The Design of a Continuous Structure made up of Precast Units

Dimensionnement d'une structure continue réalisée à l'aide d'éléments préfabriqués

Die Bemessung eines aus vorgefertigten Einfeldträgern hergestellten Durchlaufträgers

René SUTER

Dr. ès sc. techn.
Swiss Federal Institute of Technology (EPF)
Lausanne, Switzerland

SUMMARY

The problems relating to the redistribution of forces due to creep in structures built in different stages are being steadily better mastered. The usual approaches, however, neglect the possible discontinuity of deformations at joints. This nevertheless occurs in the case of connections in buildings, for which the discontinuity can reduce the moments over supports by a factor of two. This phenomenon greatly influences the serviceability state (cracking and deformations). The work presented is the result of a research projet carried out at the EPF Lausanne under the direction of Prof. R. Favre and in close collaboration with Prof. B. Thürlimann, ETH Zurich.

RÉSUMÉ

On domine toujours mieux les problèmes de la redistribution des efforts due au fluage dans des structures exécutées par étapes. On néglige toutefois dans les approches courantes une discontinuité éventuelle de la déformée au droit des joints. Tel est pourtant le cas dans les liaisons du bâtiment, où cette discontinuité peut réduire les moments sur appuis d'un facteur deux. Ce phénomène a de grandes influences sur l'état d'utilisation (fissuration et déformations). Le travail présenté est le résultat d'un projet de recherche poursuivi à l'EPF Lausanne, sous la direction du prof. R. Favre et en étroite collaboration avec le prof. B. Thürlimann, ETH Zurich.

ZUSAMMENFASSUNG

Der entwerfende Ingenieur verfügt heute über leicht anwendbare Methoden zur Abschätzung der Kräfteumlagerungen infolge Kriechen von etappenweise hergestellten Betontragwerken. Diese Bemessungsverfahren vernachlässigen jedoch im allgemeinen einen möglichen Knick der Biegelinie an der Verbindungsstelle, was im Industrie- und Hochbau kaum vermieden werden kann. Die damit verbundene Verminderung der Stützmomente beeinflusst nachhaltig das Verhalten des Tragswerks im Gebrauchszustand (Rissebildung und Verformungen). Die vorliegende Arbeit entstand im Rahmen eines Forschungsprojektes an der ETH Lausanne, unter der Leitung von Prof. R. Favre und in enger Zusammenarbeit mit Prof. B. Thürlimann, ETH Zürich.

1. DESCRIPTION OF THE STUDY

1.1 Introduction

Consider a series of n prefabricated reinforced concrete elements placed in position as freely supported simple beams and ulteriorly connected (figure 1). The initially isostatic system is transformed, as a result of the connections, into a hyperstatic system - a continuous beam of n spans -.
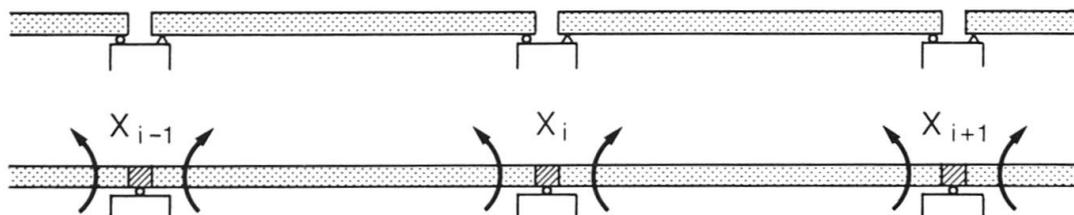

Fig. 1

Such a construction is distinguished from a monolithic structure by :
- the construction procedure in successive stages
- the presence of joints over supports

The aim of this investigation is to comprehend these two peculiarities, which have a capital influence on the distribution of internal forces along the structure, and to study their consequences. Accent is placed on the behaviour at the serviceability limit states, i.e. deformations and cracking.

1.2 Redistribution of forces in time

In a first stage we will consider the repercussions of the construction procedure. As a result of the deformations due to creep of the elements under their self weight, a bending moment occurs over the intermediate supports. This phenomenon can be explained by the fact that the deformations are impeded by the connections and thus create, in a hyperstatic system, additional forces.

Various authors have previously investigated this phenomenon and the evolution in time of the forces over supports. A first answer with respect to the intensity of the moments X_i over support (figure 1) was given by Dischinger [5]. On the basis of his fundamental hypothesis of linear creep this moment is given by :

$$X_i(t) = X_i^{mo} \cdot (1 - e^{-\varphi})$$ (1)

X_i^{mo} : moment over support i assuming a continuous monolithic system

$\varphi(t)$: coefficient of creep between the realization of the connection and the time t considered

At the same time as the development of the creep theories, this problem was taken up by other authors. The calculation models and numerical values have evolved considerably. Rüsch [5] thus gives :

$$X_i(t) = X_i^{mo} \cdot (1 - c_r) \tag{2}$$

c_r : coefficient describing the influence of creep for an instantaneously imposed deformation

According to Trost [9] the moment over a support is :

$$X_i(t) = X_i^{mo} \frac{\varphi}{1 + \rho\varphi} \tag{3}$$

ρ : aging coefficient according to Trost (for the general case Trost takes $\rho = 0,8$)

The aging coefficient has been taken up by Bazant [1], Favre [4] and the CEB [2] using the symbol χ. The consideration of more parameters enables the different characteristics of the concrete and the different loading ages to be better taken into account.

These various theories nevertheless show that the moment over support, resulting from the creep of elements due to self weight, is comparatively high and close to the total continuity moment of a monolithic beam. A number of authors even propose to calculate the whole structure as if it had been built in one single operation and to reduce the moment over support by about 15 %, as the effect of the construction procedure by different stages can practically be neglected.

1.3 Influence of the connection

The previous theoretical considerations on the redistribution of forces are based on idealized static systems, from simple supports to perfect connections. It is this absolute rigidity of the connections which explains such an important increase in negative moment over support. However, rigid connections are only carried out on constructions of a certain importance (for bridges for exemple), by means of prestressing over the supports.

For buildings or industrial constructions, on the other hand, where the spans are short or medium and the thicknesses of the elements comparatively small, prestressing over the supports is not usually considered, for economical reasons. This results in a certain flexibility of the connection, which introduces a discontinuity in the overall behaviour of the continuous structure.

The constructional imperfections of the connections influence the redistribution phenomena mentioned and the moments created over supports are decreased with respect to the theoretical values.

Consider the case of a two span continuous beam, made up of two identical beams having a common support B (figure 2).
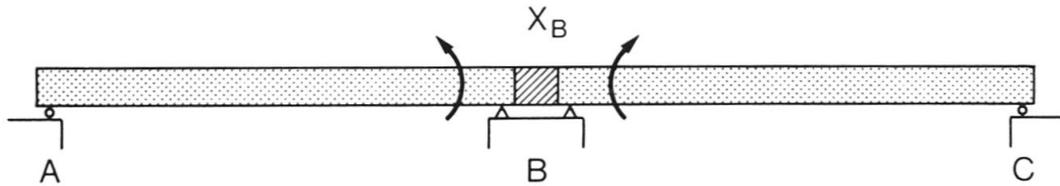
Fig. 2

For loads of short duration, applied to such a system with a perfectly rigid connection, one obtains the well known equation expressing the compatibility of the rotations :

$$\alpha_o + \alpha_1 \cdot X_B = 0 \quad \rightarrow \quad X_B = - \frac{\alpha_o}{\alpha_1} \tag{4}$$

α_o : rotation due to the loads, isostatic system

α_1 : rotation due to a unit moment $X_B = 1$, isostatic system

If the continuity through the connection is not perfect, equation (4) becomes :

$$\alpha_o + \alpha_1 \cdot X_B - \gamma = 0 \qquad X_B = - \frac{\alpha_o - \gamma}{\alpha_1} \tag{5}$$

γ : angle of discontinuity over support B

The object of this investigation is, in particular, to determine the discontinuity over support as a function of the realization of the connection and the loading pattern.

2. DISCONTINUITY OVER SUPPORT

2.1 Calculation principle

In order to take into account the quantitative influence of the connections, we propose a calculation method which allows, by comparatively simple means, a good estimation of the repercussions on the distribution of forces. This method consists in assuming the moment of inertia in naked state II over a certain length (figure 3).
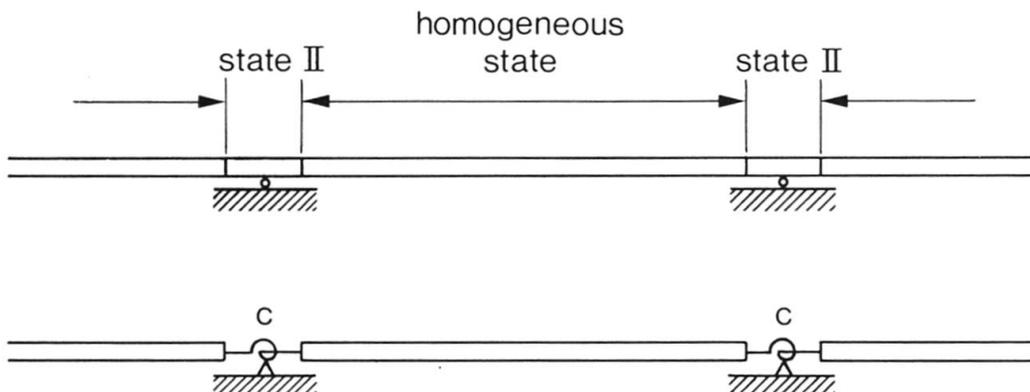

Fig. 3

To calculate the forces, the effect of this length l_a is concentrated at a single point over the support by introducing a spring with a coefficient c :

$$c = \frac{l_a \cdot k_a^{II}}{E_{cl} \cdot I_{cl}}$$ (6)

l_a : active length of the connection (chapter 2.2)

k_a^{II} : correction factor taking into account the moment of inertia in state II (figure 4)

E_{cl} : modulus of elasticity of the concrete of the connection

I_{cl} : moment of inertia of the connection in the homogeneous state
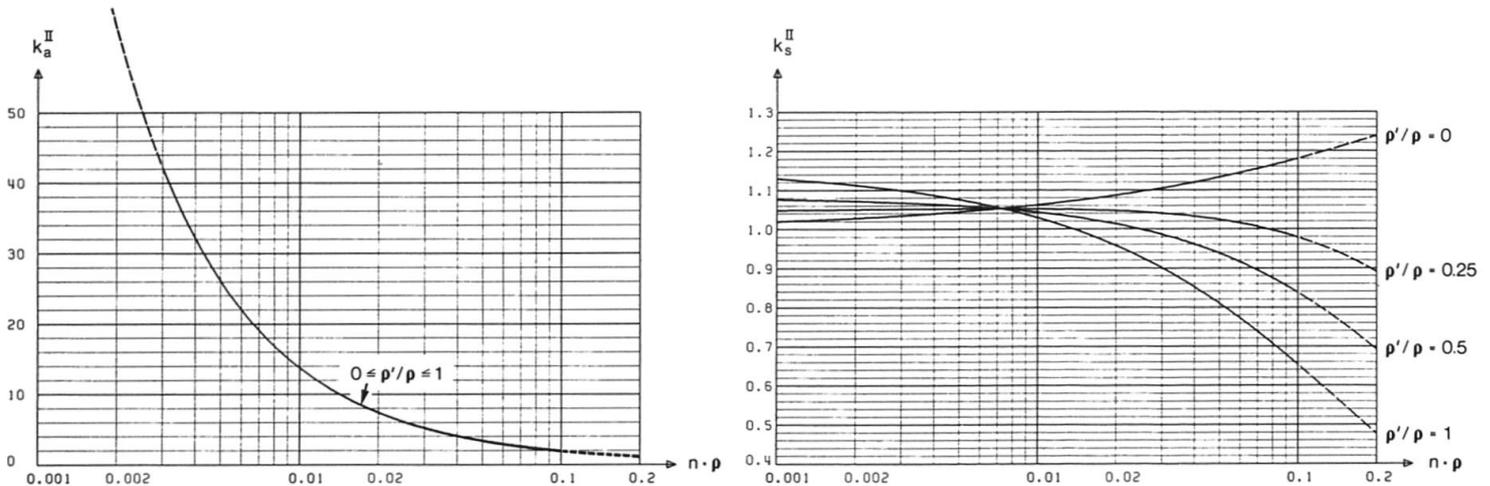
Fig. 4

In [7] it is shown that the use of a perfectly elastic spring coefficient c does not lead to any intolerable imprecision, for loads of either short or long duration.

2.2 Active length of the connection

If the terms E_{cl}, I_{cl} and k_a^{II} of equation (6) are clearly defined, the same cannot be said for the active length l_a over which the decrease in stiffness must be calculated. We base the determination of this length l_a on considerations regarding the transmission of forces by bond between the reinforcement and the concrete [7]. Moreover, it is necessary to distinguish between the various possible ways of realizing the connection.

If the transmission of tensile forces between the reinforcement bars relies on bond (bar - concrete - bar), as is the case with hook joints (figure 5) or lapping bars, the active length is given by :

$$l_a = l_1 + 12 \phi \leqslant 52 \phi$$ (7)

l_1 : length of the connection in the tension zone, limited to 40 ϕ

ϕ : diameter of the reinforcement bars at the joint
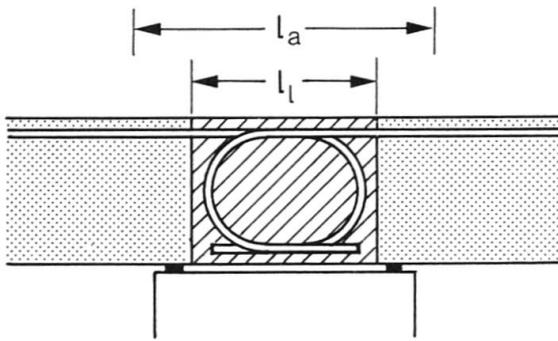
Fig. 5
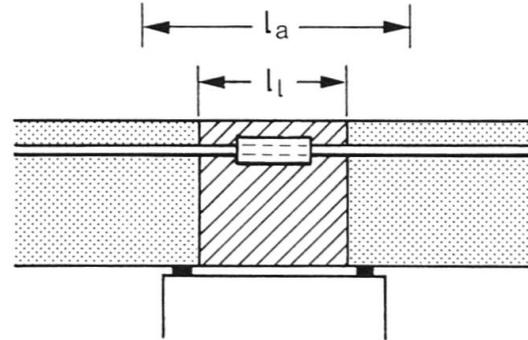
Fig. 6

In the case of direct joints, which include for example welding or other rigid coupling of bars (figure 6), the length l_1 is limited to 20 ϕ. The active length can also be given by :

$$l_a = l_1 + 12 \phi \leqslant 32 \phi \tag{8}$$

For a connection in which continuous reinforcement is cast in a covering layer of concrete (figure 7), the bond conditions are slightly modified. The active length is :

$$l_a = l_1 + 16 \phi \leqslant 32 \phi \tag{9}$$
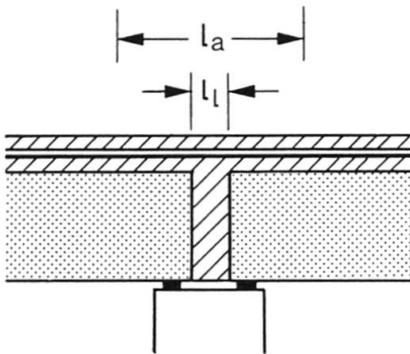
Fig. 7
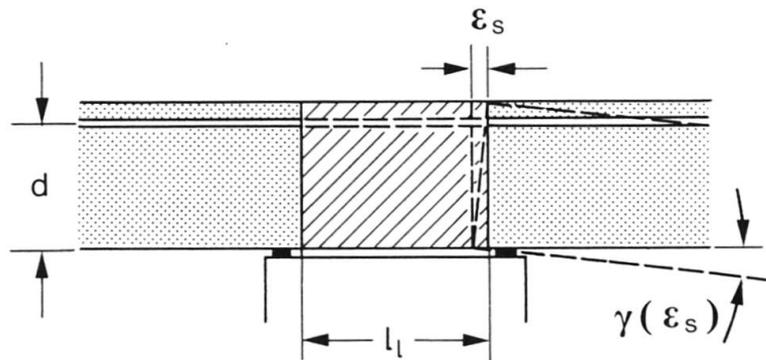
Fig. 8

For other types of connection the active length should by determined by analogy. Care shall also be taken to introduce the correct values for the moment of inertia in the case of a decrease of the support section.

One must however underline that these are estimates that are subject to a margin of error of appriximately 20 % due to the poor knowledge and very wide dispersion of the characteristics of the materials used.

2.3 Shrinkage of the concrete forming the connection

Shrinkage of the concrete forming the connection produces a rotation over support and therefore a reduction of the continuity moment. As the section over support is in state II, the resulting angle is given by the integral of the curvature along the length of the connection (figure 8) :

$$\gamma(\varepsilon_s) = \int \frac{\varepsilon_s(t) \cdot k_s^{II}}{d} \, dx = \frac{\varepsilon_s(t) \cdot k_s^{II} \cdot l_1}{d} \tag{10}$$

l_1 : length of connection in the compression zone

d : effective depth of the connection

k_s^{II} : coefficient of correction for shrinkage in state II (figure 4)

$\varepsilon_s(t)$: shrinkage strain (see [2] or [4])

Estimation (10) is valid only if the concrete of the connection has undergone a suitable treatment, which consists in wetting the neighbouring surfaces before pouring the concrete and in maintaining humidity after concreting.

2.4 Limits of application

The design of a structure assembled according to the procedure described allows a decrease in reinforcement percentage over support compared to that resulting from an elastic calculation with perfectly rigid connections. This decrease should however not be so great as to compromise the satisfactory behaviour of the structure in the serviceability state.

On the basis of the tests carried out at the EPF Lausanne [3], the following limitation would seem reasonable :

$$X_i > 0,5 \; X_i^{mo}$$

$$\rho_i > 0,5 \; \rho_i^{mo} \tag{11}$$

In addition the limitations concerning the minimum reinforcement percentage given by [2] shall be complied with.

The reduction of the moment over support decreases as the thickness of the section of the connection increases. The constructional details studied here are only suited to elements whose connection sections have an effective depth of less than 400 mm approximately. Their application is therefore limited mainly to elements of industrial constructions and buildings.

For the construction of bridges by means of prefabricated beams, one can distinguish, as a rule, the following two possibilities :

- realization of rigid or quasi-rigid connection by means of prestressing or heavy reinforcement over support [8].

- realization of a very flexible connection carrying hardly any bending moment (flexible cover slab)

One should bear in mind that every section of a hyperstatic structure should behave in a ductile manner to failure. This is all the more important as the actual distribution of internal forces in the short and especially the long term is highly uncertain, despite all the calculations performed [10]. A brittle fracture is to be feared in the connection zones, in particular in the case of joints carried out by welding, if sufficient constructional measures and precautions are not observed.

High level repetitive loads are in danger of weakening the bond between reinforcement and concrete. They can therefore produce wide cracks at joints or even premature failure of the connection in the case of insufficient anchorage lengths. Appropriate measures should thus be taken with regard to the conception of connections. In certain cases this type of structure should be abandoned.

3. ALGEBRAIC EQUATIONS OF DEFORMATIONS

3.1 Introduction of a spring over support

We start from the system of equations for a continuous monolithic beam with n redundants, subjected to an instantaneous load. Equation i corresponds to :

$$X_{i-1}\, a_{i,i-1} + X_i\, a_{ii} + X_{i+1}\, a_{i,i+1} + a_{io} = 0 \tag{12}$$

For a same beam made up of prefabricated elements the spring coefficients are introduced according to (6). The resulting static system is represented in figure 9 and, for an instantaneous load, equation i becomes :

$$X_{i-1}\, a_{i,i-1} + X_i(a_{ii}+c_i) + X_{i+1}\, a_{i,i+1} + a_{io} = 0 \tag{13}$$

Fig. 9

3.2 Introduction of deferred effects
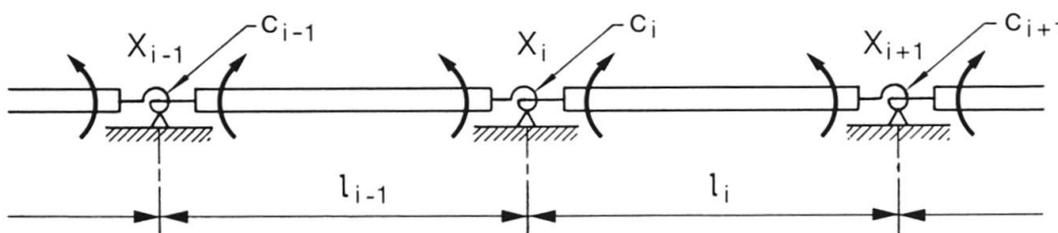
In order to be able to integrate the creep and relaxation of the concrete in the equations of compatibility of the rotations, an aging coefficient χ [4,9] is introduced. The fundamental equation relating stress, strain and time is :

$$\varepsilon(t) = \frac{\sigma(t_o)}{E}\, [1+ \varphi(t,t_o)] + \frac{\sigma(t)- \sigma(t_o)}{E}\, [1+ \chi(t,t_o)\, \varphi(t,t_o)] \tag{14}$$

$\varphi(t,t_o)$: creep coefficient at time t, starting from time t_o

$\chi(t,t_o)$: aging coefficient at time t, starting from time t_o

Figure 10 gives the evolution of coefficients $\varphi(t,t_o)$ and $\chi(t,t_o)$ for a concrete of usual quality [4].
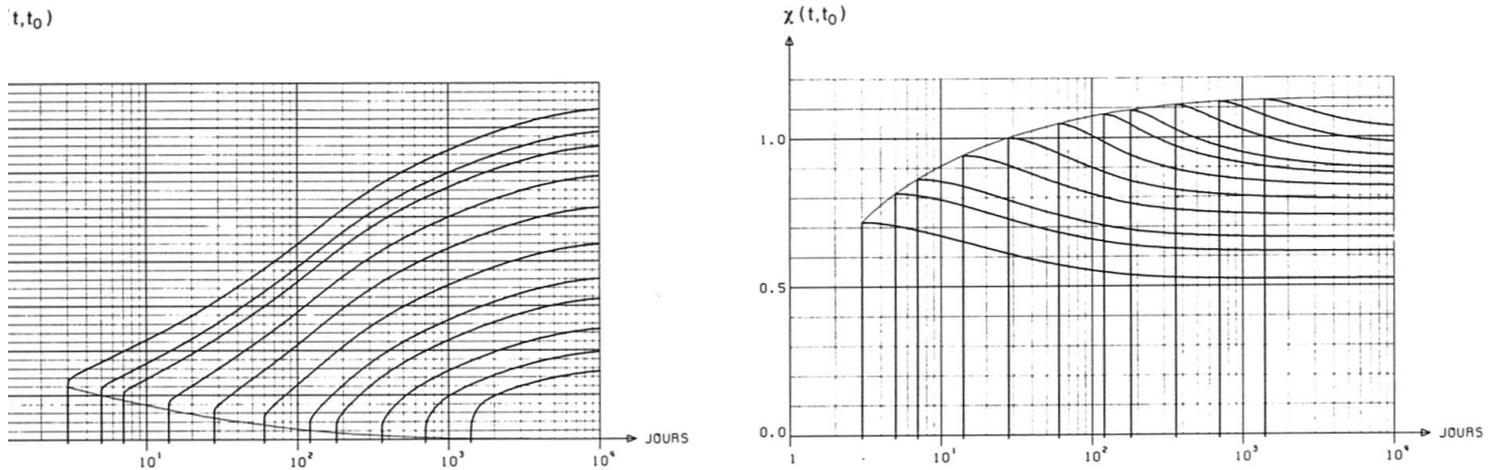
Fig. 10

The deformations due to actions that are constant in time are thus multiplied by the factor $(1+\varphi)$ and those due to variable actions by the factor $(1+\chi\varphi)$.

Consider the system of beams connected by springs as represented in figure 9. The unknown moment $X_i(t)$ over support breaks down as follows :

$$X_i(t) = X_i(t_o) + \Delta X_i(t) \tag{15}$$

$X_i(t_o)$: moment over support at time t_o, according to (13)

$\Delta X_i(t)$: variation of the moment over support in time

The assumption of purely elastic springs leads to the following equation :

$$X_{i-1}(t_o)\, a_{i,i-1}\, (1+\varphi) + \Delta X_{i-1}(t)\, a_{i,i-1}\, (1+\chi\varphi) + X_i(t_o)\, a_{ii}\, (1+\varphi) +$$

$$X_i(t_o)\, c_i + \Delta X_i(t)\, a_{ii}\, (1+\chi\varphi) + \Delta X_i(t)\, c_i + X_{i+1}(t_o)\, a_{i,i+1}\, (1+\varphi) +$$

$$\Delta X_{i+1}(t)\, a_{i,i+1}\, (1+\chi\varphi) + a_{io}\, (1+\varphi) - \gamma_i(\varepsilon_s) = 0 \tag{16}$$

3.3 Equations for the various load cases

In this paragraph we establish the equations of compatibilty of rotations over supports for three load cases :

- loads of short duration
- loads of long duration
- transformation of static system

For loads of short duration, applied to a continuous structure with connections, line i of the system of equations is given by :

$$X_{i-1} \, a_{i,i-1} + X_i (a_{ii} + c_i) + X_{i+1} \, a_{i,i+1} + a_{io} = 0 \qquad (13)$$

For loads of long duration, applied at time t, after realization of the connection, one first calculates the redundants $X_i(t_1)$. The variations in time $\Delta X_i(t)$ of these terms are then established. Taking into account (13), equation (16) can be reduced to :

$$X_{i-1} \, a_{i,i-1} \, \varphi + \Delta X_{i-1} \, a_{i,i-1} \, (1+\chi\varphi) + X_i \, a_{ii} \, \varphi + \Delta X_i \, [a_{ii} \, (1+\chi\varphi) + c_i] +$$

$$X_{i+1} \, a_{i,i+1} \, \varphi + \Delta X_{i+1} \, a_{i,i+1} \, (1+\chi\varphi) + a_{io} \, \varphi - \gamma_i(\varepsilon_s) = 0 \qquad (17)$$

In the third case we consider permanent loads applied on the isostatic beams at time t_o before the connection is made. On the basis of a possible imposed moment $X_i(t_o)$, due for example to prestressing, one calculates the moments created in time as a result of creep of the prefabricated elements as well as of shrinkage of the connection :

$$X_{i-1} \, a_{i,i-1} \, \varphi + \Delta X_{i-1} \, a_{i,i-1} \, (1+\chi\varphi) + X_i \, (a_{ii} \, \varphi + c_i) + \Delta X_i \, [a_{ii} \, (1+\chi\varphi) + c_i] +$$

$$X_{i+1} \, a_{i,i+1} \, \varphi + \Delta X_{i+1} \, a_{i,i+1} \, (1+\chi\varphi) + a_{io} \, \varphi - \gamma_i(\varepsilon_s) = 0 \qquad (18)$$

Coefficients χ and φ take into account, for the various terms of the linear equations (17) and (18), the ages of the concrete at the time of application of the actions.

4. CALCULATION PROCEDURE

4.1 Estimation of connection reinforcement

This chapter schematically presents a design and calculation procedure for a continuous structure composed of prefabricated elements. This procedure cannot systematically be applied in a hard-and-fast manner, in view of the large number of possible constructional solutions, but it constitutes a tool with which the connection can be designed and its satisfactory behaviour checked.

Consider a structure for which the following data are available :

- span lengths l

- sizes of the prefabricated elements

- self weight

- permanent and variable loads.

In a first stage the distribution of moments is determined on the assumption of a continuous monolithic structure. In order to take into account the construction procedure and a redistribution of forces due to creep of the prefabricated elements, the fixing moments are altered by about 25 % of the support moment resulting from the loads acting before the connection is made. This has the advantage of providing, from the very beginning, a good approximation for design.

If the prefabricated elements are prestressed, allowance should be made for as described in [4].

One then determines the type of connection. As a very general rule one can distinguish flexible, medium rigid, rigid and very rigid connections. Obviously the choice of connection also involves the effective depth of the elements and especially of the connection. The greater this depth, the more the connection is rigid, and must be designed in consequence. For effective depths of the connection section exceeding 400 mm approximately, prestressing of the elements or of the connection is necessary, in the majority of cases.

Depending on the type and effective depth of the connection, the percentage of reinforcement over support ρli can be estimated by means of the following table :

type of connection	d (max)	$\rho li/\rho ri$
flexible connection (hooks, lapping)	200 mm	0,5 ÷ 0,7
medium rigid connection (covering layer of reinforced concrete)	300 mm	0,6 ÷ 0,8
rigid connection (welding, sleeves)	400 mm	0,7 ÷ 0,9
very rigid connection (prestressing)	> 400 mm	0,8 ÷ 1,0

4.2 Preliminary design

Knowing the type of connection and its reinforcement percentage, the spring coefficient c can be determined (6).

The characteristics of the materials, in particular the moduli of elasticity of the concretes and coefficients χ, φ and ε_s, depend on the choice and composition of the concrete, on the working procedure and sequence, and on the atmospheric conditions. In the majority of cases it will in fact be difficult to determine these values a priori at the preliminary design stage and it will be necessary to take into account a certain margin of error by introducing minimum and maximum values.

One thus has the necessary data to establish the algebraic equations from which the actual moments over supports are obtained. If these are in good agreement with the moments initially assumed, the deformations can be checked. If not, corrections should be made, either to the conception of the prefabricated elements or to the connection :

- modification of the reinforcement lay out (percentage unchanged)
- modification of the reinforcement percentage of the connection
- alteration of the type of connection
- prestressing of the elements or of the connections

4.3 Checking deformations

As have also shown the experimental studies performed at the EPF Lausanne [3], the degree of fixity has a considerable influence on the deformations of the assembled structure. The deformations should therefore be checked, further to the preliminary design.

The angles over supports are given by :

$$\gamma_i = - c_i \cdot X_i \tag{19}$$

for loads of short duration and by :

$$\gamma_i = - (X_i + \Delta X_i) \, c_i + \gamma_i (\varepsilon_s) \tag{20}$$

for loads of long duration

The deflections of a continuous structure with connections are derived from the isostatic system, - a series of n simply supported beams -, from which one deducts the effects of the redundants X_i.

Fig. 11

The deflection at mid-span of a connected beam, due to a short duration load is given by (figure 11) :

$$a_i = a_i^{ps} + a_{ii} \, (X_i + X_{i+1}) \tag{21}$$

a_i^{ps} : deflection at the middle of span i with the loads acting on the isostatic system

a_{ii} : deflection at the middle of span i due to a unit moment applied to an adjacent support

For long duration loads, the various terms are multiplied by φ, $(1+ \varphi)$ or $(1+ \chi\varphi)$ according to the loading pattern and taking into account the ages at the time of load application.

The width of the cracks at the joints in the concrete can be estimated from the angles over supports (figure 12) :
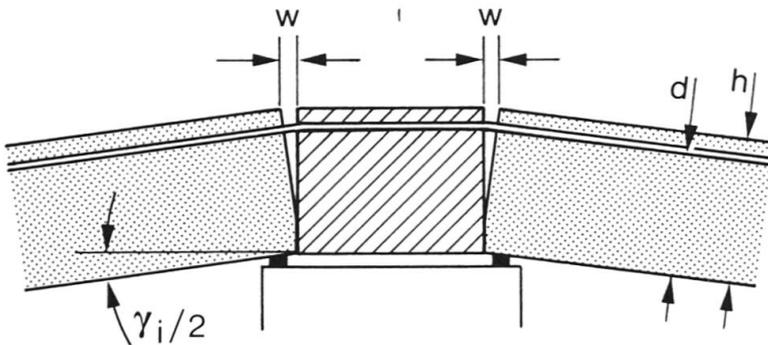
$$w = 0,4 \cdot \gamma_i \cdot d \tag{22}$$

Fig. 12

If the deformations are too great or the cracks too wide, modifications should be made, either to the conception of the prefabricated elements or to the connection.

On the basis of the moments at mid-span, the stresses in the concrete in these sections, in the homogeneous state, can be determined. One can thus check that the tensile strength of the concrete is not grossly exceeded. The tests performed at the EPF Lausanne have shown that, for good prefabricated concrete, cracking remains negligeable for tensile stresses in the concrete up to $\sigma_c = 4,0$ N/mm^2 approximately, and that the fundamental hypotheses, on which this study is based, are still valid.

5. NUMERICAL EXAMPLE

5.1 Data

Consider an industrial building covering a surface of 45,00 m x 15,00 m (figure 13). It is made up of prestressed beams at 5,00 m centres, transversely, and, in the longitudinal direction, of prefabricated reinforced concrete slab elements measuring 5,00 m x 1,00 m and 0,18 m thick.

We are concerned here with the design of the prefabricated elements, in the longitudinal direction, and with the connections between them.

The prefabricated elements, presumed identical, are placed in position at the age of 14 days. At the age of 28 days the connections are made and at the age of 60 days a permanent load is applied :

- self weight of the elements : $g = 4,50$ kN/m
- distributed permanent load : $q = 2,00$ kN/m
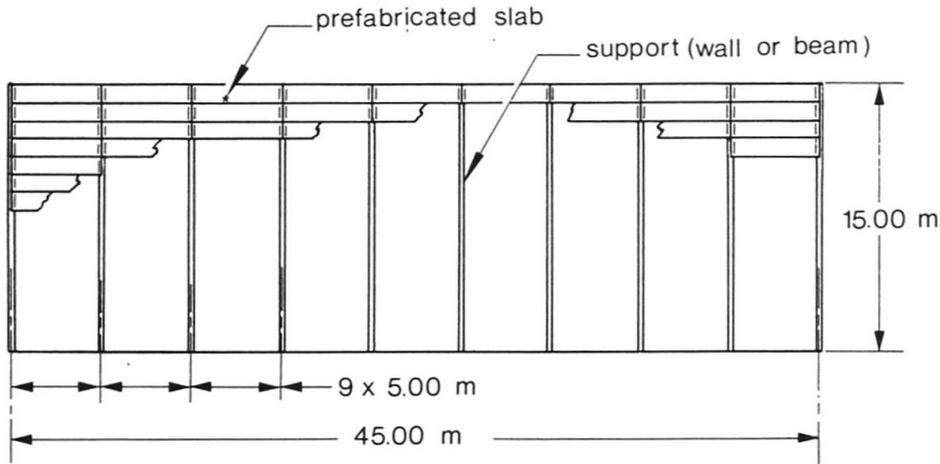- distributed live load : $p = 4,00$ kN/m

Fig. 13

5.2 Reinforcement of the connection

Should the structure be carried out in a single operation (slab cast in place), the distribution of moments would be that of a monolithic structure (figure 14). To simplify we consider element i of an infinitely long strip.
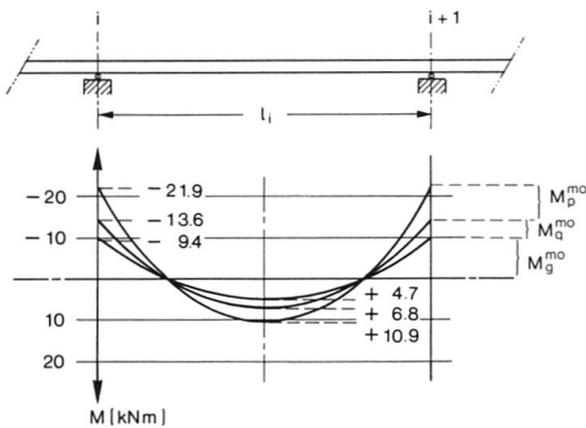

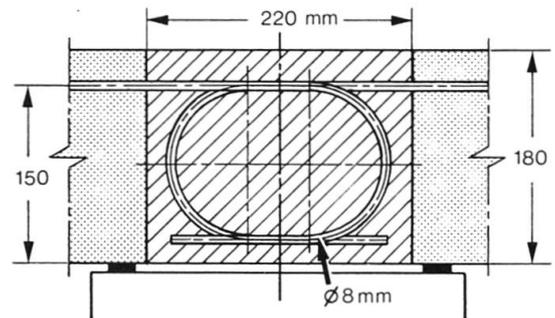Fig. 14   Fig. 15

In order to take into account the construction procedure and the redistribution of forces, 25 % is deducted from the moment M_g^{mo} over support, which gives, in the case of a perfectly rigid connection :

$$M^{ri} \approx 0{,}75\ M_g^{mo} + M_q^{mo} + M_p^{mo} = 19{,}5\ \text{kNm}$$

The connection of the prefabricated elements over support is produced by means of hooks (figure 15).

It is thus assumed, for the estimation of the reinforcement area, a reduction of approximately 30 % with respect to that of a perfectly rigid connection (chapter 4.1) :

$$A_s \approx 0{,}7\ \frac{M^{ri} \cdot 1{,}8}{0{,}9 \cdot d \cdot f_{0,2}} = 396\ \text{mm}^2$$

$8 \phi 8$ (402 mm^2) are chosen, which corresponds to a percentage of reinforcement of :

$$\rho = 0,27 \%$$

The active length l_a and the spring coefficient are :

$$l_a = l_1 + 12 \phi = 316 \text{ mm}$$

$$c_i = \frac{l_a \cdot k_a^{II}}{E_{cl} \cdot I_{cl}} = \frac{0,316 \cdot 6,8}{2,0 \cdot 10^7 \cdot 4,86 \cdot 10^{-4}} \cdot \frac{1}{\text{kNm}} = 0,221 \cdot 10^{-3} \frac{1}{\text{kNm}}$$

5.3 Distribution of moments

For self weight, which acts before the connection is made, equation (18) provides :

$$X_g(t_o) = 0$$

$$\Delta X_{i-1}(t) = \Delta X_i(t) = \Delta X_{i+1}(t) = \Delta X_g(t)$$

$$a_{i,i-1} = a_{i,i+1} = 0,25\ a_{ii}$$

$$1,5\ \Delta X_g\ a_{ii}\ (1+ \chi\varphi) + \Delta X_g\ c_i + a_{io}\ \varphi^* - \gamma_i(\varepsilon_s) = 0$$

$$\Delta X_g = -\frac{a_{io}\ \varphi^* - \gamma_i(\varepsilon_s)}{1,5\ a_{ii}\ (1+ \chi\varphi) + c_i}$$

The angle $\gamma_i(\varepsilon_s)$ at time t is, for usual outside atmospheric conditions (10) :

$$\gamma_i(\varepsilon_s) = \frac{\varepsilon_s \cdot l_1 \cdot k_s^{II}}{d} = 0,52 \cdot 10^{-3}$$

Coefficients φ and χ are taken from figure 10 :

$$\varphi(t_\infty, 28j) = 2,6$$

$$\chi(t_\infty, 28j) = 0,79$$

The term φ^* takes into consideration the creep that has already occurred in the isostatic system.

$$\varphi^* = \varphi(t_\infty, 14j) - \varphi(28j, 14j) = 2,2$$

For preliminary designs, where the characteristics of the materials are not well known, the following estimation is sufficient :

$$\varphi = 2,0 \text{ à } 3,0 \quad \text{according to the loading schedule}$$

$$\chi = 0,8$$

The rotations a_{io} and a_{ii} over support are :

$$a_{io}(g) = \frac{gl^3}{12 \cdot E_{ce} \cdot I_{ce}} = 2,76 \cdot 10^{-3}$$

$$a_{ii} = \frac{2 \cdot 1}{3 \cdot E_{ce} \cdot I_{ce}} = 0,196 \cdot 10^{-3} \frac{1}{kNm}$$

The moment over support at time $t = t_\infty$ due to self weight is thus :

$$X_g(t_\infty) = \Delta X_g(t_\infty) = -4,96 \text{ kNm}$$

If the connection were perfectly rigid one would have :

$$X_g^{ri}(t_\infty) = -\frac{a_{io} \varphi^*}{1,5 \, a_{ii} \, (1+ \chi\varphi)} = -6,77 \text{ kNm}$$

For the distributed permanent load acting on the structure after the connection is made, equation (17) applies :

$$X_{i-1}(t_1) = X_i(t_1) = X_{i+1}(t_1) = X_q(t_1)$$

$$\Delta X_{i-1}(t) = \Delta X_i(t) = \Delta X_{i+1}(t) = \Delta X_q(t)$$

$$1,5 \, X_q \, a_{ii} \, \varphi + \Delta X_q \, [1,5 \, a_{ii} \, (1+ \chi\varphi)+ c_i] + a_{io} \, \varphi = 0$$

$$\Delta X_q = -\frac{1,5 \, X_q \, a_{ii} \, \varphi + a_{io} \, \varphi}{1,5 \, a_{ii} \, (1+ \chi\varphi) + c_i}$$

$$a_{io}(q) = 1,22 \cdot 10^{-3}$$

$$X_q(t_1) = -\frac{a_{io}}{1,5 \, a_{ii} + c_i} = -2,37 \text{ kNm}$$

The terms φ and χ are again taken from figure 10 :

$$\varphi(t_\infty, 60j) = 2,2$$

$$\chi(t_\infty, 60j) = 0,84$$

The moment over support at time $t = t_\infty$ due to the distributed load is :

$$X_q(t_\infty) = X_q(t_1) + \Delta X_q(t_\infty)$$

$$= -2,37 \text{ kNm} - 1,09 \text{ kNm} = -3,46 \text{ kNm}$$

If the connection were perfectly rigid one would have :

$$X_q^{ri}(t_\infty) = -\frac{a_{io}}{1,5\ a_{ii}} = -4,17\ \text{kNm} = X_q^{mo}$$

For the distributed live load, of short duration and acting on the connected structure, equation (13) applies :

$$X_{i-1} = X_i = X_{i+1} = X_p$$

$$a_{io}(p) = 2,45 \cdot 10^{-3}$$

$$X_p = -\frac{a_{io}}{1,5\ a_{ii} + c_i} = -4,76\ \text{kNm}$$

If the connection were perfectly rigid one would have :

$$X_p^{ri} = -\frac{a_{io}}{1,5\ a_{ii}} = -8,33\ \text{kNm} = X_p^{mo}$$

In the following table the moments are given for the various static systems. Note that the actual moment over support agrees reasonably well with the moment initially estimated for the design.

Moments in kNm		monolithic	rigid connection	deformable connection	simple support
support	g	− 9,38	− 6,77	− 4,96	−
	q	− 4,17	− 4,17	− 3,46	−
	p	− 8,33	− 8,33	− 4,76	−
	Σ	− 21,88	− 19,27	− 13,18	−
span	g	4,69	7,29	9,10	14,06
	q	2,08	2,08	2,79	6,25
	p	4,17	4,17	7,74	12,50
	Σ	10,94	13,54	19,63	32,81

The maximum moment in span for the structure with deformable connections is :

$$M\ (\text{span}) = 19,63\ \text{kNm}$$

The ensuing tensile stress in the concrete (homogeneous section) is :

$$\sigma_c = +\ 3,64\ \text{N/mm}^2$$

One can therefore expect very slight cracking in the prefabricated elements and a good agreement between the calculated and actual distribution of moments.

5.4 Calculation of the deformations

From the distribution of moments for the various actions, the check of the deformations can be made.

The angle over support is (20) :

$$\gamma_i = -\Sigma X_i \cdot c_i + \gamma_i(\varepsilon_s) = 3,43 \cdot 10^{-3}$$

From the latter the width of the cracks of the joints in the concrete over support can be deduced (22) :

$$w = 0,4 \cdot \gamma_i \cdot d = 0,20 \text{ mm}$$

For the calculation of the deflections after the connection has been carried out, the various terms given by the corresponding actions are added together :

$$a_g(t_\infty) = a_g^{ps} \varphi^* + 2\, a_{ii}\, \Delta X_g\, (1 + \chi\varphi)$$

$$a_q(t_\infty) = a_q^{ps} (1 + \varphi) + 2\, a_{ii}\, X_q\, (1 + \varphi) + 2\, a_{ii}\, \Delta X_q\, (1 + \chi\varphi)$$

$$a_p = a_p^{ps} + 2\, a_{ii}\, X_p$$

$a_{g,q,p}^{ps}$: deflection at mid-span due to g, q and p in the isostatic system

a_{ii} : deflection at mid-span due to a unit moment acting on an adjacent support

$$a_g^{ps} = 2,15 \text{ mm}$$

$$a_q^{ps} = 0,96 \text{ mm}$$

$$a_p^{ps} = 1,91 \text{ mm}$$

$$a_{ii} = \frac{1^2}{16 \cdot E_{ce} \cdot I_{ce}} = 0,092 \cdot 10^{-3} \frac{1}{kN}$$

The total deflection at mid-span is :

$$a_{tot}(t_\infty) = a_g + a_q + a_p = 1,94 + 1,10 + 1,03 = 4,07 \text{ mm}$$

If the connections were perfectly rigid the deflection at mid-span would be :

$$a^{ri}(t_\infty) = a_g^{ri} + a_q^{ri} + a_p^{ri} = 0,93 + 0,63 + 0,38 = 1,94 \text{ mm}$$

In the isostatic system, the deflection would be :

$$a^{ps}(t_{\infty}) = a^{ps}_{g} \, \varphi^* + a^{ps}_{q} \, (1 + \varphi) + a^{ps}_{p} = 9{,}70 \text{ mm}$$

For the "simple supports" system the moments in span are such that cracking can no longer be neglected. The calculation of the deflections, that will be considerably greater, should take into account the procedures described in [4].

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