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***WORKING DOCUMENT***

*November 1989*

***Proposal for a Code for the Direct Use of Reliability Methods in Structural Design***

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Associations supporting the JCSS:

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# Introduction

This is the first document of an envisaged series of publications, prepared by individual authors but discussed within the Joint Committee on Structural Safety (JCSS), in particular within its Working Party. They are referred to as «working documents» since they generally will give information on the state of development of certain concepts or subjects, rather than giving approved guidelines. Where a document is officially approved by the Plenum of the JCSS, this will be identified explicitely.

This document is a first step towards a code for direct use of reliability methods in design.

Previous JCSS documents as, for example the «General Principles on Reliability for Structural Design» also published by IABSE, were mainly concerned with providing the background for a reliability based code. It is the general opinion of the JCSS Working Party, that reliability methods have advanced to an extent, that they may not only be used for deriving safety provisions in codes. A design which utilizes the full statistical information available and the advantages of a direct probabilistic modelling is possible – if only relevant for – special situations. These special situations may arise, for example, where a major part of the design information needs to be updated to account for specific conditions of the project or where detailed failure analyses are required.

It is well understood, that this type of code will never replace present (deterministic or reliability-based) codes. However, it may serve as a fundamental code which is supplemented by codes giving rules for common design.

With this document it is intended to show, how a code for direct use of reliability methods may look like. Main emphasis is given on identifying those conventions and models which need to be codified. It is far from being a complete proposal. In the present form the document addresses reliability experts only, i.e. – as a potential code – it does not intend to promote the general understanding of reliability concepts. This issue may be disputable.

This document has been discussed within the JCSS Working Party and the basic ideas and concepts are approved. Some details of modelling, e.g. concerning model uncertainties and numerical values, in particular for safety indices, mainly reflect the opinion of the authors. Also, the terminology and some conceptual details are still under discussion.

Irrespective of these reservations, publication is supported in order to initiate discussions and exchange of comments at an early stage. The document will be revised subsequently.

Marita Kersken-Bradley
for the Working Party
(General Reporter of the JCSS)

Julio Ferry Borges
(President of the JCSS)

November 1989

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# PROPOSAL FOR A CODE FOR THE DIRECT USE OF RELIABILITY METHODS IN STRUCTURAL DESIGN

## 1. PREFACE

When making considerations about structural safety it is essential to appreciate that a measure of safety based on a general probabilistic model in general does not express a pure physical property of the structure in its environments of actions. Rather the safety measure is a decision variable that embraces the applied knowledge about the strength properties of the structure in relation to the actions on the structure. The value of the safety measure therefore may change in both directions with the amount and quality of the information on basis of which it is calculated.

With this philosophy in mind the structural reliability theory becomes a design decision tool based on scientific methods rather than being a scientific theory itself aiming at a description of the "truth of nature". It may be looked upon as a formal language of rational thinking to facilitate good engineering decisions in the process of the design of structures. It should contain several formal elements and mathematical composition rules to allow for inclusion of all sorts of relevant information of sufficient confidence to let it effect the decisions. On the other hand, it should not be too rich of elements forcing the user of this reliability theory to make almost non-verifiable value assignments to which the design decisions are unreasonably sensitive. The consequence is that reliability theories for codes of practice should contain certain restrictive standardized value assignments.

In this context "code of practice" means a model universe agreed upon as a basis for design decisions. This agreement is thought of as made within the group of parties of concern (e.g. the designer, the manufacturer, the owner, the user, the last two parties possibly being represented by the public authorities). The code of practice may in this sense be specific for a given project, or it may be more general as a part of public building regulations. Thus the terminology "code of practice" as applied herein is an abstraction that should not be tied to existing types of codes of practice. In the same spirit the term "code committee" should be interpreted as the group of parties agreeing on a code of practice.

The following text aims at presenting an example (a model) of a code of practice enabling reliability methods for design. The code text is given in parallel with an explanatory text (in roman). The latter does not have the status of a code. The terminology has been discussed within the JCSS-Working Party but general agreement has not been reached. The terminology is therefore up to revision.

## 2. GENERAL

*It is a fundamental requirement of this code that the reliability measure is relative in the sense that it induces an ordering of any set of structures according to their reliability with respect to any well-defined adverse event. Furthermore, for each structure it is required that the measure induces an ordering of any set of adverse events. It must even possess sufficient generality to allow for an ordering of any set of pairs: (structure, adverse event).*

A reliability ordering relation like

(structure 1, adverse event 1) < (structure 2, adverse event 2)

may be needed for different types of structures for which the adverse events are not the same.

The question of whether there is an absolute interpretation of the reliability measure is less important for the applications. Often there is no direct physical relative frequency interpretation related to the measure. Rather such an interpretation is related to the relative frequency of no adverse event occurring in the consistent long run use of the reliability analysis methodology in the absence of gross errors (mistakes).

*This code allows for design on the basis of a reliability measure that deviates from the reliability measure defined herein provided it is within the scope of probability theory and well-documented by scientific methods and arguments.*
*If the reliability requirement is given in terms of a value of the reliability measure of this code, but an alternative reliability measure is used for the design decisions, a corresponding transformation of the requirement must be made. This transformation must be such that the alternative reliability measure when meeting the requirement leads to at least the same structural dimensions as obtained by use of the code reliability measure when both measures are applied on a sufficiently representative example structure.*
*Further details on reliability requirements are given in Section 6.*

## 3. CONCEPT OF ADVERSE STATE

*The structural performance of a whole structure or part of it should be described with reference to a specified set of adverse states beyond which the structure no longer satisfies the performance requirements. Each adverse state is the boundary of an adverse event declared to be so by the committee setting up the performance requirement. A binary description of the performance is inherent in the adverse event concept.*

Examples of adverse events are:
- loss of static equilibrium of the structure, or a part of the structure, considered as a rigid body,
- rupture of critical sections of the structure caused by exceeding the ultimate strength, possibly reduced by repeated loading, or the ultimate deformation of the material,
- transformation of the structure into a mechanism,
- loss of stability,
- progressive collapse,
- deformations which affect the efficient use or appearance of structural or non-structural elements,
- excessive vibrations producing discomfort or affecting non-structural elements or equipment,
- local damage, including cracking, which reduces the durability of a structure or affects the efficiency or appearance of structural or non-structural elements.

## 4. BASIC VARIABLES AND UNCERTAINTY MODELING

*The uncertainties of the mechanical models and their parameters as used in the process of making decisions are represented in terms of concepts from the mathematical probability theory.*
*Among the parameters of relevance some are presented as being basic variables in the sense that they are assumed to carry the entire input information to the mechanical model.*

Typically the basic variables are material parameters, external action parameters, and geometrical parameters. All other parameters are functions of these basic variables. The functions are defined by the geometrical properties of the structure, the action model and the mechanical model. Typically they are cross-section resistances, member buckling resistances, load effects, areas, volumes, safety margins, event margins, etc.
In the simplest case of modeling the basic variables may be joined into a finite-dimensional vector. Then the uncertainties of the problem is modeled by letting this vector, or a subvector of it, be a vector of random variables.
The concept of basic variables should not be confused with the probabilistic concept of mutual independent random variables. The basic variables are defined as the free input variables in the mechanical model before the probabilistic properties are defined. (In the standard mathematical analysis these variables are called the independent variables while a function of the independent variables is called the dependent variable).
The imposed probabilistic properties may imply that there is mutual stochastic dependence between the basic variables after these have been declared to be random variables. An example of a pair of basic variables is the compressive strength and the modulus of elasticity at zero stress both measured on the same concrete test cylinder. Usually these two basic variables, when considered to be random variables, are modeled as being mutually dependent.

*Within given classes of structural design problems the joint distribution types of the basic random variables are standardized in the code. These standardizations are defined in subsequent sections either directly in distributional terms or in terms of one-to-one transformations into Gaussian random variables.*

Basic variables may more generally be functions in time and space. The action history within a given time interval is an example of a basic variable. Also such functions may be uncertain. The corresponding probabilistic concept is that of a random process or a random field.

*If the mechanical model contains input variables which represent outputs from other mechanical models the joint distribution type of these input variables must be consistent with the standardized distribution types of the code after these have been transformed by the latter models.*
*If some input variables represent information from prototype testing the joint distributional type of these variables must follow from a mechanical model of the prototype test. This model relates the test results to the relevant basic variables for which the code gives distribution type specifications. Statistical uncertainty should be taken into account in this deduction (see below).*

*Uncertainties from all essential sources must be evaluated and integrated into the reliability model. Types of uncertainty to be taken into account are physical (intrinsic) uncertainty, statistical uncertainty, and model uncertainty.*

Physical uncertainty is the ubiquitous background randomness the level of which may or may not be controlled by active means. Statistical uncertainty is due to limited information as it is provided by a sample of finite size. Model uncertainty is due to the necessary idealizations on which the physical model formulation and the distributional model formulation are based. The corresponding errors are more or less unknown. This type of uncertainty may for each adverse event be described as uncertainty of the corresponding adverse state surface.

*Statistical uncertainty. If the design decisions are based on a small sample of observations of a basic variable (or a function of basic variables such as for prototype testing), statistical uncertainty must be quantified in the decision model as follows. By use of a well-documented natural conjugated prior distribution to the standardized distribution type of the actual random variable or, if a natural conjugated distribution does not exist, by use of a prior of mathematical form as the posterior, or by use of a non-informative prior, a predictive posterior distribution is calculated. This distribution must be applied in the reliability analysis.*

The prior distribution is a probabilistic model of the knowledge about the parameters of the distribution of the considered random variable (or vector) $X$ before some new independent data are available (usually in the form of an outcome of the vector $(X_1, \ldots, X_n)$ with all $X_i$ mutually independent and distributed like $X$). The posterior distribution is a conditional distribution of the parameters given the prior information and the sample data. The predictive posterior distribution is the conditional distribution of $X$ given the prior information and the sample of data.
The posterior density is obtained as being proportional to the product of the likelihood function and the prior density (according to Bayes' formula). The prior density is said to be non-informative (or diffuse) if the posterior density and the likelihood function are proportional (or almost proportional within the domain of non-zero posterior density). The likelihood function is defined by the joint distribution of $X_1, \ldots, X_n$ considered as a function of the parameters.
Corresponding to any fixed choice of a diffuse prior the family of posterior densities is closed under multiplication by the likelihood function. Thus any density from this family used as a prior density leads to a a posterior density within the family. Under certain conditions on the type of distribution of $X$ (the distribution must belong to the exponential family), the sample size parameter $n$ in the general expression for the posterior density can be extended from the positive integers to the positive real numbers. This extension leads to a larger family of densities which is also closed under multiplication by the likelihood function. This extended family is called the family of natural conjugate densities to the type of distribution of $X$. The definition reflects that the probabilistic model of uncertain knowledge formulated by the choice of the prior density needs not be restricted by the fact that sample sizes are integers.

*Model uncertainty. The reliability model must be formulated such that it contains elements which are able to reflect model uncertainty at least in a crude way. This may be done for each given adverse state surface (or each given part of a adverse state surface) by associating a judgmental random vector* $\mathbf{J}=(J_1, \ldots, J_n)$ *to the basic random vector* $\mathbf{X}=(X_1, \ldots, X_n)$. *Assuming that there is a one to one transformation by which the standardized joint distribution of* $\mathbf{X}$ *is mapped into a Gaussian vector* $\mathbf{T}(\mathbf{X})$ *the judgment random vector* $\mathbf{J}$ *is added to* $\mathbf{T}(\mathbf{X})$. *Next the sum is back-transformed into* $\mathbf{Y}=\mathbf{T}^{-1}(\mathbf{T}(\mathbf{X})+\mathbf{J})$. *The distribution of* $\mathbf{Y}$ *is determined by formally assuming that the joint distribution of* $(\mathbf{T}(\mathbf{X}),\mathbf{J})$ *is Gaussian. Finally, the random vector* $\mathbf{Y}$ *replaces* $\mathbf{X}$ *in the reliability calculation.*
*Constants in the model can also be chosen to carry model uncertainty. In that case the constants are interpreted as additional basic random variables.*

Models given in current codes of practice are often strongly biased to the conservative side. In order to make a rational reliability analysis by use of such a code-specified model, the bias should be investigated in order to remove it by assigning a proper non-zero mean vector to the judgmental random vector $\mathbf{J}$.
Some detailed recommendations about model uncertainty are referred in Appendix 3.

## 5. CONCEPT OF EVENT MARGIN

*An event margin corresponding to a specified event is defined as a function of the basic variables with the property that it takes a negative value if and only if the event occurs.*

Event margins related to adverse states are denoted safety margins.

*Information becoming available after the design of a structure can be formulated in the framework of event margins. This additional information can be utilized in reliability updating.*

During fabrication and service of a structure additional information of the performance becomes available. Actions, material parameters and geometrical parameters are realized physically and the design analysis can therefore be updated. Important additional information may arise from material compliance control, proof loading, prototype testing, vibration measurements, action measurements, etc. A part of this information is related to design parameters directly, but some information is related to a functional relation between design parameters and possibly also other parameters such as measurement and inspection errors.

## 6. RELIABILITY REQUIREMENTS

*Decision theoretical principles can be applied in order to obtain optimal reliability levels. It is required, however, that the intangible part of the cost of failure is chosen such that it is comparable in value to the population of failure costs associated with present code based engineering practice when declaring this practice to be optimal. The population of failure costs must correspond to a population of structures with similar failure consequences as for the considered structure.*

Optimal reliability levels depend on the reference period. Under stationary conditions and under due consideration of the time sequence of failure occurrences and the capitalization of costs to present value, the optimal reliability level for the entire reference period decreases with the length of the reference period.

*Required minimal reliability levels make sense only together with a specification of a reference period. The reference period should generally equal the anticipated lifetime of the structure (e.g. 100 years). For the reliability measure defined herein the required levels are obtained by calibration to structural dimensions following from present code based engineering practice.*

Transient structures are present during construction or remodeling of the structure.

*For transient structural situations shorter reference periods with corresponding special reliability requirements can be relevant.*

The principle of calibration to existing practice should be kept in operation in a reasonably long transition period during which there will be a backward correcting influence on current practice justified by the experience following from the use of this probabilistic code. After this transition period the reliability requirements (or, alternatively, the requirements on the intangible failure costs including possible risk aversion costs) associated with this code (and gradually established during the transition period) will represent superior practice.
There are structural reliability problems in which some few of the relevant basic variables are very difficult to assess by value. This shows up in the form of uncertainty distributions that are considerably more dispersed than the distributions of the other basic variables. In such cases it can be useful to report intermediate reliability analysis results in the form of fragility functions. A fragility function is a conditional probability of failure given the values of one or more basic variables and considered as a function of these values. The

fragility function gives information about the sensitivity of the reliability with respect to variation of the conditioning basic variables. Robustness is indicated by slow variation while sensitivity is indicated by steep variation. By this the fragility function indicates where to put the efforts to narrow down the uncertainty distribution of the conditioning variables.

*Direct requirements to the fragility functions of continuously varying basic variables are not given. In particular cases where a requirement to a fragility function seems to be needed, it will be indirect through a specification of a mandatory joint distribution of the conditioning basic variables.*

Another type of fragility function is obtained by conditioning on different damage states of the structure. This concept is relevant in connection with accidental events, that is, events of strong actions but occurring with such rareness that it is not economically optimal to design the structure to resist these actions without being damaged. In a discretized model each of these accidental events can be defined formally by formulating a structural model by removing one or more elements or parts of the undamaged structure. The failure probability of each of these damaged structures under the relevant actions occurring in a specified time period after the occurrence of the accidental event (evacuation period or repair period) is a fragility function over the set { A,B,C,... } of models of damaged structures. The probability distribution over the set { A,B,C,... } may be so difficult to assess in practice that code requirements to the formulation of the damaged structures (i.e. to the definitions of A, B, C, ...) and the corresponding fragility function may be needed. The purpose is to ensure structural robustness against progressive collapse, that is, to prevent that the consequences of an accidental event are out of proportion with the extend of the accidental event itself.

*Reliability levels to be used in progressive collapse investigations can be based on decision theoretical principles under the same conditions as stated in the first paragraph in this section.*

Models with non-stationarity properties are relevant when foreseeing gradual changes in environmental conditions, action history trends, creep phenomena, material deterioration or aging, soil consolidation etc.

*In case of non-stationarity modeling of resistance and action properties the reference period should be the anticipated lifetime of the structure. Alternatively a sequence of consecutive shorter than lifetime reference periods can be considered. After each reference period inspection and suitable testing of the structure should be made together with investigations about the actual actions. Upon a reliability updating analysis it can be decided if the structure can be used without changes in the next reference period or whether change of use, strengthening or even demolition should be undertaken. Decisions theoretical principles can be used under the same conditions as stated in the first paragraph in this section.*
*The same inspection and reliability updating decision strategy should be used when considering existing structures which suffer from damage or for which changes of use and environmental conditions are actual.*
*Also the information obtained from regular damage monitoring inspections can be used as the basis for decisions about reliability preserving measures applied to the existing structure.*

The occurrence of a serious adverse event sometimes raises a public (political) demand to the engineering profession of using increased reliability requirements with respect to this type of event. Such public reactions are reasonably taken into account in the long run revision of the code with respect to proper updating of the intangible costs related to the experienced adverse events.

*Decision theoretical principles should be applied to reevaluate the codified reliability levels in case of experienced adverse events causing severe public reactions.*

## 7. ACTION MODELING

*The action models set up for structural reliability analysis must be given sufficiently detailed structure to allow reasonable treatment of action effects caused by the random variation of the actions across the structure and in time. Furthermore the models should allow the study of combined action effects due to several simultaneous actions.*

*For the macro scale variation in time of an action the basic variable model element is a pulse which is characterized by at least three parameters: a level parameter (intensity), a duration parameter, and an occurrence parameter. Micro scale variations are described by random processes defined by their covariance properties or, equivalently, by their spectral properties.*

Such processes are generally derived from the family of Gaussian processes.

There are several applicable stochastic action models based on the concept of an action pulse. The Ferry Borges-Castanheta model (FBC action model) is one of the simplest and most operational of these models: The design life time is for a given action type divided into time intervals of equal length. Within each interval the action has a constant intensity level (possibly vectorial). This intensity level is an outcome of a random variable (vector). The intensities in different intervals can be mutually dependent. Such a sequence of action pulses is called an FBC action history.

Any pair of FBC action histories describing two different actions are related to each other at least in the way that the number of intervals in the one action history is an integral multiple of the number of intervals of the other action history.

The problem of combining the action effects of $n$ different FBC action histories is called an FBC $n$-combination problem.

*The FBC action model can be applied in the reliability analysis of the supporting structures of buildings.*

Appendix 2 gives an example of a table of prescribed values that specifically define the FBC action models suited for building design.

*Other models for action variation in time than the FBC action model can be applied in the reliability analysis of the supporting structures of buildings. Any other such model must be calibrated to model the essential properties of the FBC action model specified in this code, that is, to have the same distribution type for the pulse amplitude, approximately the same mean durations of the pulse, and approximately the same occurrence parameter.*

There are several practicable alternatives to the FBC action model. Here only one alternative will be mentioned. It is the Poisson pulse model for action histories with short duration pulses that are separated in time. If the pulse durations are short as compared to the mean time distance between consecutive time points for pulse starts and several such action processes with nonnegative pulse amplitudes are considered for linear combination with nonnegative influence coefficients, the load effect coincidence model of Wen is applicable for determining the distribution of the maximal load effect within a given time period.

The model of Wen assumes that the combined load effect process is a Poisson pulse process defined as the sum of several mutually independent Poisson pulse processes. Each of these processes have pulses with amplitudes that either correspond to pulses that are not overlapping in time with any other pulses, or to an overlapping of two pulses from two selected different load processes, or to overlapping of three pulses from three selected different load processes, etc. With $n$ being the number of processes for combination there are as many processes with their pulses made up of $i$ overlapping pulses as the number of ways $i$ processes can be selected out of $n$ processes (that is, $\binom{n}{i}$ ways).

The probability assignments in the Wen coincidence model is made in such a way that the model leads to a slightly conservative evaluation (that is, overestimations of the maximal combined load effect.

*Asymptotic extreme value distributions can only be applied as approximations to the exact distribution of the maximal load effect if special documentation of the validity is given. As documentation the asymptotic argument is not sufficient.*

The convergence of the exact extreme value distribution for increasing sample size depends strongly on the generating distribution. Often the convergence is extremely slow giving gross upper tail deviations between the exact extreme value distribution corresponding to a relevant sample size and the corresponding asymptotic distribution.

*For any type of structure the reliability analysis must be based on a complete set of action models that together approximately reproduce the essential probabilistic properties of all the different types of relevant action effects that can be expected to come from the future environments and uses of the structure.*
*For a given structure simple demonstrations or general experience can often be sufficient to justify the exclusion of some of these models with corresponding analyses.*

The necessary detailing of the actions models depends on the sensitivity of the considered action effect, that is, on the filtering and the amplification properties as well as the material properties of the structure. Thus different types of action models should be applied dependent on the relevant phenomenon such as long time creep effects, immediate static effects, action effects of concern in fatigue life estimation, dynamical effects that can be amplified by resonance phenomena or self-induced vibrations, impact effects, etc. Also the analysis of progressive collapse phenomena may require its own special action modeling.
For most reliability investigations it is not essential that the action models reproduce the individual action effect histories in their details. The approximate reproduction of the basic probabilistic properties of the action effect histories is often sufficient.

*Standardized distributions and process types to be used in action models for specific reliability investigations can be given in an action code to be used in parallel with this code on reliability methods. In such cases the action load model standardizations given in this code are secondary to the standardizations of the action code.*

## 8. STRUCTURAL RESISTANCE MODELING

*The reliability requirements of this code are for specific failure modes of structural elements such as bars, beams, columns, plates, walls etc. The reliability analysis of larger structural subsystems or the entire structural system must be made in order to investigate whether there are significant system effects on the reliability, and in particular whether such effects are to the side of serious decrease of the reliability.*
*This code allows the use of decision analytical principles to obtain reasonable system reliability levels provided an assessment of the intangible costs of failure has been made as required in Section 6.*
*Standardized distributions of material properties to be used in structural resistance models can be given in material oriented codes to be used in parallel with this code on reliability methods. Standardized distributions given in such material codes are superior to the standardizations given in this code. It is required that a standardized distribution of a material property assigns zero probability to any set in which no value is possible due to the physical definition of the considered material property.*

The requirement of zero probability on physically impossible sets is formulated for guidance of material code writers. It ensures against having for example negative strengths helping the reliability. However, this requirement does not prevent that calculational easier distributions that are not obeying the requirement be used as approximations provided it can be justified that the inconsistency with the physical possibilities contributes insignificantly to the calculated reliability.

Reliability analyses should always be made for each of the structural elements but also to a certain extend for the entire structural system. The structural elements can be defined as smaller or larger subsystems of the entire structural system. Required reliability levels obtained in accordance with the principles in Section 6 will depend on the element definition and will be different for the elements and for the entire system. The required system reliability should be dependent on whether the system failure is of local nature (it can be an element failure) or whether it is global implying much more severe consequences.

The fact that overestimation of the system reliability follows from the use of discretized structural models in the system reliability analyses points at the need for making sensitively analysis with respect to the fineness of the discretization.

*Reliability comparisons of different structural systems must be made on the same level of fineness of discretization of the structural system. When comparing the obtained system reliability with the element reliabilities the effect of the discretization on the system level must be taken into account.*

The present state-of-the-art of the methods of structural system reliability analysis does not yet permit formulation of very specific code requirements concerning system reliability levels.

A difficult problem is the dependency of the structural resistance on the action history to which the structure is subjected.

*System reliability analyses referred to in this code are those for which the system resistance is obtained under fixed in time but random load configurations with the load level increasing proportional with a scalar parameter starting from the self weight load situation and ending at the final random load level situation.*

The rigid ideal-plastic theory plays a particular role in the theory of structural system reliability due to the independence of the system resistance of the load history, that is, due to the existence of a load history independent adverse state of collapse. The reliability corresponding to a given reference period is then determined by the probability that the load path does not cross out through the fixed adverse state surface during the reference period.

Other difficult problems are related to the modeling of the constitutive behavior of the potential failure elements in the discretized structural system. In particular problems show up in the modeling of the post failure behavior including problems of post failure interaction between the internal generalized force components. Also here the rigid ideal-plastic theory shows substantial simplifications by adopting the associated flow rule (that is, the condition that the generalized strain vector is orthogonal to the yield condition surface).

*Rigid ideal-plastic theory can be used as the basis for system reliability analyses given that the structural system shows ductile collapse behavior. Dependent on the implied degree of idealization of the "real" constitutive behavior more or less biased and dispersed model uncertainty random variables (effectivity factors) must be introduced in the mechanical model. The evaluation of these factors must be justified by proper example studies that include the possibility of having elastic-plastic stability failures.*

For discretized systems with brittle failure elements the linear elastic-ideal brittle systems play a role as a practicable study object given that the actions grow in a fixed configuration proportionally from zero to a final random level. In the linear elastic-ideal brittle system each failure element is removed upon failure.

*For brittle systems this code conservatively defines failure of the system as occurring when the first failure of* *a single failure element occurs. The idealization to a linear elastic-ideal brittle system with suitable model uncertainty variables can be used for analysis of the conservativeness of this definition. Relaxation of the required reliability must be properly justified.*

Difficulties of taking the influence of the action history into account have motivated introduction of intuitive definitions of adverse states for structural systems. These definitions are characterized by lack of explicit concern about how the final load on the structure has been established. The system reliability analysis is made solely within a universe of a finite number of random variables describing final actions and resistances. Such adverse state definitions formulated on the basis of engineering judgment and intuition will herein be termed as "jury definitions".

*The effect of structural redundancy can be comparatively studied by use of intelligently chosen jury definitions of the adverse state. Extreme care should be taken when drawing conclusions about the reliability of the real structural system on the basis of such analysis.*

## 9. RELIABILITY MODELS

*All decreasing functions of the probability $p_f$ of some adverse event are equivalent measures of safety. They all define the same reliability ordering with respect to adverse events in the space of basic variables (Section 2).*
*A standard reliability measure may be chosen to be the generalized reliability index. It is defined as*

$$\beta = -\Phi^{-1}(p_f)$$

*Another equivalent reliability measure is the probability of the complement of the adverse event (the safe event)*

$$p_s = 1 - p_f$$

*The probability $p_f$ is calculated on the basis of the standardized joint distribution type of the basic variables and the standardized distributional formalism of dealing with both model uncertainty and statistical uncertainty (Section 5.1).*
*The standardized distribution type related to the basic variables of the action models are defined in the action code (Section 7) while the standardized distribution types related to the basic variables of the resistance models are defined in the specific material related codes (Section 8).*
*If no specific distribution type is given as standard in the action and material codes this code for the purpose of reliability evaluations standardizes the clipped (or, alternatively, the zero-truncated) normal distribution type for basic load pulse amplitudes. Furthermore, the logarithmic normal distribution type is standardized for the basic strength variables.*
*Deviations from specific geometrical measures of physical dimensions as length are standardized to have normal distributions if they act at the adverse state in the same way as load variables (increase of value implies decrease of reliability) and to have logarithmic normal distribution if they contribute to the adverse state in the same way as resistance variables (decrease of value implies decrease of reliability).*

The standardization of the logarithmic normal distribution type implies that all the corresponding basic variables of the Gaussian formulation space are obtained by logarithmic transformation of the corresponding basic variables of the original physical formulation space. For the determination of the second moment representation of the basic variables of the Gaussian formulation space the following formulae are valid:

$$\mathrm{E}[\log X] = \log \mathrm{E}[X] - \frac{1}{2}\log(1+V_X^2)$$

$$\mathrm{Var}[\log X] = \log(1+V_X^2)$$

$$\mathrm{Cov}[\log X,\log Y] = \log\left(1+ \frac{\mathrm{Cov}[X,Y]}{\mathrm{E}[X]\mathrm{E}[Y]}\right)$$

in which the pair ($\log X,\log Y$) is bivariate Gaussian, and

$$\mathrm{Cov}[\log X,Y] = \frac{\mathrm{Cov}[X,Y]}{\mathrm{E}[X]}$$

in which the pair ($\log X,Y$) is bivariate Gaussian. In these formulae 'log' is the natural logarithm.

*In special situations other than the code standardized distribution types can be relevant for the reliability evaluation. Such code deviating assumptions must be well documented on the basis of a plausible model that by its elements generates the claimed probability distribution type. Asymptotic distributions generated from the model are allowed to be applied only if it can be shown that they by application on a suitable representative example structure lead to approximately the same generalized reliability indices as obtained by application of the exact distribution generated by the model.*

*Experimental verification without any other type of verification of a distributional assumption that deviates strongly from the standard is only sufficient if very large representative samples of data are available.*

*Distributional assumptions that deviate from those of the code must in any case be tested on a suitable representative example structure. By calibration against results obtained on the basis of the standardizations of the code it must be guaranteed that the real (the absolute) safety level is not changed significantly relative to the requirements of the code.*

The reliability model of this code is a formalistic set of rules that allows engineering decision making on the basis of a mathematically rational processing of available well documented information. It is sufficient for the engineering decision making that the set of rules defines an ordering relation with respect to safety. However, such an ordering relation is not necessarily considered to be sufficient for political decision making. Even though the political decision making problem is outside the scope of this code, some comments are relevant.

Among political decision makers it is often taken for granted that the result of a probabilistic evaluation made by experts has an absolute meaning in the sense of predicting a relative frequency of the considered adverse event. In what sense it is interpretable as a relative frequency is rarely made clear. (Whether this interpretation of the concept of probability is necessary in political decision making is subject to discussions of great controversy among philosophers concerned with the scientific basis of statistics and decision making). Within the topic of structural reliability theory the practicing of this philosophy implies far reaching restrictions imposed on the probabilistic statements that can be given. These restrictions take the form of conditioning statements concerning all those uncertainty sources that are not of direct relative frequency nature. That means, for example, that the uncertainty originating from the lack of precise information about the relevant distribution types cannot be coped with except by giving a "worst case" statement. These are of types as Chebycheff bounds. A reliability measure based on such bounds can be defined. However, it is questionable as a tool for design decision making, first, because it is difficult to calculate except for some idealized particular examples, second, because it, as a worst case statement, in principle increases with more information, be it good or bad information.

Alternatively, if the worst case philosophy is not followed the decision maker is given a set of conditional probability statements which honestly can be claimed to predict the relative frequency of occurrence of the adverse event given the truth of the conditioning statements. In a structural reliability context the conditioning statement is in general a conjunction of many conditioning statements of widely different nature. In order that the decision maker can utilize the given probabilistic information he or she must weigh the different conditioning statements against each other. This means that he or she is forced into the problem of combining the conditional probabilities according to the rule of total probability using weighting probabilities that have no direct relative frequency interpretation. These probabilities are called Bayesian probabilities (or subjective probabilities). The mental process of judgment obviously calls for aiding standardizations of distribution types implying that only the values of some few parameters have to be assessed by professional judgment.

*Design by maximization of utility (minimization of total cost) can be made within the framework of this code. However, the cost consequence of some adverse event like loss of human life must be calculated on the basis of the postulate that current design practice as it is approved by the authorities is optimal.*

The target values of the generalized reliability index specified in this code (Appendix 1) have been derived by calibration to current design practice. The corresponding value of the formal failure probability $p_f$ is substituted into the cost equation for the considered structure and the failure cost $c$ is determined such that $p_f$ is the optimal failure probability.

Application of optimization design methods is relevant in the case of designing strengthening systems for an existing structure about which updated information is available. By using the failure cost $c$ obtained by calibration to current design practice of a similar new structure (no updated information available for this, naturally) it is possible to make rational decisions about the dimensions of the strengthening system including the two extreme possibilities of either making no strengthening or complete renewal of the structure.

## 10. RELIABILITY CALCULATION METHODS

The numerical value of the reliability measure is obtained by a reliability calculation method. Due to the computational complexity a method giving an approximation to the exact result is generally applied.

*Two fundamental accuracy requirements are:*

- *Overestimation of the reliability due to use of an approximative calculation method be within limits generally accepted for the specific type of structure*
- *The overestimation of the generalized reliability index must not exceed 5%.*

The accuracy of the reliability calculation method is linked to the sensitivity with respect to structural dimensions and material properties in the resulting design. General design practice has inherent rules of acceptable errors since dimensions and material properties are often only available in discrete classes. An error larger that 5% is rarely accepted.

When the modeling of the basic random variables is in terms of a random vector the first-order reliability method (FORM) in general results in a sufficiently accurate approximation to the reliability measure. The FORM analysis is based on a transformation of the basic variables $\mathbf{X}$ into standardized normal variables $\mathbf{U}$ by the transformation

$$U_i = \Phi^{-1}(F_i(X_i X_1, \ldots, X_{i-1}))$$

The distribution of $X_i$ conditioned upon the value of $(X_1, \ldots, X_{i-1})$ is thus used. The transformation simplifies when the basic random variables are mutually independent. After the transformation the adverse state surface in the normal space is approximated by one or more tangent hyperplanes at the locally most central points. The probability content in the approximation to the failure set is used as an approximation to the failure probability.

If no prior experience with the specific type of adverse state is available, the FORM result should be checked. This can be done locally around the locally most central points by an asymptotic second-order reliability method (SORM), where the adverse state surface is approximated by a second-order surface at the locally most central points, or by an importance sampling around the locally most central points. Globally it should be checked that the most central point has been identified. This can be done by a Monte Carlo simulation, e.g., using directional sampling.

Besides computing the reliability measure it is recommended to check the sensitivity of this reliability measure to all input parameters, i.e., the deterministic basic variables and distribution parameters for the random basic variables. The asymptotic results for the sensitivity of the generalized reliability index are in general sufficiently accurate for this task.

## 11. LIST OF SYMBOLS

| | |
|---|---|
| Cov [ , ] | : covariance |
| D [ ] | : standard deviation |
| E [ ] | : mean value |
| $F(\,)$ | : distribution function |
| $I$ | : judgmental random factor |
| $J$ | : judgmental random variable |
| $n$ | : number of shifts per year |
| $p_f$ | : failure probability |
| $p_s$ | : survival probability |
| $q_c$ | : characteristic value of velocity pressure |
| $s_k$ | : characteristic value of ground snow load |
| $T$ | : transformation of random vector into a Gaussian vector |
| $U$ | : standardized normal variable |
| $V_X$ | : coefficient of variation of $X$ |
| $X$ | : random variable |
| $Y$ | : random variable |
| $\beta$ | : reliability index |
| $\mu$ | : distributional location parameter |
| $\Phi$ | : standard normal distribution function |
| $\rho[\,,\,]$ | : correlation coefficient |
| $\sigma$ | : distributional dispersion parameter |

## 12. GENERAL BIBLIOGRAPHY


**Books on Reliability Methods:**

1. Ang, A.H.-S. and Tang, W.H., *Probability Concepts in Engineering Planning and Design*, Vol. I&II, John Wiley, 1984.
2. Augusti, G., Baratta, A. and Casciati, F., *Probabilistic Methods in Structural Engineering*, Chapman and Hall, 1984.
3. Benjamin, J.R. and Cornell, C.A., *Reliability, Statistics and Decision for Civil Engineers*, Mc-Graw Hill, 1970.
4. Bolotin, V.V., *Wahrscheinlichkeitsmetoden zur Berechnung von Konstruktionen*, VEB Verlag fur Bauwesen, 1981.
5. Borges, J.F. and Castanheta, M., *Structural Safety*, Laboratorio Nacional de Engenharia Civil Lisbon, 1971.
6. Ditlevsen, O., *Uncertainty Modeling with Applications to Multidimensional Civil Engineering Systems*, Mc-Graw Hill, 1981.
7. Ditlevsen, O., *Structural Reliability Methods (under translation)*, SBI, 1990.
8. Madsen, H.O., Krenk, S. and Lind, N.C., *Methods of Structural Safety*, Prentice-Hall, 1986.
9. Melchers, R.E., *Structural Reliability and Predictions*, Ellis Horwood/J. Wiley, 1987.
10. Schueller, G., *Einfuhrung in der Sicherheit und Zuverlassigkeit von Tragwerken*, Verlag Wilhelm Ernst & Son, Berlin, 1981.
11. Thoft-Christensen, P. and Baker, M., *Structural Reliability Theory and Its Applications*, Springer Verlag, 1982.
12. Thoft-Christensen, P. and Murotsu, Y., *Application of Structural Systems Reliability Theory*, Springer Verlag, 1986.

**Present Codes and Model Codes:**

13. American Institute of Steel Construction (AISC), "Load and Resistance Factor Design Specifications," AISC, Chicago, 1986.
14. American National Standards Institute (ANSI), "American National Standard Minimum Design Loads for Buildings and other Structures," ANSI A58.1, ANSI, New York, 1982.
15. CEB, "First Order Concepts for Design Codes," CEB Bulletin No.112, Munich, 1976.
16. CEB, "Common Unified Rules for Different Types of Construction and Material, Vol. I," CEB Bulletin No.116, Paris, 1976.
17. CIRIA, "Rationalisation of Safety and Serviceability Factors in Structural Codes," CIRIA Report No. 63, London, 1977.
18. ISO, "General Principles on Reliability for Structures," ISO/DIS 2394, 1984.
19. NBS, "Development of a Probability Based Load Criterion for American National Standard A58," U.S. Department of Commerce, NBS Special Publication 577, 1980.
20. Nordic Committee for Building Structures (NKB), 'Recommendation for Loading and Safety Regulations for Structural Design,' NKB Report No. 36, 1978.
21. Ontario Ministry of Transportation (OMTC), "Ontario Highway Bridge Design Code,", OMTC Downsview, Ontario, 1983.

**Appendix 1. Example of Reliability Requirements.**

The following table gives an example of required values of the generalized reliability index (Section 9). The values are obtained by calibration to Danish practice for design of buildings and similar structures (source: The Nordic Committee on Building Regulations, Ref.20)

| REQUIRED RELIABILITY INDICES reference period 1 year | | type of failure: ductile with reserves | ductile without reserves | brittle |
|---|---|---|---|---|
| safety class | low | 3.1 | 3.7 | 4.2 |
| | normal | 3.7 | 4.2 | 4.7 |
| | high | 4.2 | 4.7 | 5.2 |

**Table 1** Example of reliability index requirements

The table shows a dependency of the required values of both the reliability class and the type of failure. Both classifications refer to the consequences of failure and reflect a calibration in accordance with decision theoretical principles. The reliability class solely refers to the use of the structure and the nature of the nearest surroundings of the structure (densely populated surroundings or rural surroundings). The type of failure classification refers to possible warnings of failure and less dramatic development of the failure. Moreover, the table column marked "ductile with reserves" refers to substantial carrying capacity reserves not utilized in the mathematical model of the adverse state.

Required reliability index values for other reference periods than the 1 year period must be determined by use of a suitable action model as for example the FBC action model specifically defined by the table in Section 7.

**Appendix 2. Example of action model parameters.**

The following table of scalar data is an example of prescribed values (up to revision) suited for design in Denmark. The numbers may be different in different geographical regions. (Alternatively, such kinds of tables are placed in an action code).

The table contains prescribed values for building design. For loads on different floors a suitable model formulation should include the possibility of having correlation between intensities. This correlation is particularly important for parking houses.

Drift of snow must be considered in the roof load model. This model contributes to a part of the mathematical definition of the actual adverse state in the physical formulation space. The model is formulated by use of form factors defined in the action code under due consideration of the topography of the building and its landscape surroundings. In principle it may contain both the random snow load variable $\max\{0,X_{snow}\}$ the random wind load variable $\max\{0,X_{wind}\}$, and the random wind direction. The wind direction is discretely or continuously distributed across 8 sectors with a probability distribution derived from the wind roset for the actual locality. Within the same pulse interval of the wind load the wind direction is kept constant. Snow load, wind load, and wind direction may be considered stochastically independent of the loads on the floors. For the wind load the values of $\mu$ and $\sigma$ in the table corresponds to an equivalent uniform velocity pressure (constant influence function =1) over a square with side length 50 m.

It is emphasized that $\mu$ and $\sigma$ are not the mean value and the standard deviation respectively of the load

| Data for the Ferry Borges-Castanheta load model for buildings valid for a 1 year time period (calibrated crudely to Danish codes) | | | | |
|---|---|---|---|---|
| Load intensity = max $\{0,X\}$ (clipped Gaussian) $X$ Gaussian, $E[X] = \mu$, $D[X] = \sigma$ | | | | |
| | | No. of shifts per year $n$ | $\mu$ $kN/m^2$ | $\sigma$ $kN/m^2$ |
| self weight | | 1/100 | | |
| floor loads: | | | | |
| residences | long term | 1/20 | 0.5 | 0.15 |
| | short term | 400 | -0.75 | 0.42 |
| offices and schools | long term | 1/20 | 0.60 | 0.20 |
| | short term | 400 | -2.00 | 1.00 |
| hotels | long term | 1/40 | 0.30 | 0.12 |
| | short term | 200 | -1.50 | 0.86 |
| parking houses | | 400 | -3.50 | 1.80 |
| nature loads: | | | | |
| snow load (ground)* | | 5 | -0.20 $s_k$ | 0.45 $s_k$ |
| wind load** | | 400 | -0.32 $q_c$ | 0.34 $q_c$ |

* $s_k$ is the characteristic value of the ground snow load given in the action code. Snow load is only occurring in the half year of winter.
** $q_c$ is the characteristic value of the velocity pressure given in the action code.
NOTE: Values are up for revision.

**Table 2** Example of action model parameters
intensity. The parameters $\mu$, $\sigma$, and $n$ are determined by requiring (1) that

$$\Phi(\frac{x-\mu}{\sigma})^n = 0.98$$

in which $x$ is the 98-percentile in the distribution of the annual extreme for the considered load which means a return period of 50 years, (2) that $\Phi(\mu/\sigma)$ is the average fraction of the season period with the considered action type acting, and (3) that $n\Phi(\frac{\mu-x_0}{\sigma})$ is the average number of periods per year in the season period with the load intensity larger than $x_0$.

For the wind load $x_0$ is put to that fraction of the characteristic velocity pressure $q_c$ that corresponds to a mean number of exceedances of 20 per year. For other types of actions $x_0$ is put to zero.

The conditions are fulfilled with the degree of approximation which is enforce by the restrictions in the FBC model about the interval divisions.

## Appendix 3. Example of model uncertainty specifications

Since a basic strength variable according to this code is transformed logarithmically an additive model uncertainty judgmental random variable J in the transformed space (Section 4) corresponds to a judgmental random factor $I$ on the basic strength variable $X$ itself. The expectation and the coefficient of variation of $XI$ can be determined from the formulae

$$E[XI] = E[X]E[I](1+\rho[X,I]V_X V_I)$$

$$1+V_{XI}^2 = (1+V_X^2)(1+V_I^2)(1+\rho[X,I]V_X V_I)^2$$

It is on the basis of calibrations recommended to split the judgmental factor $I$ into three mutually independent lognormally distributed factors $I_1, I_2, I_3$ for which the following table is given (source: The Nordic Committee on Building Regulations, Ref.20)

| | | | |
|---|---|---|---|
| $j$=1 | good | normal | bad |
| $j$=2 | small | medium | large |
| $j$=3 | strict | normal | gentle |
| $V_{I_j}$ | 0.04 | 0.06 | 0.09 |
| $\rho[X,I_j]$ | -0.3 | 0 | 0.3 |

**Table 3** Example of judgmental factor statistics

The classifications in the table are as follows:

$j$=1: Degree of realism in the prediction of failure by the idealized failure criterion.

$j$=2: Uncertainty concerning the relation between the strength parameter in the structure and the specified substitute of the parameter defined in the description of the structure.

$j$=3: Extent of control on site of the identity of materials and of the building process.

These values of $V_{I_j}$ and $\rho[X,I_j]$ (open to revision) are used in the formulae

$$E[I] = E[I_1]E[I_2]E[I_3]$$

$$1+V_i^2 = (1+V_{I_1}^2)(1+V_{I_2}^2)(1+V_{I_3}^2)$$

$$1+\rho[X,I]V_X V_I = \prod_{j=1}^{3}(1+\rho[X,I_j]V_X V_I)$$

For the load pulse amplitudes in the FBC action model a model uncertainty correction of the form max $\{0,X_i+J_i+J\}$ can be applied. The index $i$ refers to the $i$th pulse in a given action history and $J$ is common for the entire action history. The random variables $J, J_1, \ldots, X_1, \ldots$ can be assumed to be mutually independent unless there are strong reasons to assume otherwise. Calibration studies indicate that the standard deviations of all the judgmental random variables can reasonably be put to (up to revision)

$$0.15[\mu+\sigma\Phi^{-1}(0.98^{1/n})]$$

in which $n,\mu,\sigma$ are the values given in the table defining the FBC action model in Section 7.

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