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An Experimental and Theoretical Study of the Behaviour of Folded Plates Supported by Intermediate Columns

Etude expérimentale et théorique du comportement de stuctures plissées appuyées sur des colonnes intermédiaires

Eine experimentale und theoretische Untersuchung des Verhaltens von Faltwerken, die auf Zwischenstützen aufgelegt sind

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SUMMARY:

The paper describes an experimental and theoretical study of the behaviour of folded plate structures supported by intermediate columns. It is shown that the introduction of columns at the quarter-span positions underneath the longitudinal edges is an effective way of reducing the structural displacements under load. The results presented also confirm the accuracy of the finite element method of folded plate analysis, whereas the elasticity method is shown to lead to considerable errors, in certain circumstances, due to end support assumptions.

RÉSUMÉ:

L'article traite d'une étude expérimentale et théorique du comportement de structures plissées appuyées sur des colonnes intermédiaires. L'étude montre que la présence de colonnes aux quarts de la portée longitudinale permet de réduire efficacement les déplacements sous l'effet de charges.
Les résultats confirment aussi la précision de l'analyse des structures plissées par la méthode des éléments finis. La méthode élastique, par contre, peut donner des erreurs considérables, résultant des conditions aux limites admises.

ZUSAMMENFASSUNG:

Der Artikel behandelt eine experimentale und theoretische Untersuchung des Verhaltens von Faltwerken, die auf Zwischenstützen aufgelegt sind. Es wird gezeigt, dass ins Viertelfeld eingebrachte Stützen effektvoll wirken, um die durch die Belastung verursachten Verschiebungen zu vermindern.
Die Ergebnisse bestätigen auch die Genauigkeit der Methode der finiten Elemente bei der Analyse von Faltwerken. Die Elastizitätsmethode hingegen kann wegen ihrer Annahmen bezüglich der Endstützung zu grossen Ungenauigkeiten führen.

1. INTRODUCTION

In recent years, folded plates have become increasingly popular as roofing structures because they provide an economical and aesthetically pleasing design. Their popularity has led to the publication of numerous papers dealing with folded plate analysis and the various methods that have been proposed were critically reviewed by Evans and Rockey |1| in a previous publication. The majority of the proposed methods are only applicable to simply supported structures and very few |2,3,4| are capable of dealing with structures supported by randomly placed intermediate columns, such as those shown in Fig. 1. Such columns are frequently employed in practice since they can greatly increase the span of the roof without interfering to any great extent with the access to or the utilization of the covered area.

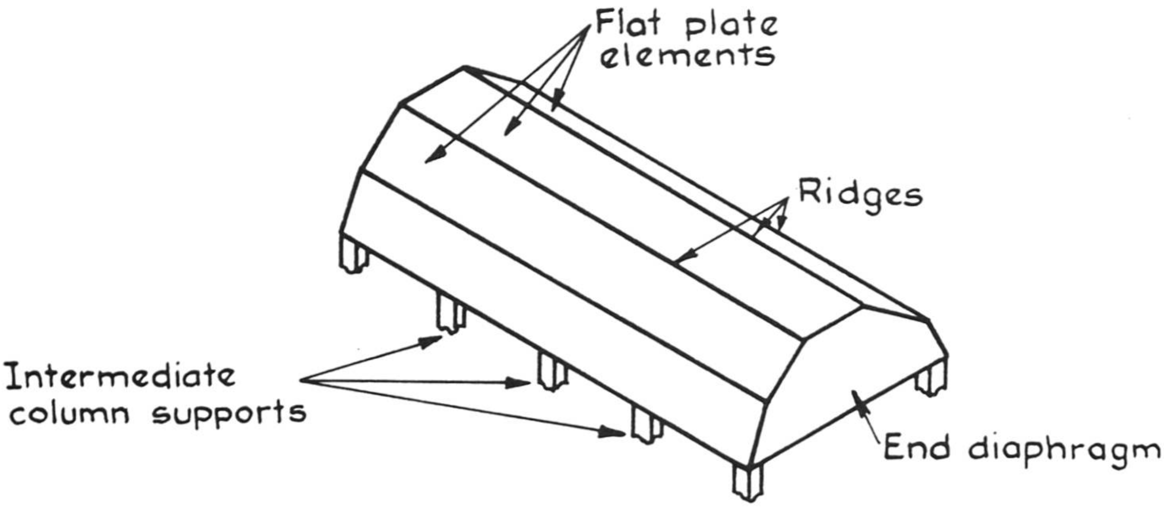

FIG. 1 A TYPICAL FOLDED PLATE STRUCTURE

The few theories that have been proposed for the analysis of folded plates supported by columns are rather limited in their range of application. They are based on superposition techniques which require several re-analyses of the structure within any one solution and the extensive calculations involved necessitate the use of a digital computer. Furthermore, since they are based on either the "ordinary" |2| or "elasticity" |3| methods of folded plate analysis, they assume that each supporting end diaphragm, such as that illustrated in Fig. 1, is infinitely stiff in its own plane and perfectly flexible normal to its plane. The end cross-section of the folded plate is thus assumed to be free to warp in the longitudinal direction.

In fact, most end diaphragms provided in practice will be far too stiff to allow this longitudinal warping to occur freely and, in previous publications, Rockey and Evans |5,6| have shown, both experimentally and theoretically, the great effect that this warping restraint has upon the overall structural behaviour. In this previous study, errors of up to 200% were observed in the values predicted by the elasticity method, and it was proved that these errors were primarily due to the way in which the end support conditions were idealized in the method. A finite element solution, which had been developed earlier by Rockey and Evans |7|, was applied in this previous study, this method being capable of accurately representing the end support conditions provided in the experiments, and good agreement was obtained between the experimental and finite element results. In this present investigation, this finite element technique will be applied to the analysis of folded plates supported by intermediate columns.

In contrast to the large number of papers published on the analysis of folded plates, very few experimental studies of folded plates have been reported |5,6,8|. This is particularly true for folded plates supported by intermediate columns and in this report, the results of eight tests on such structures will be presented. The effects of varying both the type of column support and the location of the columns will be illustrated and the experimental results will in each case be compared to theoretical values obtained by the finite element method. In addition, theoretical results obtained by the elasticity method will be included in the comparison where possible. A similar series of tests upon folded plates containing large openings was described by Rockey and Evans |9| in 1976.

2. DETAILS OF MODELS TESTED AND COLUMN SUPPORTS

The folded plate model tested was of the vault type of cross-section shown in Fig. 2, the cross-sectional dimensions being as shown in the figure. The longitudinal span of the model between end diaphragms was $71\frac{1}{4}$ in (1810 mm), the length/width ratio of the internal plates thus being 11.2. The model was formed from HS30/P aluminium sheet having a thickness of 0.08 in (2 mm), and the model was folded from a single flat sheet, thus ensuring full continuity between adjacent plates at each longitudinal ridge. A series of material tests established the modulus of elasticity and the Poisson's ratio of the aluminium as 10.2×10^6 lbf/in^2 (70.33×10^6 N/mm^2) and 0.32, respectively.

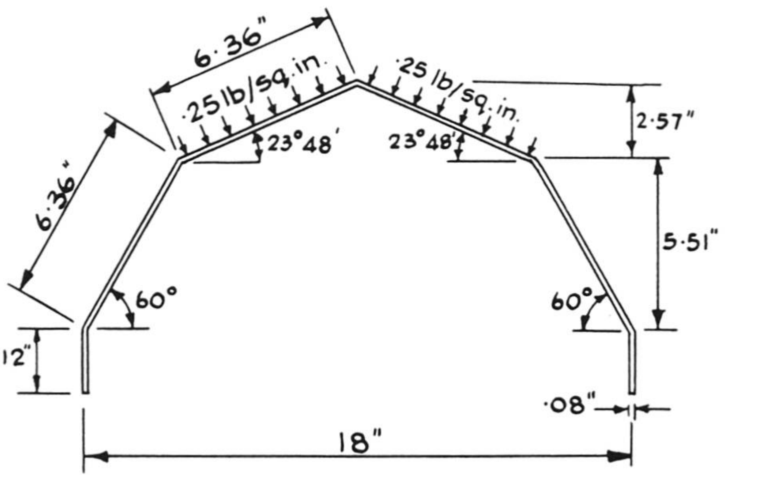

FIG. 2 DETAILS OF VAULT-TYPE CROSS-SECTION CONSIDERED IN THE EXPERIMENTS

Having first tested the model with a clear span between end diaphragms, column supports were then introduced underneath both longitudinal edges of the model. Two different types of column supports were considered, columns that restrained all displacements at their point of application being considered in the first instance, and then columns that only restrained the vertical and transverse horizontal deflections at a point being introduced. These two types of columns will be further referred to as "clamping" and "simply supporting" columns, respectively, and they are illustrated diagrammatically in Fig. 3.

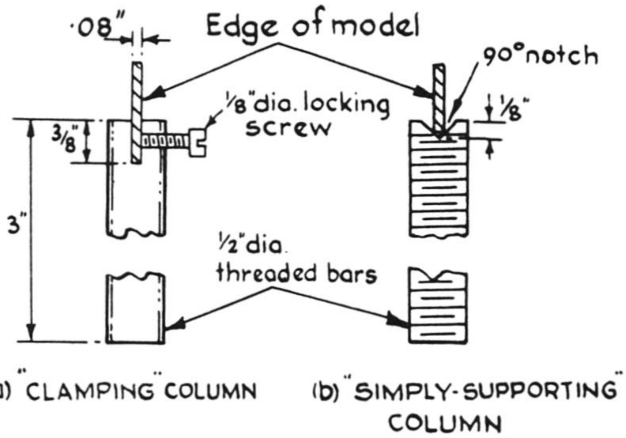

FIG. 3 DETAILS OF COLUMN SUPPORTS

To assess the effects of column spacing, four different arrangements of each column type were considered. The columns were introduced under the edges of the model, first of all at the mid-span position only, then at the quarter-span positions, then at the eighth-span positions and, finally, at the sixteenth-span positions, as shown in Fig. 4.

Nine tests were thus carried out in all and they may be summarized as follows:

TEST 1 - model with clear span between end supports.

TEST 2 - clamping columns introduced at mid-span positions.

TEST 3 - clamping columns introduced at quarter-span positions.

TEST 4 - clamping columns introduced at eighth-span positions.

TEST 5 - clamping columns introduced at sixteenth-span positions.

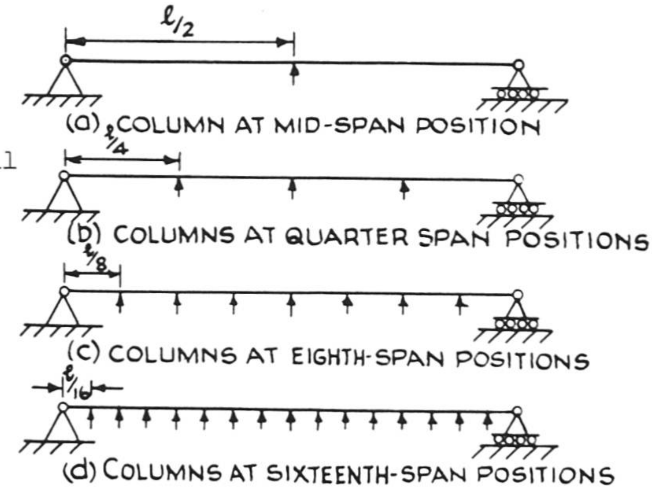

FIG. 4 LOCATIONS OF EDGE SUPPORTING COLUMNS.

TEST 6 - simply supporting columns introduced at mid-span positions.

TEST 7 - simply supporting columns introduced at quarter-span positions.

TEST 8 - simply supporting columns introduced at eighth-span positions.

TEST 9 - simply supporting columns introduced at sixteenth-span positions.

3. GENERAL EXPERIMENTAL DETAILS

In all tests, symmetrical loads of uniform intensity were applied perpendicular to the two top plates of the model, as shown in Fig. 2. These loads were applied by means of an air pressure-bag system, the air-bag being confined to bear against the top surface of the model so that the pressure in the bag was transmitted as a uniformly distributed load perpendicular to the plane of each loaded plate.

The model was supported by end diaphragms machined from $\frac{3}{8}$ in (9.5 mm) thick mild steel plates to the shape of the vault-type cross-section, the model being rigidly bolted to the diaphragms. These diaphragms were then mounted on special bearings, designed so that free rotation and translation of the diaphragms perpendicular to their planes could take place.

In the tests, the displacements of the model were measured at the mid-span and quarter-span cross-sections, using dial gauges graduated in units of 0.0001 in (0.0025 mm), positioned underneath the model. Seven values of ridge displacements were measured at the mid-span cross-section, as shown in Fig. 5(a). In addition, the displacements perpendicular to the plane of the six plates at the quarter-span cross-section were measured, as shown in Fig. 5(b). The system of ridge and plate numbering adopted and the notation and positive directions to be used for the measured displacements are also defined in these diagrams.

The maximum load applied in each test was chosen such that the model was not loaded beyong its elastic range and that the membrane action within the individual plates did not become significant. Within this load range, 14 increments of load were applied in each test and the deflections recorded at each load level. The resulting load-deflection relationships all exhibited good linear characteristics and enabled accurate values of the slope of the load/deflection graphs to be obtained. Using this data, the experimental deflections corresponding to an applied load intensity of 0.25 lbf/in^2 (1.7 N/mm^2) were determined and adopted for comparison with theoretical results.

In all tests, the required symmetry of the measured deflections about the longitudinal centre line was obtained. At the end of each test, the pressure in the air-bag was released and the deflection of the model in its final unloaded state recorded. In each case, it was observed that the final unloaded position was almost identical to the initial position of the model, thus confirming that the model had not been loaded beyond its elastic range.

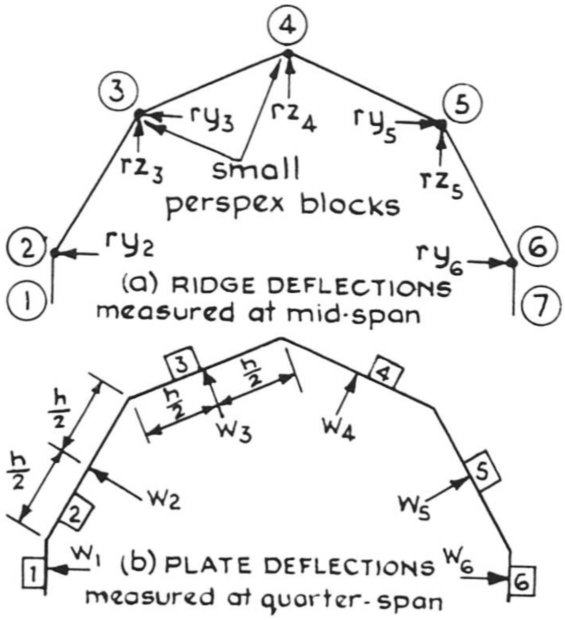

FIG. 5 POSITIONS OF DEFLECTION MEASUREMENTS DURING TESTS.

4 THEORETICAL SOLUTIONS

Each of the models tested was analysed by the finite element method. The application of this method to the analysis of folded plates has been considered in detail by Rockey and Evans |7| in a previous publication and the solution established employs rectangular elements, five degrees of freedom, viz. the two displacements associated with plane stress and the three displacements associated with plate bending, being considered at each node.

In each finite element solution, the folded plate was divided into 640 elements, as shown in Fig. 6, since previous experience |6| had shown that a convergent solution would be obtained with such a mesh. Because of the symmetry of the model and of the loading and support conditions, it was only necessary to consider one-quarter of the structure in the analysis, as shown in Fig. 6.

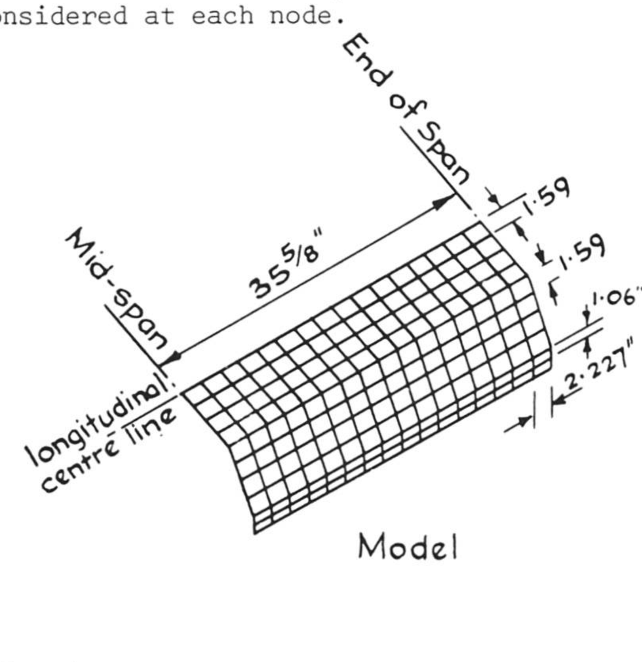

FIG. 6 DISTRIBUTION OF ELEMENTS FOR FINITE ELEMENT SOLUTION

One of the main advantages of the finite element method is the ease with which it can be adapted to deal with different support conditions. The intermediate column support conditions of the present investigation were dealt with conveniently, simply by ensuring that a node point of the mesh coincided with each support position and specifying that the relevant components of displacement were zero at these points. Also, as discussed in detail elsewhere |6|, the rigid body rotation and translation of the end diaphragms in the longitudinal direction were conveniently and accurately simulated in the finite element solution.

In addition to the finite element solutions, the models were also analysed, when possible, by the elasticity method. Since this method in its basic form is unable to take discreet point supports into account, only three elasticity solutions were obtained. In the first of these solutions, the longitudinal edges of the model were assumed to be free, as in Test 1. In the second solution, each edge was assumed to be fully and continuously clamped along its length, thus providing a limiting theoretical solution for Tests 2, 3, 4 and 5 in which the number of clamping columns underneath each edge was progressively increased. Then, in the final elasticity solution, each edge was assumed to be simply supported along its length, thus providing a limiting theoretical solution for Tests 6, 7, 8 and 9 in which the simply supporting columns were introduced.

Both the finite element and elasticity solutions were obtained by means of computer programs written in FORTRAN for the ATLAS computer. Neither method is amenable to a hand solution because of the extensive calculations involved.

5. RESULTS

Both the finite element and elasticity solutions give comprehensive pictures of the stresses and displacements set up within a structure. However, in the present report, only those displacements that were also measured experimentally, as illustrated in Fig. 5, will be discussed.

In Table 1, the displacements measured in Tests 1, 2, 3, 4 and 5 are presented and compared to finite element values, the notation used for the deflections being as defined in Fig. 5. The behaviour of the models supported by clamping columns at the mid, quarter, eighth and sixteenth-span positions, i.e. in Tests 2, 3, 4 and 5 respectively, may thus be compared to the behaviour of the model with unsupported edges in Test 1. Corresponding values for the models supported by the simply-supporting columns in Tests 6, 7, 8 and 9 are compared to the values for the unsupported model in Test 1 in Table 2.

In Figs. 7(a) and 7(b), the values of two typical mid-span ridge displacements, viz. the vertical displacement of ridge 5 (from symmetry, $rz_5 = rz_3$) and the horizontal displacement of ridge 6 (from symmetry, $ry_6 = ry_2$), are plotted against the number of columns introduced under each edge. The effects of both the clamping and simply supporting columns are illustrated in these diagrams. Similar curves are presented in Figs. 8(a) and 8(b) for two typical mid-plate displacements at the quarter-span cross-section, the displacements perpendicular to plates 4 (from symmetry, $w_4 = w_3$) and 6 (from symmetry, $w_6 = w_1$) being illustrated.

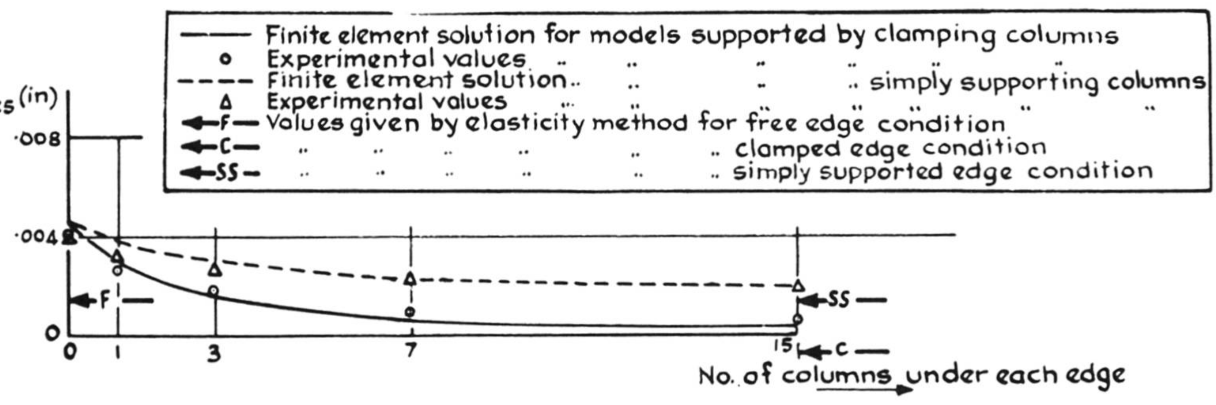

FIG.7(a) VERTICAL DEFLECTIONS (in.) OF RIDGE 5 (r_{z5}) AT MID-SPAN

Column Positions	Solution	Ridge deflections at mid-span ins (mm) x 10^{-5}				Plate deflections at quarter-span ins (mm) x 10^{-5}		
		rz_4	$ry_3 = ry_5$	$rz_3 = rz_5$	$ry_2 = ry_6$	$w_3 = w_4$	$w_2 = w_5$	$w_1 = w_6$
Free Edges Test 1	Finite Element	7.1(180)	-1.2(/o.5)	4.6(117)	-13.3(338)	7.6(193)	-4.2(107)	- 8.7(221)
	Experimental	7.6(193)	-1.7(43.2)	4.0(102)	-16.9(429)	7.7(196)	-6.3(160)	-11.2(284)
Mid-Span Test 2	Finite Element	7.0(178)	-1.8(45.7)	3.0(76.2)	- 6.8(173)	6.9(175)	-3.5(81.3)	- 4.8(122)
	Experimental	7.4(188)	-2.3(58.4)	2.6(66.0)	- 6.5(165)	6.6(168)	-4.0(102)	- 5.6(142)
1/4-Span Test 3	Finite Element	6.2(157)	-2.1(53.3)	1.6(40.6)	- 4.8(122)	6.3(160)	-4.6(117)	- 1.5(38.1)
	Experimental	6.8(173)	-2.4(61.0)	1.8(45.7)	- 5.2(132)	6.2(157)	-5.1(130)	- 1.8(45.7)
1/8-Span Test 4	Finite Element	5.1(130)	-2.0(50.8)	0.6(15.2)	- 3.1(78.7)	5.5(140)	-3.9(99.1)	- 1.0(25.4)
	Experimental	6.0(152)	-2.6(66.0)	0.9(22.9)	- 4.0(102)	6.2(157)	-5.0(127)	- 1.4(35.6)
1/16-Span Test 5	Finite Element	4.5(114)	-1.9(48.3)	0.3(7.6)	- 2.4(61.0)	5.1(130)	-3.6(91.4)	- 0.7(17.8)
	Experimental	5.8(147)	-2.6(66.0)	0.5(12.7)	- 3.1(78.7)	6.1(155)	-4.8(122)	- 0.9(22.9)

TABLE 1 : Deflections of models supported by clamping columns.

Column Positions	Solution	Ridge deflections at mid-span ins (mm) x 10^{-5}				Plate deflections at quarter-span ins (mm) x 10^{-5}		
		rz_4	$ry_3 = ry_5$	$rz_3 = rz_5$	$ry_2 = ry_6$	$w_3 = w_4$	$w_2 = w_5$	$w_1 = w_6$
Free Edges Test 1	Finite Element	7.1(180)	-1.2(30.5)	4.6(117)	-13.3(338)	7.6(193)	-4.2(107)	- 8.7(221)
	Experimental	7.6(193)	-1.7(43.2)	4.0(102)	-16.9(429)	7.7(196)	-6.3(160)	-11.2(284)
Mid-Span Test 6	Finite Element	7.1(180)	-1.5(38.1)	3.7(94.0)	- 7.9(201)	7.2(183)	-3.5(88.9)	- 5.9(150)
	Experimental	7.4(188)	-2.0(50.8)	3.2(81.3)	- 7.5(191)	6.9(175)	-4.6(117)	- 6.2(157)
1/4-Span Test 7	Finite Element	7.1(180)	-1.8(45.8)	3.1(78.7)	- 7.1(180)	7.2(183)	-4.9(124)	- 3.0(76.2)
	Experimental	7.4(188)	-2.4(61.0)	2.7(68.6)	- 6.9(175)	6.9(175)	-5.3(135)	- 2.9(73.7)
1/8-Span Test 8	Finite Element	6.9(175)	-2.0(50.8)	2.3(58.4)	- 6.0(152)	6.8(173)	-4.8(122)	- 2.5(63.5)
	Experimental	7.3(185)	-2.6(66.0)	2.3(58.4)	- 5.7(145)	6.5(165)	-5.4(137)	- 2.6(66.0)
1/16-Span Test 9	Finite Element	6.7(170)	-2.2(55.9)	1.9(48.3)	- 5.4(137)	6.6(168)	-4.8(122)	- 2.2(55.9)
	Experimental	7.5(191)	-2.7(68.6)	1.9(48.3)	- 5.5(140)	6.9(175)	-5.6(142)	- 2.5(63.5)

TABLE 2 : Deflections of models supported by simply-supporting columns

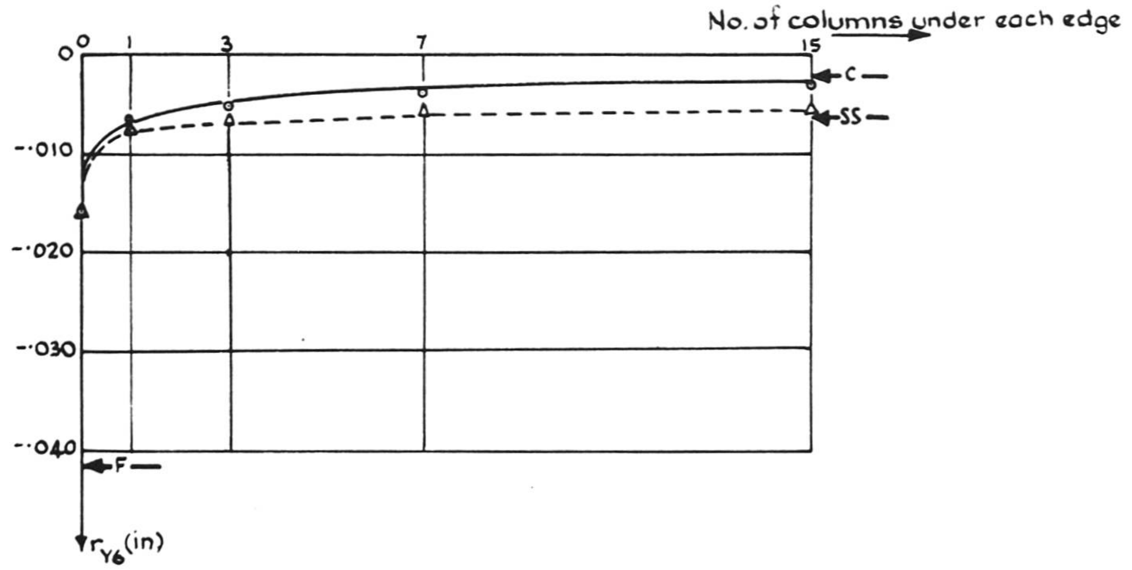

FIG. 7(b) HORIZONTAL DEFLECTIONS (in.) OF OUTER RIDGE (r_{Y6}) AT MID-SPAN

In addition to the finite element and the experimental values, the limiting theoretical values obtained by the elasticity method, first of all assuming the edges of the model to be free, then assuming the edges to be fully clamped, and, finally, assuming the edges to be simply supported, are included in Figs. 7(a), 7(b), 8(a) and 8(b).

Finally, in Figs. 9(a) and 9(b), the longitudinal movements of the end diaphragms are plotted. The values shown in Fig. 9(a) are for the free edge model of Test 1 and, in Fig. 9(b), values for the model supported by simply-supporting columns at the sixteenth-span positions in Test 9 are shown. Finite element and elasticity values are included on both diagrams for comparison with the measured values.

6. DISCUSSION OF RESULTS

From the results, it is seen that the introduction of the clamping columns has a considerably greater influence on the behaviour of the structure than the introduction of the simply-supporting columns. It is also apparent that, whereas the introduction of a column of either type at the mid-span position significantly increases the structural stiffness, as more columns are introduced, the rate of increase of structural stiffness with the number of columns decreases.

In fact, little gain in stiffness is obtained by increasing the number of colums under each edge from 7 to 15, so that a structure supported by columns at the eighth-span positions along each edge is, effectively, as stiff as a structure in which the edges are continuously supported along the complete span. Even introducing columns at the quarter-span positions only, i.e. positioning 3 columns under each edge, is an effective way of reducing the structural deflections, and, from a design point of view, such an arrangement has the great advantage of offering very little interference to the access to the area covered by the folded plate roof.

For all models tested, Tables 1 and 2 show reasonable agreement between the experimental values and the results obtained by the finite element method and, in

——	Finite element solution for models supported by clamping columns
o	Experimental values " " " "
- - - -	Finite element solution " " " " simply supporting columns
Δ	Experimental values " " " " " " "
←F—	Values given by elasticity method for free edge condition
←C—	" " " " " " clamped edge condition
←SS—	" " " " " " simply supported edge condition

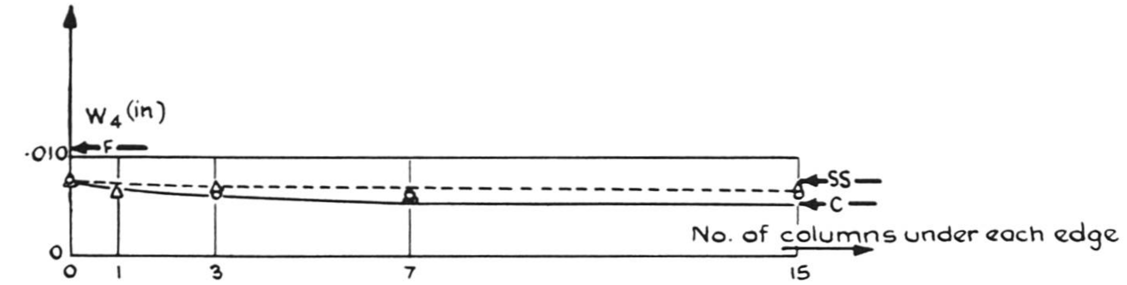

FIG. 8(a) NORMAL DEFLECTIONS (in.) AT MID-POINT OF PLATE 4 (W_4) AT 1/4-SPAN

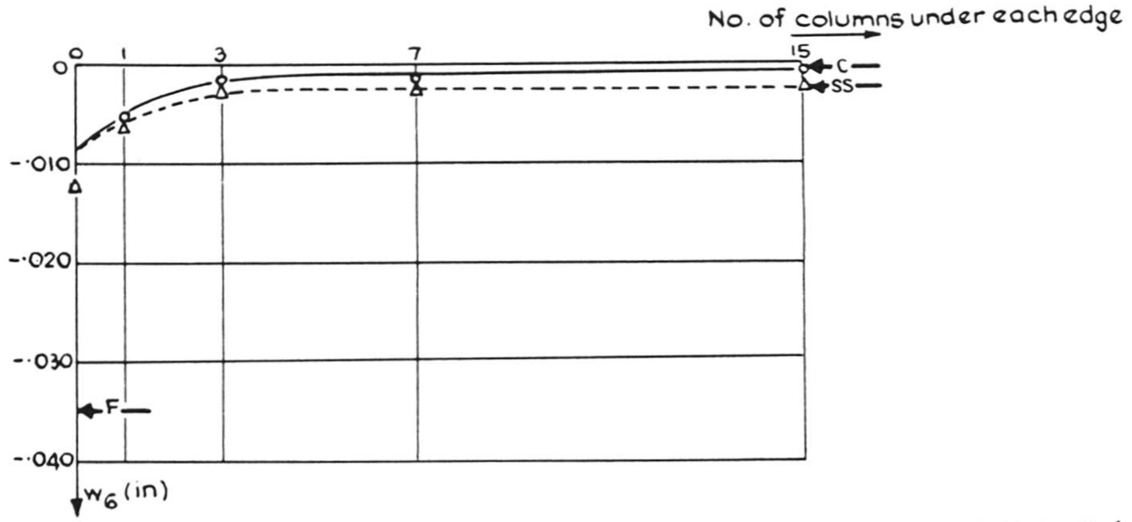

FIG. 8(b) NORMAL DEFLECTIONS (in.) AT MID-POINT OF PLATE 6 (W_6) AT 1/4-SPAN.

Figs. 7 and 8, the experimental points are seen to lie close to the finite element curves. A slight tendency is observed for the method to underestimate the deflections of those models supported by the clamping columns, this being due to the fact that the experimental columns were not capable of completely eliminating all the displacements at the points of support, as assumed in the theory.

The values obtained from the three elasticity solutions for the different edge support conditions are included in Figs. 7 and 8 and confirm that, as suggested previously by Rockey and Evans |5,6|, large errors may occur in the values given by this method because of the inherent assumption that the end diaphragms impose no restraint on the longitudinal warping of the end cross-sections. The graphs show that extremely large errors occur in the values predicted for the model with free edges (denoted by symbol F), but that the elasticity solution shows greater accuracy for the models in which the edges are supported (denoted by symbols C and SS).

The reasons for the errors in the elasticity solution are shown clearly in Figs. 9(a) and 9(b), where the longitudinal movements of the end diaphragm are plotted for the models of Tests 1 and 9. In Fig. 9(a), it is seen that the elasticity method predicts large longitudinal warping displacements of the end cross-section of the model with unsupported edges in Test 1 and that these predicted displacements differ greatly from those actually occurring during the test. Figure 9(b), on the other hand, shows that for the model with simply supported edges in Test 9, far less warping of the end cross-section is predicted by the elasticity method and the theoretical end movements are much closer to those measured experimentally. Consequently, the elasticity values of ridge and plate displacements at the mid and quarter-span cross-sections of the models with supported edges are in much better agreement with the experimental values than are the elasticity values for the model with unsupported edges.

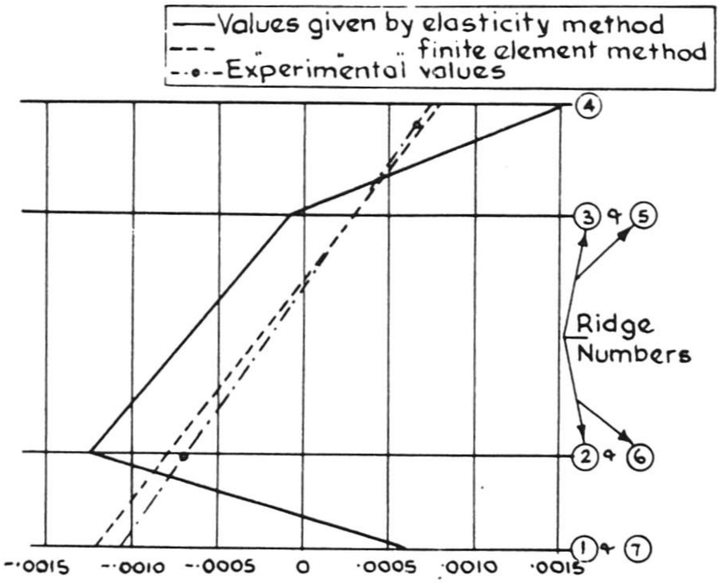

FIG. 9(a) END MOVEMENTS (in) OF MODEL WITH FREE EDGES.

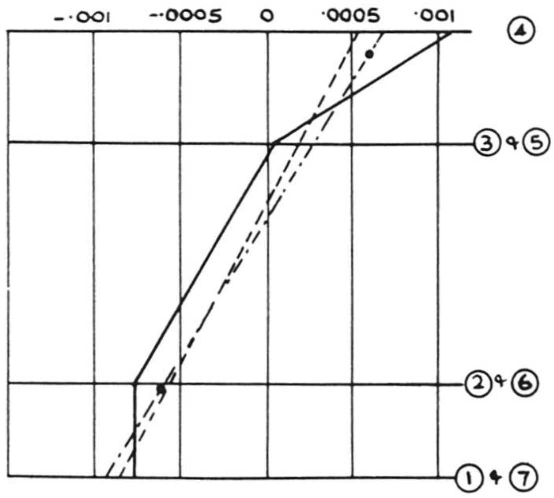

FIG. 9(b) END MOVEMENTS (in.) OF MODEL WITH ALL SIMPLY SUPPORTING COLUMNS IN POSITION

Figures 9(a) and 9(b) also show that, for the model with unsupported edges in Test 1 and for the model with the simply-supporting columns at the sixteenth-span positions in Test 9, the end movements predicted by the finite element method are in good agreement with the experimentally measured movements. These diagrams thus illustrate once more the adaptability of the finite element method in the specification of support conditions.

7. CONCLUSIONS

On the basis of both the experimental and finite element results, it may be concluded that the introduction of intermediate column supports at the quarter-span positions underneath the edges of a folded plate is an effective way of reducing the structural displacements under load. The slight increase of stiffness obtained by introducing more columns is not sufficient to justify the increased cost of providing the additional columns or the greater interference with the access to the covered area that the additional columns would offer.

Secondly, it has been shown that, if the columns introduced can be made to provide clamped conditions at the support points, i.e. a restriction on all the displacement and rotation components at the points, then their efficiency is significantly increased.

The results obtained have also confirmed the accuracy of the finite element method when applied to the analysis of folded plates and it has been established that the method can be successfully adapted to the analysis of folded plates supported by intermediate columns.

Finally, the inaccuracies that can arise from the application of the elasticity method have again been illustrated. It has been shown that these errors are much greater for models with unsupported longitudinal edges since the introduction of edge supports reduces the tendency for the cross-section of the structure to warp longitudinally. The difference between the end support conditions assumed in the elasticity method and those provided by normal end diaphragms are then no longer critical.

NOTATION

All terms are defined in Figs. 5(a) and (b).

rz_n vertical deflection of ridge n
ry_n horizontal deflection of ridge n
w_n deflection perpendicular to plate n at the mid-width position on the plate.

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