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Autor: Rajaraman, A. / Loganathan, K. / Raman, N.V.

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Nonlinear Analysis of Cable-Stayed Bridges

Analyse non linéaire des ponts haubannés

Nichtlineare Analysis von seilverspannten Brücken

A. RAJARAMAN
Scientist

K. LOGANATHAN
Scientist

N.V. RAMAN
Deputy Director

Structural Engineering Research Centre, CSIR
Madras, India

SUMMARY

The inherent nonlinear behavioural aspects of cable-stayed bridges are discussed, componentwise, and the analysis of a typical profile of a bridge is detailed with particular reference to modifications needed in the conventional stiffness method. The nonlinear effects are presented quantitatively to bring out their influences – individually and in a combined manner.

RÉSUMÉ

L'article décrit d'une façon précise les diverses non-linéarités rencontrées dans les ponts haubannés. Le calcul statique est effectué pour un cas typique de pont haubanné, considérant les modifications à apporter par rapport à la méthode classique des déplacements. Les effets non linéaires sont présentés quantitativement, seuls ou combinés, pour mettre en évidence leurs influences.

ZUSAMMENFASSUNG

Die massgebenden nichtlinearen Einflüsse in seilverspannten Brückensystemen werden einzeln diskutiert. Anschliessend wird eine typische Seilverspannung rechnerisch erfasst und die bei der konventionellen Steifigkeitsmethode anzubringenden Korrekturen dargestellt. Die nichtlinearen Einflüsse werden sowohl einzeln als auch in kombinierter Form quantitativ ermittelt, um deren Bedeutung klarer darzulegen.

1. INTRODUCTION

Cable-stayed bridges are increasingly being built for bridging medium and long spans. They possess several outstanding advantages such as economy, stiffness, superior aesthetic qualities, ease of erection without falsework, and freedom in selecting the structural arrangement. Consequently, they offer competitive alternatives and a large number of such bridges, both in steel and concrete, have been constructed in many countries since 1955.

2. BEHAVIOUR OF CABLE-STAYED BRIDGES

The principal components of a cable-stayed bridge system at the superstructure level are (i) decking with stiffening, (ii) towers, and (iii) cables.

The stress resultants coming on to these components vary. The decking is dominated by bending with axial forces - introduced by prestressing - being secondary. In the towers, the axial forces are significant but bending also contributes to the deformations. The cables can carry only axial loads. Hence, for the complete system, a discrete finite element formulation - either a plane frame or space frame model - suggests itself and the design phase for the components can be easily organised using these forces from the system level.

A major departure in the behaviour of this sytem from conventional systems is the effect of deformation on the three principal components. In decking and towers, due to axial and bending loads being present, it is necessary to account for the beam-column effect requiring the interplay of deformations with stress resultants. In the cables, even though only axial forces are present, their magnitude is dependent both on the end deformations and their weight effect, namely, sag. So, in the analysis of this system, the effects of these can be studied only through nonlinear analysis. The present study details this aspect with particular reference to an analytical model of a prototype bridge.

3. NONLINEARITIES IN CABLE-STAYED BRIDGES

Cable-stayed bridges possess the advantages of both suspension and girder-slab bridges. In suspension bridges, nonlinearity is mainly due to the presence of cables. Cables as mentioned earlier, possess, two types of geometric nonlinearities, viz.

(i) Due to large deformations, and

(ii) Due to sag-effect.

Even though these effects are interrelated, with suitable assumptions of the cable behaviour, they may be separated. In girder-slab bridges nonlinearities are mostly absent; but in exceptional cases where prestressing is used,

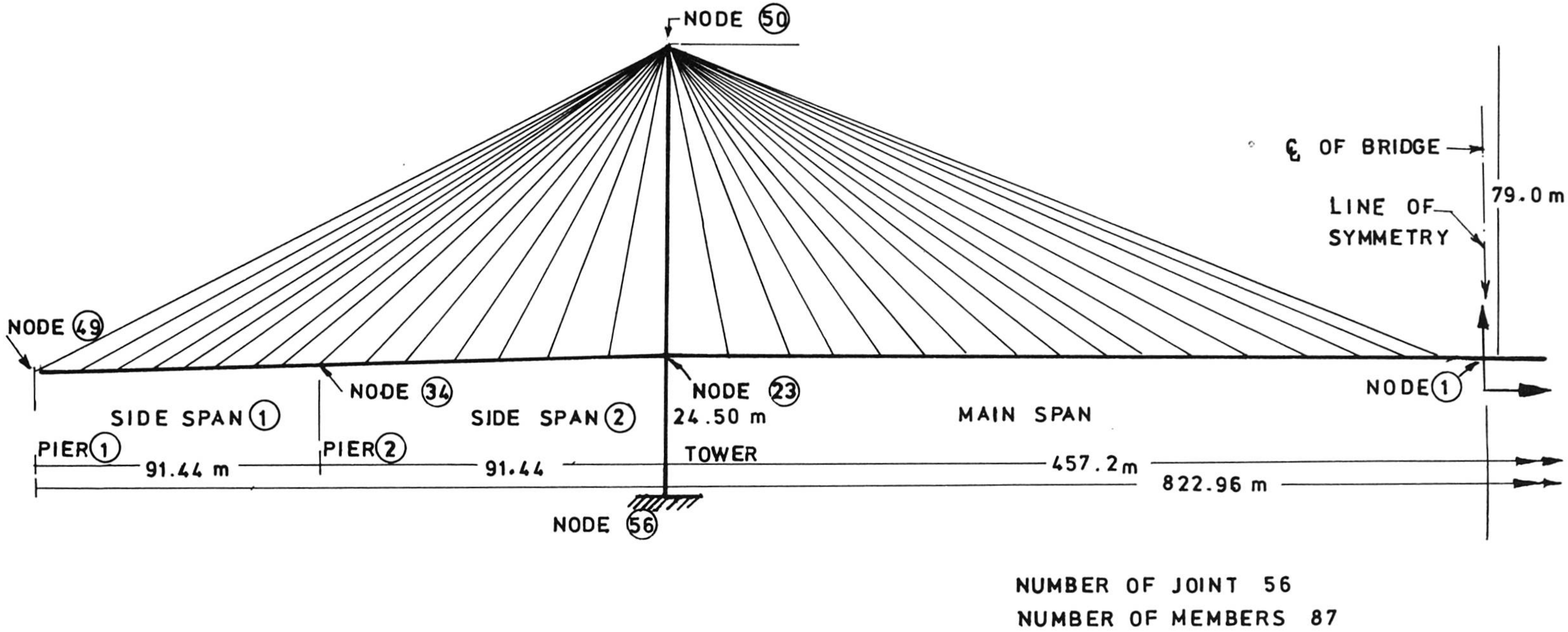

FIG. 1 GENERAL ELEVATION OF THE BRIDGE USED FOR NON LINEAR ANALYSIS

beam-column nonlinearities may become a dominant factor.

Cable-stayed bridge behaviour is affected by a combination of both these nonlinearities due to the presence of cables. Because of the axial forces, introduced due to the staying effect of the cables, beam-column nonlinearity assumes considerable importance both in tower and deck portions of the structure.

In the present investigation, the following nonlinearities have been included in the analysis:

(a) Geometric nonlinearity due to deformation
(b) Sag effect in cables
(c) Beam-column effect in deck and tower.

In the plane frame model chosen for the analysis, nonlinear effects can be readily incorporated and it has been found to give reasonably accurate results. The more accurate space frame analysis considering nonlinearities would be prohibitive in terms of computational effort and time. No nonlinear space frame analysis appears to have been done for a structure of this type so far.

4. BRIEF DESCRIPTION OF THE ANALYTICAL MODEL

The bridge consists of two box beams supporting the deck, which in turn are supported through cable-staying by the towers in the central span and piers in the end spans. The elevation of the bridge is given in Fig. 1. The cables form four distinct groups symmetrical with respect to the centre-line of the bridge as well as the longitudinal centre line of the deck. Since the dead and live loadings are taken as uniformly distributed only, one quarter of the bridge is considered for the analysis. Further, the object of the investigation being the study of nonlinear effects, plane frame analysis of one quarter of the bridge - with and without nonlinearities - is presented. Hence, the taper in the towers and the consequent out-of-plane action with the cable systems are neglected. Results pertaining to three stages of loading viz.,

(a) Dead load only
(b) Live load in all spans + Dead load
(c) Live load in central span + Dead load

are presented here. The structure has been analysed assuming no pretension in the cables.

The loads have been assumed to be uniformly distributed over the length of the bridge. The actual loads, however, are applied as concentrated loads at the cross girder points.

5. INCLUSION OF NONLINEARITIES

The analysis uses the stiffness method to solve for deformations. Since plane frame analysis is used, any member of the structure will have only three degrees of freedom at its ends:

(i) Axial deformation
(ii) Lateral deformation, and
(iii) Rotation.

Consequently, these deformations give rise to three force resultants, viz., axial force, shear force and moment. Before assembling the member stiffnesses at the nodes to get the total stiffness of the structure, the nonlinearities have to be included in the force-deformation relationships. The nonlinearities to be included, as mentioned before, are:

(i) Geometric nonlinearity due to large deformations
(ii) Geometric nonlinearity due to sag in the cables
(iii) Beam-column nonlinearity.

All three of the above nonlinearities are not present in all the members. Cables will not have beam-column effects as they are incapable of taking moments and the deck and tower members will not have sag effects. Hence depending on the type of members, distinguished by its moments of inertia, the nonlinear effects are to be incorporated.

The force-deformation relations of a plane frame member has been given in matrix form in Eqn. (1). The quantities given in the equation refer to the end member forces, deformations and elastic properties of the member as given in Fig. 2.

$$\begin{Bmatrix} P_i \\ S_i \\ M_i \\ P_j \\ S_j \\ M_j \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & (\text{SYM.}) & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_i \\ \omega_i \\ \theta_i \\ u_j \\ \omega_j \\ \theta_j \end{Bmatrix} \qquad (1)$$

i.e. $\{P\} = [K]\{\delta\}$

The quantities given in the above equation refer to the end member forces, deformations and elastic properties of the member as given in Fig. 2.

The coefficients in matrix K are suitably modified to account for the nonlinear effects.

$$L_c = \sqrt{(L + x_j - x_i)^2 + (y_j - y_i)^2} - L$$

$$\alpha_{ij} = \tan^{-1} \frac{(y_j - y_i)}{(L + x_j - x_i)} \tag{2}$$

The change in length and rigid body rotation are suitably introduced in matrix K to account for this type of nonlinearity.

5.1 Nonlinearity due to large deformations

Fig. 3 shows, to a large scale, a typical member, before and after deformations. It may be seen from the figure that large deformations introduce change in length of the member, a rigid body rotation and bowing action. Since bowing action is not quite significant in normal structures, it is not considered in this analysis. The change in length and rigid body rotation can be easily expressed in terms of end displacements as follows:

5.2 Nonlinearity due to sag in cables

As mentioned earlier, self-weight of cables introduces nonlinearity in cable forces since the tensions and deflections are interrelated. Though the nonlinearity due to large deformations and that due to sag are dependent on each other, the latter effect may be estimated separately using an approximate formula suggested by Leonhardt [1]. Sag effect may thus be calculated independently and included suitably in the coefficient matrix of the cable members. The formula estimates the value of Young's modulus at any stage of deformations and is given below:

$$A_c E_i = \frac{A_c E_o}{1 + \frac{\gamma^2 L^2 E_o}{12 \sigma^3}} \tag{3}$$

where

A_c = Area of cable steel

E_i = Young's modulus of cable with sag

E_c = Young's modulus of straight cable

γ = Specific weight of cable

L = Length of horizontal projection of the cable

σ = Tensile stress in the cable.

5.3 Beam-column Nonlinearity

In a beam-column, lateral deformations and axial loads are inter-related and hence at any stage of deformation the end forces are estimated using stability functions. These stability functions denoted as s_1, s_2, s_3 and s_4 are dependent on the axial force. While incorporating these functions, the nature of axial force, i.e., whether tensile or compressive, has to be ascertained and proper functions have to be chosen. These functions are given in Ref. [2]. They are introduced in the stiffness matrix given in Eq. (1), suitably.

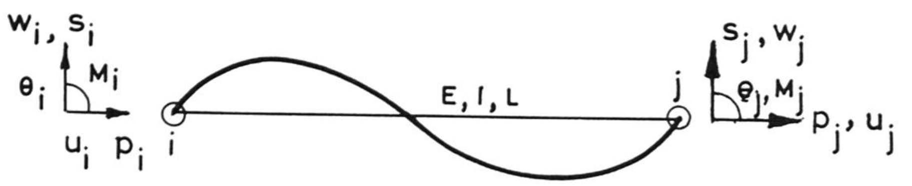

FIG. 2 FORCES AND DEFORMATIONS IN A MEMBER

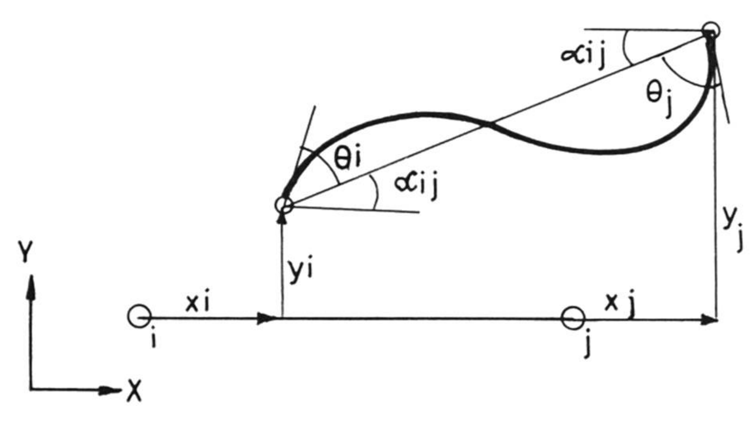

FIG. 3 LARGE DEFORMATIONS

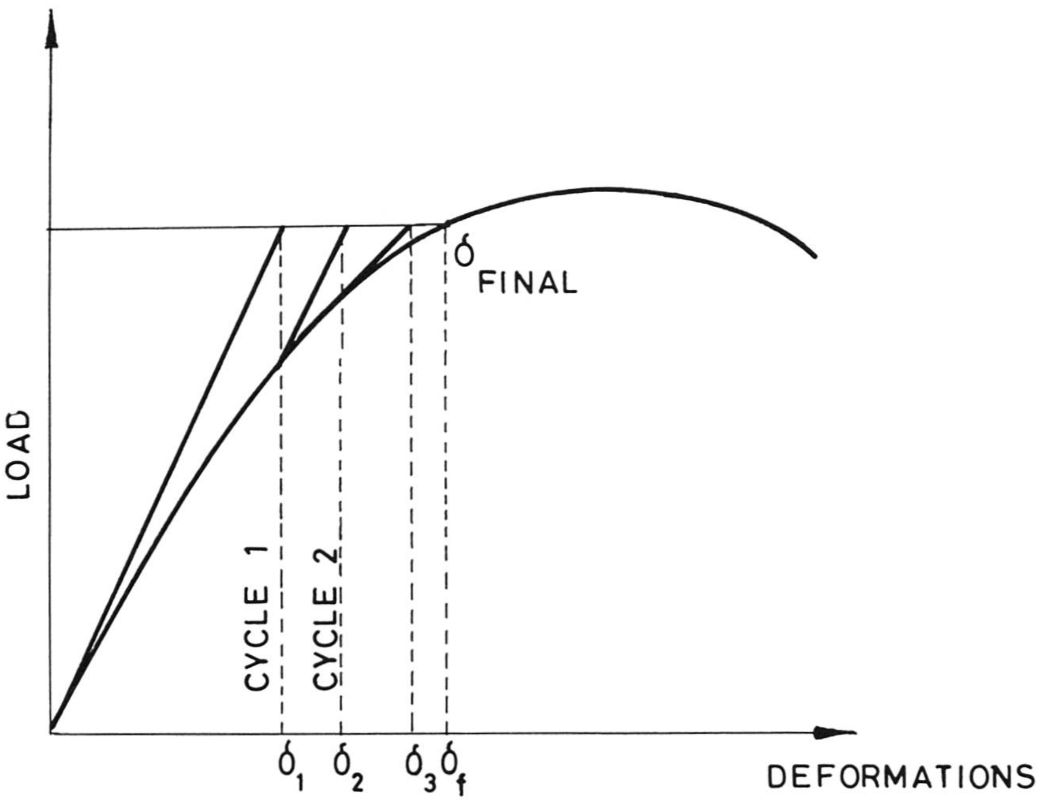

FIG. 4 NONLINEAR ANALYSIS

6. NONLINEAR ANALYSIS OF THE BRIDGE

Having suitably modified the member stiffness matrix to include nonlinearities, the overall stiffness matrix of the structure can be obtained by assembling and transforming to a common coordinate system. Here the coordinate system is chosen in such a way that the entire line of the bridge-span forms the Y axis and the line joining the tops of end piers forms the X axis. All member end forces are transformed with respect to these axes. Nodes are introduced along the deck and tower at points where there is change in property or geometry. In one quarter of the bridge, 55 nodes (restrained or free) and one fixed node at the base of the tower have been introduced. The central high point in the deck is the point of symmetry. The total number of members is 87. These have been grouped on the basis of their flexural properties. The two pier points are restrained vertically. With these boundary conditions and the overall stiffness of the structure, the bridge can be analysed for any particular type of loading. Here again, the loadings and their types are grouped so that member-end forces and moments may be suitably evaluated if loadings are different from one member to another.

Generally nonlinear analysis is of an iterative type. Hence for the initial geometry and loadings, the problem is solved using the initial stiffness and loadings. This gives the deformations and using these deformations, the stiffness matrix including the nonlinearities, is revised. The problem is solved for the revised geometry and the procedure is repeated. This process is continued till two subsequent deformations are within a prescribed tolerance limit. This is explained graphically in Fig. 4. In this figure, the actual load deformation curve is nonlinear; for the first iteration, a linear relation has been used resulting in fictitious joint restraints, which are corrected gradually in subsequent iterations. This procedure has been chosen since the applied loads are constant.

The computer program using this procedure needed only a small number of cycles for convergence. Further flexibility in the program was introduced so that the nonlinearities could be included individually or jointly. Linear analysis can be done with the help of this program by suppressing the nonlinear effects.

7. RESULTS FOR DEAD LOAD ANALYSIS

The bridge was first analysed for dead load. Here the girders alone were considered for the analysis and the influence of stringers and cross beams on the stiffness of the boxes was completely neglected. The analysis was done for the following cases:

i) Linear behaviour
ii) Geometric nonlinearity due to deformation
iii) Nonlinearity due to sag
iv) Nonlinearity due to beam-column effect
v) Combination of (ii) and (iii)
vi) Combination of (ii) and (iv)
vii) Combination of (iii) and (iv)
viii) Combination of (ii), (iii) and (iv)

The maximum values of deflections, moments and axial forces for the various cases of nonlinearity listed above are given in Table 1.

It may be observed from the table that the effect of sag and geometric nonlinearity are predominant on the deformations of the deck individually and in a combined manner as compared to the beam-column nonlinearity. The effect of sag alone influence to the extent of a 10% increase over linear analysis values. Further the convergence rate for this type of nonlinearity is relatively slower, with a slightly better rate for beam-column effects. These two combined together take more time to converge as compared to even all three types, put together. The tower deflections show only a marginal change over linear analysis values as compared to deck. And in general, there is a reduction in the values of maximum moments and axial forces due to increased deformations.

8. RESULTS FOR LIVE LOAD ANALYSIS

The analysis for live load has been done for two cases:

i) Dead and live loads over the full span, and
ii) Dead load, and live loads on the middle span only.

Table 2 shows a comparison of the maximum values of deflection, moments and axial forces obtained in the linear and nonlinear analysis.

Here again, it may be seen that the effects of nonlinearity are pronounced to the extent of an increase of 9.6 per cent in deformations, 6.2% increase in negative moments, 13.2% decrease in positive moments and 21.2% increase in axial forces in the deck alone, whereas in the tower the deformation increases by 16.8%, while there is 6.2% increase in negative moments, 16.1% decrease in positive moments and 2.9% increase in axial forces.

9. CONCLUSIONS

Results presented in Tables 1 and 2 bring out the effects of nonlinearity of the three different types clearly. These will become more significant during erection stages as well, and are to be assessed taking into account the sequence of construction.

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REFERENCES

1. LEONHARDT, F. and ZELLNER, W., "Cable Stayed Bridges - Report on latest developments". Proceedings, Canadian Structural Engineering Conference, University of Toronto, February 1970.

2. GERE, J.M. and WEAVER, W., "Analysis of Framed Structures". Van Nostrand Co., 1965.

Table 1

MAXIMUM VALUES OF DEFLECTIONS, MOMENTS AND AXIAL FORCES FOR VARIOUS NONLINEARITIES

		Case 1	Case 2 (4 cycles)*	Case 3 (8 cycles)	Case 4 (5 cycles)	Case 5 (7 cycles)	Case 6 (9 cycles)	Case 7 (9 cycles)	Case 8 (6 cycles)
Maximum Deflection (m)	Deck	2.57461	2.59278	2.73811	2.59432	2.76139	2.62537	2.7595	2.79131
	Tower	0.484281	0.50119	0.49614	0.49028	0.51799	0.510903	0.50405	0.52764
Maximum Moment (tm)	Deck (Neg.)	-2583.26	-2620.71	2994.55	-2550.02	-2997.02	-2569.20	-2926.70	-2936.12
	Deck (Pos.)	6341.80	6265.88	6349.84	6174.62	6275.04	6091.90	6183.37	6100.76
	Tower	4059.41	3644.20	4158.40	4014.22	3763.97	3569.44	4113.82	3676.19
Maximum Axial Force (t)	Deck	4162.47	4162.20	4137.77	4205.77	4155.77	4202.60	4178.20	4203.05
	Cable	615.25	613.41	638.17	623.55	639.41	626.10	648.52	651.71
	Tower	6266.14	6233.47	6278.02	6286.73	6266.54	6283.07	6318.43	6324.10

Case	Description
1.	Linear behaviour
2.	Geometric nonlinearity
3.	Nonlinearity due to sag
4.	Nonlinearity due to beam-column effect
5.	Combination of 2 and 3
6.	Combination of 2 and 4
7.	Combination of 3 and 4
8.	Combination of 2, 3 and 4

* Number of cycles for convergence.

Table 2

COMPARISON OF RESULTS OF LINEAR AND NONLINEAR ANALYSIS

		D.L.			D.L.+L.L. IN ALL SPANS			D.L.+L.L. IN CENTRAL SPAN		
		Linear	Nonlinear	Ratio	Linear	Nonlinear	Ratio	Linear	Nonlinear	Ratio
Maximum Deflection (m)	Deck	2.57461	2.79131	1.084	3.58403	3.93627	1.096	3.67983	3.96830	1.081
	Tower	0.48428	0.52764	1.092	0.67414	0.78637	1.168	0.70789	0.78878	1.113
Maximum Moment (tm)	Deck (Sag)	2583.26	2936.12	1.134	3596.10	3819.44	1.062	3661.86	3886.02	1.062
	Deck (Hog)	6341.80	6100.76	0.963	8828.37	8909.52	1.001	7824.67	7780.61	0.994
	Tower	4059.41	3676.19	0.906	5650.94	4901.74	0.868	5933.81	4966.63	0.839
Maximum Axial Forces (t)	Deck	4162.57	4205.03	1.008	5166.85	6264.92	1.212	5770.10	6009.71	1.041
	Tower	6266.14	6324.10	1.008	8722.86	9292.33	1.065	8562.56	8821.10	1.029
	Cables	615.25	651.71	1.058	856.46	956.74	1.117	928.56	998.21	1.078