An Experimental and Theoretical Study of the Behaviour of Folded Plates Supported by Intermediate Columns

Etude expérimentale et théorique du comportement de stuctures plissées appuyées sur des colonnes intermédiaires

Eine experimentale und theoretische Untersuchung des Verhaltens von Faltwerken, die auf Zwischenstützen aufgelegt sind

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SUMMARY:

The paper describes an experimental and theoretical study of the behaviour of folded plate structures supported by intermediate columns. It is shown that the introduction of columns at the quarter-span positions underneath the longitudinal edges is an effective way of reducing the structural displacements under load. The results presented also confirm the accuracy of the finite element method of folded plate analysis, whereas the elasticity method is shown to lead to considerable errors, in certain circumstances, due to end support assumptions.

RÉSUMÉ:

L'article traite d'une étude expérimentale et théorique du comportement de structures plissées appuyées sur des colonnes intermédiaires. L'étude montre que la présence de colonnes aux quarts de la portée longitudinale permet de réduire efficacement les déplacements sous l'effet de charges.
Les résultats confirment aussi la précision de l'analyse des structures plissées par la méthode des éléments finis. La méthode élastique, par contre, peut donner des erreurs considérables, résultant des conditions aux limites admises.

ZUSAMMENFASSUNG:

Der Artikel behandelt eine experimentale und theoretische Untersuchung des Verhaltens von Faltwerken, die auf Zwischenstützen aufgelegt sind. Es wird gezeigt, dass ins Viertelfeld eingebrachte Stützen effektvoll wirken, um die durch die Belastung verursachten Verschiebungen zu vermindern.
Die Ergebnisse bestätigen auch die Genauigkeit der Methode der finiten Elemente bei der Analyse von Faltwerken. Die Elastizitätsmethode hingegen kann wegen ihrer Annahmen bezüglich der Endstützung zu grossen Ungenauigkeiten führen.

1.   INTRODUCTION

In recent years, folded plates have become increasingly popular as roofing structures because they provide an economical and aesthetically pleasing design. Their popularity has led to the publication of numerous papers dealing with folded plate analysis and the various methods that have been proposed were critically reviewed by Evans and Rockey|1| in a previous publication. The majority of the proposed methods are only applicable to simply supported structures and very few|2,3,4| are capable of dealing with structures supported by randomly placed intermediate columns, such as those shown in Fig. 1. Such columns are frequently employed in practice since they can greatly increase the span of the roof without interfering to any great extent with the access to or the utilization of the covered area.
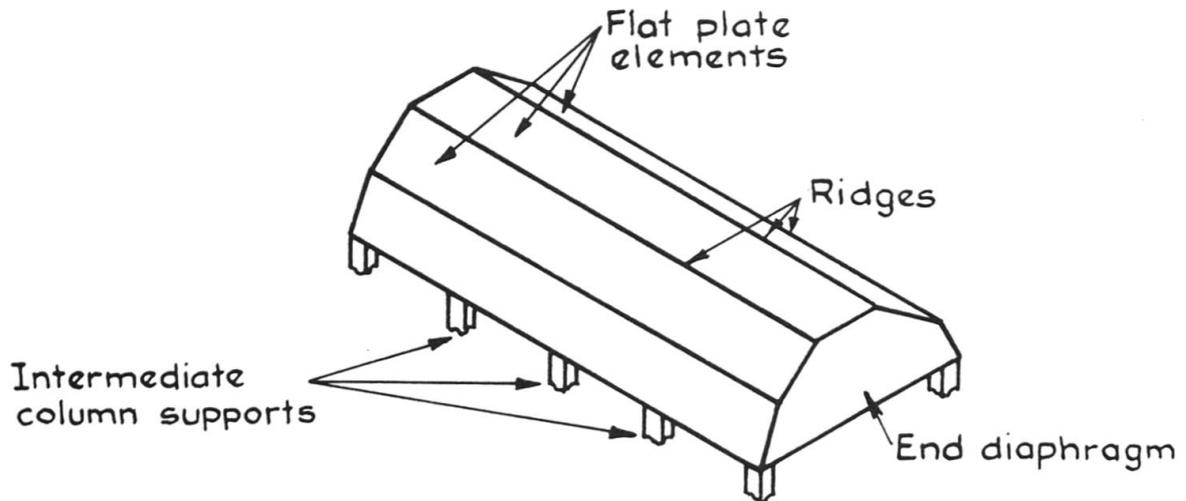

FIG. I   A TYPICAL FOLDED PLATE STRUCTURE

The few theories that have been proposed for the analysis of folded plates supported by columns are rather limited in their range of application. They are based on superposition techniques which require several re-analyses of the structure within any one solution and the extensive calculations involved necessitate the use of a digital computer. Furthermore, since they are based on either the "ordinary"|2| or "elasticity"|3| methods of folded plate analysis, they assume that each supporting end diaphragm, such as that illustrated in Fig. 1, is infinitely stiff in its own plane and perfectly flexible normal to its plane. The end cross-section of the folded plate is thus assumed to be free to warp in the longitudinal direction.

In fact, most end diaphragms provided in practice will be far too stiff to allow this longitudinal warping to occur freely and, in previous publications, Rockey and Evans|5,6| have shown, both experimentally and theoretically, the great effect that this warping restraint has upon the overall structural behaviour. In this previous study, errors of up to 200% were observed in the values predicted by the elasticity method, and it was proved that these errors were primarily due to the way in which the end support conditions were idealized in the method. A finite element solution, which had been developed earlier by Rockey and Evans|7|, was applied in this previous study, this method being capable of accurately representing the end support conditions provided in the experiments, and good agreement was obtained between the experimental and finite element results. In this present investigation, this finite element technique will be applied to the analysis of folded plates supported by intermediate columns.

In contrast to the large number of papers published on the analysis of folded plates, very few experimental studies of folded plates have been reported|5,6,8|. This is particularly true for folded plates supported by intermediate columns and in this report, the results of eight tests on such structures will be presented. The effects of varying both the type of column support and the location of the columns will be illustrated and the experimental results will in each case be compared to theoretical values obtained by the finite element method. In addition, theoretical results obtained by the elasticity method will be included in the comparison where possible. A similar series of tests upon folded plates containing large openings was described by Rockey and Evans|9| in 1976.

2.   DETAILS OF MODELS TESTED AND COLUMN SUPPORTS

The folded plate model test-
ed was of the vault type of
cross-section shown in Fig.
2, the cross-sectional
dimensions being as shown
in the figure. The longit-
udinal span of the model
between end diaphragms was
$71\frac{1}{4}$ in (1810 mm), the
length/width ratio of the
internal plates thus being
11.2. The model was formed
from HS30/P aluminium sheet
having a thickness of
0.08 in (2 mm), and the model
was folded from a single flat
sheet, thus ensuring full
continuity between adjacent
plates at each longitudinal
ridge. A series of material
tests established the modulus of
elasticity and the Poisson's ratio of
the aluminium as 10.2×10^6 lbf/in^2
(70.33×10^6 N/mm^2) and 0.32,
respectively.

FIG.2   DETAILS OF VAULT-TYPE CROSS-SECTION CONSIDERED IN THE EXPERIMENTS

Having first tested the model with a
clear span between end diaphragms,
column supports were then introduced
underneath both longitudinal edges of
the model. Two different types of
column supports were considered,
columns that restrained all displace-
ments at their point of application
being considered in the first instance,
and then columns that only restrained
the vertical and transverse horizontal
deflections at a point being introduced.
These two types of columns will be

FIG.3   DETAILS OF COLUMN SUPPORTS

further referred to as "clamping" and "simply supporting" columns, respectively, and they are illustrated diagrammatically in Fig. 3.

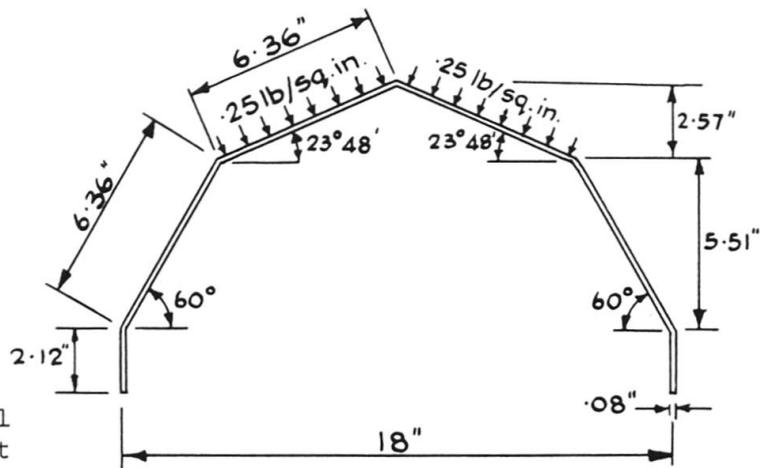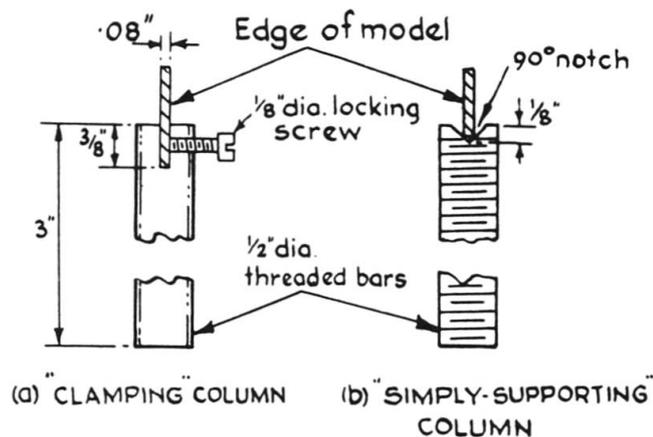To assess the effects of column spacing, four different arrangements of each column type were considered. The columns were introduced under the edges of the model, first of all at the mid-span position only, then at the quarter-span

positions, then at the eighth-span
positions and, finally, at the
sixteenth-span positions, as shown in
Fig. 4.

Nine tests were thus carried out in all
and they may be summarized as follows:

TEST 1 - model with clear span
between end supports.

TEST 2 - clamping columns introduced
at mid-span positions.

TEST 3 - clamping columns introduced
at quarter-span positions.

TEST 4 - clamping columns introduced
at eighth-span positions.

TEST 5 - clamping columns introduced
at sixteenth-span positions.

FIG. 4 LOCATIONS OF EDGE SUPPORTING COLUMNS.

TEST 6 - simply supporting columns introduced at mid-span positions.

TEST 7 - simply supporting columns introduced at quarter-span positions.

TEST 8 - simply supporting columns introduced at eighth-span positions.

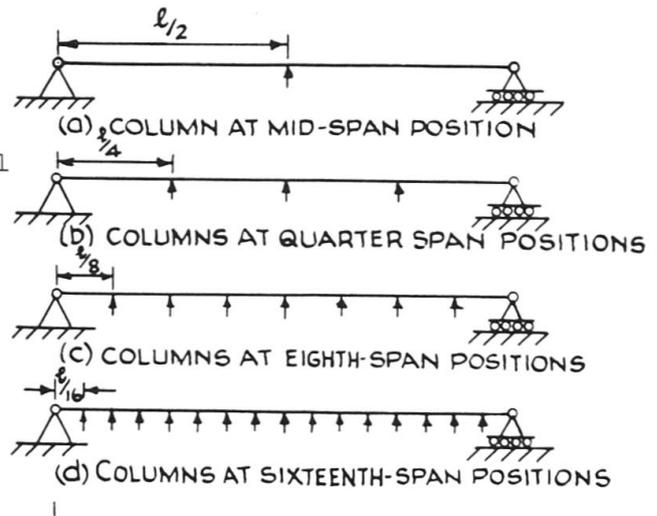TEST 9 - simply supporting columns introduced at sixteenth-span positions.

3.　　GENERAL EXPERIMENTAL DETAILS

In all tests, symmetrical loads of uniform intensity were applied perpendicular
to the two top plates of the model, as shown in Fig. 2. These loads were
applied by means of an air pressure-bag system, the air-bag being confined to
bear against the top surface of the model so that the pressure in the bag was
transmitted as a uniformly distributed load perpendicular to the plane of each
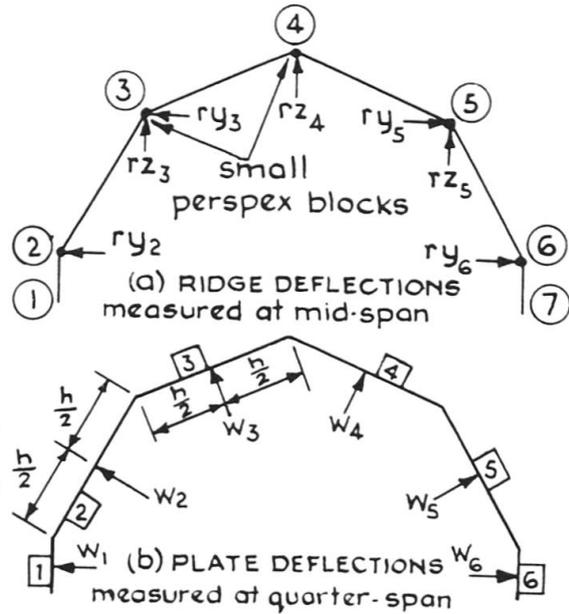loaded plate.

The model was supported by end diaphragms machined from $\frac{3}{8}$ in (9.5 mm) thick mild
steel plates to the shape of the vault-type cross-section, the model being
rigidly bolted to the diaphragms. These diaphragms were then mounted on special
bearings, designed so that free rotation and translation of the diaphragms
perpendicular to their planes could take place.

In the tests, the displacements of the model were measured at the mid-span and
quarter-span cross-sections, using dial gauges graduated in units of 0.0001 in
(0.0025 mm), positioned underneath the model. Seven values of ridge displace-
ments were measured at the mid-span cross-section, as shown in Fig. 5(a). In
addition, the displacements perpendicular to the plane of the six plates at the
quarter-span cross-section were measured, as shown in Fig. 5(b). The system of
ridge and plate numbering adopted and the notation and positive directions to be
used for the measured displacements are also defined in these diagrams.

The maximum load applied in each test was chosen such that the model was not
loaded beyong its elastic range and that the membrane action within the
individual plates did not become significant. Within this load range, 14
increments of load were applied in each test and the deflections recorded at
each load level. The resulting load-deflection relationships all exhibited
good linear characteristics and enabled accurate values of the slope of the

load/deflection graphs to be obtained.
Using this data, the experimental
deflections corresponding to an applied
load intensity of 0.25 lbf/in^2
(1.7 N/mm^2) were determined and
adopted for comparison with theoretical
results.

In all tests, the required symmetry of the
measured deflections about the longitudinal
centre line was obtained. At the end of
each test, the pressure in the air-bag was
released and the deflection of the model in
its final unloaded state recorded. In each
case, it was observed that the final un-
loaded position was almost identical to the
initial position of the model, thus
confirming that the model had not been
loaded beyond its elastic range.

FIG. 5 POSITIONS OF DEFLECTION
MEASUREMENTS DURING
TESTS.

4   THEORETICAL SOLUTIONS

Each of the models tested was analysed by
the finite element method. The
application of this method to the analysis
of folded plates has been considered in
detail by Rockey and Evans|7| in a previous publication and the solution
established employs rectangular elements, five degrees of freedom, viz. the two
displacements associated with plane stress and the three displacements
associated with plate bending, being considered at each node.

In each finite element solution, the
folded plate was divided into 640
elements, as shown in Fig. 6, since
previous experience|6| had shown that
a convergent solution would be
obtained with such a mesh. Because of
the symmetry of the model and of the
loading and support conditions, it was
only necessary to consider one-quarter
of the structure in the analysis, as
shown in Fig. 6.

One of the main advantages of the
finite element method is the ease with
which it can be adapted to deal with
different support conditions. The
intermediate column support conditions
of the present investigation were
dealt with conveniently, simply by
ensuring that a node point of the
mesh coincided with each support
position and specifying that the

FIG. 6 DISTRIBUTION OF ELEMENTS
FOR FINITE ELEMENT SOLUTION

relevant components of displacement were zero at these points. Also, as
discussed in detail elsewhere|6|, the rigid body rotation and translation of
the end diaphragms in the longitudinal direction were conveniently and
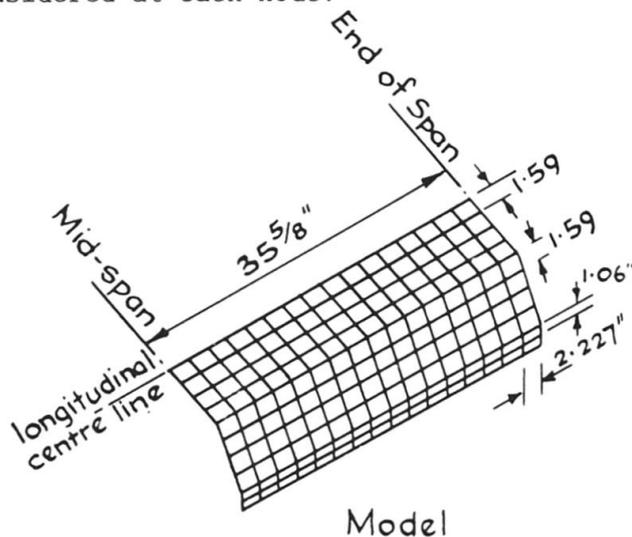accurately simulated in the finite element solution.

In addition to the finite element solutions, the models were also analysed, when possible, by the elasticity method. Since this method in its basic form is unable to take discreet point supports into account, only three elasticity solutions were obtained. In the first of these solutions, the longitudinal edges of the model were assumed to be free, as in Test 1. In the second solution, each edge was assumed to be fully and continuously clamped along its length, thus providing a limiting theoretical solution for Tests 2, 3, 4 and 5 in which the number of clamping columns underneath each edge was progressively increased. Then, in the final elasticity solution, each edge was assumed to be simply supported along its length, thus providing a limiting theoretical solution for Tests 6, 7, 8 and 9 in which the simply supporting columns were introduced.

Both the finite element and elasticity solutions were obtained by means of computer programs written in FORTRAN for the ATLAS computer. Neither method is amenable to a hand solution because of the extensive calculations involved.

5.   RESULTS

Both the finite element and elasticity solutions give comprehensive pictures of the stresses and displacements set up within a structure. However, in the present report, only those displacements that were also measured experimentally, as illustrated in Fig. 5, will be discussed.

In Table 1, the displacements measured in Tests 1, 2, 3, 4 and 5 are presented and compared to finite element values, the notation used for the deflections being as defined in Fig. 5. The behaviour of the models supported by clamping columns at the mid, quarter, eighth and sixteenth-span positions, i.e. in Tests 2, 3, 4 and 5 respectively, may thus be compared to the behaviour of the model with unsupported edges in Test 1. Corresponding values for the models supported by the simply-supporting columns in Tests 6, 7, 8 and 9 are compared to the values for the unsupported model in Test 1 in Table 2.

In Figs. 7(a) and 7(b), the values of two typical mid-span ridge displacements, viz. the vertical displacement of ridge 5 (from symmetry, $rz_5 = rz_3$) and the horizontal displacement of ridge 6 (from symmetry, $ry_6 = ry_2$), are plotted against the number of columns introduced under each edge. The effects of both the clamping and simply supporting columns are illustrated in these diagrams. Similar curves are presented in Figs. 8(a) and 8(b) for two typical mid-plate displacements at the quarter-span cross-section, the displacements perpendicular to plates 4 (from symmetry, $w_4 = w_3$) and 6 (from symmetry, $w_6 = w_1$) being illustrated.
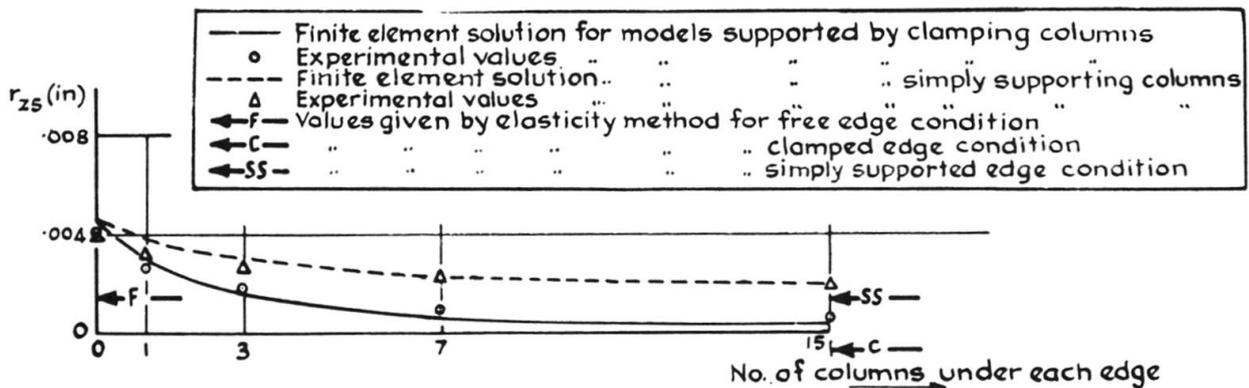

FIG.7(a) VERTICAL DEFLECTIONS (in.) OF RIDGE 5 (r_{z5}) AT MID-SPAN

Column Positions	Solution	Ridge deflections at mid-span $\cdot$ ins (mm) x 10^{-5}				Plate deflections at quarter-span ins (mm) x 10^{-5}		
		rz_4	ry_3 $= ry_5$	rz_3 $= rz_5$	ry_2 $= ry_6$	w_3 $= w_4$	w_2 $= w_5$	w_1 $= w_6$
Free Edges Test 1	Finite Element	7.1(180)	-1.2(/o.5)	4.6(117)	-13.3(338)	7.6(193)	-4.2(107)	- 8.7(221)
	Experimental	7.6(193)	-1.7(43.2)	4.0(102)	-16.9(429)	7.7(196)	-6.3(160)	-11.2(284)
Mid-Span Test 2	Finite Element	7.0(178)	-1.8(45.7)	3.0(76.2)	- 6.8(173)	6.9(175)	-3.5(81.3)	- 4.8(122)
	Experimental	7.4(188)	-2.3(58.4)	2.6(66.0)	- 6.5(165)	6.6(168)	-4.0(102)	- 5.6(142)
$\frac{1}{4}$-Span Test 3	Finite Element	6.2(157)	-2.1(53.3)	1.6(40.6)	- 4.8(122)	6.3(160)	-4.6(117)	- 1.5(38.1)
	Experimental	6.8(173)	-2.4(61.0)	1.8(45.7)	- 5.2(132)	6.2(157)	-5.1(130)	- 1.8(45.7)
$\frac{1}{8}$-Span Test 4	Finite Element	5.1(130)	-2.0(50.8)	0.6(15.2)	- 3.1(78.7)	5.5(140)	-3.9(99.1)	- 1.0(25.4)
	Experimental	6.0(152)	-2.6(66.0)	0.9(22.9)	- 4.0(102)	6.2(157)	-5.0(127)	- 1.4(35.6)
$\frac{1}{16}$-Span Test 5	Finite Element	4.5(114)	-1.9(48.3)	0.3(7.6)	- 2.4(61.0)	5.1(130)	-3.6(91.4)	- 0.7(17.8)
	Experimental	5.8(147)	-2.6(66.0)	0.5(12.7)	- 3.1(78.7)	6.1(155)	-4.8(122)	- 0.9(22.9)

TABLE 1 : Deflections of models supported by clamping columns.

Column Positions	Solution	Ridge deflections at mid-span ins (mm) x 10^{-5}				Plate deflections at quarter-span ins (mm) x 10^{-5}		
		rz_4	ry_3 $= ry_5$	rz_3 $= rz_5$	ry_2 $= ry_6$	w_3 $= w_4$	w_2 $= w_5$	w_1 $= w_6$
Free Edges Test 1	Finite Element	7.1(180)	-1.2(30.5)	4.6(117)	-13.3(338)	7.6(193)	-4.2(107)	- 8.7(221)
	Experimental	7.6(193)	-1.7(43.2)	4.0(102)	-16.9(429)	7.7(196)	-6.3(160)	-11.2(284)
Mid-Span Test 6	Finite Element	7.1(180)	-1.5(38.1)	3.7(94.0)	- 7.9(201)	7.2(183)	-3.5(88.9)	- 5.9(150)
	Experimental	7.4(188)	-2.0(50.8)	3.2(81.3)	- 7.5(191)	6.9(175)	-4.6(117)	- 6.2(157)
$\frac{1}{4}$-Span Test 7	Finite Element	7.1(180)	-1.8(45.8)	3.1(78.7)	- 7.1(180)	7.2(183)	-4.9(124)	- 3.0(76.2)
	Experimental	7.4(188)	-2.4(61.0)	2.7(68.6)	- 6.9(175)	6.9(175)	-5.3(135)	- 2.9(73.7)
$\frac{1}{8}$-Span Test 8	Finite Element	6.9(175)	-2.0(50.8)	2.3(58.4)	- 6.0(152)	6.8(173)	-4.8(122)	- 2.5(63.5)
	Experimental	7.3(185)	-2.6(66.0)	2.3(58.4)	- 5.7(145)	6.5(165)	-5.4(137)	- 2.6(66.0)
$\frac{1}{16}$-Span Test 9	Finite Element	6.7(170)	-2.2(55.9)	1.9(48.3)	- 5.4(137)	6.6(168)	-4.8(122)	- 2.2(55.9)
	Experimental	7.5(191)	-2.7(68.6)	1.9(48.3)	- 5.5(140)	6.9(175)	-5.6(142)	- 2.5(63.5)

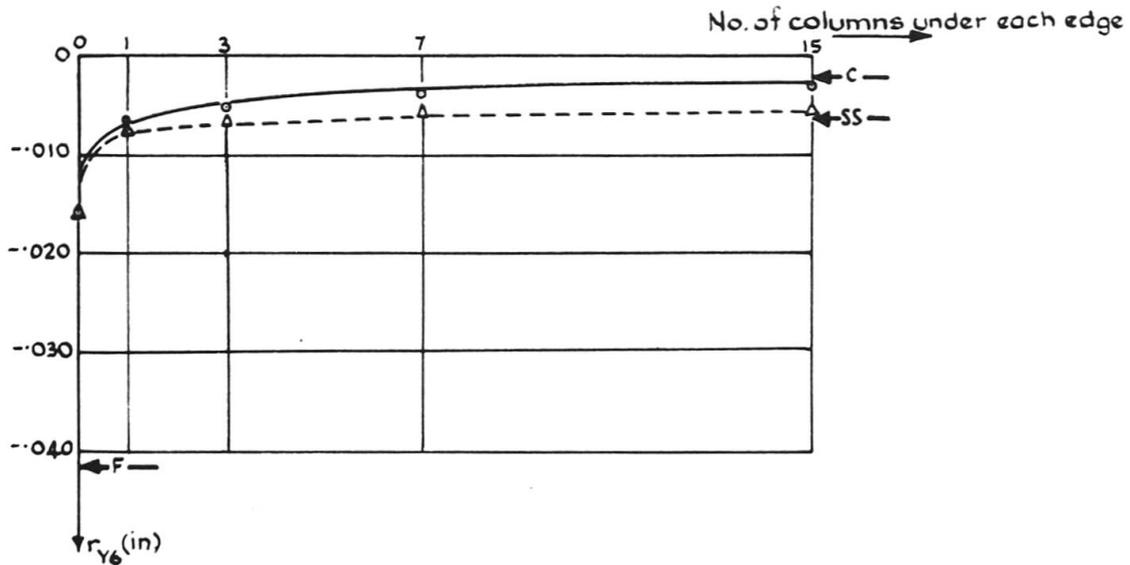TABLE 2 : Deflections of models supported by simply-supporting columns

FIG. 7(b) HORIZONTAL DEFLECTIONS (in.) OF OUTER RIDGE
(r_{Y6}) AT MID-SPAN

In addition to the finite element and the experimental values, the limiting
theoretical values obtained by the elasticity method, first of all assuming the
edges of the model to be free, then assuming the edges to be fully clamped,
and, finally, assuming the edges to be simply supported, are included in
Figs. 7(a), 7(b), 8(a) and 8(b).

Finally, in Figs. 9(a) and 9(b), the longitudinal movements of the end
diaphragms are plotted. The values shown in Fig. 9(a) are for the free edge
model of Test 1 and, in Fig. 9(b), values for the model supported by simply-
supporting columns at the sixteenth-span positions in Test 9 are shown.
Finite element and elasticity values are included on both diagrams for
comparison with the measured values.

6.   DISCUSSION OF RESULTS

From the results, it is seen that the introduction of the clamping columns has
a considerably greater influence on the behaviour of the structure than the
introduction of the simply-supporting columns. It is also apparent that,
whereas the introduction of a column of either type at the mid-span position
significantly increases the structural stiffness, as more columns are
introduced, the rate of increase of structural stiffness with the number of
columns decreases.

In fact, little gain in stiffness is obtained by increasing the number of colums
under each edge from 7 to 15, so that a structure supported by columns at the
eighth-span positions along each edge is, effectively, as stiff as a structure
in which the edges are continuously supported along the complete span. Even
introducing columns at the quarter-span positions only, i.e. positioning 3
columns under each edge, is an effective way of reducing the structural
deflections, and, from a design point of view, such an arrangement has the great
advantage of offering very little interference to the access to the area
covered by the folded plate roof.

For all models tested, Tables 1 and 2 show reasonable agreement between the
experimental values and the results obtained by the finite element method and, in

Finite element solution for models supported by clamping columns
o   Experimental values   „   „   „   „   „
- - - - Finite element solution „   „   „   „ simply supporting columns
Δ   Experimental values   „   „   „   „   „   „   „
←F—   Values given by elasticity method for free edge condition
←C—   „   „   „   „   „   „ clamped edge condition
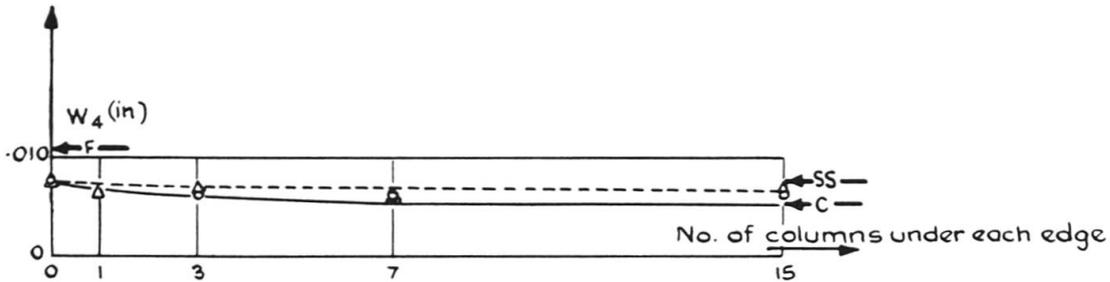←SS—   „   „   „   „   „   „ simply supported edge condition

FIG. 8(a) NORMAL DEFLECTIONS (in.) AT MID-POINT OF PLATE 4
(W₄) AT ¼-SPAN
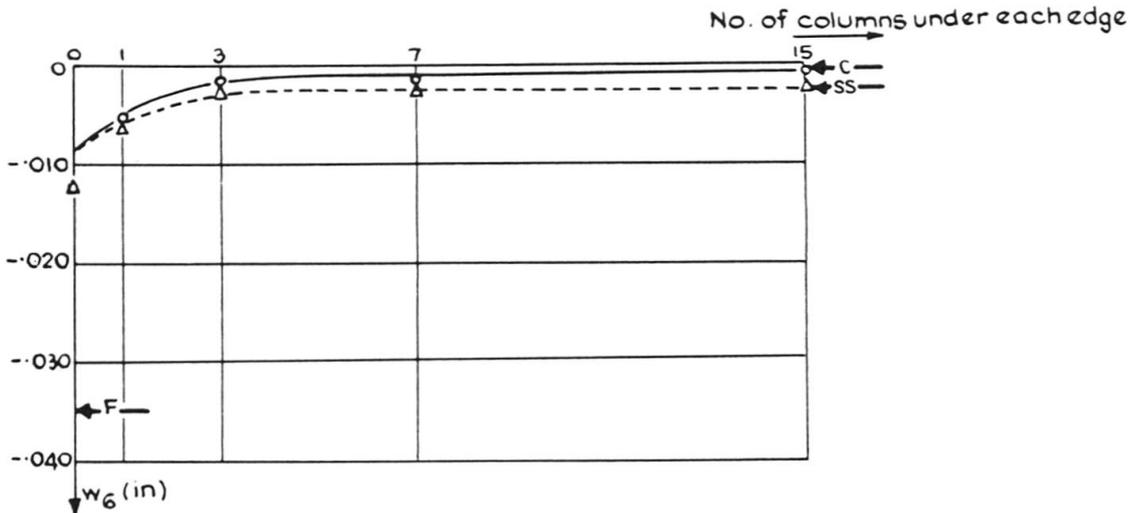
FIG. 8(b) NORMAL DEFLECTIONS (in.) AT MID-POINT OF PLATE 6
(W₆) AT ¼-SPAN.

Figs. 7 and 8, the experimental points are seen to lie close to the finite
element curves. A slight tendency is observed for the method to underestimate
the deflections of those models supported by the clamping columns, this being
due to the fact that the experimental columns were not capable of completely
eliminating all the displacements at the points of support, as assumed in the
theory.

The values obtained from the three elasticity solutions for the different edge
support conditions are included in Figs. 7 and 8 and confirm that, as suggested
previously by Rockey and Evans |5,6|, large errors may occur in the values given
by this method because of the inherent assumption that the end diaphragms
impose no restraint on the longitudinal warping of the end cross-sections. The
graphs show that extremely large errors occur in the values predicted for the
model with free edges (denoted by symbol F), but that the elasticity solution
shows greater accuracy for the models in which the edges are supported (denoted
by symbols C and SS).

The reasons for the errors in
the elasticity solution are
shown clearly in Figs. 9(a)
and 9(b), where the
longitudinal movements of
the end diaphragm are plotted
for the models of Tests 1
and 9. In Fig. 9(a), it is
seen that the elasticity
method predicts large
longitudinal warping
displacements of the end
cross-section of the model
with unsupported edges in
Test 1 and that these
predicted displacements
differ greatly from those
actually occurring during
the test. Figure 9(b),
on the other hand, shows
that for the model with
simply supported edges in
Test 9, far less warping
of the end cross-section
is predicted by the elasticity
method and the theoretical
end movements are much
closer to those measured
experimentally. Consequently,
the elasticity values of ridge
and plate displacements at the
mid and quarter-span cross-sections
of the models with supported edges
are in much better agreement with the
experimental values than are the
elasticity values for the model with
unsupported edges.

Figures 9(a) and 9(b) also show that, for
the model with unsupported edges in Test 1
and for the model with the simply-
supporting columns at the sixteenth-
span positions in Test 9, the end
movements predicted by the finite
element method are in good agreement
with the experimentally measured
movements. These diagrams thus illustrate
once more the adaptability of the finite
element method in the specification of
support conditions.

FIG. 9(a) END MOVEMENTS (in)
OF MODEL WITH
FREE EDGES.
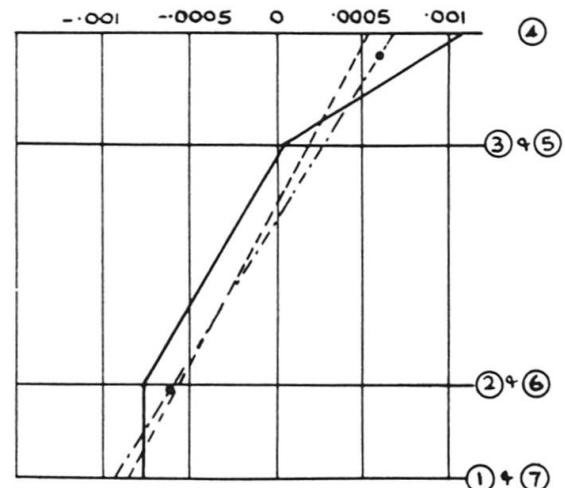
FIG. 9(b) END MOVEMENTS (in.)
OF MODEL WITH ALL
SIMPLY SUPPORTING
COLUMNS IN POSITION

7.   CONCLUSIONS

On the basis of both the experimental and finite element results, it may be
concluded that the introduction of intermediate column supports at the quarter-
span positions underneath the edges of a folded plate is an effective way of
reducing the structural displacements under load. The slight increase of
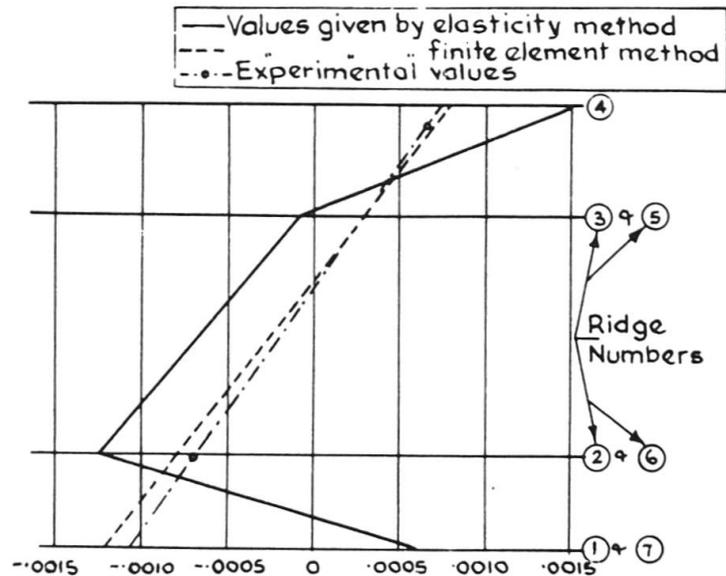stiffness obtained by introducing more columns is not sufficient to justify the

increased cost of providing the additional columns or the greater interference with the access to the covered area that the additional columns would offer.

Secondly, it has been shown that, if the columns introduced can be made to provide clamped conditions at the support points, i.e. a restriction on all the displacement and rotation components at the points, then their efficiency is significantly increased.

The results obtained have also confirmed the accuracy of the finite element method when applied to the analysis of folded plates and it has been established that the method can be successfully adapted to the analysis of folded plates supported by intermediate columns.

Finally, the inaccuracies that can arise from the application of the elasticity method have again been illustrated. It has been shown that these errors are much greater for models with unsupported longitudinal edges since the introduction of edge supports reduces the tendency for the cross-section of the structure to warp longitudinally. The difference between the end support conditions assumed in the elasticity method and those provided by normal end diaphragms are then no longer critical.

NOTATION

All terms are defined in Figs. 5(a) and (b).

rz_n vertical deflection of ridge n
ry_n horizontal deflection of ridge n
w_n deflection perpendicular to plate n at the mid-width position on the plate.

ACKNOWLEDGEMENT

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