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## Dynamic Response of Concrete Railway Bridges

Comportement dynamique des ponts-rails en béton armé

Dynamisches Verhalten von Eisenbahnbrücken aus Stahlbeton

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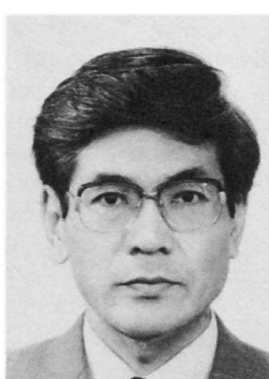
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### SUMMARY

This paper outlines the present status of investigations of dynamic response of concrete railway bridges under running trains. The dynamic response of a railway bridge represents phenomena involving a number of related factors. These factors are individually introduced. Next, systematic investigations of dynamic factor are presented. This paper also touches the other problems related to the dynamic response of bridges under running trains.

### RÉSUMÉ

L'article présente les études actuelles sur le comportement dynamique des ponts-rails en béton armé lors de passages de train. Le comportement dynamique est fonction de plusieurs facteurs qui sont considérés séparément. Les études systématiques de facteurs dynamiques sont présentées. L'article traite d'autres problèmes relatifs au comportement de ponts lors de passages de train.

### ZUSAMMENFASSUNG

Der Aufsatz umreißt die laufenden Studien über das dynamische Verhalten von Eisenbahnbrücken unter rollendem Verkehr. Das dynamische Verhalten von Brücken ist von vielen Faktoren beeinflusst. Diese werden einzeln berücksichtigt und eine systematische Untersuchung des Stosszuschlages wird dargestellt. Der Beitrag befasst sich ebenfalls mit weiteren Problemen, die mit dem dynamischen Verhalten von Eisenbahnbrücken zusammenhängen.



## 1. INTRODUCTION

It is qualitatively predictable that vibrations in a bridge will be greater as the speed of vehicles running over the bridge increases. The extent of the effect of these vibrations on the strength of the bridge is evaluated in terms of a dynamic factor, which in practical design is usually treated as an impact factor. An increase of bridge vibrations and deflections calls for greater attention to the running safety and riding comfort of the vehicles running over the bridge.

This paper outlines the present status of investigations of vibration of concrete railway bridges under running vehicles. Since there is much in common between steel and concrete bridges, some topics on steel bridges may be discussed for the convenience of explanation.

## 2. BASIC STUDIES

Investigations of the dynamic response of railway bridges to running vehicles date back to many years ago. Early studies mention the analyses of the inertia effect of a load moving on a beam illustrated in Fig.1 (A), or the forcing effect of a concentrated load moving on a beam illustrated in Fig.1 (B), where the beam mass was neglected in the former and the vehicle mass was neglected in the latter([1]).

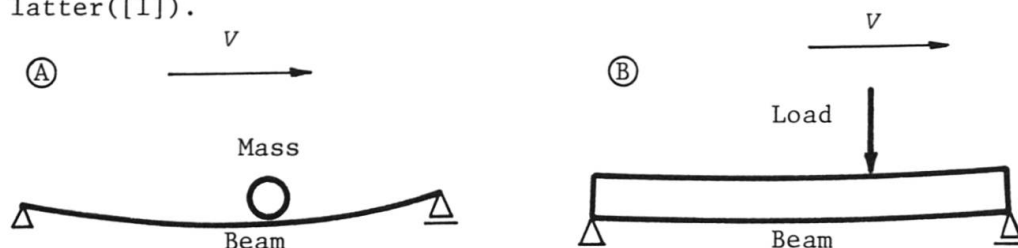


Fig. 1 Simple dynamic models for calculation of dynamic response of bridge in early studies

A detailed study has been published on the hammer-blow effect of the eccentric driving wheels of a steam locomotive as well as the mass of the beam and the vehicle([2]). This is an analysis of the phenomena using differential equations and the problems are so complicated for the technology in these days that their solutions are limited to only specific conditions. Actual measurements of dynamic response of bridges are equally old, and a number of reports were published([3],[4]). Modern advances in electronic computers have enabled rapid analysis of so far unsolvable complex problems([5],[6]).

As the result of such practical tests and theoretical studies it is now accepted that the dynamic response of railway bridge to running vehicles is affected by various factors. These factors are as follows([7],[8]);

- 1) Sudden deflection of the bridge under oncoming vehicles.
- 2) Periodic forces caused by vehicles.
- 3) Irregularities of tracks or wheels.

These items may change depending on the aims of the study, but the following explanations will generally cover effects of all these factors.

First, when a vehicle just come in a bridge, the deflection of the bridge is short of reaching static balance and causes an instantaneous unbalance, resulting in vibration of the beam. The vibration, however, not only depends on the beam inertia, but also on the vehicle mass on the bridge. The vibration is compounded by the effect of successive loading of vehicles on the bridge. The effect of coupling vibrations of beam and vehicle and that of wheelbase are covered under the item 1.

Second, the effect of periodic forces generated by vehicles(item 2) arises from responses of the bridge to the unbalanced weights of the driving wheels of a steam locomotive or to the periodic forces caused by engines such as those in a diesel locomotive.

Third, the effects of irregularities in tracks or wheels(item 3) are replaced by those of vehicle vibration or wheel load variation. This item also includes the vibrations of vehicles just before coming on the bridge and periodic forces generated by vehicles passing over deflections of the rail between sleepers, because all of them are attributable to track irregularities.

The magnitude of the dynamic response of a bridge to each of these factors is determined by combinations of conditions such as bridge span, vehicle speed, vibrational characteristics of the bridge and vehicle and the state of maintenance of the track and vehicle. Accordingly, the extent of the effects of each factor will differ depending on these conditions.

In the following sections, the phenomena will be discussed more elaborately and the present status of studies on these factors will be written. Because of the nature of the problem, discussions will be confined to the fundamentals, presuming that the bridge beam is one with a constant sectional area which obeys the Bernoulli-Euler laws.

## 2.1 Dynamic Effect of a Bridge Suddenly Deflecting under Oncoming Vehicles

### 2.1.1 Fundamental Characteristics

Fig.2 illustrates dynamic models and some parametric results of theoretical analysis of the passing of load on a simple supported beam. It is rearranged to make it easier to understand the fundamental characteristic of the dynamic response of the bridge.

In Fig.2 (A), the vehicles are simplified as one concentrated load. The equation of motion for the dynamic response of a bridge to a moving concentrated load is ordinarily obtained by the deflection mode method. The analysis usually starts from the assumption that the beam deflection is the product of the deflection profile, i.e., a function of only the bridge axis coordinates by an unknown time function, and from this assumption is derived an equation for equilibrium of the dynamic forces. Thus in the most simplified approximation the deflection profile of the beam is equated to a half sine wave as expressed by Eq.(1) and from the condition of dynamic equilibrium is derived Eq.(2).

$$y = Y \sin \frac{\pi x}{L} \quad (1)$$

$$\frac{d^2 Y}{dt^2} + 2\zeta\omega \frac{dY}{dt} + \omega^2 Y = \frac{P}{M_b} \varepsilon \sin \frac{\pi vt}{L} \quad (2)$$

Where,

- $t$  : Time counted from the moment when the moving load come on the bridge.
- $x$  : Coordinate in right direction of girder originated from left side support of girder.
- $y$  : Deflection of girder, the function of  $x$  and  $t$ .
- $Y$  : Deflection function depend only on time.
- $L$  : Span of girder.
- $P$  : Moving concentrated load.
- $v$  : Velocity of the moving concentrated load.
- $\varepsilon$  :  $\varepsilon=1$  when  $0 < x_p < L$ ,  $\varepsilon=0$  others.  $x_p = vt$ .
- $\omega$  : Fundamental circular frequency of girder.
- $\zeta$  : Damping factor of girder.
- $M_b$  : Mass of girder.



Solution to Eq.(2), when  $\zeta$  is ignored as negligible small, will become Eq.(3) and the maximum value of Eq.(3) will be expressed by Eq.(4). Finally the dynamic factor will be expressed by Eq.(5).

$$y = \frac{y_s}{1 - K_O^2} (1 - K_O \sin 2\pi f_O t) \quad (3)$$

$$y_{max} = \frac{y_s}{1 - K_O} \quad (4)$$

$$\phi = \frac{y_{max} - y_s}{y_s} = \frac{K_O}{1 - K_O} \quad (5)$$

Where,

$\phi$  : Dynamic factor of girder.

$y_s$ : Static deflection of girder at mid-span.

$f_O$ : Natural frequency of girder,  $f_O = \frac{\omega}{2\pi}$ .

$K_O$ : Speed parameter. This is defined by the next formula.

$$K_O = \frac{V}{2f_O L} \quad (6)$$

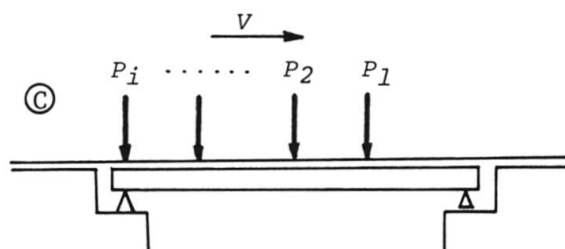
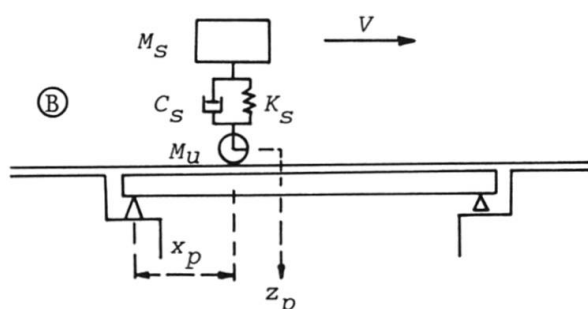
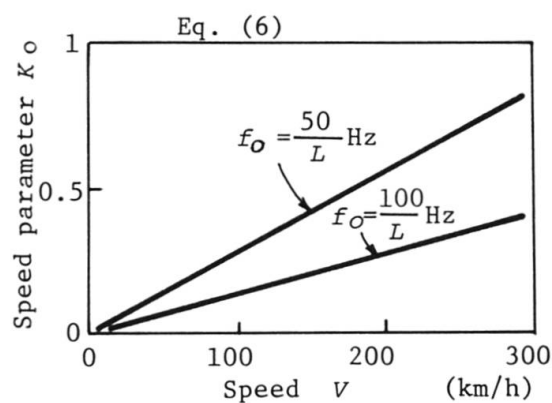
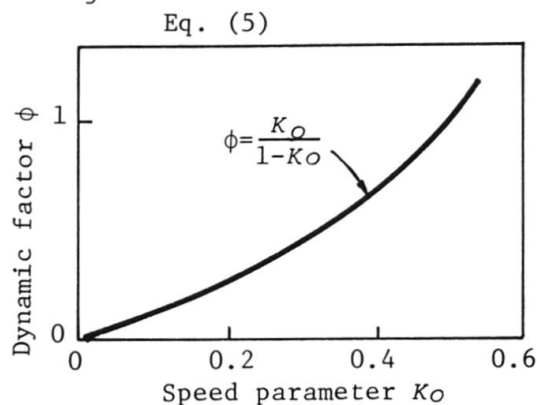
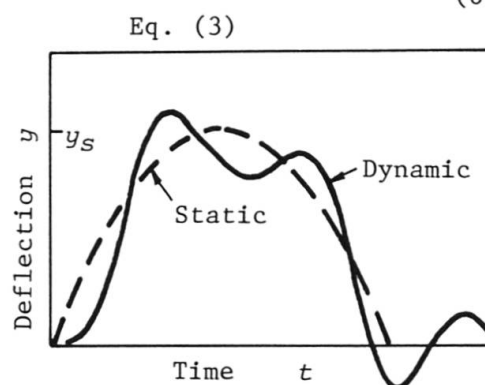
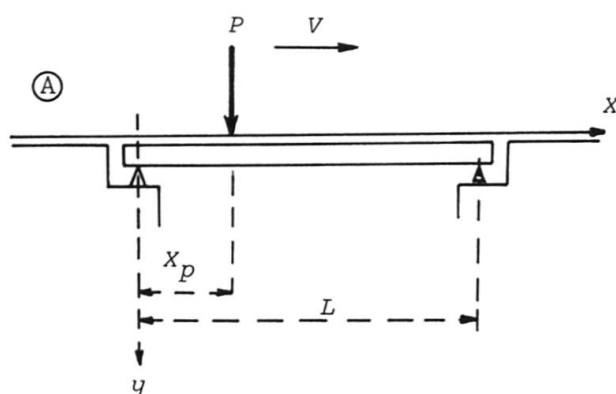


Fig. 2 Dynamic models and some parametric results of theoretical analysis

Taking a typical case of one concentrated load passing over a simple supported beam, where only the fundamental frequency  $f_0$  of the beam is taken into account disregarding the damping of the beam. The dynamic factor  $\phi$  can be expressed using Eq.(5) in terms of the speed parameter  $K_0$  alone in Eq.(6). These are graphically shown in Fig.2.

Fig.2 (B) shows a running vehicle and considers the car body mass, the wheel mass and the supporting structure such as the springs. In this case, an equation for vehicle response to the vertical displacement of the track surface under a deflection of the beam will be derived and analyzed with the dynamic deflection of the beam remaining as an unknown function. A solution will be obtained by an integration of Eq.(2) coupled with Eq.(7). Eq.(7) is for vehicle movement and it is related to Eq.(2) for beam motion through Eq.(8).

$$M_S \frac{d^2 z}{dt^2} + C_S \left( \frac{dz}{dt} - \frac{dz_p}{dt} \right) + K_S (z - z_p) = 0 \quad (7)$$

$$P = M_V g + M_S \frac{d^2 z}{dt^2} + M_U \frac{d^2 z_p}{dt^2} \quad (8)$$

Where,

$z$  : Vertical displacement of sprung mass.

$z_p$  : Vertical displacement of wheel (When there is no irregularity in track,

$z_p = y_p = Y \sin \frac{\pi x_p}{L}$ ,  $x_p$  is the position of load).

$M_S$  : Sprung mass of vehicle.

$M_U$  : Unsprung mass of vehicle ( $M_V = M_S + M_U$ ).

$C_S$  : Damping factor of vehicle spring.

$K_S$  : Spring constant of vehicle spring.

$g$  : Acceleration of gravity.

Thus, in the vehicle model illustrated in Fig.2 (B) the beam response may be analyzed using the three equations Eqs.(2),(7) and (8). The equation for beam motion will be rather complicated a differential equation involving a time-variant parameter, but at present a direct application of numerical integration by computer makes the analysis very easy and even a solution to these equations involving non-linear terms or an analysis under special conditions of a sudden off-loading as the result of the wheels separating from the rails will not be so difficult. The three equations for beam response to a running vehicle considering supporting structure is generally more simplified through the introduction of the natural frequency of the beam under loaded conditions.

The natural frequency of the beam under loaded conditions will be obtained from the assumption that the vehicles are standing still on the beams and the supporting springs are infinitely rigid. In the case of a vehicle as illustrated in Fig.2 (B), the natural frequency of the loaded beam is expressed by Eq.(9) supposing that the vehicle stops at mid-span.

Fig.2 (C) shows the case of concentrated loads passing over the beams in succession, which is close to the real condition of loading with a train. The equation for beam motion in this case will be obtained by merely replacing the right side of Eq.(2) with expression (10).

$$f = f_0 \sqrt{\frac{M_b}{M_b + 2M_V}} \quad (9)$$

$$\sum_{i=1}^n \frac{P_i}{M_b} \varepsilon_i \sin \frac{\pi v t_i}{L} \quad (10)$$



Where,

$f$  : Natural frequency of girder under loaded condition.

$i$  : Number of moving concentrated load.

$n$  : Total number of moving concentrated load.

$P_i, \epsilon_i, t_i$  :  $P, \epsilon, t$ , defined each other in relation to  $i$ .

Even in the case of several sprung vehicles running in succession as illustrated in Fig.2 (B) an equation of motion considering the beam-vehicle interaction may be derived by formulating Eqs.(7) and (8) for each vehicle and combining them with Eq.(2). Similar analysis will be possible for bridge beams with the more generalized conditions and for vehicles of different types. Referring to these fundamental characteristics, past studies will be reviewed concerning various factors.

### 2.1.2 Effect of Wheelbase

At low speeds, the effect of wheel base is generally not as large as those of other factors, but it can be large for high speed trains consisting of a number of similar types of vehicles. Fig.3 shows the results of analysis in [9] on the dynamic factor of a 10m span reinforced concrete slab beam by the running of a detailed model strictly simulating the vehicle with the dynamic characteristics and loads passing over the beam without and with application of the natural frequency of beam under loaded conditions.

Differences between the three cases are minor and the most simplified analysis (Unloaded model) of a series of moving loads is found to yield sufficient accuracy. A very large value for dynamic response appears near 300 km/h. Nearly the same tendency is recognized in the results of measurement. For steel bridges where the mass ratio between vehicle and beam is greater than concrete bridges, high accuracy cannot be usually expected from the analysis of a series of moving loads using the unloaded natural frequency of beam.

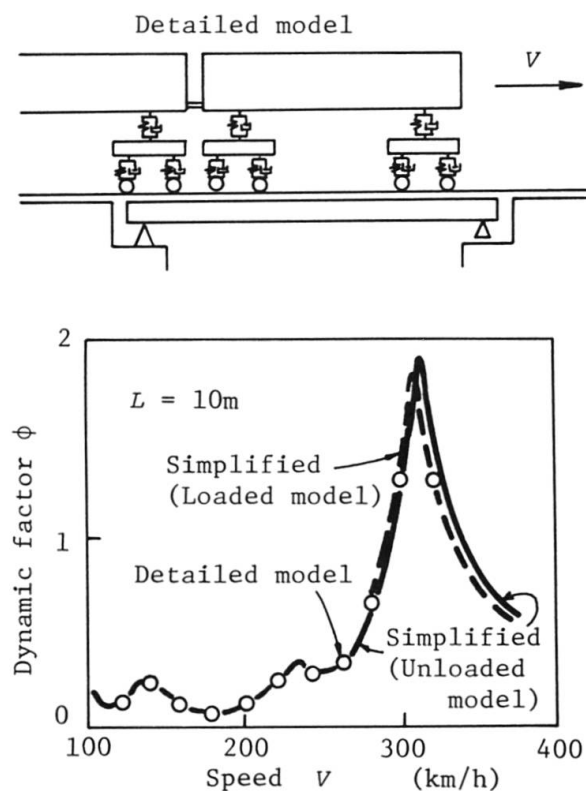


Fig. 3 Effect of wheel base of vehicles and loaded frequency of girder

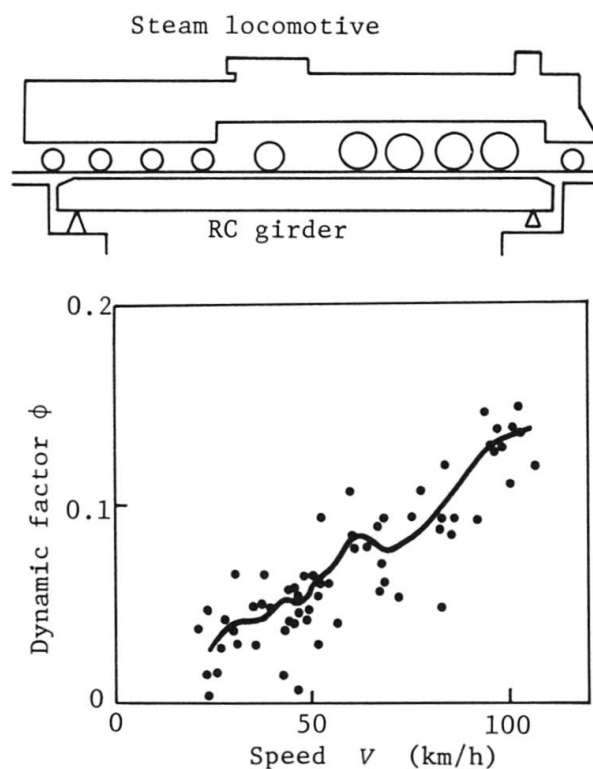


Fig. 4 Measured dynamic factor of concrete bridge to steam locomotive



In [9] the condition for the dynamic factor to have a maximum is as indicated in Fig.3, in the agreement of a prominent frequency component contained in the load term Eq.(10) with the natural frequency of the beam. In this case, under regular loading, a frequency component with a period equal to 1/3 of the time in which one vehicle moves becomes prominent in Exp.(10) and 300 km/h is the speed at which this frequency component agrees with the natural frequency of the beam. This condition has been still more generalized by a resonance in a series of moving loads that occurs when Eq.(11) holds.

$$K = \text{Car body length} / (2 \times \text{Span} \times \text{Integer}) \quad (11)$$

## 2.2 Effect of Periodic Forces Generated by Vehicles

Periodic forces generated by vehicles are shown typical in the hammer-blow of a steam locomotive. When the period of the hammer-blow accompanying the wheel rotation matches the natural frequency of the beam, the dynamic response of the beam will reach a maximum.

Fig.4 shows a practical example of the dynamic response of a reinforced concrete bridge to a running of a steam locomotive ([10]). The peak value of the dynamic response to the hammer-blow appears near 60 km/h, but it is smaller than the dynamic factor recorded at higher speeds.

Past studies of the hammer-blow effect of a steam locomotive dealt mostly with iron or steel beams and it seems that there is no recent study using a detailed model. In principle, however, numerical analysis would be possible when a term for hammer-blow is added to the right side of Eq.(8) and this is coupled with Eq.(2) or Eq.(7).

In one study ([11]), a parametric analysis of the effect of periodic forces has been accomplished using real data on a diesel railcar. This study is also included in the following discussions on the effects of track irregularities. The analysis was made of the cases of one axle ( $i=1$ ) and two axles ( $i=2$ ) taking Eq.(12) for the load expressed by Eq.(10) and the results shown in Fig.5 were obtained.

$$P_i = P_{Si} (1 + 0.15 \cos 2\pi\beta f t) \quad (12)$$

Where,

$P_{Si}$ : Static load of  $i$ th load.

$\beta$ : Ratio of frequency of periodic force to the fundamental frequency of girder in loaded condition.

$K$ : Speed parameter,

$$K = \frac{V}{2fL} \quad (13)$$

As seen from Fig.5, when  $\beta=1$ , that is, when the frequency of the periodic forces and the frequency of the beam are the same, a large value of the dynamic factor  $\phi$  appears even  $K$  is small, but little difference in the dynamic factor depending on the frequency of periodic forces is recognized where  $K$  is large. Moreover, in the case of two loads ( $i=2$ ), a dynamic factor larger than in the case of one load ( $i=1$ ) illustrated here does not appear where  $K$  is large.

## 2.3 Effect of Track or Wheel Irregularities

Track irregularities include longitudinal level difference, misalignment and cross-level difference. Longitudinal level difference causes a vertical motion and pitching of the running vehicle. Misalignment causes a lateral motion and yawing of the vehicle. And the cross-level difference causes rolling of the vehicle. These irregularities also affect bridge vibrations. On bridges where the vibrations in vertical direction to the track are predominant, attention has to be paid to only the level differences.



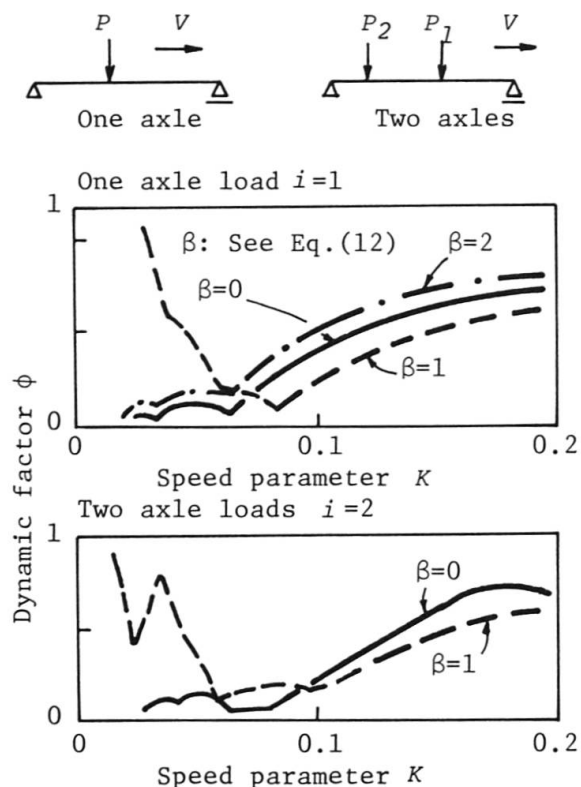


Fig. 5 Parametric analysis of the effect of periodic forces

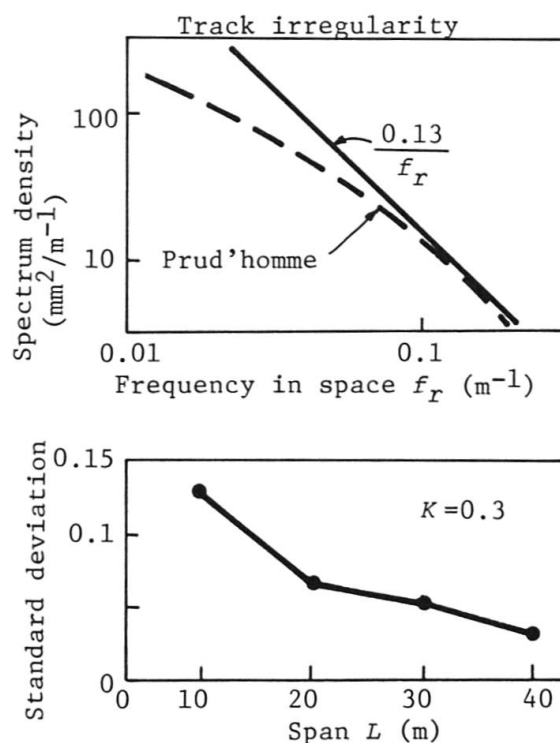


Fig. 6 Direct simulation by using spectrum density of track irregularity

With the profile of the level difference in track as  $w(x)$ , the running surface of wheel of vehicle movement can be expressed by Eq.(14). Thus, when Eq.(14) is used instead of  $z_p$  in Eqs.(7) and (8) the dynamic response of a beam with track irregularities can be analyzed.

$$z_p = Y \sin \frac{\pi x_p}{L} + w(x) \quad (14)$$

Level difference of ordinary track is indeterminate and this is also true on bridges. Naturally, the dynamic response of a beam when a vehicle moves on a bridge with track irregularity turns out to be highly complicated.

A detailed study on the dynamic response of a highway bridge with irregular surfaces has been made using the theory of non-stationary random vibrations and the procedure adopted might be applied to investigations of railway bridges. In this case, however, the results would not be directly available as those of railway bridges on account of differences in characteristic values, etc. As for the railway bridges, there is an analysis directly simulating a railway bridge ([9]), wherein an appropriate number of track irregularity profiles are sampled from the power spectrum density of track irregularities as shown in Fig.6; using Eq.(14), the dynamic factors of beams and their distribution are found; and from them it was concluded that the standard deviation of variance in the dynamic factor is large on a short span and small on a long span.

On the other hand, there is a study which examined the effect of track irregularities without using a probabilistic method ([12]). In this study, the level difference in track is replaced by a sine wave and accordingly the analytic method employed is similar to the one adopted in studying the dynamic effect of periodic forces generated by vehicles in 2.2.

Next, a study taking into account the characteristics of the track and the bridge will be cited that dealt with the track irregularities of elastic track and wheel flats ([13]). Fig.7 illustrates the model for analysis used in this study and the results obtained.

The features of this study are that the track elasticity is assumed linear with provision of springs and the non-contact condition between the wheel and rail is also considered. The equation of motion for this model, being somewhat complicated, is omitted here, but it is characterized by taking an equation of the vertical motion for sprung mass of the vehicle, one of rotational motion for sprung mass of the vehicle, one of vertical motion for front wheels and one of vertical motion for rear wheels instead of Eq.(7) and taking Eqs.(15) and (16) for the wheel load which depends on the track elasticity instead of Eq.(8). Wheel flat is analyzed by substituting an equivalent track irregularity for it.

$$P_i = K_i u_i \quad (15)$$

$$z_{pi} = Y \sin \frac{\pi x p_i}{L} + w(x) + u_i \quad (16)$$

Where,

$K_i$ : Spring constant of track at the position of  $i$ th load.

$u_i$ : Deflection of spring of track at the position of  $i$ th load.

In [13], by changing the parameters in a vehicle-beam model considering the track elasticity, the effects on the dynamic response of the beam have been investigated with the following findings.

As indicated in Fig.7, the effect of wheel flats on the dynamic response of a beam is large at a low  $K$  value; the stiffer the track elasticity and the heavier the track irregularities, the larger the dynamic response; the effect of an increase in the sprung mass is larger than that of an increase in the unsprung mass; and the effect of irregularities on the bridge is greater than that of the initial vibration of an oncoming vehicle.

#### 2.4 Other Problems

In these above discussions, the subject has been narrowed down to the very fundamentals to avoid complexity, but the following problems remain in the dynamic response of bridge beams to a moving load:

- 1) Effect of a higher order frequency of the beam.
- 2) Non-linear behavior of the vehicle and the beam.
- 3) Dynamic response of statically indeterminate structures such as continuous beam and rigid frame.
- 4) Dynamic response of members in truss bridges, etc.
- 5) Dynamic response of curved bridges.
- 6) Dynamic response of flat slab bridges and skew bridges.
- 7) Dynamic response of a beam of variable section.

Some studies have dealt with these problems in detail, but these results are not always incorporated in the dynamic design of railway bridges.

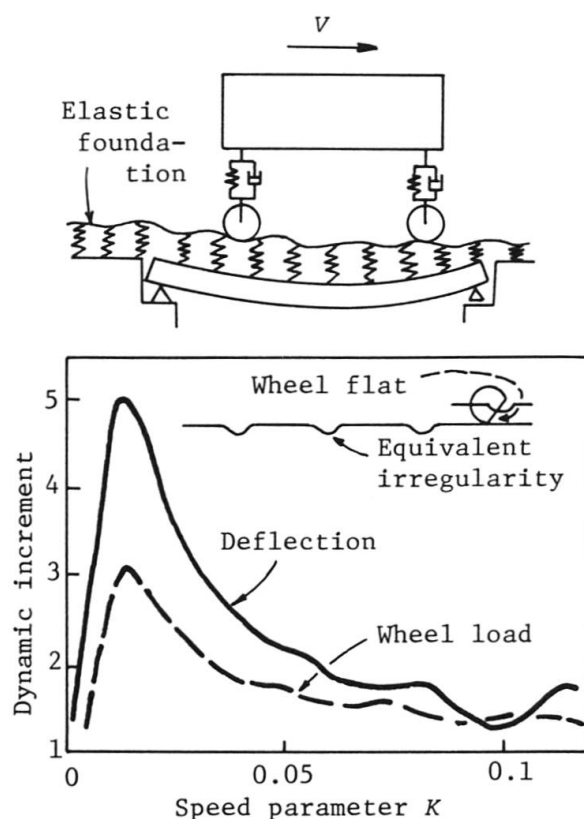


Fig. 7 Simulation taking account of irregularity of elastic track and wheel flat

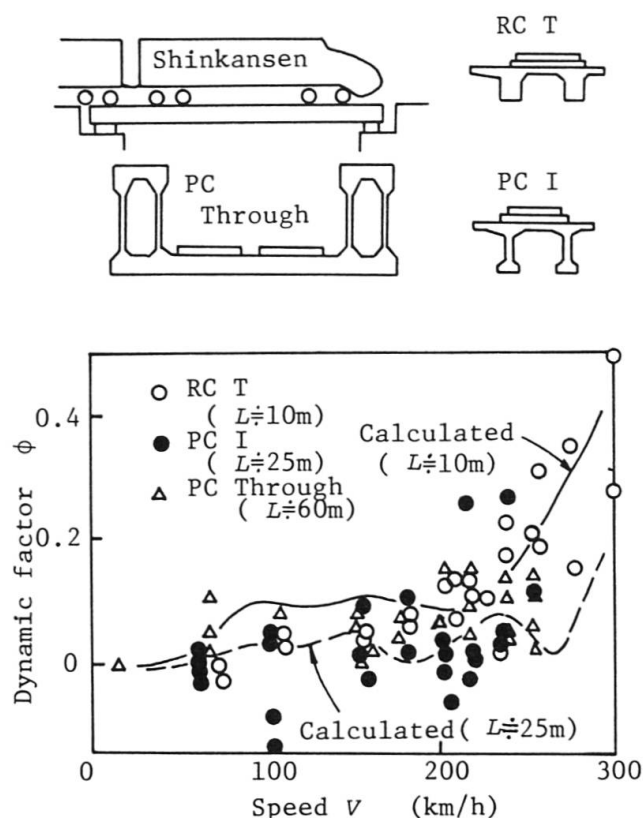


Fig. 8 Examples of practical measurements of high speed run

### 3. SYSTEMATIC INVESTIGATION

The dynamic response of a railway bridge represents phenomena involving a number of related factors. These factors have been individually investigated and yield results which are universally acceptable to a certain extent. Nevertheless much remains unknown about the contribution of individual factors to the whole system. In the present design practice of railway bridges a reasonable impact factor will not be determined without systematic investigation using abundant practical data that contains various factors which affect the dynamic response of bridges.

From this standpoint, systematic investigations based on practical measured data, in addition to studies on effect of the respective factors are considered important. Here, past studies, the latest results of testing on high speed trains and new concepts about design are summarized.

#### 3.1 Systematic Studies Undertaken by the JNR

##### 3.1.1 Basic Investigation

In Japan, many of the studies have been on steel bridges. In these studies ([7]) the dynamic factor of a bridge Eq.(17) under train speeds of 100 km/h and spans shorter than 30 m was proposed. The respective terms in Eq.(17) correspond to respective factors classified in the preceding sections.

$$\phi = C_a K + \phi_1 + \phi_2 \quad (17)$$

Where,

$C_a$ : Constant value fixed by vehicle types or girder types.

Generally  $C_a = 1 \sim 4$ ,  $K < 0.15$ .

$\phi_1$ : Dynamic factor affected by the hammer-blow of steam locomotive.

This is the ratio of static wheel load to maximum periodical load considered in design of locomotive.

$\phi_2$ : Dynamic factor affected by the track irregularity or by the vehicle vibration. This was found that this value is about  $\phi_2 = 0.1 \sim 0.2$ , being interpreted the initial vibration of vehicle coming on bridge is the most affecting factor.

Subsequently practical measurements and studies of the dynamic response of concrete bridges to train running at more than 200 km/h have been carried out. High speed trains operated at 200 km/h, in Japan, are composed of electric railcars of the same type, each car has relatively positive dynamic characteristics. JNR has formulated an equation for dynamic interaction between the bridge and vehicle using a dynamic model of the Shinkansen vehicle as illustrated in Fig.3. The dynamic response of a bridge to the real characteristic conditions has been investigated and from the results the following conclusions have been drawn about concrete bridges ([9]):

- 1) On concrete bridges, the dynamic interaction between the vehicle and bridge is not influenced by the vehicle components and the difference in natural frequency of beam between the loaded and unloaded conditions is negligible.
- 2) The dynamic factor of a bridge tends to increase with an increase in speed, i.e., an increase in the speed parameter  $K$ . At a specific speed where the loading frequency of the series of wheel loads and the natural frequency of the bridge are correlated, a resonance occurs and the dynamic factor has a maximum value. It should be noted that the value of the dynamic factor may be extremely large when  $K > 1/3$ .
- 3) The influence of the track level difference on the dynamic factor is greater on a short span than on a long span.

These conclusions come from analysis of the models shown in Fig.6 as well as Fig.3, the conditions given in Eq.(11) correspond to resonance conditions mentioned under 2.1.2.

### 3.1.2 Examples of Practical Measurements and a Proposal for a Dynamic Factor

On Shinkansen line, test runs of trains were recently conducted over more than ten bridges involving RC T type girders ( $L \approx 10$  m), RC hollow girders ( $L \approx 15$  m), PC I type girders ( $L \approx 25$  m), PC box girders ( $L \approx 50$  m) and PC through girders ( $L \approx 60$  m) ([14]). The train consisted of 6 prototype vehicles and data were collected on runs at 300 km/h maximum.

The measured natural frequency of the beam was read from the residual stress curve after the passage of a train. This can be expressed in relation to the bridge span as Eq.(18). Fig.8 shows examples of the results of measurements, which can be covered by Eq.(19). This equation holds for  $K < 1/3$ .

$$f = 70 L^{-0.78} \quad (\text{Hz}) \quad (L : \text{m}) \quad (18)$$

$$\phi = K + \left( 0.12 - \frac{L}{1400} \right) \quad \left( K < \frac{1}{3} \right) \quad (19)$$

The JNR tentatively proposes Eq.(20) for the design impact factor of concrete bridges based on the results of these investigations ([14]). The first term in Eq.(20) is derived from Eq.(18), and the second term in Eq.(20) represents the effect of an indefinite factor due to track irregularities.

$$\phi = \frac{V}{504 L^{0.22}} + 0.12 - \frac{L}{1400} \quad (L : \text{m}, V : \text{km/h}) \quad (20)$$



### 3.2 Investigations in Europe

The Office for Research and Experiment (ORE) of the International Union of Railways (UIC) has made a review of the design impact formulas adopted in different countries; measured numerous bridges; analyzed results; and proposed design formulas ([18]). These activities are outlined with future problems in the following.

#### 3.2.1 Field Test and Model Experiment

ORE has measured a number of bridges including steel bridges, concrete bridges; bridges with ballasted track, bridges with non-ballasted track; and single-track and double-track bridges. Among the concrete bridges, seven were reinforced concrete and steel-embedded slab girders ( $L=4.1\sim 17.2$  m) and six were prestressed concrete ( $L=7.6\sim 37$  m).

To fully investigating dynamic behavior, ORE prepared a 1/8 scale model of a prestressed concrete bridge and conducted test runs of a model vehicle on it. In the model testing, a detailed parametric study was made using the widely variable characteristics of the running load and bridge girder mass, while for direct comparison with measured test runs of a model locomotive simulating the real vehicle were performed.

#### 3.2.2 Analysis of Measured Results and Agreement with Theoretical Results

In the early stage of sorting out the collected data in investigations, the reinforced concrete girders and the prestressed concrete girders were treated separately, but the concept changed in favor of unifying the two because the concrete girders had fine surface cracks and the elasticity of concrete was hard to estimate, the dynamic factor was directly treated without examining the envelope in terms of maximum stress. ORE has found that the mean value of the dynamic factor and standard deviation of variance can be expressed by Eqs.(21) and (22).

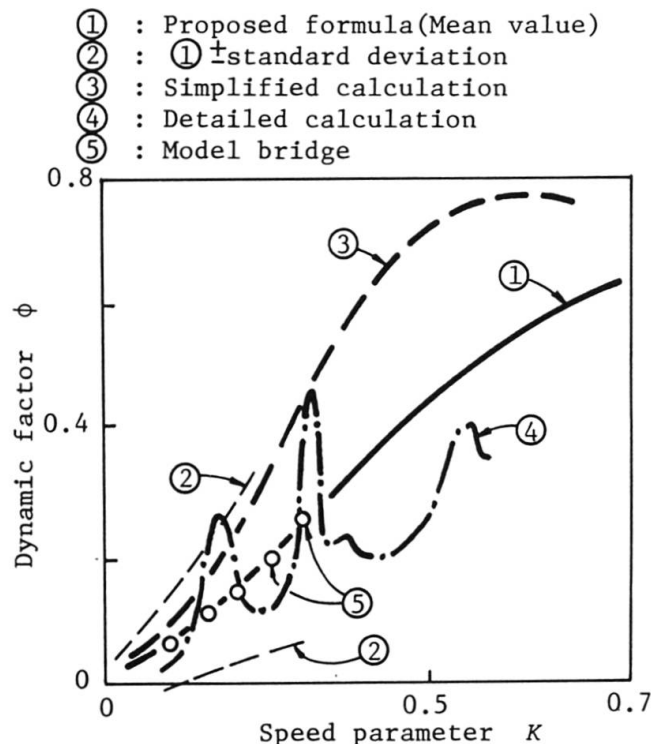


Fig. 9 Results of ORE's research

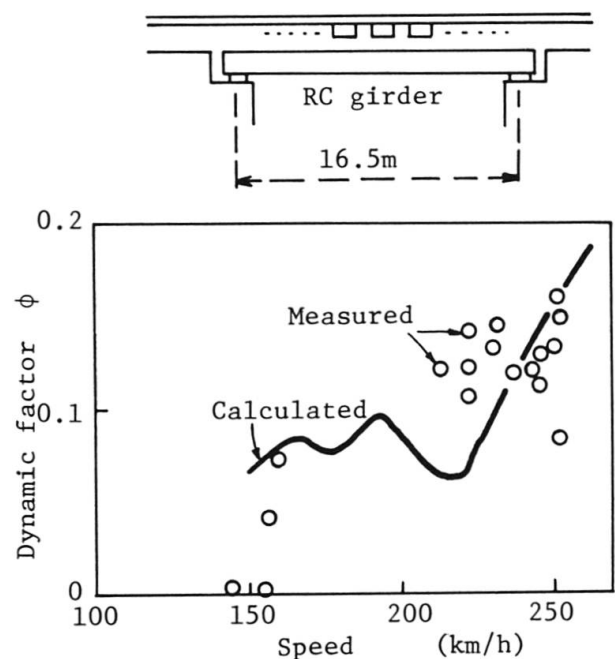


Fig. 10 Example of practical measurements of high speed run

$$\phi = 0.533 \frac{K}{1 - K} - 0.0009 \quad (K < 0.2) \quad (21)$$

$$s = 0.014 \left( 1 + 21 \frac{K}{1 - K} \right) \quad (\text{Standard deviation}) \quad (22)$$

ORE analyzed the results of these measurement and model tests in detail using an analytical model with the following findings:

Maxima obtained from analysis of a detailed model in the case of 0.25 mm deep depressions on rail of an elastic track as indicated in Fig.7 turned out to have about the same variance as in the measured data. These maxima occurred at a relatively small value of  $K$  and at high speeds they were less affected by track irregularities. The results of model tests came relatively close to the mean value of the data (Fig.9).

### 3.2.3 Proposal of Design Formula

The values adjusted by Eq.(21) may hold for the bending stress or deflection in the main girder of a steel bridge. In the case of a concrete bridge, where  $K$  is larger than 0.2, this equation will not hold. Therefore a design formula is required that has a large value of  $K$ , so it can hold up in high speed operation.

In view of the theoretical dynamic factor of one axle run being 0.77 maximum and for a detailed model of the concrete slab beam being 0.55 maximum, ORE proposed the following formula Eq.(23) so that  $\phi$  could have a maximum at  $K=0.7$ , the standard deviation of variance being expressed by Eq.(24).

$$\phi = 0.65 \frac{K}{1 - K + K^2} \quad (23)$$

$$s = 0.025 \left( 1 + 18 \frac{K}{1 - K + K^2} \right) \quad (24)$$

Conclusions of the ORE's research may be summarized as follows;

- 1) The dynamic factor depends on the basic parameter  $K$ .
- 2) The track condition on the bridge has a great influence on the dynamic response.
- 3) Between steel bridges and concrete bridges no difference is recognized in the mean value of the  $K$ -adjusted dynamic factor in the main girder. Variance in the measured value, however, is wider on steel bridges than on concrete bridges.

As the result of these investigations ORE has proposed Eq.(25) as the design formula. Eq.(25) is a safe-side transformation of Eqs.(23) and (24) into the design formula considering the real characteristic of bridge, wherein  $\phi_1$  and  $\phi_2$  have been adjusted in the form of Eqs.(26) and (27).

$$\phi = \phi_1 + 0.5 \phi_2 \quad (25)$$

$$\phi_1 = \frac{K}{1 - K + K^2} \quad (26)$$

$$\phi_2 = \frac{a}{100} \left[ 56 e^{-0.01L^2} + 50 \left( \frac{f}{80} - 1 \right) e^{-0.0025L^2} \right] \quad (27)$$

Where,

$$e = 2.728 \dots$$

$$a : a = 1.0, \quad v \geq 22 \text{ m/s},$$

$$a = \frac{v}{22}, \quad v < 22 \text{ m/s}.$$





### 3.2.4 Measurements of High Speed Runs and Influence of Track Irregularities

The proposed formula Eq.(23) has been arranged for high speed runs. To verify this formula in practical measurements, ORE performed test runs at up to 250 km/h and analyzed the results ([15],[16]). Test bridges were comprised on steel-concrete composite girder ( $L=26.4$  m), steel girder ( $L=16$  m) and pre-stressed concrete girder ( $L=16.5$  m). The vehicles tested included turbo-train RTG01, electric locomotive TGV001; and loco+coaches.

The measurements produced a very wide variance in the dynamic factor, but even in the high speed range the value was less than UIC formula Eq.(25) derived from the proposed formula. The value for prestressed concrete girders was found to be very small and an example of the measurement is given in Fig.10.

The influence of track irregularities has been investigated by ORE. As stated (see Figs.7 and 9), theoretical analysis and test runs conducted using real vehicles on the track of a real bridge with rail gaps and depressions artificially created on the rail top have revealed that track irregularities have a great influence on the dynamic response of a bridge, but no value exceeding the UIC design formula have been registered.

Further, ORE examined the effect of fatigue due to repeated loads and concluded that the vibration of a bridge, though their amplitude is small, should not be neglected because they are repeated a large number of times. The fatigue life was estimated by a frequency analysis of measured data by the rainfall method under Palmgren-Miner hypothesis.

In the dynamic response of a bridge the effect of an increased amplitude due to the fundamental natural frequency of the bridge is small, whereas the effect of vibrations due to track irregularities is large.

## 4. OTHER STUDIES RELATED TO THE DYNAMIC RESPONSE OF BRIDGE

### 4.1 Concerning the Deflection Limit

The deflection limit of a bridge was an example of a girder depth ratio restriction suppressing steel girder vibrations under running vehicles. When a high speed train crosses a bridge, the deflection is regarded as the track depression over the entire length of the bridge. The deflection limit is considered if it adversely affects the running safety or riding comfort of the vehicle. When the natural frequency of each element in the vehicle is the same the frequency generated when the vehicle passes over the bridge deflection, resonance occurs and amplitude increases. From this standpoint, [17] to [19] discuss the deflection limit of a bridge and the results obtained are used more often in practical design.

In the case of concrete bridges whose magnitude of deflection is usually small, the deflection will have no great influence on the designing.

### 4.2 Pertaining to Long Spanned Bridge

In various countries, construction of suspension bridges or cable-stayed bridges is being planned and some of them are railway bridges. Designing long spanned bridges for a railway involves specific problems associated with the running of vehicles. Theoretical analysis and model tests have been carried out concerning the dynamic factor of a long suspension bridge ([17]), and a cable-stayed bridge ([20]). A design impact factor has been proposed.

These investigations suggest that there is no need to provide for a very high value of the impact factor, because in the case of a suspension bridge the propagation of the deflection wave in the bridge axis direction is usually faster than the running train; and in the case of cable-stayed bridge the system



damping effect is great. Problems such as the lateral vibrations of cables in cable-stayed bridges, however, remain to be solved in the future.

#### 4.3 Magnetic Levitation Railway

With many countries vying with one another in the development of magnetic levitation railway systems, attention is being focused on vigorous research into the dynamic interaction between the guideway (bridge) and levitated vehicle ([21],[22]).

The items in such research include; the basic problem of a moving load at super high speed; the speed limit on continuous elastically supported beams; the interaction between a dynamic model of the levitated vehicle and the bridge girders; and problem of an impact load in case of a sudden dropout of the levitating force ([23]).

#### 5. CONCLUDING REMARKS

Concerning the dynamic response of concrete railway bridges to running vehicles, the present technology permits advanced analysis and facilitates parametric studies over a wide range. There is considerable data on model tests and running tests on real bridges. Thus elaborate analyses and numerous experiments are making clear the dynamic response of railway bridges, and the results of this phenomenon on high speed runs. The dynamic response of concrete bridges, however, which is characterized by a strong probabilistic element, is hard to estimate quantitatively. Therefore, determination of a more reasonable dynamic factor or fatigue life from this standpoint will remain a problem to be solved in the future.



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Remark: Not all dynamic responses of railway bridge are listed here.

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