Multistage Decision Models for the Assessment of Structural Defects

Modèles de décision, à phases successives, pour l'analyse de dommages structuraux

Mehrstufen-Entscheidungsmodelle für die Beurteilung von Bauschäden

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SUMMARY

The assessment of defective structures usually has to be carried out in a climate of uncertainty and relatively high risk of error. A multistage decision model is proposed as a systematic and logical framework for the assessment process. Monitoring and testing are treated as important experimental options for improving the reliability of the assessment decisions. To illustrate the decision procedure, the case is considered of a buried conduit which undergoes continuing large deformations while in service.

RÉSUMÉ

L'analyse de structures endommagées est généralement réalisée, dans un cadre d'incertitude et avec un risque d'erreur relativement élevé. Un modèle de décisions, à phases successives, est proposé comme outil systématique et logique dans le processus d'analyse. Le calibrage et les tests sont des éléments importants pour augmenter la fiabilité de l'analyse. Pour illustrer la procédure de décision, l'article traite d'une canalisation souterraine en service, soumise à de grandes déformations.

ZUSAMMENFASSUNG

Die Beurteilung schadhafter Bauten muss meistens in einem Klima der Ungewissheit und mit einem relativ hohen Fehlerrisiko erfolgen. Es wird ein Mehrstufen-Entscheidungsmodell als systematisches Gerüst für den Beurteilungsprozess vorgeschlagen. Die Überwachung und der Versuch werden als wichtige experimentelle Möglichkeiten für die Erhöhung der Zuverlässigkeit der Beurteilung dargestellt. Um den Entscheidungsprozess zu illustrieren, wird der Fall einer unterirdisch verlaufenden Rohrleitung, welche im Betriebszustand andauernde grosse Verformungen zeigt, behandelt.

1. INTRODUCTION

Most structures display symptoms of inadequacy at some stage in their service life. If inherent inadequacies have been caused by errors in design or construction, the symptoms may exist even before the structure is brought into service. In some cases the symptoms and the underlying inadequacies may appear suddenly when the in-service structure is damaged. In other cases, inadequacies may develop gradually as the result of normal deterioration and ageing. Whenever evidence of significant structural inadequacy is discovered, a careful program of inspection and diagnosis needs to be undertaken. Depending on the diagnosis, it may be necessary to undertake corrective work to allow the structure to perform in a satisfacory manner in the future.

The processes of assessment and treatment of defective structures involve considerable risk. Indeed, the risk levels associated with this type of work are often much higher than those normally associated with the routine, code-based design of a new structure. Although systematic diagnostic and evaluative procedures can be used in the assessment of structural adequacy and in the planning and execution of the corrective work [1], they need to be supplemented by procedures for managing the risks that are involved. Several alternative approaches are possible. These include the use of Bayesian decision models [3,8], and the introduction of subjective, fuzzy treatments of belief [4], as already used in automated medical diagnosis [5,6].

In the case of an existing, possibly defective, structure which is not in danger of imminent collapse, two valuable options are available for reducing the risk levels. These are in-service monitoring and load testing. Monitoring of the performance of the in-service structure is undertaken over a limited period of time to provide additional information on service load behaviour. Load testing is usually undertaken on the structure after it has been taken temporarily out of service, and can therefore provide information on structural adequacy in both the service load and overload ranges.

2. MODELLING THE ASSESSMENT PROCESS

2.1 The Preliminary Step, N=0

The assessment process commences with a thorough physical inspection of the structure and a review of the relevant available information, including the design documents, the details of the construction procedures and the history of construction and subsequent in-service performance. When evaluated, the information obtained in the initial investigation will be considered to fall into one of the following three categories:

$Z(1,0)$: the information suggests that the structure is adequate, ie will perform in a satisfactory manner;

$Z(2,0)$: the information obtained is inconclusive and suggests neither adequacy nor inadequacy;

$Z(3,0)$: the information suggests that the structure is inadequate.

The preliminary step in the assessment is one of information gathering and involves no decision. It is represented in Fig. 1 by a single branch with three sub-branches showing the possible categories of the information obtained.
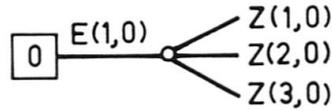
$$0 \;\; \overset{E(1,0)}{\longrightarrow} \; \begin{cases} Z(1,0) \\ Z(2,0) \\ Z(3,0) \end{cases}$$

Fig. 1   Preliminary step, N = 0

2.2   Typical N-th Step

The final outcome of the assessment process is assumed to be a decision, made on the best possible grounds, either to undertake corrective work or to do nothing with the in-service structure. Either of these courses of action might be taken during the first step in the decision process if the information obtained initially is sufficiently conclusive. Alternatively, it may be decided to defer action and to gather more information, by testing or monitoring. These four options will be available at any subsequent step in the decision process. The typical N-th step in the process is thus shown schematically in Fig 2, with A(1,N) and A(2,N) representing the two courses of action available. To maintain the traditional decision theory terminology, monitoring and testing are represented in Fig 2 as "experiments" E(1,N) and E(2,N), respectively. It will be convenient to refer generally to the actions as A(I,N) and to the experiments as E(K,N) and to show the N-th step as in Fig 2b, where the generalised choice is either to take the final decision (to do nothing or to do corrective work) or to defer the final decision and carry out an experiment in order to obtain additional information. To choose the most appropriate of the four available options, the possible consequences have to be evaluated. In the present study, three-way outcomes are assumed to follow each possible option. To consider the assessment process further, each action and experiment will be considered in turn, together with the possible outcomes.
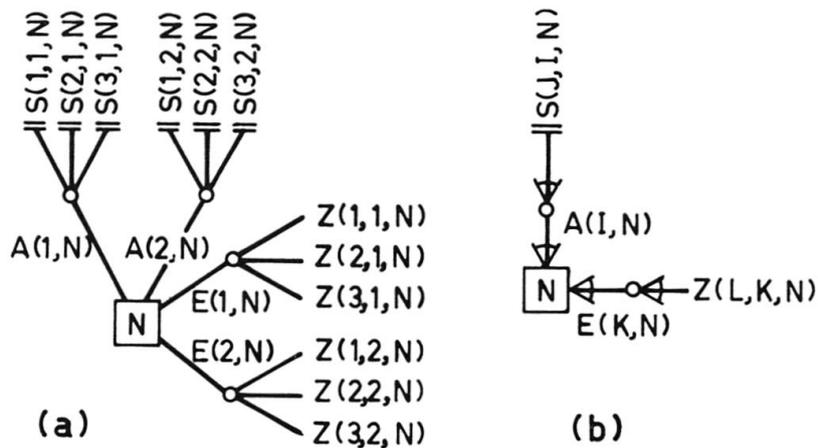

Fig. 2   Typical N-th step

2.3   Action A(1,N), Do-Nothing

With this course of action, the assessment process is discontinued, and the structure is allowed to continue in-service with no further attention than is appropriate for normal husbandry (such as the inspection, maintenance and

repair program appropriate for this type of structure). Re-assessment at some future date is not therefore precluded, should further evidence of defect or inadequacy arise. Associated with the do-nothing option is the possibility of a Type II assessment error, ie the error of incorrectly assessing the structure to be adequate [1,2], with the possible consequences of structural damage, and, in extreme cases, property damage and even loss of life. In Fig. 2 outcome S(1,1,N) represents satisfactory future behaviour; outcomes S(2,1,N) and S(3,1,N) represent future states of unserviceability and failure, respectively, both being consequences of a Type II Error. Idealisation of the possible outcomes into three lumped categories is an obvious but often acceptable simplification. In many situations the outcome S(J,I,N) will not vary with N. Nevertheless, it is useful to maintain the extended indexing because the estimates of the costs and probabilities of outcomes are likely to change at each step and the indices prevent ambiguity.

2.3 Action A(2,N), Corrective Action

Corrective action can result in a range of possible outcomes, but in Fig 2 these are simplified to just three terminating branches, S(J,2,N), which are taken to be similar in nature to the S(J,1,N). Although structural unserviceability and failure are both possibile consequences of corrective work, the probability of such occurrences are likely to be much lower than when no action is taken. It will be noted that the Type I assessment error (incorrectly assessing the structure to be inadequate) is hidden in the terminating branches, so that the cost of this type of error is included in the cost of these branches.

2.4 Monitoring, E(1,N)

Monitoring and testing are both non-terminating options which must be followed by at least one further step in the decision process. The information gathered during these actions is of prime importance in the subsequent decision step, and will be considered in some detail in the quantitative analysis of the process in Section 3 below.

Monitoring provides information on structural behaviour under in-service conditions which actually occur over the limited monitoring period. The information obtained may not be helpful in evaluating response to overload or load capacity. At the expense of some oversimplification, three possible outcomes of the monitoring option will be considered:

Z(1,1,N): performance observed throughout the monitoring period is judged to be acceptable and gives no indication of defect in the structure;

Z(2,1,N): monitoring does not produce useful information and does not indicate whether the structure is satisfactory or unsatisfactory;

Z(3,1,N): performance of the structure during the monitoring period is not satisfactory and suggests that the structure is defective.

2.5 Testing, E(2,N)

If the concern is with strength rather than service load behaviour, it is possible to choose test loads which represent an overload condition. As in the monitoring option, the results of the testing option will be represented in a simplified manner by three categories:

Z(1,2,N):   response to the load test is satisfactory and gives no
            evidence of structural defect;
Z(2,2,N):   the adequacy of the structure cannot be evaluated from the
            tests conducted;
Z(3,2,N):   the tests suggest that the structure may be defective.

It should be clear that Z(1,2,N) and Z(2,2,N) do not necessarily lead to the
conclusion that the structure is adequate and hence that the do-nothing option
will be chosen in the next step of the decision process. Likewise, Z(3,2,N)
does not necessarily lead to A(2,N).

2.6   The Complete Assessment Process

The typical N-th step is depicted in Fig. 2 with four possible options, ie two
experiments and two actions. The process is brought to a conclusion by either
action, A(I,N); it is continued to a further step by either experiment, E(K,N).
The complete process is achieved by concatenating a sequence of decision steps,
as shown in Fig 3.
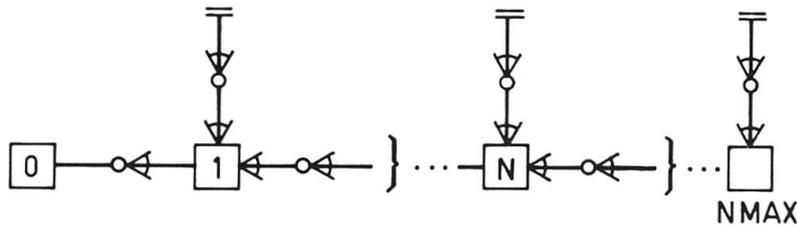
Fig. 3   Complete decision process

When it becomes necessary to trace a chain backwards from an action or an
outcome, the appropriate sequence of indices will be indicated. For example,
S(L,K,J,I,N+1) is the J-th outcome of action A(I,N+1) in step (N+1), following
on from experiment E(K,N) in step N, with consequence Z(L,K,N). The notation
thus indicates the chain E(K,N)-Z(L,K,N)--A(I,N+1). The indices are dropped
when there is no chance of ambiguity or where the particular chain is not of
concern. Fig 4 shows the indices which apply to actions, experiments and
outcomes in any two successive steps N and (N+1).
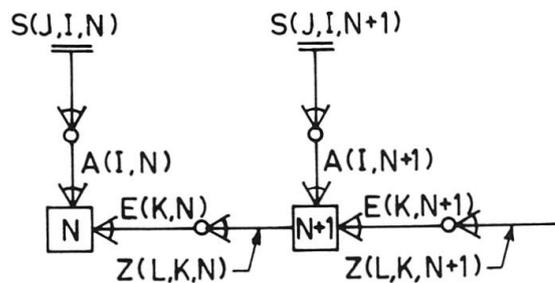
Fig. 4   Notation for two successive steps of the decision process

3. ANALYSIS OF THE DECISION PROCESS

The purpose of the analysis is to identify the best chain of decisions. At
each decision node (represented by a rectangular box in the figures) one of
four possible paths is chosen on the basis of the information available at the
time when the decision is made. The appropriate decision criterion is minimum
expected cost. Procedures exist for transforming dollar costs into units of
utility in such a way that risk aversion and risk acceptance can both be taken
into account in a simple manner [3]. However, to simplify the presentation
here, aversion to and acceptance of risk will be ignored and dollar terms will
be used for all costs.

3.1 Costs of Actions, Experiments and Final Outcomes

The total cost of a potential experiment, of an action and of a final outcome
at the N-th step will be expressed as $C[E(K,N)]$, $C[A(I,N)]$ and $C[S(J,I,N)]$,
respectively. The prime component of $C[A(I,N)]$ is obviously the corrective
work itself and in many cases it may be appropriate to take $C[A(1,N)]$ to be
zero. The costs of the final outcomes $C[S(J,I,N)]$ include the consequences of
serviceability failure or collapse, for $J = 2$ or 3. To simplify the
calculations, all previous costs incurred on the branch leading to $S(J,I,N)$,
such as $C[A(I,N)]$ and $C[E(K,N-1)]$, will be included directly in $C[S(J,I,N)]$.

3.2 Probabilities of Outcomes of Experiments and Actions

At the commencement of the 1st step in the process, the only information
available is that obtained from the result $Z(K,0)$ of the initial investigation.
It is necessary, using this information, to make estimates of the relative
probabilities of the outcomes of the actions, $P[S(J,I,1)]$, and of the
experimental outcomes, $P[Z(L,K,1)]$.

At the Nth step, prior to any final action having been undertaken, additional
information will have been gathered during the intervening experimental steps,
so that improved estimates can be made of the probabilities. The terms
$P[S(J,I,N)]$ and $P[Z(L,K,N)]$ signify the best estimates that can be made in step
N on the basis of available information. Terminology for probabilities in
steps N and N+1 are shown in Fig. 5. The question of how to evaluate these
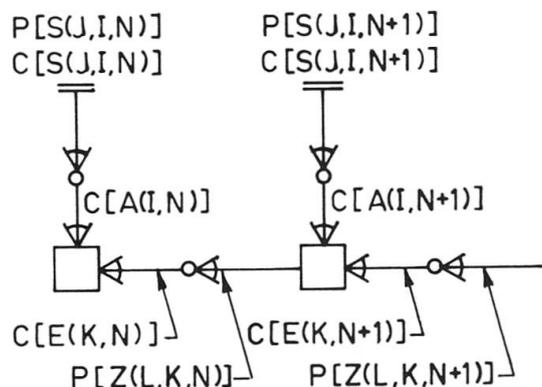probabilities will be considered in Section 3.4.

Fig. 5   Costs and Probabilities for two successive steps of the
          Decision Process.

3.3 Expected Costs

The expected cost of an action A(I,N) is calculated as:

$$EC[A(I,N)] = \sum_{J=1,2,3} P[S(J,I,N)]*C[S(J,I,N)] \qquad (1)$$

The expected cost of an experiment is more difficult to calculate because the process continues for at least one more step. The cost of option E(K,N) is thus made up not only of C[E(K,N)] but also the costs incurred of every future step in the process which will follow on from this experiment. The estimation of the probability P[Z(L,K,N)] also requires consideration of the future possible decision chain.

3.4 Numerical Analysis of the Process

The mathematical problem underlying the decision process is, in effect, a stochastic linear program which may be terminated after any step, but which may also extend indefinitely, ie NMAX is undetermined. The solution of the problem can require considerable numerical calculation. A solution can be obtained in principle by solving a series of problems with NMAX = 2, 3, etc and taking the case which yields minimum cost. Each problem with fixed NMAX can be solved by the process of backwards induction or "averaging out and folding back" [10]. A simplified approach can also be developed in which the complex chains following on from any step N are cut at the outcomes of experiments in step (N + 1) by introducing intuitive estimates of the probabilities and costs of each cut branch and hence estimating their expected costs. With such an approach, an initial analysis can be made for NMAX = 2 with cut branches, and the approximate costs for the cut branch are compared with the costs of action in steps 1 and 2, in order to decide whether it is advisable to proceed to a solution for NMAX = 3, and so on.

In the present discussion, attention will be restricted to a two-step process and illustrated with an example. The case of NMAX = 2 is shown in Fig 6. The expected cost of the terminating actions A(I,1) in step 1 are calculated using Eq 1. These calculations are straight forward, as estimates will have been made of the relevant costs and probabilities.
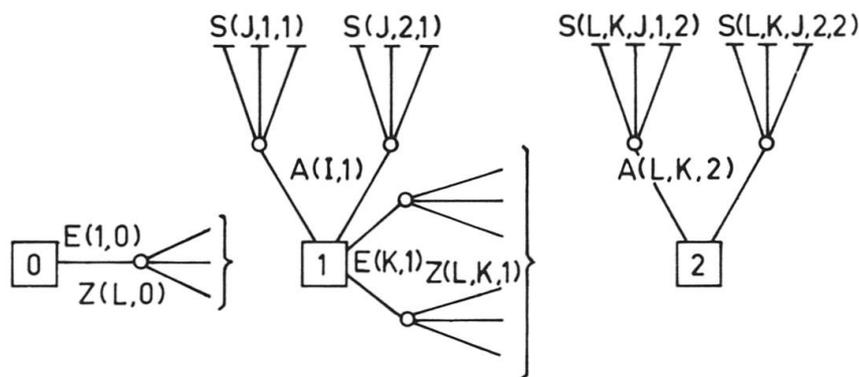

Fig. 6 Two-step process

In calculating the expected cost of an experiment, $EC[E(K,1)]$, all subsequent chains leading from $E(K,1)$ to termination of the process must be followed, ie all values of path L in Fig 6. For K=1 and L=1, say, chains through $A(1,1,1,2)$ and $A(1,1,2,2)$ must be considered and, in each case, their three possible outcomes. The preferred action minimises the expected cost of the end of the chain which is

$$\min_{I=1,2} [EC[A(1,1,I,2)]] \qquad (2)$$

In calculating $EC[A(L,K,I,2)]$ for each chain we have

$$EC[A(L,K,I,2)] = \sum_{J=1,2,3} P[S(L,K,J,I,2)] * C[S(L,K,J,I,2)] \qquad (3)$$

Bayes' Theorem [8,10] can be used to obtain an improved estimate of the required probabilities following an experimental result. The following equations apply:

$$P[S(L,K,J,I,2)] = \frac{1}{A} P[Z(L,K,1) / S(L,K,J,I,2)] * P[S(J,I,1)] \qquad (4)$$

$$A = P[Z(L,K,1)] = \sum_{J=1,2,3} P[Z(L,K,1) / S(L,K,J,I,2)] * P[S(J,I,1)] \qquad (5)$$

The expected cost of experiment $E(K,1)$ is

$$EC[E(K,1)] = \sum_{L} P[Z(L,K,1)] * \min_{I} [EC[A(L,K,I,2)]] \qquad (6)$$

The probability value to be used, $P[Z(L,K,1)]$, is given by Eq 5.

The use of these expressions will be illustrated by an example in the following section.

4. EXAMPLE: BURIED FLEXIBLE CONDUIT

The case of a buried flexible conduit underpass is considered in which large and asymmetric deflections have been discovered within two years of construction during a routine inspection. A study of available documentation has not uncovered any serious errors or inadequacies, and the deflections are attributed to an inadequate design procedure coupled with unequal side pressures resulting from uneven compaction of the fill. With no adequate basis for theoretical analysis and prediction of future behaviour, an assessment has to be made without adequate information to make reliable decisions.

In the present case, testing is not feasible and three available options are considered:

$A(1,1)$: do nothing, at zero cost;
$A(2,1)$: undertake extensive reconstruction; at a cost of \$300,000;
$E(1,1)$: monitor the conduit for a period of six months at a cost of \$15,000.

The possible outcomes of both actions are taken to be as follows:

S(1,I,1): satisfactory structural performance, despite some increase in deflections;

S(2,I,1): continued increase in deflections, with visual inadequacies and concern to the public, but without structural failure;

S(3,I,1): collapse of the underpass.

The possible results of a monitoring program are placed into three categories:

Z(1,1,1): observed deflections remain relatively small, and appear to be cyclic and seasonal in nature;

Z(2,1,1): deflections increase significantly, but tend to level out, ie the second derivative of the deflection with respect to time is negative.

Z(3,1,1): deflections continue to increase, and accelerate, ie the second derivative is positive.

In this discussion, only one step of monitoring will be considered. Fig. 7 therefore shows only actions following from the second decision step.
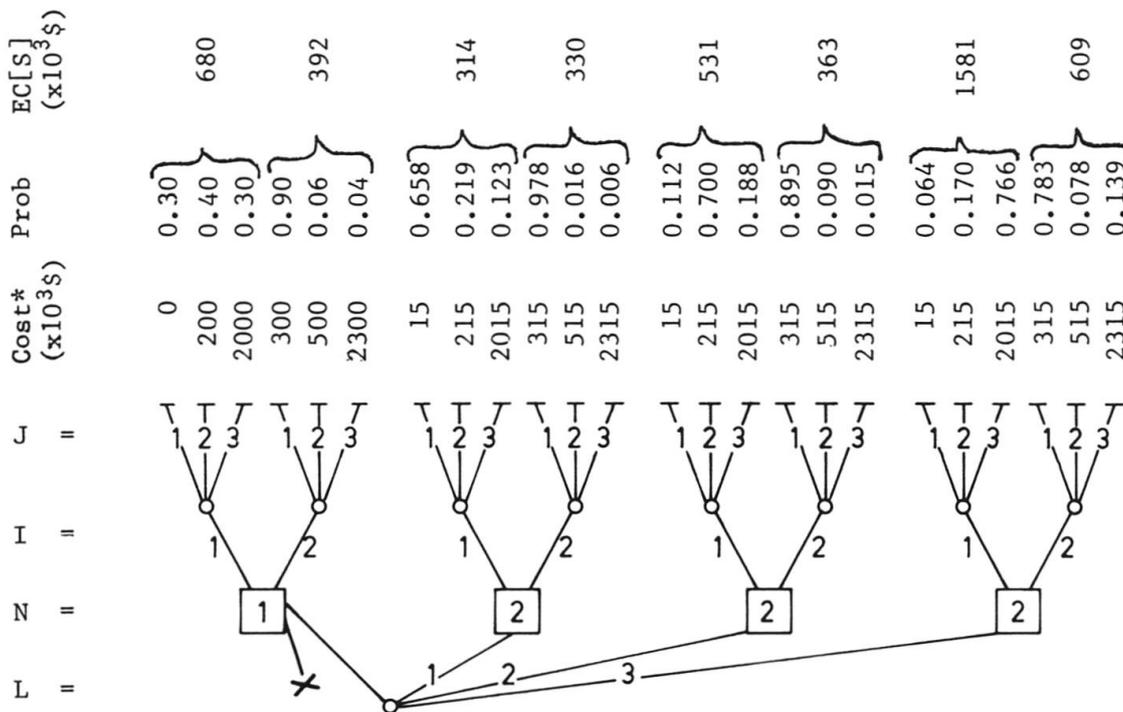
Fig. 7 Structural Assessment of Conduit Underpass

Note: EC[S] = EC[S(L,J,I,2)]; Prob = P[S(L,J,I,2)]; Cost = C[S(L,J,I,2)]

The estimated costs of the various action outcomes in Fig. 7 are expressed in thousands of dollars and include all monitoring and strengthening cost. The cost of excessive deflections in the unstrengthened structure, C[S(2,1,1)], is thus estimated to be $200,000, while the estimated cost of collapse C[S(3,1,1)] is $2,000,000. The complete decision tree in Fig. 7 contains three decision boxes for step 2 to allow for the three outcomes (L=1,2,3) of the monitoring

			(1)	(2)	(3a)
L	J	I	P[Z/S]	P[S(1)]	P[Z/S]*P[S]
1	1	1	0.80	0.30	0.240
2	1	1	0.15	0.30	0.045
3	1	1	0.05	0.30	0.015
1	2	1	0.20	0.40	0.080
2	2	1	0.70	0.40	0.280
3	1	1	0.10	0.40	0.040
1	3	1	0.15	0.30	0.045
2	3	1	0.25	0.30	0.075
3	3	1	0.60	0.30	0.180
1	1	2	0.60	0.90	0.540
2	1	2	0.30	0.90	0.270
3	1	2	0.10	0.90	0.090
1	2	2	0.10	0.06	0.006
2	2	2	0.60	0.06	0.036
3	2	2	0.30	0.06	0.018
1	3	2	0.05	0.04	0.002
2	3	2	0.15	0.04	0.006
3	3	2	0.80	0.04	0.032

			(3b)	(4)	(5)
L	J	I	P[Z/S]*P[S]	A	P[S(2)]
1	1	1	0.240	0.365	0.658
1	2	1	0.080		0.219
1	3	1	0.045		0.123
1	1	2	0.540	0.548	0.985
1	2	2	0.006		0.011
1	2	3	0.002		0.004
2	1	1	0.045	0.400	0.112
2	2	1	0.280		0.700
2	3	1	0.075		0.188
2	1	2	0.270	0.312	0.866
2	2	2	0.036		0.115
2	3	2	0.006		0.019
3	1	1	0.015	0.235	0.064
3	2	1	0.040		0.170
3	3	1	0.180		0.766
3	1	2	0.090	0.140	0.643
3	3	2	0.032		0.128
3	3	2	0.032		0.229

Explanation of column headings:

(1) $P[Z/S] = P[Z(L,1,1)/S(L,J,I,2)]$

(2) $P[S(1)] = P[S(J,I,1)]$

(3a), (3b) $P[Z/S] * P[S] = P[Z(L,1,1,)/S(L,J,I,2)] * P[S(J,I,1)]$

(4) $A = P[Z(L,1,1)] = \sum_{J=1,2,3} P[Z(L,1,1)/S(L,J,I,2)] * P[S(J,I,1)]$

(5) $P[S(2)] = P[S(L,J,I,2)]$

Table 1   Calculations for P[S(L,J,I,2)] using Bayes' Theorem

operation. Initial estimates of the probabilities of the possible outcomes of
the actions for step 1 (N=1;I=1,2) are shown in Fig. 7.

Using the data contained on the branch N=1, I=1 in Fig. 7, we obtain the
expected cost of A(1,1):

$$EC[A(1,1)] = 0.3(0) + 0.4(200) + 0.3(2000) \qquad = 680$$

Likewise, for N=1, I=2:

$$EC[A(2,1)] = 0.9(300) + 0.06(200) + 0.04(2000) = 392$$

To evaluate the cost of the monitoring option, probability estimates are
required for all outcomes, $S(L,J,I,2)$, emanating from $E(1,1)$. In Table 1, the
term $P[Z(L,1,1)/S(L,J,I,2)]$ represents a subjective estimate of the probability
of achieving the result $Z(L,1,1)$ from the monitoring program, if it is known
that the final outcome of subsequent action $A(L,I,2)$ is $S(L,J,I,2)$. Estimates
of these probabilities are given in Table 1 for all possible chains L-I-J.
Other data required for application of Eqs 4 and 5 are $P[S(J,I,1)]$ which are
also entered in Table 1. The values have been estimated from the information
available and previous experience, largely on an intuitive basis. Calculations
to estimate the probabilities $P[S(L,J,I,2)]$ are carried out in Table 1
according to Eqs 4 and 5. The normalising terms, A, listed in column 4 are the
sum, over J, of the items in column 3. As shown in Eq 5, these terms also
represent the probabilities of observing a particular monitoring result,
$P[Z(L,1,1)]$.

For consistency, the estimated value of $P[Z(L,1,1)$ should be independent of the
actions taken following monitoring, ie independent of $A(L,I,2)$. This is not
the case in Table 1. For example, in column 4 values of 0.365 and 0.548 are
given for $P[Z(L,1,1)]$, depending on whether or not the structure will be
strengthened after monitoring (ie for I=1 and 2). This reflects inconsistency
in the underlying probability estimates $P[Z(L,1,1)/S(L,J,I,2)]$ and $P[S(J,I,1)]$
and is not surprising given their intuitive origin. To overcome this
inconsistency, the probability estimates in columns 1 and 2 of Table 1 need to
be adjusted iteratively, with recalculation of $P[Z(L,1,1)]$ using Eqs 4 and 5,
until the set is consistent.

A revised set of probabilities is given in column 1 of Table 2 together with
recalculated values of A in column 4. The consistency between the two
estimates of $P[Z(L,1,1)]$ for I=1 and 2 is good, but not perfect. Due to the
subjective nature of the data, it is difficult to achieve perfect agreement,
and the values in Table 2 are considered adequate for the present analysis.
The average values for $P[Z(L,1,1)]$, obtained from the two estimates in column 4
in Table 2, will be used in the following calculations, ie:

$$P[Z(1,1,1)] = 0.367$$
$$P[Z(2,1,1)] = 0.401$$
$$P[Z(3,1,1)] = 0.232$$

The probabilities and consequences of each outcome, based on these values, are
shown in Fig. 7. The expected cost of each outcome, calculated by substituting
cost and probability values into Eq 1, is also shown in Fig. 7.

For step 2, the minimum expected cost for each chain L=1,2,3 is obtained simply
by comparing expected costs:

			(1)	(2)	(3a)
L	J	I	P[Z/S]	P[S(1)]	P[Z/S] * [S]
1	1	1	0.80	0.30	0.240
2	1	1	0.15	0.30	0.045
3	1	1	0.05	0.30	0.015
1	2	1	0.20	0.40	0.080
2	2	1	0.70	0.40	0.280
3	2	1	0.10	0.40	0.040
1	3	1	0.15	0.30	0.045
2	3	1	0.25	0.30	0.075
3	3	1	0.60	0.30	0.180
1	1	2	0.40	0.90	0.360
2	1	2	0.40	0.90	0.360
3	1	2	0.20	0.90	0.180
1	2	2	0.10	0.06	0.006
2	2	2	0.60	0.06	0.036
3	2	2	0.30	0.06	0.018
1	3	2	0.05	0.04	0.002
2	3	2	0.15	0.04	0.006
3	3	2	0.80	0.04	0.032

			(3b)	(4)	(5)
L	J	I	P[Z/S]*P[S]	A	P[S(2)]
1	1	1	0.240	0.365	0.658
1	2	1	0.080		0.219
1	3	1	0.045		0.123
1	1	2	0.360	0.368	0.978
1	2	2	0.006		0.016
1	3	2	0.002		0.006
2	1	1	0.045	0.400	0.112
2	2	1	0.280		0.700
2	3	1	0.075		0.188
2	1	2	0.360	0.402	0.895
2	2	2	0.036		0.090
2	3	2	0.006		0.015
3	1	1	0.015	0.235	0.064
3	2	1	0.040		0.170
3	3	1	0.180		0.766
3	1	2	0.180	0.230	0.783
3	2	2	0.018		0.078
3	3	2	0.032		0.139

Explanation of column headings:

(1)   P[Z/S] = P[Z(L,1,1)/S(L,J,I,2)]

(2)   P[S(1)] = P[S(J,I,1)]

(3a), (3b)   P[Z/S] * P[S] = P[Z(L,1,1)/S(L,J,I,2)]* P[S(J,I,1)]

(4)   A = P[Z(L,1,1)] = $\sum_{J=1,2,3}$ P[Z(L,1,1)/S(L,J,I,2)]* P[S(J,I,1)]

(5)   P[S(2)] = P[S(L,J,I,2)]

Table 2   Calculations for P[S(L,J,I,2)] using Bayes' Theorem (Revised)

$$L = 1: \quad \min_{I} [\ EC[A(I,2)]\ ] \ = \ \min\ [314,330] \ = \ 314 \ (I=1)$$

$$L = 2: \quad \min_{I} [\ EC[A(I,2)]\ ] \ = \ \min\ [531,363] \ = \ 363 \ (I=2)$$

$$L = 3: \quad \min_{I} [\ EC[A(I,2)]\ ] \ = \ \min\ [1581,609] \ = \ 609 \ (I=2)$$

If monitoring is carried out, reconstruction should be undertaken if the result L = 2 or 3 is obtained, otherwise no action should be initiated.

The expected cost of carrying out the monitoring and subsequent actions can now determined:

$$EC[E(1,1)] \ = \ \sum_{L=1,2,3} P[Z(L,1,1)] * \min_{I} [\ EC[A(I,2)]\ ]$$

$$= \ .367 \text{x} 314 \ + \ .401 \text{x} 363 \ + \ .232 \text{x} 609 \ = \ 402$$

In this example, the expected cost of reconstruction is 392 and this therefore appears to be the best course of action. However, as the expected cost of monitoring (402) is only several per cent higher, a sensitivity study of the results might well be appropriate, before final action is taken.

5. CONCLUDING REMARKS

The multistage decision model of structural assessment provides a systematic and logical framework within which the alternative courses of action, and their possible consequences, can be considered. Furthermore, it provides an evaluative procedure for choosing the most appropriate courses of action from those available.

Although emphasis has been given here to monitoring and testing as means of obtaining additional information to improve the reliability of the assessment decisions, the multistage decision model is general and can be applied to a variety of other processes for the assessment of structural defects.

In order to keep the presentation at a simple level, detailed consideration has been given only to a two-stage process with two-way alternatives for action and experiment, and three-way outcome categories. Such simplifications may well be appropriate in a preliminary analysis of an assessment problem and are achieved by the lumping of logical alternatives. Clearly, more complex categories of options and alternative outcomes will often be used in the detailed analysis. The multi-stage model can of course be used to analyse more complex processes, even though the numerical calculations become more complicated.

Additional work is required to develop more effective analysis techniques. Cutting and terminating need to be developed into a simplified, approximate method of analysis, preferably with upper and lower bound estimates of the basic expected costs and probabilities. A general solution procedure for the variable length decision process is also needed in the form of an efficient computational algorithm.

Nevertheless, the multistage decision model is useful even now in providing a

logical and organised approach to structural assessment. It also gives a useful basis for quantifyng the associated decisions.

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