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Initial Elastoplastic Buckling of Stiffened Plate Assemblies

Flambement en régime élastoplastique de panneaux raidis

Elastoplastisches Beulen ausgesteifter Platten

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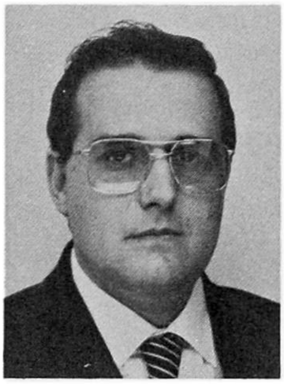

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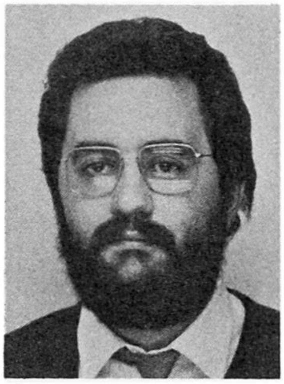

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SUMMARY

A matrix technique using harmonic analysis is shown which allows the study of the elastoplastic buckling of stiffened panels subjected to longitudinal loading and residual stresses. The bifurcation of equilibrium method is used. Therefore the edge loads are supposed to be acting in the middle plane of the panel and geometrical imperfections are ignored.

RÉSUMÉ

Les auteurs exposent une technique matricielle d'analyse harmonique, qui permet d'analyser le flambement en régime élastoplastique de panneaux raidis sollicités par des charges longitudinales et contraintes résiduelles de la soudure. La méthode de la "bifurcation" est appliquée avec les hypothèses suivantes: les charges latérales sont supposées agir au centre, les imperfections géométriques sont négligées.

ZUSAMMENFASSUNG

Es wird gezeigt, wie mittels harmonischer Analyse das elastisch-plastische Beulen ausgesteifter Platten unter Längsbelastung erfasst werden kann. Dabei werden Eigenspannungen berücksichtigt. Das Problem wird als Verzweigungsproblem behandelt, weshalb zentrische Belastung angenommen wird und geometrische Imperfektionen ausser Betracht bleiben.

1. INTRODUCTION

This paper presents a matrix technique using harmonic analysis which allows the study of the elastoplastic buckling of stiffened panels subjected to longitudinal loading. Residual stresses (welding stresses) are taken into account in calculation but geometrical imperfections are not. Therefore, this theory can be used for stiffened plates with low width - thickness ratios because such plates have little postbuckling strength and small geometrical imperfections have not a great influence on their ultimate strength.

2. ANALYSIS

As it is known, there are two different basic methods to study the buckling of steel structural elements under compression: the bifurcation of equilibrium method, which determines the critical loads of ideal structural systems; and the divergence of equilibrium method, which obtains the ultimate loads in real structural systems. The basic difference between these two methods is that while the first one studies buckling as a state of neutral equilibrium (i.e. an infinite number of solutions), the second considers buckling as a problem of strength (i.e. the highest point in the load -deflection diagram).

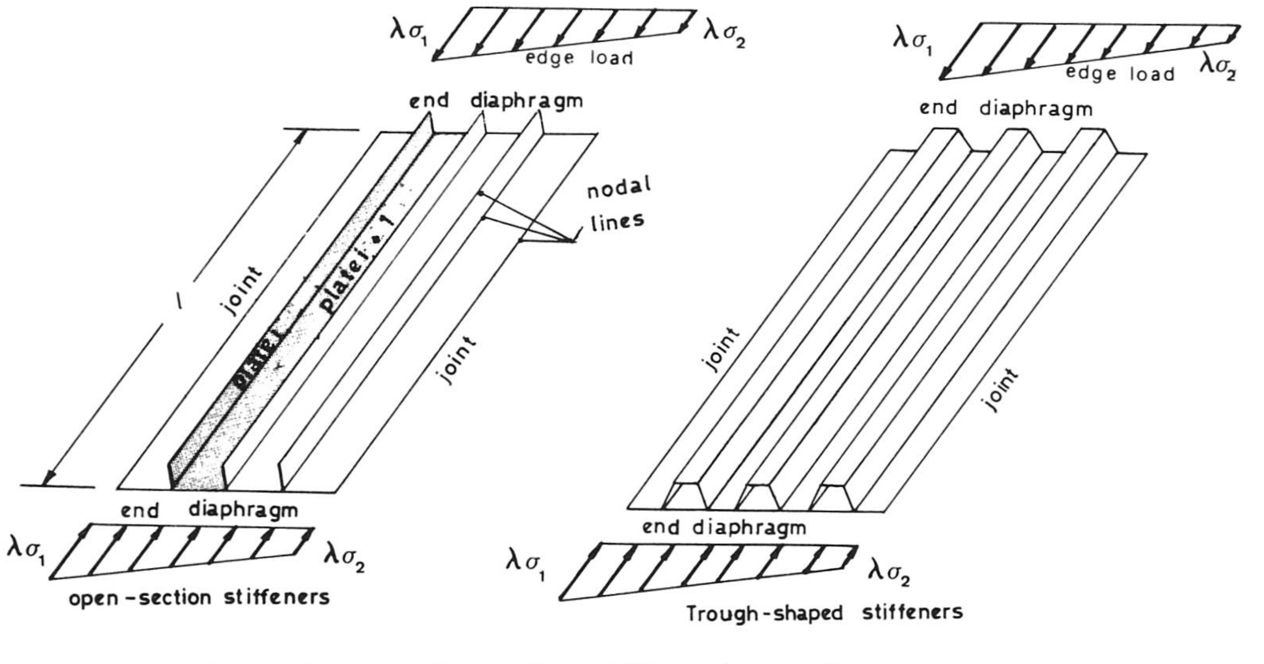

Fig. 1. Tipical examples of stiffened panels

In order to study the panel buckling (figure 1) using the bifurcation method, the problem must be idealized. The assumptions are:

- The material has an elastic-perfectly plastic stress-strain relationship (figure 2).

- Each one of the plates that make up the panel remains plane before buckling.

- The idealized residual stress platterns (figure 3) satisfy the self-equilibrating conditions.

- Lastly, it is assumed that no strain reversal takes place at the instant of buckling.

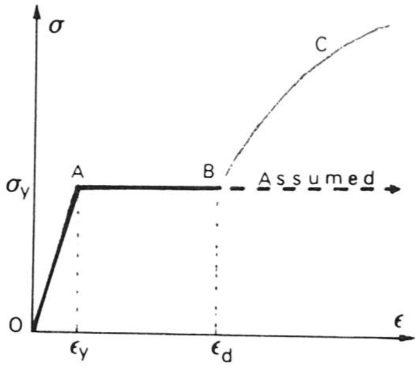

Fig. 2.
Stress-strain curve

To simplify terminology, the term INITIAL GEOMETRY is used throughout for the state of deformed equilibrium due to edge loads and residual stresses. Likewise the forces induced by these loads will be called PRIMARY FORCES.

The question we are addresing when studying the buckling of panels using the bifurcation of equilibrium method is the following Is it posible for the stiffened panel to be also in equilibrium in an infinite number of deformed configurations that are very close to the so-called INITIAL GEOMETRY?

To answer this question, the differential equations of equilibrium of each plate in a slightly deformed configuration must be integrated.

To make this integration easier, we postulate the existence of diaphragms in both ends of the stiffened panel (Levy-type solutions) and we suppose too that the plates that make up the panel are divided across its width into a number of strips in a way such that in any of these strips the following conditions must be met:

- If the strip behaves elastically the PRIMARY longitudinal FORCE must be approximately constant in the transverse direction.

- If the strip behaves plastically the PRIMARY longitudinal DEFORMATION must be approximately constant in the transverse direction.

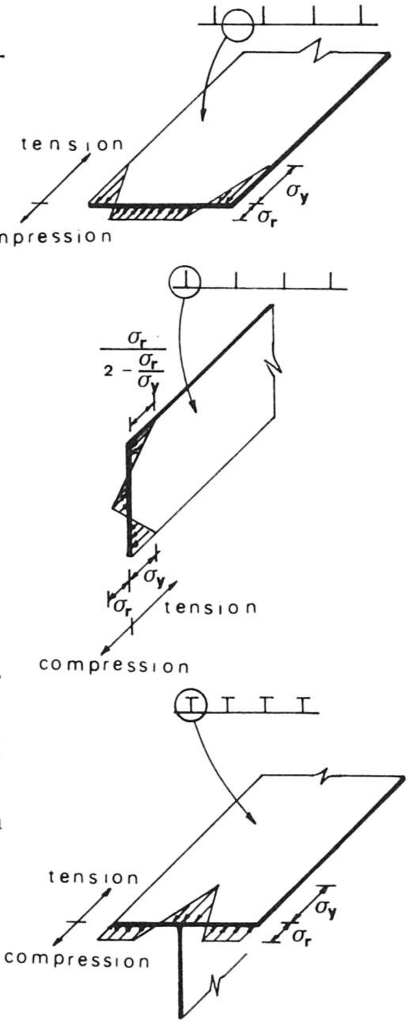

Fig. 3. Assumed welding residual stresses

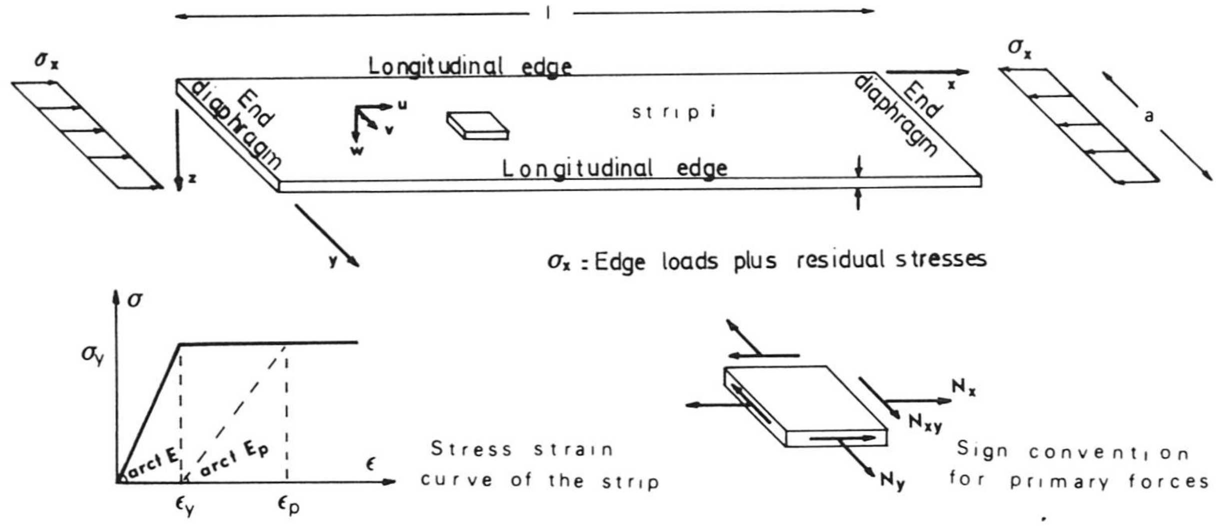

Fig. 4. One of the strips that make up the plates of the stiffened pane

When imposing boundary conditions for the longitudinal edges where the different strips are assembled (equilibrium and compatibility conditions), a homogeneous set of equations is obtained. Some of the coefficients are transcendental functions of the load parameter λ and others are constant. This set of equations admits infinite solutions for those values of λ that make the determinant of its coefficients vanish. Our objective is, precisely, to obtain these values of $\lambda = \lambda_{cr}$ (critical values).

2.1. Governing differential equations for equilibrium of the strips in a slightly deformed position.

At buckling, the initially plane strips (figure 4) that make up the plates of the stiffened panel are subject to small displacements that in turn induce SECONDARY FORCES.

Let be $w^*(x,y)$ the deflections of the middle plane of the strip. Then, the SECONDARY FORCES due to flexure are:

$$M_x^* = -\frac{Et^3}{12}\left(k_1 \frac{\partial^2 w^*}{\partial x^2} + k_2 \frac{\partial^2 w^*}{\partial y^2}\right)$$

$$M_y^* = -\frac{Et^3}{12}\left(k_2 \frac{\partial^2 w^*}{\partial x^2} + k_3 \frac{\partial^2 w^*}{\partial y^2}\right)$$

$$M_{xy}^* = M_{yx}^* = \frac{Et^3}{6} k_4 \frac{\partial^2 w^*}{\partial x \partial y}$$

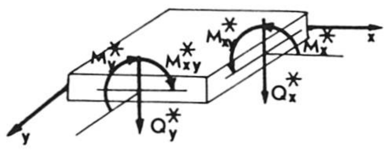

Fig. 5. Secundary forces in flexural behaviour (1)

$$V_x^* = Q_x^* - \frac{\partial M_{xy}^*}{\partial y} = -\frac{Et^3}{12}\left(k_1 \frac{\partial^3 w^*}{\partial x^3} + (k_2 + 4\,k_4) \frac{\partial^3 w^*}{\partial x \partial y^2}\right)$$

$$V_y^* = Q_y^* - \frac{\partial M_{xy}^*}{\partial x} = -\frac{Et^3}{12}\left(k_3 \frac{\partial^3 w^*}{\partial y^3} + (k_2 + 4\,k_4) \frac{\partial^3 w^*}{\partial x^2 \partial y}\right)$$

whereas if the in-plane displacements of the middle plane are $u^*(x,y)$, $v^*(x,y)$, the induced SECONDARY MEMBRANE-FORCES will be:

$$
\begin{aligned}
N_x^* &= Et\left(k_1 \frac{\partial u^*}{\partial x} + k_2 \frac{\partial v^*}{\partial y}\right) \\
N_y^* &= Et\left(k_2 \frac{\partial u^*}{\partial x} + k_3 \frac{\partial v^*}{\partial y}\right) \qquad (2) \\
N_{xy}^* &= Et\, k_4 \left(\frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x}\right)
\end{aligned}
$$

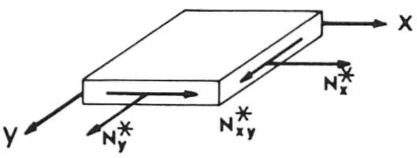

Fig. 6. Secondary forces in-plane behaviour

where

$$k_1 = \frac{1}{1-\nu^2} \; ; \quad k_2 = \nu k_1 \; ; \quad k_3 = k_1 \text{ and } k_4 = \frac{1}{2(1+\nu)}$$

if the strip behaves elastically, and

$$k_1 = \frac{1}{3E/Ep+5-4\upsilon} \; ; \quad k_2 = 2k_1 \; ; \quad k_3 = 4k_1 \text{ and } k_4 = \frac{1}{3E/Ep+2(1+\upsilon)}$$

if the strip behaves plastically (assuming that the deformation theory of Bijlaard [1] is valid in the plastic range).

The governing differential equations of equilibrium in a slightly deformed position, when both primary and secondary forces are present, take the form:

-Flexural behaviour [2]

$$k_1 \frac{\partial^4 w^*}{\partial x^4} + (2k_2+4k_4)\frac{\partial^4 w^*}{\partial x^2 \partial y^2} + k_3 \frac{\partial^4 w^*}{\partial y^4} + \frac{12\,\sigma_x}{Et^2}\frac{\partial^2 w^*}{\partial x^2} = 0 \qquad (3)$$

-Extensional behaviour [2]

$$\left(k_1 - \frac{\sigma_x}{E}\right)\frac{\partial^2 u^*}{\partial x^2} + (k_2+k_4)\frac{\partial^2 v^*}{\partial x\,\partial y} + k_4 \frac{\partial^2 u^*}{\partial y^2} = 0 \qquad (4)$$

$$\left(k_4 - \frac{\sigma_x}{E}\right)\frac{\partial^2 v^*}{\partial x^2} + (k_2+k_4)\frac{\partial^2 u^*}{\partial x\,\partial y} + k_3 \frac{\partial^2 v^*}{\partial y^2} = 0 \qquad (5)$$

The above expressions have been obtained assuming that at the beginning of buckling the middle plane deflections are small. This assumption allows us to apply the linear theory for thin plates under flexure and to uncouple the aforementioned two types of behaviour of the strip.

2.2 Integration of the set of equations.

A series solutions of the Levy-type of the form

$$w^*(x,y) = \sum_{n=1}^{\infty} W_n(y) \sin \frac{n\pi x}{L} \qquad (6)$$

$$u^*(x,y) = \sum_{n=1}^{\infty} U_n(y) \cos \frac{n\pi x}{L} \qquad (7)$$

$$v^*(x,y) = \sum_{n=1}^{\infty} V_n(y) \sin \frac{n\pi x}{L} \qquad (8)$$

automatically satisfy the boundary conditions in the ends of the plate with transverse diaphragms.

Substituting Eq. (6) into Eq. (3) and integrating one obtains

$$W_n(y) = \sum_{i=1}^{4} A_n^i e^{r_i y} \qquad (9)$$

where

$$r_{i\,(i=1,4)} = \pm \sqrt{\frac{\frac{n^2\pi^2}{L^2}(k_2+2k_4) \pm \frac{n\pi}{L}\sqrt{((k_2+2k_4)^2 - k_3k_1)\frac{n^2\pi^2}{L^2} + \frac{12\,\sigma_x\,k_3}{Et^2}}}{k_3}}$$

Substituting Eq. (7) and Eq. (8) into Eq. (4) and Eq. (5) and integrating one obtains

$$U_n(y) = \sum_{i=5}^{8} A_n^i e^{r_i y} \qquad (10)$$

$$V_n(y) = \sum_{i=5}^{8} L_n^i A_n^i e^{r_i y} \qquad (11)$$

where

$$r_{i\,(i=5,8)} = \pm \frac{n\pi}{L}\sqrt{\frac{-S \pm \sqrt{S^2 - 4RT}}{2R}}$$

$$L_n^i = \frac{(k_1 - \frac{\sigma_x}{E})\frac{n^2\pi^2}{L^2} - k_4 r_i^2}{\frac{n\pi}{L} r_i (k_2 + k_4)}$$

$$R = k_3k_4 \;;\; T = (k_1 - \frac{\sigma_x}{E})(k_4 - \frac{\sigma_x}{E}) \;;\; S = (k_2^2 + 2k_2k_4 - k_1k_3 + (k_3+k_4)\frac{\sigma_x}{E})$$

The infinite series expressions for w^*, v^*, u^* given in equations (6), (7) and (8) are functions of a set of constants $A_n^{i\,(i=1,8)}$. These constants are obtained from the boundary conditions on the longitudinal edges ($y = o$, $y = a$) where the different strips are assembled.

These boundary conditions are: equilibrium equations and compatibility of displacements on the nodal lines. As there are no loads applied in the nodal lines, the boundary conditions translate into a homogeneous set of equations. This set of equations (eight for each one of the constitutive strips) takes the general form

$$
\begin{aligned}
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-7,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-6,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-5,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-4,\,8(k-1)+i}\; A_{n,k}^{i}\cos\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-3,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-2,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j-1,\,8(k-1)+i}\; A_{n,k}^{i}\sin\frac{n\pi x}{l}\\
&\sum_{n=1}^{\infty}\sum_{k=1}^{r}\sum_{i=1}^{8} C_n^{8j,\,8(k-1)+i}\; A_{n,k}^{i}\cos\frac{n\pi x}{l}
\end{aligned}
\qquad (12)
$$

where: $j = 1, 2, \ldots, r$ (r = number of constitutive strips)

The coefficients C_n are, in general, transcendental functions of the load parameter λ and of the integer n. For arbitrary values of n and λ the set of equations is only satisfied when the coefficients $A_{n,k}^{i}$ are zero, that is, the stiffened panel is only in equilibrium if the initial geometry is maintained. However, the homogeneous set, as it is readily proved, admits an infinity of solutions for those values of λ that make the determinant of the coefficients associated with each n vanish.

$$
\begin{vmatrix}
C_n^{1,1}\sin\frac{n\pi x}{l} & \cdots & C_n^{1,8r}\sin\frac{n\pi x}{l}\\
\vdots & & \vdots\\
C_n^{8r,1}\cos\frac{n\pi x}{l} & \cdots & C_n^{8r,8r}\cos\frac{n\pi x}{l}
\end{vmatrix}
= \left[\sin\frac{n\pi x}{l}\right]^{6r}\left[\cos\frac{n\pi x}{l}\right]^{2r}
\begin{vmatrix}
C_n^{1,1} & \cdots & C_n^{1,8r}\\
\vdots & & \vdots\\
C_n^{8r,1} & \cdots & C_n^{8r,8r}
\end{vmatrix}
$$

In fact, for each integer n there is an infinity of such values of $\lambda = \lambda_{cr}$. What we are interested in is the least value of them all, $\lambda = \lambda_{cl}$, as it can be shown that for $\lambda < \lambda_{cl}$, the so-called INI-

TIAL GEOMETRY is a stable equilibrium configuration whereas for $\lambda > \lambda_{c1}$ it is an unstable one.

The equilibrium deflected shape at the instant of buckling (buckling mode), is longitudinally defined by n and, transversely, by the coefficients $A^{i}_{n,k}$.

The number n (equations 6, 7 and 8) defines the number of sinusoidal half-waves in which the total length of the panel is divided at the instant of buckling.

The coefficients $A^{i}_{n,k}$ can be obtained substituting $\lambda = \lambda_{cr}$ in the set of equations (12). Since this is a homogeneous set and its determinat is zero, all the coefficients can be given as functions of one of them that may be taken as an arbitrary constant. This is the case, then, of a deformed configuration whose shape is determined but its magnitude is not (infinite solutions).

3. STIFFNESS HARMONIC METHOD FOR THE STUDY OF ELASTOPLASTIC BUCKLING OF STIFFENED PANELS.

In the previous paragraph it has been shown that the stiffened panels, longitudinally loaded in their middle plane, buckle sinusoidally, in a way such that the total length of the panel is divided into an integer number of half-waves. Therefore, both the deflected shape of each one of the strips that make up the panel and the interaction forces between the strips also vary in a sinusoidal way.

This fact allows us to apply the stiffness harmonic method for the study of the buckling of the stiffened panels.

First, we analyze each individual strip, looking for a relation between the applied sinusoidal loads and the corresponding sinusoidal displacements. Once the previous relation has been obtained (stiffness matrix of a single strip), we shall proceed to assemble the strips along the nodal lines, establishing the equilibrium and compatibility conditions. In this way, the sinusoidal displacements of the nodal lines (line joints) are related to the external loads acting on those nodal lines. But, since there are no external loads applied on the nodal lines, the set of equations is a homogeneneous one. The non-trivial solutions define the critical loads and the associated buckling modes.

3.1 The sinusoidal stiffness matrix for a flat strip.

Firstly, the problem will be analyzed assuming that the loads that are applied on the longitudinal edges induce flexural forces only. The case where the forces induced are membrane forces only, will be considered later on.

- The out-of planes stiffness matrix.

With reference to figure 7 and making use of Eqs. (1, 6 and 9), one obtains

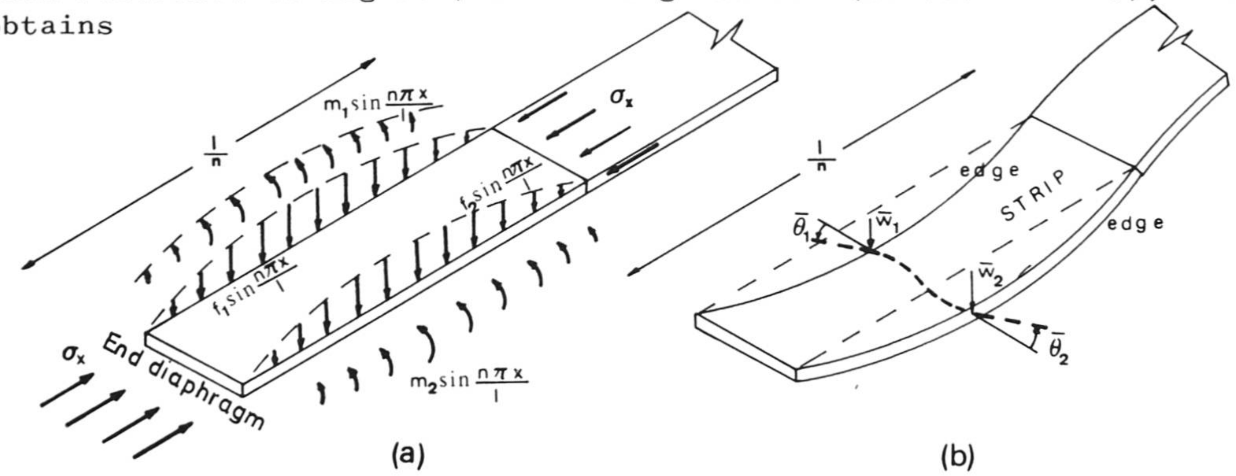

Fig. 7. The out-of-plane stiffness matrix

$$\begin{vmatrix} m_1 \\ f_1 \\ m_2 \\ f_2 \end{vmatrix} = \begin{vmatrix} \bar m_1 \\ \bar f_1 \\ \bar m_2 \\ \bar f_2 \end{vmatrix} \sin \frac{n\pi x}{l} = \frac{Et^3}{12} \begin{vmatrix} k_3 r_i^2 - k_2 \frac{n^2\pi^2}{l^2} 1^i \\ r_i \left(k_3 r_i^2 - (k_2+4k_4) \frac{n^2\pi^2}{l^2}\right) \\ -e^{r_i a} \left(k_3 r_i^2 - k_2 \frac{n^2\pi^2}{l^2}\right) \\ -r_i e^{r_i a} \left(k_3 r_i^2 - (k_2+4k_4) \frac{n^2\pi^2}{l^2}\right) \end{vmatrix} \begin{vmatrix} A_n^1 \\ A_n^2 \\ A_n^3 \\ A_n^4 \end{vmatrix} \sin \frac{n\pi x}{l} \tag{13}$$

$$\begin{vmatrix} \theta_1 \\ w_1 \\ \theta_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} \bar\theta_1 \\ \bar w_1 \\ \bar\theta_2 \\ \bar w_2 \end{vmatrix} \sin \frac{n\pi x}{l} = \begin{vmatrix} -r_i \\ 1^i \\ -r_i e^{r_i a} \\ e^{r_i a} \end{vmatrix} \begin{vmatrix} A_n^1 \\ A_n^2 \\ A_n^3 \\ A_n^4 \end{vmatrix} \sin \frac{n\pi x}{l} \qquad i=1,2,3,4 \tag{14}$$

Equations (13) and (14) can be written in matrix form as

$$\bar f_p = \bar{\bar D}_p \bar A_p \tag{15}$$

$$\bar d_p = \bar{\bar G}_p \bar A_p \tag{16}$$

Finally, the out-of-plane stiffness matrix of the strip $\bar{\bar{K}}_p$ is obtained from equations (15) and (16)

$$\bar{f}_p = \bar{\bar{K}}_p \, \bar{d}_p \qquad (17) \qquad \text{where} \qquad \bar{\bar{K}}_p = \bar{\bar{D}}_p \, \bar{\bar{G}}_p^{-1}$$

- The in-plane stiffness matrix.

With reference to figure 8 and making use of Eqs. (2, 7, 8, 10 and 11), one obtains.

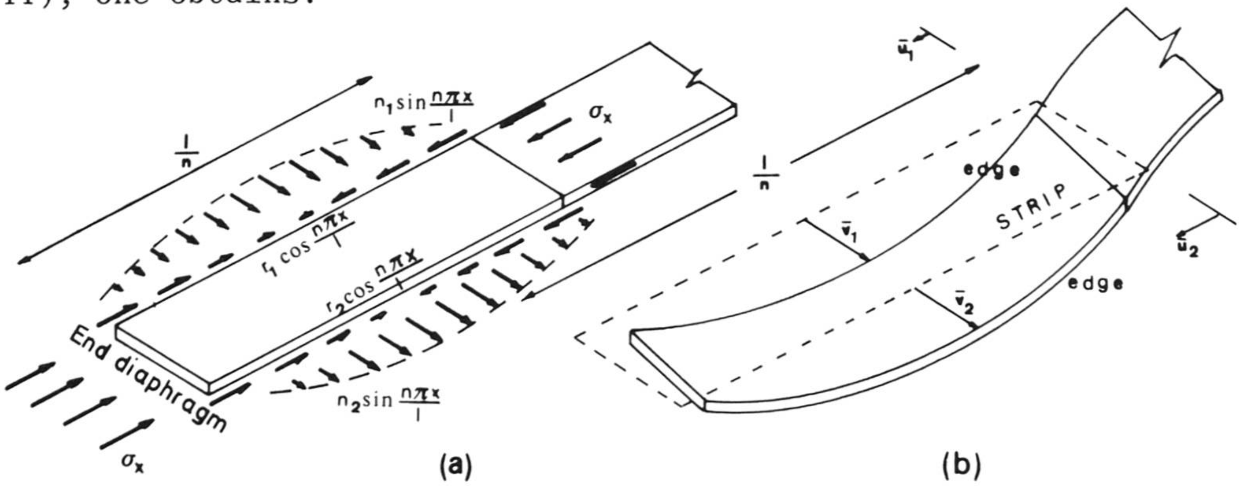

Fig. 8. The in-plane stiffness matrix

$$\left|\begin{matrix} n_1 \\ r_1 \\ n_2 \\ r_2 \end{matrix}\right| = \left|\begin{matrix} \bar{n}_1 \sin \frac{n\pi x}{l} \\ \bar{r}_1 \cos \frac{n\pi x}{l} \\ \bar{n}_2 \sin \frac{n\pi x}{l} \\ \bar{r}_2 \cos \frac{n\pi x}{l} \end{matrix}\right| = Et \left|\begin{matrix} (k_2 \frac{n\pi}{l} 1^i - k_3 L^i_n r_i) \sin \frac{n\pi x}{l} \\ (-k_4 (r_i + \frac{n\pi}{l} L^i_n)) \cos \frac{n\pi x}{l} \\ (-e^{r_i a} (k_2 \frac{n\pi}{l} - k_3 L^i_n r_i)) \sin \frac{n\pi x}{l} \\ (k_4 e^{r_i a} (r_i + \frac{n\pi}{l} L^i_n)) \cos \frac{n\pi x}{l} \end{matrix}\right| \left|\begin{matrix} A^5_n \\ A^6_n \\ A^7_n \\ A^8_n \end{matrix}\right| \qquad (18)$$

$$\left|\begin{matrix} v_1 \\ u_1 \\ v_2 \\ u_2 \end{matrix}\right| = \left|\begin{matrix} \bar{v}_1 \sin \frac{n\pi x}{l} \\ \bar{u}_1 \cos \frac{n\pi x}{l} \\ \bar{v}_2 \sin \frac{n\pi x}{l} \\ \bar{u}_2 \cos \frac{n\pi x}{l} \end{matrix}\right| = \left|\begin{matrix} L^i_n \sin \frac{n\pi x}{l} \\ 1^i \cos \frac{n\pi x}{l} \\ e^{r_i a} L^i_n \sin \frac{n\pi x}{l} \\ e^{r_i a} \cos \frac{n\pi x}{l} \end{matrix}\right| \left|\begin{matrix} A^5_n \\ A^6_n \\ A^7_n \\ A^8_n \end{matrix}\right| \qquad (19)$$

$i = 5, 6, 7, 8$

Equations (18) and (19) can be written in matrix form as

$$\bar{f}_\ell = \bar{\bar{D}}_\ell \, \bar{A}_\ell \qquad (20)$$

$$\bar{d}_\ell = \bar{\bar{G}}_\ell \, \bar{A}_\ell \qquad (21)$$

Finally, the in-plane stiffness matrix of the strip $\bar{\bar{K}}_\ell$ is obtained from equations (20) and (21)

$$\bar{f}_\ell = \bar{\bar{k}}_\ell \, \bar{d}_\ell \qquad (22) \qquad \text{where} \qquad \bar{\bar{K}}_\ell = \bar{\bar{D}}_\ell \, \bar{\bar{G}}_\ell^{-1}$$

- Global behaviour.

The stiffness matrix of the strip considering flexural and extensional behaviour simultaneously, is obtained putting together the expressions (17) and (22).

$$\begin{Vmatrix} \bar{m}_1 \\ \bar{f}_1 \\ \bar{m}_2 \\ \bar{f}_2 \\ \bar{n}_1 \\ \bar{r}_1 \\ \bar{n}_2 \\ \bar{r}_2 \end{Vmatrix} = \begin{Vmatrix} p_{1,1} & p_{1,2} & p_{1,3} & p_{1,4} & & & & \\ p_{2,1} & p_{2,2} & p_{2,3} & p_{2,4} & & 0 & & \\ p_{3,1} & p_{3,2} & p_{3,3} & p_{3,4} & & & & \\ p_{4,1} & p_{4,2} & p_{4,3} & p_{4,4} & & & & \\ & & & & L_{1,1} & L_{1,2} & L_{1,3} & L_{1,4} \\ & & & & L_{2,1} & L_{2,2} & L_{2,3} & L_{2,4} \\ & 0 & & & L_{3,1} & L_{3,2} & L_{3,3} & L_{3,4} \\ & & & & L_{4,1} & L_{4,2} & L_{4,3} & L_{4,4} \end{Vmatrix} \begin{Vmatrix} \bar{\theta}_1 \\ \bar{w}_1 \\ \bar{\theta}_2 \\ \bar{w}_2 \\ \bar{v}_1 \\ \bar{u}_1 \\ \bar{v}_2 \\ \bar{u}_2 \end{Vmatrix}$$

Where the coefficients $p_{i,j}$ and $L_{i,j}$ $(i,j=1,2,3$ and $4)$ are transcendental functions of the strip geometry, of the longitudinal force or deformation and of the integer n (n = 1,2,3 ...).

3.2. The process of setting up the sinusoidal stiffness matrix of the stiffned panel.

The matrix expression (stiffness matrix of the strip) has just been obtained that relates the loads acting on the sides of the strip to the displacements of those sides.

The next step is to assemble all the strips along the nodal lines of the stiffened panel. For this, compatibility of displacements must be established first along all the nodal lines, that is, the displacements along the sides of the strips must be compatible with the displacements of the nodal lines to wich these strips are attached. Equilibrium equations for loads are then set up imposing that the sum of all the loads acting on the sides of the strips that meet along a nodal line be equal to the external loads acting there. After this process, the following set of equations is finally obtained

$$\bar{F}_n = \bar{\bar{K}}_n \, \bar{D}_n \qquad (n = 1, 2, 3, \ldots)$$

where

$\bar{F}_n$ = array defining the amplitudes of the sinusoidal loading applied on the nodal lines.

$\bar{\bar{K}}_n$ = stiffness matrix of the stiffened panel.

$\bar{D}_n$ = array defining the amplitudes of the sinusoidal displacements of the nodal lines.

As there are no external loads applied on the nodal lines

$$\bar{0} = \bar{\bar{K}}_n \; \bar{D}_n \qquad (n = 1, 2, 3, \ldots)$$

the critical loads and their associated buckling modes being defined by the non-trivial solutions.

Thus, to determine the critical loads of the sttiffened panel, the values of $\lambda = \lambda_{cr}$ that make the determinant vanish must be evaluated for each n.

It should be born in mind that the terms of the stiffeness matrix are transcendental functions of λ and n. Therefore for each n there are, in general, infinite values of λ that make the determinant of $\bar{\bar{K}}_n$ vanish.

In fact we are only interested in the least value of λ . As the terms of the stiffness matrix vary with λ in a complex way, taking zero or infinity values at irregular intervals, a suitable iterative technique has to be used that insures convergence to the least eigenvalue. For this, it proves convenient to use an algorithm, initially proposed by Lord Rayleigh to analyze structural dynamics problems, and that has been reelaborated later by Wittrick [3], in order to apply it to instability problems.

3.3. Computer analysis.

Applying these criteria a computer program has been written that studies the influence of welding residual stresses in the value of the least critical load of a stiffened panel, assuming elastic or elastoplastic buckling.

The input data for the program is: the geometry of the cross-section of the panel, the edge longitudinal loading of each plate, the distribution of welding residual stresses, the number of longitudinal bands in which we wish to discretize the zones of the plates in which there is a transverse variation of the longitudinal force, and the number of longitudinal bands in which we wish to discretize the plastified zone of the plates in which there is a transverse variation of the longitudinal deformation.

The stiffened panel of figure 9 has been analysed by means of

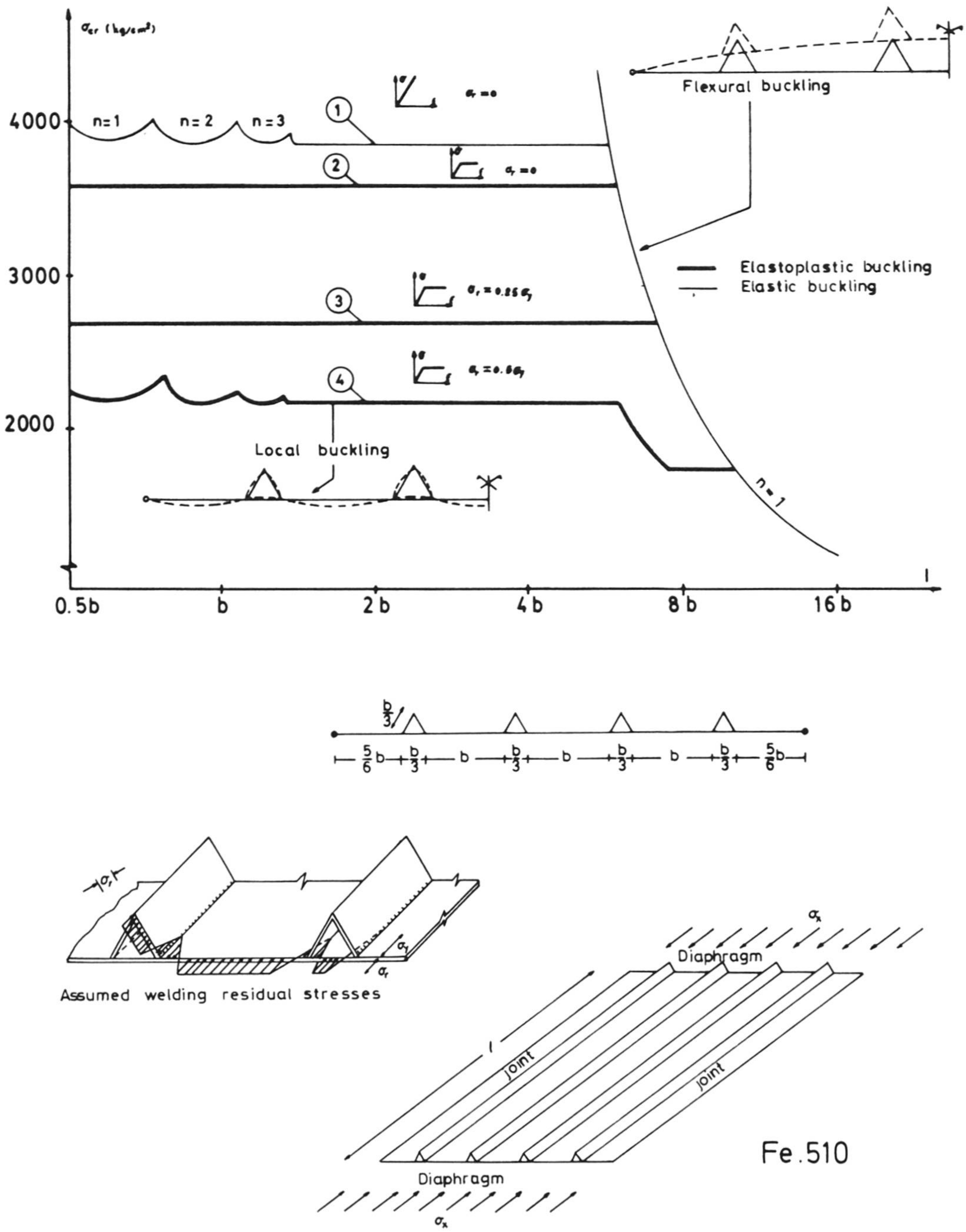

Fig. 9. Variation with "l" of the lowest critical stress

this program, for different values of the length of the panel.

Curves 1 and 2 have been obtained assuming that there are no welding residual stresses and, in the first case, linear elastic behaviour, of the structural material, whereas in the second case elastoplastic behaviour has been assumed.

Curves 3 and 4 have been obtained supposing elastoplastic behaviour of the structural material and considering that there are welding residual stresses whose values are $\sigma_r / \sigma_y = 0.25$ and $\sigma_r / \sigma_y = 0.5$ respectively.

Examining the figure 9, we can conclude that the residual stresses, together with the elastoplastic behaviour of the structural material, have a marked influence on the value of the least critical load corresponding to a local buckling mode. On the other hand, it can also be seen that when the panel buckles by flexure of its stiffeners, the welding stresses only have an influence if the primary loading (edge loads plus residual stresses) causes plasticity to occur in the stiffeners, thereby decreasing its flexural and torsional stiffness. This last phenomenon can be observed in the part AB of curve 4.

4. A COMPARISON OF THIS THEORY WITH THE AVAILABLE TEST RESULTS.

Figs. 10 and 11 show a comparison of the available test results with the theoretical results obtained for the buckling strength of unstiffened and stiffened plates subjected to longitudinal loading and residual stresses.

In both cases we can see that, in the range $0{,}6 < \beta < 1{,}2$, there is a good agreement between the theoretical curves and the test results.

When β is greater than 1,6 the "ultimate strengths" are higher than the "critical stresses" because these plates have a high postbuckling strength; when β is less than 0,6 the "ultimate strengths" are higher too than the "critical stresses" because these plates buckle when the material is working in the CD zone of the stress-strain diagram (figure 2).

5. SUMMARY.

The elastic buckling of stiffened panels has been studied by Wittrick-Williams [4], Smith [5], Przemieniecki [6], etc.

These authors analyze the panel by breaking it up into a set of

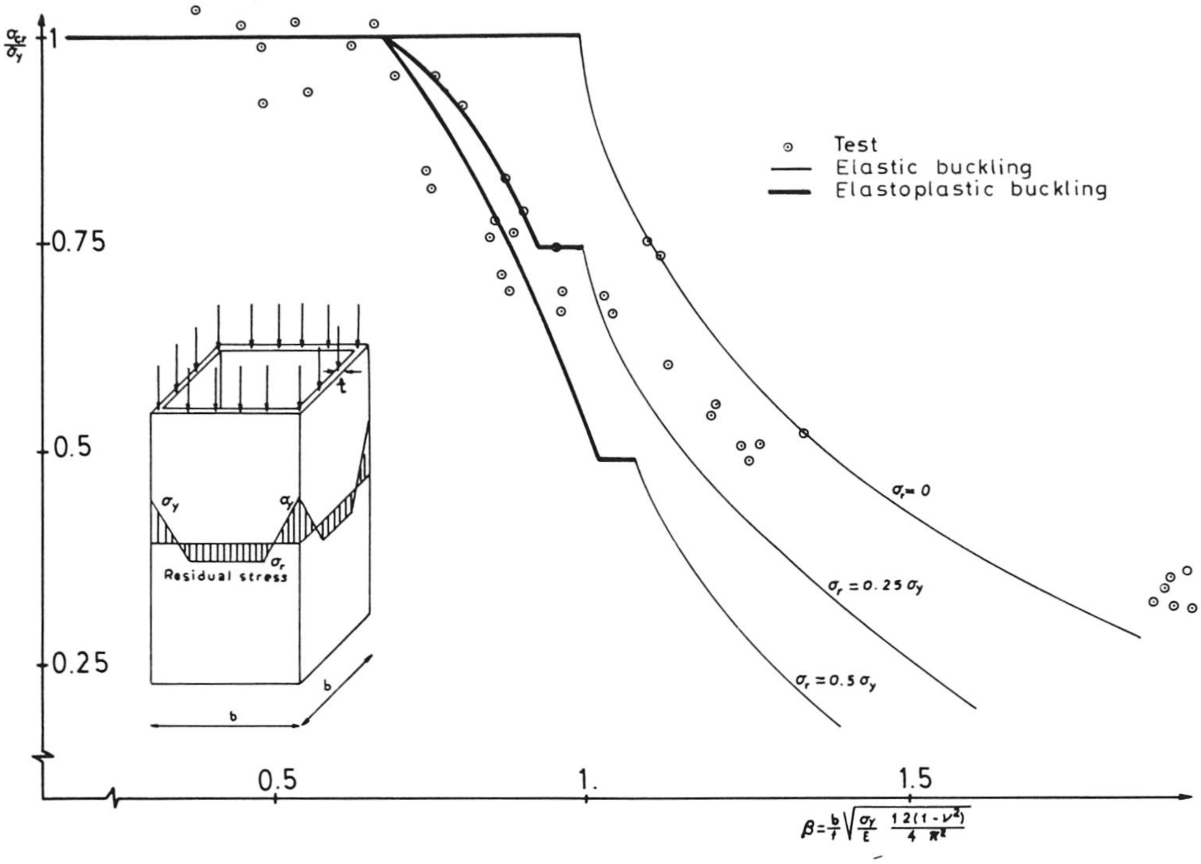

Fig. 10. Buckling strength curves of unestiffened plates

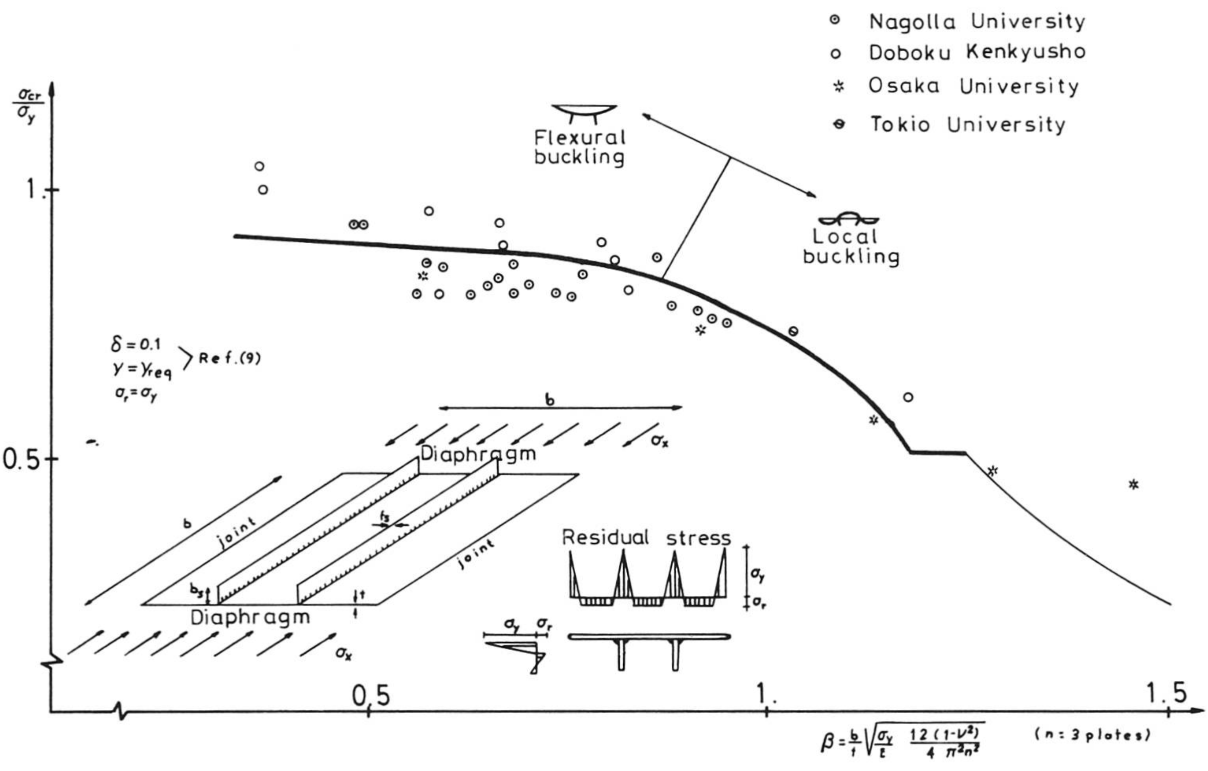

Fig. 11. Buckling strength curve of stiffened plates

thin flat rectangular plates that are joined along odal lines. The interaction of the plates connected along nodal lines is studied using a matrix technique of harmonic analysis. In some cases [4], the stiffness matrix of each plate is obtained by folded plate analysis, whereas in other instances [5], the stiffness matrix of each plate is obtained by finite-strip method.

This matrix technique can be extended, as it is shown in this paper, to handle the elastoplastic buckling of stiffened panels. As far as the authors know, no previous publication has tackled this problem in all of its aspects. However, it must be pointed out that several papers have been written in that direction. Among them reference should be made to Fukumoto-Usami-Okamoto [6] and Hasegawa-Ota-Nishino 7 . In these papers certain simplifications have been made that confine the analysis to elastoplastic buckling of panels with uniformly spaced open-section stiffeners.

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