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Vereinsnachrichten: Discussion of IABSE proceedings

5. Discussion of IABSE PROCEEDINGS

«Statistical Model for Fatigue Analysis of Wires, Strands and Cables» E. Castillo, A. Fernández Canteli, V. Esslinger, B. Thürlimann, P-82/85, published in February 1985 in IABSE PERIODICA 1/1985

A Discussion by Dr. J.E. Spindel, British Railways Board, London, UK

In principle the problem treated by the authors, the fatigue strength of long cables, arises not only in similar situations, such as long welds, but in any situation where the strength required is that of the least of a number of components at equal risk. The authors' arguments and methods thus apply to all evaluations of fatigue data and should fit them all equally well – if they are justified.

It is, therefore, interesting to compare and contrast the authors' methods with those used in one such evaluation, the strength of plates containing varying numbers of rivets or rivet holes at equal risk [1].

In that case the obvious unit was one rivet or rivet hole while other such holes in the same specimen at equal risk were treated as runouts. Thus all specimens, irrespective of the number of holes they contained, could be, and were, analysed as one set of results, after checking that they gave similar results when analysed separately.

The method used, like the authors', was based on maximum likelihood. Runouts, however, were directly treated since their contribution to the likelihood function can be derived from the cumulative distribution function, just as that of failures is derived from the probability density [2]. There is no need to «assign» fictitious failure values to the runouts.

The analysis was based on the assumption that the logarithms of the endurance, N, were normally distributed, not because it was believed that this is an accurate representation of reality but because it is simple and it is known that there is little, if any, difference between this distribution and the Weibull distribution within a range of about ± 2 standard deviations. Since very few results are available outside this range, it is not possible to distinguish between these distributions in practical cases. An argument that results should «logically» belong to one of these cannot be used to extrapolate beyond known data. In particular, such an argument cannot be used to determine a lower bound fatigue limit as the authors seem to suggest.

The analysis was further based on determining the logarithm of N as a function of the logarithm of the stress range, thus taking the latter as the independent variable. Until someone can perform the authors' impossible experiment of repeatedly testing the same specimen to failure at various stress levels, or until someone invents a machine which allows a number of cycles to be set and then produces the corresponding stress range to cause failure, this is the only realistic course to follow. The fact that, after analysis, one requires to infer a stress range to ensure with reasonable safety a given number of cycles of endurance is no reason to depart from this procedure. The problem of deriving the independent variable from the dependent is not new and at least one method exists for doing this [3]. It does not require the complex mathematical model used by the authors and it certainly does not justify it.

The investigation also showed that a single straight line regression was a totally inadequate representation of the available data which included tests at stress levels sufficient to cause yield round rivet holes. In fact, the best representation was found to be a set of three straight lines, no doubt an approximation to a more elegant continuous curve. The first, in the high stress range, had a slope of –14, the second, in the medium stress range, a slope of –3 and the third, in the low stress range, a slope of –7.

In this particular case, the standard deviation of the logarithm of N was found to be sensibly constant over the whole range. This is not consistent with observations on fatigue test results of welded connections where the standard deviation varies along the length of the S/N curve insuch a way that parallel lines represent the varying probability levels, in other words, it appears as though the standard deviation of the logarithms of the stress range were constant throughout the range of the data [5]. There are indications that this is also true of the data represented in fig. 2b of the authors' paper but it is not true of the percentile curves shown in fig. 7 of the paper. It is, therefore, doubtful whether the latter family of curves applies generally to fatigue data. Indeed, the results shown in figs. 14, 15 and 16 of the paper seem insufficient to provide any evidence to support any assumptions as to either the shape of the curve or the probability distribution. Those shown in fig. 2b might. The fact is that all sorts of different curves can be fitted to limited data with equal «justification» [3]. A suggestion for arriving at a better definition of curves by combining suitable sets of data is discussed elsewhere [2], as is the choice of the mathematical model [4].

Whether data are sufficient to define a curve or not usually becomes apparent when confidence limits for the various constants needed to describe the curve and probability distribution are determined. It is unfortunate that the authors' analysis does not include information on such confidence limits.

The fact that there are confidence limits to be attached to these constants means that the percentile curves shown by the authors should be further apart, just as the non-central t distribution would give the 95 % limit as the mean minus 1.64 standard deviations if means and standard deviations had been determined from an infinitly large sample, but by means minus 2.22 standard deviations with 95 % confidence if these two parameters had been determined from only 30 results.

None of what has been said above concerning the authors' methods detracts from the main principle which they have stated in their paper, namely that the least fatigue strength of a large number of «small» samples, or one very «long» sample, is the lower bound of fatigue strengths. This follows from the authors' equation [1], even for a modest survival probability of 0.75 for the lowest of 250 results. The fact that the standard deviation then also tends to zero, means that for this particular case the one to one relationship between stress range and endurance sought by the authors is obtained automatically, and I would say, by respectable means. The difficulty lies in determining that lower bound on a basis of fact rather than extrapolation. The authors' rather unlikely looking hyperbola for this limit lies soome 2.5 to 2.8 standard deviations below the median for «short» specimens. All I can offer, is that the analysis of a large number of sets of fatigue data for welded joints of various kinds seems to show that a line with a constant slope down to the «fatigue limit» and horizontal beyond this, drawn 2 standards deviations below a similar mean line, is such a lower bound. (The standard deviation is that of log N divided by the slope). This then represents the commonly used design curve.

References:

[1] Cheesewright, P. R., «Analysis of Fatigue Data from Riveted Wrought Iron Girders using the Method of Maximum Likelihood». British Rail Research Report TM MF 26, Derby, March 1982.

[2] Spindel, J. E., and Haibach, E., International Journal of Fatigue Vol.1, No. 2, 1979 pp. 81 – 88.

[3] Hald, A., «Statistical Theory with Engineering Applications», Wiley, New York, 1952.

[4] Spindel, J. E., and Haibach, E., «Some Considerations in the Statistical Determination of the Shape of S/N Curves», ASTM STP 744, 1981 pp. 89 – 113.

[5] Nishijima, S., «Statistical Fatigue Properties of Some Heat-Treated Steels for Machine Structural Use» ASTM STP 744 1981 pp. 75 – 88.

Note: ASTM STP 744 reference is:
«Statistical Analysis of Fatigue Data», R. E. Little and J. C. Ekvall, Eds., American Society for Testing and Materials.

Reply of the Authors to Dr. Spindel's Discussion

The validity of the model goes indeed beyond the particular analysis of wires, strands and cables. There are many real cases in which the weakest link principle can be applied. The derivation procedure shows clearly the assumptions required for the model to be valid. An application of the model to the case of plates containing a varying number of rivets or rivet holes is possible, provided that equal stresses are acting on the rivets. If not, some modifications are needed [1].

The simultaneous study of all specimens, as suggested by Dr. Spindel, is a consequence of the consideration of the length as a parameter of the model, which for the case of plates is equivalent to the number of rivets or rivet holes. If this approach is used, then there is no need to consider other holes as run-outs.

The E-M algorithm with assigned fictitious failure values for the run-outs is one of the possible variants of the maximum likelihood method and gives a better feeling of the goodness of fit for the model to experimental data. The standard method of maximum likelihood method, as indicated by Dr. Spindel, has also been applied by the authors elsewhere.

The assumption of normality used by Dr. Spindel is not consistent with the weakest link principle, because the minimum of a series of normal laws is not normal anymore. If normality is assumed for an arbitrary length l_o, any other different length must show non-normality.

Contrary to Dr. Spindel's opinion the authors feel that the adopted distribution plays a relevant role in evaluating the fatigue strength of very long wires or cables, where extrapolation from short specimens is necessary.

As an example, if the 5 %-fatigue resistance of an element is derived from the fatigue information of a specimen 300 times shorter, the 0,02 % fatigue resistance for the latter has to be evaluated (see figure 1).

The authors are not defending a Weibull distribution in an arbitrary way. The model arises from both physical and statistical considerations, (stability, limit and compatibility considerations), as the only solution of the functional equation. Therefore the choice of the distribution was more than just «logical».

The only reason for using «the impossible experiment» of repeatedly testing the same specimen to failure is to prove that the stress level follows a Weibull distribution too. Anyway, the experiment could be possible in the case of non-destructive testing. What is new in the model is not the derivation of the independent variable $\Delta\sigma$, from the dependent one, N, but the enforcing of the compatibility condition for both, the Weibull distribution for $\Delta\sigma$ given N and the Weibull distribution for N given $\Delta\sigma$, in order to derive a functional equation from which the proposed model arises.

The authors' model is restricted to high-cycle fatigue. An extended model taking into account the low cycle fatigue is currently being developped.

The model proposed by the authors is able to explain the two different cases discussed by Dr. Spindel (constant and varying standard deviation) depending on the region where the results are fitted: Figure 2 shows a schematic representation of four regions showing different trends including an almost constant law (case 2) for the standard deviation of log $\Delta\sigma$ or of log N.

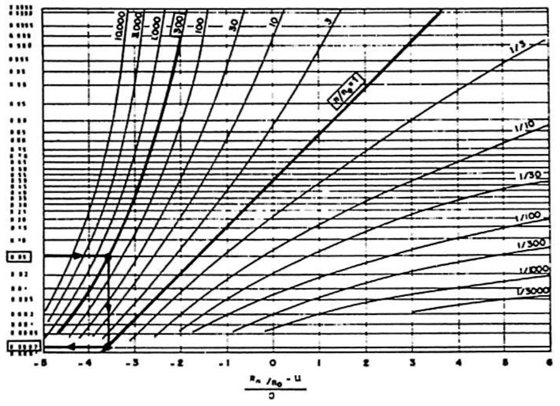
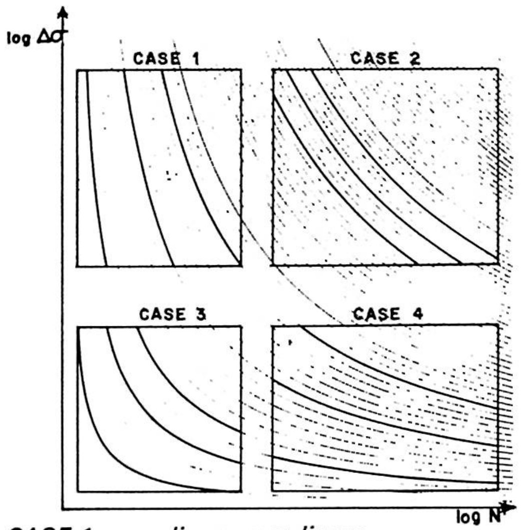


Figure 1. Interdependance of failure probability and length.



CASE 1: m ≅ linear, s ≅ linear
CASE 2: m ≅ linear, s ≅ constant
CASE 3: m ≠ linear, s ≠ constant
CASE 4: m ≅ linear, s ≅ linear

Figure 2: Schematic representation of different trends for median line and standard deviation depending of the considered region.

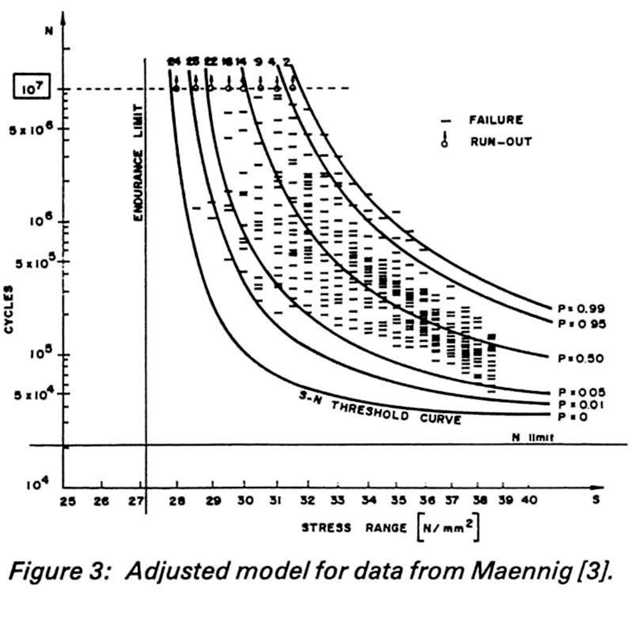


Figure 3: Adjusted model for data from Maennig [3].

The reference to Nishijima's paper by Dr. Spindel is unfortunate because he assumed a constant coefficient of variation for the stress level with the number of cycles. This assumption has not been inferred from data. Contrary to Dr. Spindel's opinion, neither the standard deviation of the stress range nor that of the logarithm of the stress range in Figure 2 of the authors' paper is constant. In addition, the authors do not state that the results shown in Figures 14, 15 and 16 are sufficient to support their assumptions. They simply state that the model is not contradicted by the experimental data. To the contrary, data in Figures 18, 19 and 20 were adjusted as a whole (72 specimens) and constitute therefore more than just «limited data».

Finally, the model has been fitted to the data from Maennig in Figure 3.

The comments about confidence limits are certainly justified: confidence limits must be given in order to complete a statistical analysis.

An analytical derivation of the confidence limits for the proposed model is hard to achieve. However, Monte Carlo methods have been used by the authors to this end. Satisfactory results, to be published elsewhere, were obtained mainly for the endurance limit, whose variance became unexpectedly small.

The method proposed by Dr. Spindel [4] to obtain the endurance limit by drawing a horizontal line two standard deviations below a median for a not clearly defined number of cycles (2×10^6, 10^7, 10^8,...?) is completely arbitrary. The authors' model, derived from physical and statistical considerations, seems to be more appropriate if extrapolation to large number of cycles or different lengths is needed.

References

[1] Mora E., Castillo E., Ascorbe A. «Influence of the Stress Distribution on the Strength of Wires. Statistical Analysis. 44th Session of ISI, Madrid, 1983.

[2] Spindel J.E., Haibach E. «Some Considerations in the Statistical Determination of the Shape of S/N Curves», ASTM STP 744, 1981, pp. 89 – 113.

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[4] Spindel J.E., Haibach E. «The Method of Maximum Likelihood Applied to Statistical Analysis of Fatigue Data». Int. J. Fatigue Vol 1, No. 2, April 1979, pp. 81 – 88.