{failure in } (0 - t)\}$ is identical to the event $\{L < t\}$ , equation (7) gives the distribution function $F_L(t)$ (see Fig. 2) for the service life L. By differentiating once, the probability density function $f_L(t)$ is obtained (see Fig. 3).
When R and S are monotonic functions of the time the equality R = S can sometimes be transformed into an explicit expression for the service life L. In that case L may be concieved as a special type of resistance and the time t as a special type of load. This method is also used in the example calculations that will be presented further on.

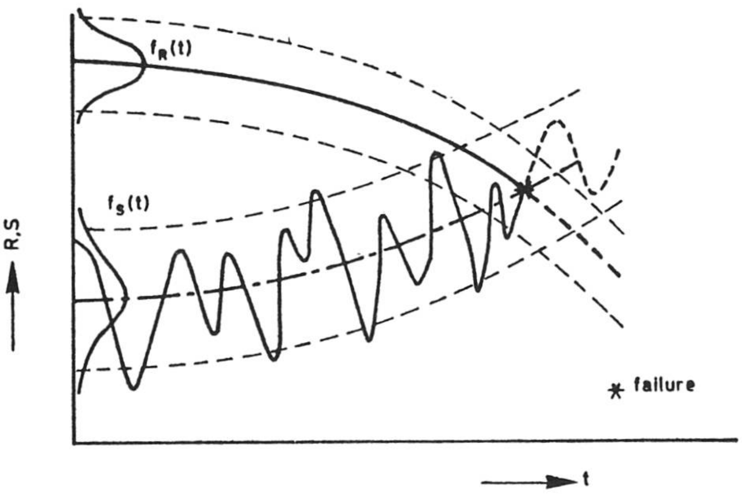
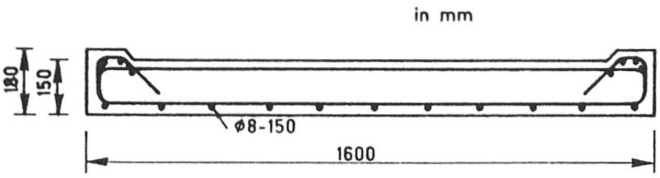



Fig. 4 R and S are both time dependent




length 6000 mm

cover 15/30 mm

w/c ratio 0,50-0,55

blast furnace cement (60% slag)

Fig. 5 Cross section of the gallery slab

## 4. DURABILITY OF AN OUTDOOR GALLERY SLAB

As a demonstration an outdoor gallery slab (see Fig. 5) made of reinforced concrete will be optimized. All assumptions in this example are as realistic as possible, but are still open to discussion.
From a FMEA of the gallery slab, it follows that in the Netherlands the main risk is governed by neutralization of the concrete cover as a result of carbon dioxyde. As an effect of carbonation the passivation layer on the reinforcement bars will be destroyed and corrosion becomes possible. Corrosion products have more volume then the original steel. The expansion results in tensile stresses and subsequent spalling of the concrete cover (see table 1). An impression of this type of damage is given in Fig. 6.
In this example four design alternatives will be distinguished:

1. cover 15 mm, no coating
2. cover 30 mm, no coating
3. cover 15 mm, coating with maintenance every 20 years
4. cover 15 mm, coating with maintenance every 10 years.

Of course the number of alternatives can be extended without additional theoretical difficulty. The target service life of the slabs is equal to 60 years. When damage occurs various repair measures are possible, such as replacement of the gallery slab, removing the carbonated zone, placing shotcrete and local repair with synthetic mortar. The kind of repair to be carried out depends upon the amount of damage and upon the point in time that the damage occurs. Extensive repair is for instance only meaningful if the building structure has still to function for a relative long period. In table 2 a cost survey is given for both initial investments, maintenance and repair.

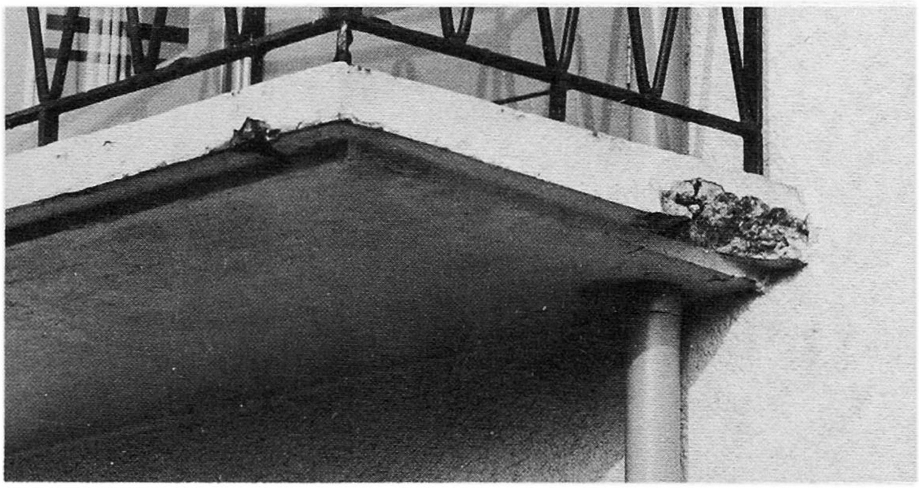


Fig. 6 Corrosion damage due to carbonation

| | | |
|---|---|---|
| new slabs (d=depth in [m]) | guilder/m2 | 40 + 0,4.d |
| replacement for d = 150 mm | guilder/m2 | 275 |
| repair with shotcrete | guilder/m2 | 225 |
| repair with synthetic mortar | guilder/m2 | 250 |
| coating on fresh concrete | guilder/m2 | 50 |
| coating on old concrete | guilder/m2 | 80 |
| maintenance of coating | guilder/m2 | 40 |
| repair of coating | guilder/m2 | 80 |

Table 2 Cost survey

From reference [13] a service life relation for the carbonation is derived:

$$L=\left[\frac{(c-\Delta)}{R.K}\left\{\frac{2.7}{46w-17.6}\right\}\right]^2+\frac{C.c}{\phi.v_c}+\frac{(c-\Delta)}{180\,f_o}\left\{\frac{\frac{T}{T_o}\ln f_o}{1-e^{-\frac{T}{T_o}\ln f_o}}\right\} \quad ....(9)$$

The first term in this equation describes the actual carbonation process for unprotected concrete [8,9]. In this term expresses, that the extreme peaks in the carbonation front that reach the reinforcement bars, determine the first moment of corrosion and not the mean carbonation depth (see figure 7). The second term represents the time gap until the visibility of the corrosion [10,11] and the last term is the prolongation due to the coating [12,13].

In literature there is an ever lasting discussion on the right model for the rate of carbonation in concrete. It is not the intention of this study to take a stand in the right formulation. The only intention is to demonstrate, that reliability analysis can be used for solving durability problems. Equation (9) is therefore more or less arbitrarily chosen.

In table 3 a survey is given of all the variables in equation (9) and their stochastic properties, viz. the type of the distribution, the mean value and the coefficient of variation (c.o.v). The type of the distribution is denoted with LN for log-normal and D for deterministic. In reality the deterministic variables will also be scattered. The coefficient of variation is, however, so small that it is possible to treat those variables as deterministic.

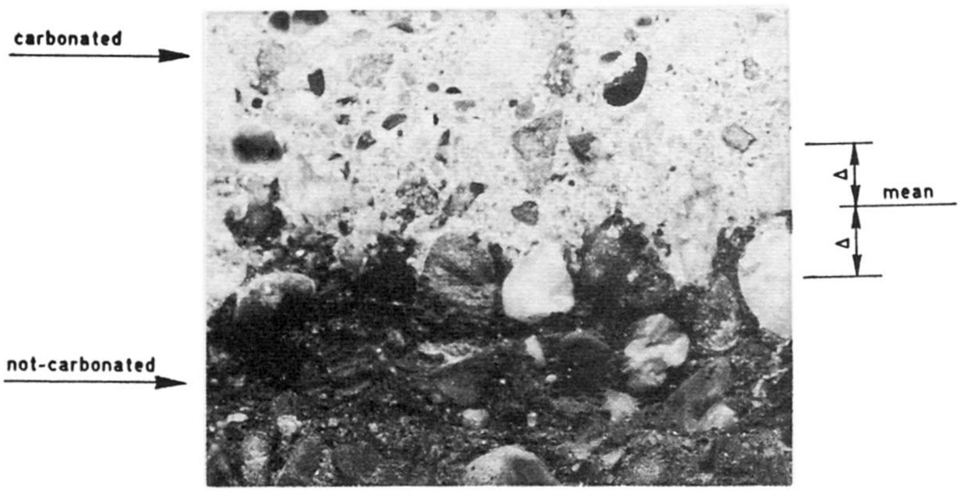



Fig. 7 Irregular carbonation front in concrete

| description | | | type | mean | c.o.v. |
|---|---|---|---|---|---|
| c | concrete cover nominal 15 | mm | LN | 20 | 0.25 |
| c | concrete cover nominal 30 | mm | LN | 35 | 0.14 |
| Δ | distance maximum-mean carb. depth | mm | LN | 5 | 0.20 |
| R | cement type parameter | - | LN | 2.0 | 0.15 |
| K | climate parameter | - | LN | 0.7 | 0.20 |
| w | water-cement ratio | - | LN | 0.5 | 0.05 |
| C | constant | mm | D | 0.08 | - |
| $\phi$ | diameter of reinforcement bar | mm | D | 8 | - |
| $v_c$ | corrosion rate | mm/year | LN | 0.04 | 0.50 |
| s | thickness of coating | mm | D | 0.18 | - |
| $f_o$ | damage coefficient for coating | mm/year | LN | 0.00001 | 1.00 |
| $T_o$ | durability parameter | year | LN | 50 | 0.50 |
| T | maintenance period | year | D | 10/20 | - |

Table 3 Survey of the carbonation variables

| design alternative | | mean | standard deviation | probability $P\{L<60 \text{ year}\}$ |
|---|---|---|---|---|
| 1 | year | 34 | 28 | 0.76 |
| 2 | year | 123 | 86 | 0.13 |
| 3 | year | 103 | 144 | 0.34 |
| 4 | year | 417 | 474 | 0.05 |

Table 4 Resulting service life distributions

The distribution of the service life is calculated on the basis of equation (9) and the parameters in table 3. This is done with the "First Order Second Moment" method (FOSM) [3]. In table 4 the main results, as have been calculated in [13], have been collected. First, the mean service life and the standard deviation have been determined, then the failure probability within a target service life of 60 years is calculated. In [13] it is demonstrated, that with these means and standard deviations almost the same failure probabilities are found if it is assumed that the service lifes have log-normal distributions. In figure 8, the calculated service life distributions for the design alternatives 1 and 2 are shown.

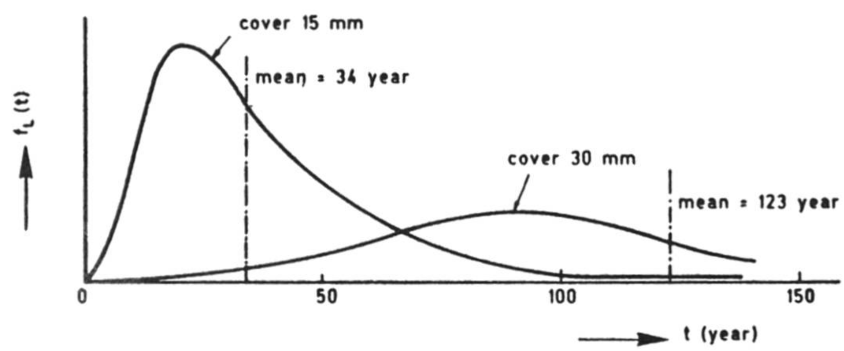



Fig. 8 Service life distribution of the alternatives 1 and 2

| | description | design alternative 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| c | cover | 52% | 21% | 9% | 10% |
| Δ | distance maximum-mean carb. depth | 2 | 1 | 0 | 0 |
| R | cement type parameter | 9 | 16 | 0 | 0 |
| K | climate parameter | 17 | 29 | 1 | 0 |
| w | water-cement ratio | 19 | 33 | 1 | 0 |
| $v_c$ | corrosion rate | 1 | 0 | 0 | 0 |
| $f_o$ | damage coefficient for coating | - | - | 11 | 50 |
| $T_o$ | durability parameter | - | - | 78 | 40 |
| | total | 100% | 100% | 100% | 100% |

Table 5 Relative contributions to the scatter

In accordance with the expectations it is found, that the scatter in the service life is very high. The standard deviation is of the same order of magnitude as the mean value. In table 5, a survey is given of the contributions of each of the main variables to this scatter. The contributions result directly from the FOSM-calculations. The table shows that, as stated in the introduction, the reliability analysis is in fact also a sensitivity analysis for the influence of the scatter of the individual variables on the scatter in the service life.

In the absence of a coating (the alternatives 1 and 2), the dominating variables happen to be the thickness of the cover, the water-cement ratio and the climate parameter. When a coating is applied (the alternatives 3 and 4), the quality of the coating is the most important variable.

The uncertainty in the service life can be reduced, if the scatter or the uncertainty in these variables will be reduced. Furthermore, especially for these important variables, it is worth-while to gather more information with respect to the correctness of the statistical properties.

Once the service life distributions are known, the expected capitalized costs of each design alternative can be calculated with the aid of equation (1). In table 6 the three terms of equation (1) are presented. They are based on the cost information from table 2 and the service life distributions from table 4. Repair measures that can be considered are replacement, shotereting of damaged parts or locacal repair with epoxy resin repair mortar. Which methd will be optimal, depends on the moment of occurence of the damage. An intensive repair is only meaningful if the structure has to last for a longer period.

The optimum design proves to be the gallery slab with a concrete cover of 30 mm. This result corresponds well with the development in the Dutch Concrete Code. In this Code the minimum concrete cover for outdoor slabs was recently increased, in connection with extensive damages. The calculated failure probabilities are obvious realistic, although maybe a little bit too high in relation to the practice.

It is clear from table 6, that the application of a coating is even more expensive then the non-optimal design alternative 1, despite the longer service lifes. This does not alter the fact that the application of a coating can be a good remedy to improve a defective concrete cover, or to improve the aesthetic properties.

| cost term | | design alternative 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| extra concrete cover | guilder | – | 60 | – | – |
| coating | guilder | – | – | 500 | 500 |
| maintenace costs of coating | guilder | – | – | 860 | 1250 |
| repair cost expectation | guilder | 860 | 170 | 260 | 40 |
| $E\{C_{kap}\}$ | guilder | 860 | 230 | 1620 | 1790 |

Table 6 Expected capitalised costs

## 5. CONCLUSIONS

The optimization analysis of the outdoor concrete gallery slab shows, that it is in principle possible to use the techniques of the reliability analysis for the solving of durability problems. The reliability analysis provides an estimate of the mean service life and the scatter. From these results a rational choise can be made out of a number of design alternatives, based on the criterion of lowest cost expectation.

From a sensitivity analysis the variables that are mainly responsible for the scatter in the service life can be identified. For the practice this means, that controlling the scatter of these variables will lead to a service life with a lower scatter. Extreme low service lifes can be avoided this way.

An extension of the research into other durability problems seems to be promising. In the Netherlands it has been decided to prepare a probability based code of practice for durability. Of course this does not mean, that an design decision from now on has to be based on extensive probabilistic and economic calculations. The aim is, just as in the case for standard safety and serviceability design problems, to derive a set of simple design rules for the common cases in every day practice. Only for special cases explicit optimization calculations will be encouraged.

ACKNOWLEDGEMENT

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