| Zeitschrift: | IABSE proceedings = Mémoires AIPC = IVBH Abhandlungen     |
|--------------|---|
| Band:        | 5 (1981)  |
| Heft:        | P-48: Numerical analysis of continuous helicoidal girders |
|              |   |
| Artikel:     | Numerical analysis of continuous helicoidal girders       |
| Autor:       | Pulmano, V.A. / Kabaila, A.P.                             |
| DOI:         | https://doi.org/10.5169/seals-35892                       |

#### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. <u>Mehr erfahren</u>

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. <u>En savoir plus</u>

#### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. <u>Find out more</u>

## Download PDF: 22.08.2025

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

## **Numerical Analysis of Continuous Helicoidal Girders**

Calcul numérique de poutres continues spirales Numerische Berechnung durchlaufender spiralförmiger Träger

V.A. PULMANO

A.P. KABAILA

Associate Professor

Associate Professor

University of New South Wales Kensington, Sydney, Australia

## SUMMARY

A numerical analysis of continuous helicoidal girder by the stiffness method is presented. The full stiffness matrix of a helicoidal member is derived by first evaluating the flexibility matrix at one end which when inverted gives the end stiffness matrix. The integrals associated with the end flexibility matrix and equivalent load vector, are numerically evaluated by Simpson's rule using the computer. The analysis of a helicoidal girder of varying crosssection can easily be treated. Numerical examples are presented to illustrate the proposed method.

## RÉSUMÉ

Une méthode numérique pour le calcul de poutres continues spirales est présentée. La matrice complète de rigidité d'un élément spiral est dérivée en évaluant la matrice de flexibilité à une extrémité puis en déterminant la matrice inverse. L'intégrale correspondant à la matrice de flexibilité et au vecteur charge est calculée numériquement à l'aide de la méthode de Simpson. Le calcul de poutres spirales à section variable est également possible. Des exemples numériques illustrent la méthode.

## ZUSAMMENFASSUNG

Eine numerische Methode zur Berechnung durchlaufender spiralförmiger Träger wird dargestellt. Die vollständige Steifigkeitsmatrix eines spiralförmigen Elementes ergibt sich durch zahlenmässige Festlegung der Flexibilitätsmatrix an einem Ende und anschliessender Bestimmung der Kehrmatrix. Die der Flexibilitätsmatrix und dem äquivalenten Lastvektor zugeordneten Integrale werden zahlenmässig mittels Simpsonmethode festgelegt. Variable Querschnitte sind einfach zu berücksichtigen. Numerische Beispiele erläutern die vorgeschlagene Berechnungsmethode.

#### 1. INTRODUCTION

Earlier works on helicoidal girders [1,2,3,4] treated mainly the problem of helicoidal staircases which are indeterminate to the sixth degree. The analysis of continuous helicoidal girder has been attempted by Abdul-Baki and Shukair [5] by deriving explicitly the end flexibility matrix of a helicoidal segment which, when inverted, gives the end stiffness matrix. However, the resulting expressions for the different terms in the end flexibility matrix are very lengthy, and therefore inconvenient. Derron and Jirousek [6] presented a method of analysis of helicoidal girders, which was based on an assumed displacement field. In this respect their analysis is similar to the customary finite element formulation leading to approximate solution. The accuracy of solution usually improves as the number of elements to model a given problem is increased.

In this paper the integrals associated with the derivation of the basic matrices, viz., flexibility matrix, equivalent nodal loads are derived in the exact form, but the integration is carried out numerically. The method, therefore, is substantially different from those described in the above references. The effects of bending, shear, axial and torsional deformations are included without difficulty, however the effects of warping are excluded. In the analysis, the structure is assumed to be linearly elastic, and to obey other basic assumptions related to small deflection theory. Numerical examples are presented.

#### THEORETICAL CONSIDERATIONS

#### 2.1 Geometry and Axis Transformation

In Fig. 1(a), a helicoidal member AB is shown with the nodal coordinates in the positive directions and their numbering. The nodal axes are taken parallel to the orthogonal reference axes  $x_j$ . At any section C, defined by angle  $\beta$ , the member axes are defined by three mutually perpendicular axes  $\overline{x_1}$ ,  $\overline{x_2}$ , and  $\overline{x_3}$  which coincide respectively with the tangent to the helix, normal to the helix (which is also normal to the generator of the cylinder), and the binormal to the helix. The  $\overline{x_2}$  and  $\overline{x_3}$  axes are assumed to coincide with principal axes of the cross-section of the helicoidal member.

The member axis  $\overline{x}_i$  and the reference axis  $x_i$  are related by the equations

$$\{\bar{\mathbf{x}}\} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \{\mathbf{x}\} = [\lambda] \{\mathbf{x}\}$$
(1)

in which  $[\lambda]$  is the rotation matrix. The term  $\ell_{ij}$  in Eq. 1 is the cosine of the angle between the member axis  $\bar{x}_i$  and the reference axis  $x_j$ . Considering the geometry of the helicoidal member of Fig. 1, it can be shown that the terms of the rotation matrix,  $\lambda$ , are as follows:

$$[\lambda] = \begin{bmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & \sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ -\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha \end{bmatrix}$$
(2)





Angle  $\alpha$  defines the inclination of the helix, angle  $\beta$ , the position of the point considered. Both  $\alpha$  and  $\beta$  are shown in Fig. 1.

#### 2.2 End Flexibility of Helicoidal Member.

As shown in Fig. 1(b), end A of the helicoidal member AB is considered to be fixed. The end actions  $p_B$ , applied at B, are expressed in terms of the nodal axes. The internal actions,  $\{\sigma_C\}$  at any section C are in terms of end actions  $p_B$  and are given by the matrix expression

$$\sigma_{\rm C} = H \{ p_{\rm R} \} \tag{3}$$

in which  $\{\sigma_{C}\} = \{N \ S_{x_{2}} \ S_{x_{3}} \ T \ M_{x_{2}} \ M_{x_{3}}\}$ ,  $\{p_{B}\} = \{p_{B_{1}} \ p_{B_{2}} \ p_{B_{3}} \ p_{B_{4}} \ p_{B_{5}} \ p_{B_{6}}\}$ , H = transformation matrix, N = axial force,  $S_{\overline{x}_{2}}, S_{\overline{x}_{3}}$  = shear forces in the  $\overline{x}_{2}$  $\overline{x}_{3}$  directions, respectively, T = twisting moment, and  $M_{\overline{x}_{2}}, M_{\overline{x}_{3}}$  = bending moments about  $\overline{x}_{2}$  and  $\overline{x}_{3}$  axes, respectively.

In Eq. 3, the transformation matrix H, as shown elsewhere [7], is simply the product of the rotation matrix,  $\hat{R}$  and the translation matrix  $\hat{T}$ , that is,

$$H = \hat{R} \hat{T}$$
(4)

in which

$$\hat{\mathbf{R}} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \quad (5) \qquad \hat{\mathbf{T}} = \begin{bmatrix} \mathbf{I} & 0 \\ \hat{\mathbf{X}} & \mathbf{I} \end{bmatrix} \quad (6) \qquad \hat{\mathbf{X}} = \begin{bmatrix} 0 & -\mathbf{r}_3 & \mathbf{r}_2 \\ \mathbf{r}_3 & 0 & -\mathbf{r}_1 \\ -\mathbf{r}_2 & \mathbf{r}_1 & 0 \end{bmatrix}$$



FIG. 1 HELICOIDAL MEMBER AB SHOWING ITS REFERENCE AXES, MEMBER AXES AND NUMBERING OF NODAL COORDINATES.

167

(7)

R = radius of circular cylinder defining the helix

I = (3x3) identity matrix

Substitution of Eqs. 5 and 6 to Eq. 4 yields

$$H = \begin{bmatrix} \lambda & 0 \\ \hat{\lambda} \\ \lambda X & \lambda \end{bmatrix}$$
(8)

Using the principle of virtual work, the end flexibility,  $f_{BB}$ , of the helicoidal member AB is evaluated from the equation

$$f_{BB} = \int_{A}^{B} H^{T} D^{-1} H ds$$
(9)

where  $D = [EA_1 GA_2 GA_3 GJ EI_2 EI_3]$ , in which the symbol denotes a diagonal matrix

E = modulus of elasticity

 $A_1, A_2, A_3$  = the area of section, the shear area in  $\bar{x}_2$  - direction, and the shear area in  $\bar{x}_3$  direction, respectively.

 $I_2, I_3 =$  moment of inertia about  $\bar{x}_2$  and  $\bar{x}_3$  axes, respectively ds = R d\beta/cos  $\alpha$ 

In evaluating Eq. 9, a numerical procedure is used. This approach entails the evaluation of the various matrices at discrete points along the member and the integration is performed numerically by Simpson's rule.

#### 2.3 Stiffness Matrix of Member AB

The inversion of matrix fBB defined by Eq. 9 gives the end stiffness kBB, i.e.

$$k_{BB} = f_{BB}^{-1}$$
(10)

such that

$${p_{B}} = [k_{BB}] \{\bar{u}_{B}\}$$
 (11)

in which  $\{p_B\}$  and  $\{u_B\}$  are the forces and displacements at the end of the cantilevered helicoidal member. If the helicoidal member is a part of a structure, then in addition to the displacements of one end relative to the other, it may also undergo rigid body motion defined by the displacement  $u_A$  at end A. In order to establish a member stiffness, k, these rigid body displacements must be taken into account. The stiffness matrix, k, which relates the end forces and end displacements in terms of nodal axes is given by the following equation,

$$\{p_A p_B\} = [k] \{u_A u_B\}$$
(12)

The end forces  $p_A$  at A are in equilibrium with  $p_B$ , thus

IABSE PERIODICA 4/1981

IABSE PROCEEDINGS P-48/81

$$p_{A} = -\hat{T} p_{B}$$
(13)

The end actions at A and B in terms of the  $p_{R}$  are therefore given by

$$\begin{bmatrix} \mathbf{p}_{A} \\ \mathbf{p}_{B} \end{bmatrix} = \begin{bmatrix} -\hat{T} \\ I \end{bmatrix} \begin{bmatrix} \mathbf{p}_{B} \end{bmatrix}$$
(14)

and by contragredience,

$$[\bar{u}_B] = [-\hat{T}^T I] \{u_A u_B\}$$
(15)

Since the strain energy, U, in the helicoidal member remains the same with or without the inclusion of the rigid body motion, the strain energy in terms of  $k_{BB}^{}$  is given by

$$U = \frac{1}{2} \overline{u}_{B}^{T} k_{BB} \overline{u}_{B}$$
$$= \frac{1}{2} \left[ u_{A}^{T} u_{B}^{T} \right] \begin{bmatrix} -\widehat{T} \\ I \end{bmatrix} \begin{bmatrix} k_{BB} \end{bmatrix} \begin{bmatrix} -\widehat{T}^{T} & I \end{bmatrix} \begin{bmatrix} u_{A} \\ u_{B} \end{bmatrix}$$
(16)

and in terms of the full stiffness matrix, k, of helicoidal member,

$$U = \frac{1}{2} \begin{bmatrix} u_A^T & u_B^T \end{bmatrix} \begin{bmatrix} k \end{bmatrix} \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$
(17)

Comparing Eqs. 16 and 17, it follows that

$$k = \begin{bmatrix} -\hat{T} \\ I \end{bmatrix} \begin{bmatrix} k_{BB} \end{bmatrix} \begin{bmatrix} -\hat{T}^{T} & I \end{bmatrix}$$
(18)  
$$\begin{bmatrix} \hat{T} & k_{BB} \hat{T}^{T} & -\hat{T} & k_{BB} \end{bmatrix}$$

or

$$\mathbf{k} = \begin{bmatrix} \hat{\mathbf{T}} \mathbf{k}_{BB} \hat{\mathbf{T}}^{T} & -\hat{\mathbf{T}} \mathbf{k}_{BB} \\ & & \\ -\mathbf{k}_{BB} \hat{\mathbf{T}}^{T} & \mathbf{k}_{BB} \end{bmatrix}$$
(19)

Equation 19 is the stiffness matrix of the helicoidal member AB expressed in terms of the nodal axes.

#### 2.4 Equivalent Nodal Loads {Q}

#### 2.4.1 Concentrated load on AB

Let the load vector at D, expressed in terms of the nodal axes, be denoted  $\{p_D\} = \{p_{D_1}, p_{D_2}, p_{D_3}, \dots, p_{D_6}\}$ . The angle  $\Omega_D$  defines the location of load  $p_D$  as shown in Fig. 2(a). It can be shown that the displacement vector at the release B, caused by  $p_D$  at point D, is given by

$$u_{BO} = \int H^T D^{-1} H_o ds$$
 (20)

169

where  $H_0$  is the transformation matrix of the loads at D to the internal actions  $\{\sigma_C\}$ . To calculate  $H_0$ , the expression given by Eq. 8 applies, but  $\Omega_D$  replaces  $\Omega_B$ , coordinates of point D replace that of B, and for  $\beta > \Omega_D$ ,  $H_0$  = 0.

Having found  $u_{\text{BO}}$  and  $f_{\text{BB}},$  the reaction  $p_{\text{B}}$  are found considering the compatibility condition,

$$u_{BO} + f_{BB} p_B = 0$$
 (21)

or

$$p_{B} = -f_{BB}^{-1} u_{BO} = -k_{BB} u_{BO}$$
(22)





FIG. 2 CONCENTRATED AND DISTRIBUTED LOADINGS ON

HELICOIDAL MEMBER AB.



By statics, the reactions at A are given by

$$\mathbf{p}_{A} = -\hat{\mathbf{T}}_{DA} \mathbf{p}_{D} - \hat{\mathbf{T}}_{BA} \mathbf{p}_{B}$$
(23)

The equivalent nodal loads,  $\{Q\} = \{Q_A | Q_B\}$  which are numerically equal to the end reactions but act in the opposite directions, are given by,

$$\{Q\} = \{Q_A Q_B\} = -\{p_A p_B\}$$
 (24)

#### 2.4.2 Distributed Loading on AB

Consider the cantilevered helicoidal member AB subjected to a distributed loading,  $w = w(\theta)$ , in which  $\theta$  is a horizontal angle as shown in Fig. 2(b). The components of the distributed loading are considered positive in the positive directions of the  $x_i$  - axes. The internal forces at any section C, due to distributed loading on portion CB, are given by

$$\{\sigma_{C}\} = \hat{R} \int_{\Omega_{B}}^{\beta} \hat{T} dF$$
(25)

where R and T are the rotation and translation matrices, respectively, and dF is the elemental load vector defined by angle  $\theta$ .

For a uniformly distributed load with intensity  $w_0$  per unit length of the horizontal projection, and acting parallel to the  $x_3$  - axis,

$$\{dF\} = \{0 \quad 0 \quad w_0 R \, d\theta \quad 0 \quad 0 \quad 0\}$$
(26)

Substitution of Eq. 26 into Eq. 25, and evaluation of the resulting integral yields,

$$\{\sigma_{c}\} = \begin{bmatrix} \lambda & o \\ o & \lambda \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & W_{0}R & (\beta - \Omega_{B}) \\ -w_{0}R^{2} & [\sin \beta - \sin \Omega_{B} - (\beta - \Omega_{B}) \cos \beta] \\ +w_{0}R^{2} & [\cos \beta - \cos \Omega_{B} + (\beta - \Omega_{B}) \sin \beta] \end{bmatrix} \equiv H_{w}$$
(27)

The end displacements  $u_{BO}$  due to a uniformly distributed load  $w_0$  is then given by,  $\Omega_p$ 

$$u_{BO} = \int_{\Omega_A}^{B} H D^{-1} H_w^T ds$$
(28)

and hence, the end reactions at B are

$$p_{B} = -k_{BB} u_{BO}$$
(29)

From equilibrium conditions, the reactions at A are

$$\mathbf{p}_{\mathbf{A}} = -\hat{\mathbf{T}}_{\mathbf{B}\mathbf{A}} \mathbf{p}_{\mathbf{B}} - \int_{\Omega_{\mathbf{B}}}^{\Omega_{\mathbf{A}}} \hat{\mathbf{T}}_{\mathbf{C}\mathbf{A}} d\mathbf{F}$$
(30)

The second term of the right-hand side of Eq. 30 is simply equal to second matrix of the right-hand side of Eq. 27 but  $\beta$  is now replaced by  $\Omega_A$ .

#### 3. NUMERICAL EXAMPLES

#### 3.1 Example 1 Deflections at Free End of Cantilever Helicoidal Girders

The vertical deflections at the free end of cantilevered helicoidal girders have been evaluated for girders with a uniform cross-section, linearly varying depth and parabolically varying depth. The geometric and material properties of these girders are given in the figure of Table 1.

The girder of a uniform section has been analysed considering two load cases, namely; (a) a point load at free end, and (b) a uniformly distributed load of intensity  $w_0$  per unit length of horizontal projection acting on the whole girder. Analyses were made with different values of the inclination angle,  $\alpha$ , and the number of segments, n, which sub-divide the girder. The numerical results tabulated in Table 1 are in excellent agreement with the closed form solutions presented by Gerstle [8]. Note that Gerstle's solutions included only the effects of bending and torsional deformations. This accounts partially for the small differences between the two solutions.

The treatment of helicoidal girders of varying cross-section is easily done, and to illustrate this point two cantilever helicoidal girders, subjected to a point load at free end, were analysed. One is a girder with linearly varying depth, and the other, with a parabolically varying depth. Both girders have constant width. Numerical results for  $\alpha = 15^{\circ}$  and n = 32 are given in Table 1.

#### 3.2 Example 2 Fixed End Actions in Helicoidal Girders

A helicoidal girder of uniform section is analysed for equivalent nodal loads of two load cases, namely: (a) a point load at the midpoint of the girder, and (b) a uniformly distributed load of intensity  $w_0$  per unit length of horizontal projection, acting on the whole girder. Using 32 segments to sub-divide the girder, numerical values for equivalent nodal loads of each load case were obtained for two values of the inclination angle,  $\alpha = 0^{\circ}$  and  $\alpha = 20^{\circ}$ , and are summarized in Table 2.

#### 3.3 Example 3 Two-Span Helicoidal Girder

A continuous helicoidal girder consisting of two spans, as shown in the figure of Table 3, is fixed at the two ends, and supported at the midspan. The intermediate support prevents displacement in the vertical direction only. The girder was analysed for three load cases, namely: (a) a point load at midpoint of span AB only; (b) a uniformly distributed load of intensity  $w_0$  per unit length of horizontal projection acting on span BC only; and (c) load cases (a) and (b) combined. Numerical values for reactions at supports were obtained for two values of the inclination angle,  $\alpha = 0^{\circ}$ , and  $\alpha = 20^{\circ}$ , and are tabulated in Table 3. Note that the numerical results obtained for load case (c) are equal to the super-position of results obtained for load cases (a) and (b). This serves as a partial check to the solution.

#### 4. CONCLUSIONS

In this paper a numerical procedure for the analysis of continuous helicoidal girder by the stiffness method is presented. The full stiffness matrix of a



173

# TABLE 1VERTICAL DEFLECTION COMPONENTS (mm) AT FREE END<br/>OF CANTILEVERED HELICOIDAL GIRDERS (EXAMPLE 1)

| Load          | No. of<br>Segments   | Inclination Angle, $\alpha$             |                                   |                                   |
|---------------|----------------------|---|-----------------------------------|-----------------------------------|
|               | n                    | 0 <sup>0</sup>                          | 15 <sup>0</sup>                   | 30 <sup>0</sup>                   |
|               |                      | Girder with Uniform Section             |                                   |                                   |
| Point<br>Load | 8<br>16<br>, Ref.[8] | -36.38673<br>-36.38673<br>-36.319       | -37.80223<br>-37.80223<br>-37.750 | -42.56490<br>-42.56490<br>-40.406 |
| u.d.1         | 16<br>Ref.[8]        | -9.69262<br>-9.638                      | -10.06906<br>-                    | -11.33578<br>-                    |
|               |                      | Girder with Linearly Varying Depth      |                                   |                                   |
| Point         | 32                   | -                                       | -15.99359                         | -                                 |
| Load          |                      | Girder with Parabolically Varying Depth |                                   |                                   |
|               | 32                   | -                                       | -17.16251                         | -                                 |



| TABLE 2 | EQUIVALENT NODAL LOADS FOR A POINT LOAD        | AND  | A UNIFORMLY |
|---------|--|------|-------------|
|         | DISTRIBUTED LOAD ON A HELICOIDAL MEMBER        | FOR  | VARIOUS     |
|         | VALUES OF THE INCLINATION ANGLE, $\alpha$ (EXA | MPLE | 2).         |

| EOF   | Inclination Angle, $\alpha$   |  |   |
|---|---|--|---|
| Components  | 0 <sup>0</sup>  | 10 <sup>0</sup>  | 20 <sup>0</sup>   |
|   | Point Load, P = -100 kN, at B   |  |   |
| $ \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6 \\ 6 \\ 6 \\ 1 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12$ | $\begin{array}{c} 0.\\ 0.\\ -50.00009\\ -45.42277\\ 125.00030\\ 0.\\ 0.\\ 0.\\ -49.99991\\ 45.42233\\ 124.99970\\ 0\end{array}$ | $\begin{array}{r} -2.18725\\ 0.00014\\ -50.00005\\ -45.09989\\ 123.48570\\ 5.46836\\ 2.18725\\ -0.00014\\ -49.99995\\ 45.09942\\ 123.48530\\ 5.46790\end{array}$ | -4.30213<br>0.00028<br>-49.99996<br>-44.12459<br>118.85100<br>10.75576<br>4.30213<br>-0.00028<br>-50.00004<br>44.12400<br>118.85090<br>10.75487 |
|   | Uniformly distributed load (udl),<br>w_ = -10 kN/m over whole span  |  |   |
| $ \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \end{cases} kN-m $  | 0.<br>0.<br>-39.26991<br>-18.59739<br>62.50000<br>0.  | -0.86175<br>-0.00000<br>-39.26991<br>-18.47017<br>61.90330<br>2.15438  | -1.69479<br>-0.00000<br>-39.26991<br>-18.08595<br>60.07763<br>4.23697   |
| $ \begin{cases} 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ \end{cases} kN-m $   | 0.<br>0.<br>-39.26991<br>18.59739<br>62.50000<br>0.   | 0.86175<br>0.00000<br>-39.26991<br>18.47017<br>61.90330<br>2.15438   | $\begin{array}{r} 1.69479 \\ 0.00000 \\ -39.26991 \\ 18.08595 \\ 60.07763 \\ 4.23697 \end{array}$   |

EQF = equivalent nodal force

E =  $200 \times 10^{6} \text{ kPa}$   $\mu$  = 0.3 b = 50 mm; d = 100 mm

5



10

| Reaction  | Only span AB  | Only Span BC  | Both Spans AB   |
|---|---|---|---|
| Components  | Loaded  | Loaded  | and BC Loaded   |
|   | Inclination Angle, $\alpha = 0^{\circ}$   |   |   |
| $ \begin{cases} 1\\2\\3 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$  | 0.  | 0.  | 0.  |
|   | 0.  | 0.  | 0.  |
|   | 64.48747  | - 5.84984   | 58.63763  |
|   | 9.55895   | - 1.53322   | 8.02573   |
|   | -80.45739   | 10.53136  | -69.92603   |
|   | 0.  | 0.  | 0.  |
| $     \begin{cases}             7 \\             8 \\           $   | 0.  | 0.  | 0.  |
|   | 0.  | 0.  | 0.  |
|   | -11.22502   | 46.47837  | 35.25335  |
|   | 2.94558   | - 4.28729   | - 1.34170   |
|   | 20.52457  | -40.75315   | -20.22858   |
|   | 0.  | 0.  | 0.  |
| 13 kN   | 46.73755  | 37.91129  | 84.64884  |
| $     \begin{bmatrix}       1 \\       2 \\       3     \end{bmatrix} kN     $ $     \begin{cases}       4 \\       5 \\       6     \end{bmatrix} kN-m $ | Incli<br>0.43957<br>- 2.13174<br>63.64783<br>10.19532<br>-78.19164<br>- 4.46240 | nation Angle, α<br>- 0.13842<br>0.45869<br>- 5.61513<br>- 1.65617<br>9.82650<br>1.07285 | 0.30115<br>- 1.67305<br>58.03270<br>8.53915<br>-68.36514<br>- 3.38954 |
| $ \begin{cases} 7\\8\\9 \end{cases} kN \\ 10\\11\\12 \end{cases} kN-m $   | - 0.43957   | 0.13842   | - 0.30115   |
|   | 2.13174   | -0.45869  | 1.67305   |
|   | -11.07113   | 46.49947  | 35.42834  |
|   | 2.92330   | -4.29691  | -1.37361  |
|   | 20.61198  | -40.88896   | -20.27697   |
|   | 2.26453   | - 0.38074   | 1.88380   |
| 13 kN   | 47.42330  | 37.65548  | 85.07878  |

## TABLE 3REACTION COMPONENTS AT THE SUPPORTS OF A<br/>TWO-SPAN HELICOIDAL GIRDER (EXAMPLE 3).



helicoidal member is derived by first evaluating the flexibility matrix at one end which when inverted gives the end stiffness matrix. The Simpson's rule is used to evaluate the integrals associated with the flexibility matrix and equivalent load vector. The effects of axial, bending, shear and torsional deformations are included.

The results from the numerical examples indicate the high degree of accuracy of the proposed numerical method of analysis.

The analysis of helicoidal girders of varying cross-section can be treated quite easily. However, in order to obtain sufficiently accurate solutions more segments to subdivide the girder are generally required.

- 5. REFERENCES
- BERGMAN, V.R.: "Helicoidal Staircases of Reinforced Concrete", Journal ACI, Vol. 28, No. 4, Oct., 1956.
- [2] COHEN, J.S.: "Design of Helical Staircases", Concrete and Construction Engineering, Vol. 50, No. 5, London, England, May, 1955.
- [3] YOUNG, Y.F. and SCORDELIS, A.C.: "An Analytical and Experimental Study of Helicoidal Girders", Journal of the Structural Division, ASCE, Vol. 84, No. ST5, Proc. Paper 1756, Sept., 1958.
- [4] McMANUS, P. et al.: "Horizontally Curved Girders State-of-the-Art", Journal of the Structural Division, ASCE, Vol. 95, No. ST5, Proc. Paper 6556, May, 1969.
- [5] ABDUL-BAKI, A. and SHUKAIR, A.: "Continuous Helicoidal Girders", Journal of the Structural Division, ASCE, Vol. 99, No. ST10, Proc. Paper 10108, Oct., 1973.
- [6] DERRON, M. and JIROUSEK, J. : "Elements Spatiaux de Barres Courbes", Publications, International Association for Bridge and Structural Engineering, Zurich, Vol. 36-II, 1976.
- [7] HALL, A.S. and WOODHEAD, R.W. : Frame Analysis, John Wiley and Sons, Inc., New York, N.Y., 1967.
- [8] GERSTLE, K.H.: <u>Basic Structural Analysis</u>, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974.