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Thermomechanical Creep of Aging Concrete — A Unified Approach

Fluage thermomécanique de béton vieillissant — Une approche synthétisant plusieurs méthodes

Thermomechanisches Kriechen von alterndem Beton — Zur Vereinheitlichung der Kriechverfahren

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1. Introduction

There is an extensive body of literature describing experimental results for creep of concrete subjected to prescribed histories of stress. Tests are typically conducted under conditions of uniaxial compression at constant temperature and humidity. Under these conditions, and with the restriction that the maximum stress in a given loading program does not exceed approximately 30–50 percent of the nominal ultimate strength of the concrete, it appears that a linear, viscoelastic material model adequately predicts creep behaviour. Typical results can be found in work reported in [1], [2], [3], [4].

The increasing importance of concrete as a material for reactor containment vessels has necessitated widening the scope of experimental and modeling studies beyond that noted above. In particular there has arisen the need for predicting creep response under non-isothermal conditions, multi-axial stress states and non-uniform moisture content. In addition the effect of aging in creep response, primarily as a result of the accelerative effect of high temperatures under operating conditions, has become an important factor [30], [31], [32], [33].

In the first instance, which we shall refer to as mechanical creep, mathematical structure of the linear viscoelastic model (non-aging) is unambiguous, i.e. such mechanical creep models are always expressible by a hereditary (superposition) integral [2]. However, when aging is considered, this result is largely a matter of formal interest. When one passes to the practical questions of parameterizing the kernels of these hereditary integrals (which are the so-called creep functions of the material) and calculating the optimal values of the parameters for best fit of prescribed data, there immediately arise differences of opinion as to what constitutes the "best" parameterization, what experiments should be performed to determine the parameters and what numerical schemes are "best" for calculation. The abundance of published work in this area is adequate testimony to the state-of-art of these questions and the resulting absence of any consensus among researchers. For representative viewpoints see [3], [4], [5], [6].

A much more difficult problem of modeling occurs when concrete is simultaneously subjected to histories of stress and temperature, especially when aging is of practical importance. To construct models for such thermomechanical creep on a purely empirical basis is virtually impossible and even when satisfactory agreement of an hypothetical model with a given data set is achieved, its predictive value remains unknown. The difficulty in constructing thermomechanical creep models stems from the fact that while creep strain as a good approximation may be assumed to depend linearly on stress history, dependence on temperature history and age at loading is nonlinear. Consequently, in addition to the previously described difficulties associated with the definition of mechanical creep functions, one must face the fact that there is no general mathematical structure for thermomechanical creep comparable to that available for linear, isothermal viscoelasticity. Accordingly, the modeling problem must be imbedded as strongly as possible in an appropriate theoretical framework which incorporates at least the following qualities:

(1) adequacy of the constitutive postulates must be supported by qualitative testing with experiments,
(2) structure of the theory should permit a systematic method for improving the flexibility of the model through parameterization,
(3) the resulting constitutive model for creep, when incorporated into a stress analysis program for complex structures, should lend itself to economically feasible computational algorithms.

The objectives of this paper are directly related to these matters. Considerations are limited to plain concrete at constant moisture content subjected to prescribed histories of stress. We wish to develop a creep model capable of exhibiting the following features:

creep strain is a linear functional of stress history, or in other terms superposition (modified by temperature and aging) of stress effects is possible; instantaneous application or removal of stress produces corresponding instantaneous (elastic) strain; upon removal of stress, creep recovery ("delayed elasticity") occurs, reaching a limiting value called irrecoverable creep ("viscous flow"). The mechanisms of each of the above qualitative creep strains will of course be affected by temperature and aging.

In Section 2 we propose as a point of departure a constitutive equation resulting from non-equilibrium thermodynamics based on internal variables. This approach has already been used with some success to provide the general structure of constitutive equations for viscoelasticity and viscoplasticity. Representative summaries of recent work can be found in [7], [8], [9], [10]. First, an internal variable model is developed for non-aging, isothermal conditions at reference temperature based on a first order law of evolution which is subsequently extended to account for variable loading age and temperatures.

In Section 3 the general structure of the constitutive model is restricted to the case in which loading age and temperature effects are simulated via equivalent transformations of time scale yielding a convolution integral law. The effective time characterization of creep is then examined with regard to identification of the governing internal variable parameters from underlying test programs. Particular attention is given to the evaluation of recoverable and irrecoverable creep deformations as function of time, loading age and temperature.

In Section 4 the proposed creep formulation, which is well adapted to automatic computation because of its "degenerate structure", is reduced to the aging flow model (rate of creep method), to the aging delayed elastic model (generalized Arutyunyan's method) and to the combined flow-delayed elastic model (generalized rate of flow method).

In Section 5 two load-unload examples illustrate application of the creep model to three code specifications for mass concrete as well as three creep laws of concrete literature for the Wylfa reactor concrete.

2. Constitutive Equation for an Aging Linear Viscoelastic Model

In this section we will present a constitutive equation whose structure is determined from thermodynamics of solids based on internal variables. Inasmuch as our purpose here is one of application and illustration of various approaches to modeling creep, only the necessary results will be stated; the interested reader is referred to works such as [7], [8], [9], [10] for additional details. For simplicity of presentation the development will be limited to uniaxial (one-dimensional) equations which are readily generalized to multiaxial stress states.

The starting point of our discussion is the concept of thermomechanical state of the material and an associated measure of energy. Here we choose as state variables stress σ, temperature T (relative to an unstressed reference state of the material) and a set of internal variables q_α, $\alpha = 1, \ldots, n$. Each of these variables may be a function of time. The internal variables describe internal (microscopic) creep mechanisms and may be related to elements of mechanical models in linear viscoelasticity [11]. However, here they are introduced solely for the purpose of developing a mathematical structure for the creep model and therefore need not have an immediate physical interpretation. We introduce the complementary free energy of the state as

$$X = X(\sigma, T, q_\alpha) \tag{2.1}$$

It then follows [10] that the constitutive equation for total strain $\gamma(t)$ can be written

$$\gamma(t) = \frac{\partial X}{\partial \sigma} \tag{2.2}$$

and every creep process must satisfy the dissipation inequality

$$\sum_{\alpha=1}^{n} \frac{\partial X}{\partial q_\alpha} \dot{q}_\alpha \geq 0 \tag{2.3}$$

It should be emphasized that these statements are valid for materials in general, irrespective of their constitution, when subjected to mechanical stress and temperature changes. To define a specific class of materials one must introduce a constitutive equation for the internal variables. Here we use

$$\dot{q}_\alpha = f_\alpha(\sigma, T, q_\beta); \alpha, \beta = 1, \ldots n \tag{2.4}$$

At this point we must adopt explicit constitutive postulates so that equations (2.1), (2.3) and (2.4) acquire mathematical structure. First, (2.3) will be satisfied if we assume

Postulate 1

$$\frac{\partial X}{\partial q_\alpha} = G_\alpha \dot{q}_\alpha \tag{2.5}$$

provided that the coefficients G_α are positive. Further, since we are concerned here with a linear theory, we postulate a quadratic form for the complementary energy

Postulate 2

$$X = \frac{\sigma^2}{2E} + \sum_{\alpha=1}^{n} B_\alpha q_\alpha \sigma - \frac{1}{2}\sum_{\alpha=1}^{n} C_\alpha q_\alpha^2 + DT\sigma + \frac{1}{2}FT^2 \tag{2.6}$$

Combining (2.6) and (2.2) gives the basic "creep law"

$$\gamma(t) = \frac{\sigma}{E} + DT + \sum_{\alpha=1}^{n} B_\alpha q_\alpha \tag{2.7}$$

Combining (2.5) and (2.6) leads to

$$G_\alpha \dot{q}_\alpha + C_\alpha q_\alpha = B_\alpha \sigma \tag{2.8}$$

which defines a rate equation, or equation of evolution, Eq. (2.4), for the internal variables. We observe here that the forms chosen for Eqs. (2.5) and (2.6) are ad hoc constitutive assumptions whose suitability can only be examined in an *a fortiori* manner, and that the coefficients B_α, C_α, G_α, D, E, F as yet are arbitrary, apart from the restriction of positivity of G_α to satisfy Eq. (2.3). More general forms, or other mathematical structures, can indeed be employed. We shall now show, however, that those chosen are adequate to describe phenomena observed in thermomechanical creep of concrete. For algebraic simplicity and clarity we will begin with isothermal, non-aging models.

2.1 *Non-aging Model at Reference Temperature* T_0

In this instance $T = T_0 = 0$ and B_α, C_α, G_α and E are constants. Anticipating the need to permit both delayed elasticity and permanent flow in the model, we divide the rate equation (2.4) into two parts by setting

$$\begin{aligned} C_\alpha &= 0, \alpha = 1 \\ C_\alpha &= \text{const.}, \alpha > 1 \end{aligned} \tag{2.9}$$

The solutions of (2.8) corresponding to (2.9) are

$$\begin{aligned} q_1 &= \frac{B_1}{G_1}\int_0^t \sigma(\tau)d\tau, \alpha = 1 \\ q_\alpha &= \frac{B_\alpha}{G_\alpha}\int_0^t e^{-\lambda_\alpha(t-\tau)}\sigma(\tau)d\tau, \alpha > 1 \end{aligned} \tag{2.10}$$

where

$$\lambda_\alpha = \frac{C_\alpha}{G_\alpha}, \alpha > 1 \tag{2.11}$$

and where $t = 0$ is the state of rest for the internal variables. Substituting Eq. (2.10) into Eq. (2.7), remembering that $D = 0$, gives the creep constitutive equation

$$\gamma(t) = \underbrace{\frac{\sigma(t)}{E}}_{\substack{\text{instantaneous}\\ \text{elastic}}} + \underbrace{\frac{B_1^2}{G_1}\int_0^t \sigma(\tau)d\tau}_{\substack{\text{irreversible}\\ \text{flow}}} + \sum_{\alpha=2}^{n} \frac{B_\alpha^2}{G_\alpha}\underbrace{\int_0^t e^{-\lambda_\alpha(t-\tau)}\sigma(\tau)d\tau}_{\substack{\text{delayed}\\ \text{elastic}}} \tag{2.12}$$

Equation (2.12) corresponds to a creep phenomenon comprised of an initial elastic response, delayed elastic recovery and permanent flow. This is easily established by recalling that the creep compliance is defined as the total strain at time t produced by a unit stress at time $t = 0$. Introducing this condition in Eq. (2.12) and integration yields for the unit creep function

$$\gamma(t)\Big|_{\sigma=1} = J(t) = J_0 + J_1 \cdot t + \sum_{\alpha=2}^{n} J_\alpha\left(I - e^{-\lambda_\alpha t}\right) \tag{2.13}$$

where

$$\begin{aligned} J_0 &= \frac{I}{E} \\ J_1 &= \frac{B_1^2}{G_1}, \alpha = 1 \\ J_\alpha &= \frac{B_\alpha^2}{G_\alpha}, \alpha > 1 \end{aligned}$$

and λ_α is defined by Eq. (2.11). Note that the creep function is parameterized by the constants J_0, J_1, J_α, λ_α, and that the specific creep strain at time t consists of three components:

$$\varepsilon = J_0 = J(o) = \frac{1}{E} \qquad \text{– instantaneous elastic strain}$$

$$\eta_F = J_1 \cdot t \qquad \text{– permanent flow}$$

$$\eta_{DE} = \sum_{\alpha=2}^{n} J_\alpha(I - e^{-\lambda_\alpha t}) \qquad \text{– delayed elastic strain.}$$

Using the definition of the creep function, Eq. (2.13), in the constitutive equation (2.12), the following alternative hereditary creep formulas are easily obtained:

$$\gamma(t) = J(o)\sigma(t) - \int_0^t \sigma(\tau)\frac{\partial J(t-\tau)}{\partial \tau}d\tau$$

$$\gamma(t) = J(t)\sigma(o) + \int_0^t J(t-\tau)\frac{\partial \sigma(\tau)}{\partial \tau}d\tau \tag{2.14}$$

where $J(t-\tau)$ denotes the creep strain at time t due to a unit stress applied at time τ, and is obtained from Eq. (2.13) by inserting $(t-\tau)$ in place of t.

Figure 1 qualitatively illustrates the strain response associated with the creep function, Eq. (2.13), resulting from application and removal of a unit stress. Note that the delayed elastic strain asymptotically approaches a limiting value equal to the permanent flow developed at the time of unloading. We note in passing that the creep model of Eq. (2.13) defined by our constitutive assumptions is equivalent to a generalized Burgers body consisting of a Maxwell element in series with a chain of Kelvin elements, thus furnishing a mechanical representation of the internal variable formulation. This equivalence has been noted by Valanis [29].

2.2 Non-isothermal, Aging Model

We have shown that the constitutive assumptions (2.5), (2.6) built into the mechanical creep model described above, lead to qualitatively correct behaviour for concrete. We will now indicate how the model can be generalized to incorporate thermomechanical creep and aging. The rate equation (2.8) for internal variables has been shown to control the evolution of inelastic creep components, i.e., delayed elasticity and flow. Accordingly, we postulate initially that the coefficients B_α, C_α, G_α in Eq. (2.8) are time-dependent, which may result both from temperature and aging effects; further, in the complementary energy equation (2.6) we assume that E is similarly dependent on time while D is associated with the temperature-dependent coefficient of thermal expansion, $\alpha(T)$, such that

$$DT = \int_0^T \alpha(T')dT' \tag{2.15}$$

The division of internal variables in Eq. (2.9) is retained in the modified form

$$C_\alpha = 0,\ \alpha = 1$$
$$C_\alpha = C_\alpha(t),\ \alpha > 1 \tag{2.16}$$

Solutions of Eq. (2.8) corresponding to Eq. (2.10) with the assumptions above are:

$$q_1 = \int_0^t \frac{B_1(\tau)}{G_1(\tau)}\sigma(\tau)d\tau$$
$$q_\alpha = \int_0^t \frac{B_\alpha(\tau)}{G_\alpha(\tau)}e^{-[\xi_\alpha(t)-\xi_\alpha(\tau)]}\sigma(\tau)d\tau \tag{2.17}$$

where

$$\xi_\alpha(t) = \int_0^t \frac{C_\alpha(s)}{G_\alpha(s)}ds;\ \xi_\alpha(\tau) = \int_0^\tau \frac{C_\alpha(s)}{G_\alpha(s)}ds \tag{2.18}$$

Substituting (2.15), (2.17) and (2.18) into Eq. (2.7) gives the thermomechanical creep equation

$$\begin{aligned}\gamma(t) = {} & \frac{\sigma(t)}{E(t)} + \int_0^T \alpha(T')dT' \\ & + B_1(t)\int_0^t \frac{B_1(\tau)}{G_1(\tau)}\sigma(\tau)d\tau \\ & + \sum_{\alpha=2}^{n} B_\alpha(t)\int_0^t \frac{B_\alpha(\tau)}{G_\alpha(\tau)}e^{-[\xi_\alpha(t)-\xi_\alpha(\tau)]}\sigma(\tau)d\tau\end{aligned} \tag{2.19}$$

which constitutes a generalization of Eq. (2.12). To proceed further it is evident that additional constitutive assumptions are required. Instantaneous elastic response is assumed to be temperature and age dependent, thus

$$E = E[T(t), t] \tag{2.20}$$

Temperature and aging effects on delayed elastic response are assumed to be reflected *only* in the coefficients G_α in the internal variable rate equations with the further provision that all coefficients are *equal.* Accordingly, B_1 and B_α are constants and we can define

$$\frac{1}{G_\alpha(\tau)} = \phi[T(\tau), \tau],\ \alpha > 1 \tag{2.21}$$

and rewrite Eq. (2.18) in the form

$$\xi_\alpha(t) = C_\alpha \xi(t) \tag{2.22}$$

where

$$\xi(t) = \int_0^t \phi[T(s),s]ds; \; \xi(\tau) = \int_0^\tau \phi[T(s),s]ds \tag{2.23}$$

In a similar fashion the flow term coefficient is written

$$\frac{1}{G_1(\tau)} = \psi[T(\tau),\tau] \tag{2.24}$$

Incorporating these constitutive assumptions and definitions in Eq. (2.19) leads to the following form of thermomechanical creep equation

$$\begin{aligned}\gamma(t) = &\frac{\sigma(t)}{E[T(t), t]} + \int_0^T \alpha(T')dT' \\ &+ B_1^2 \int_0^t \psi[T(\tau), \tau]\sigma(\tau)d\tau \\ &+ \sum_{\alpha=2}^{n} B_\alpha^2 \int_0^t \phi[T(\tau), \tau]e^{-c_\alpha[\xi(t)-\xi(\tau)]}\sigma(\tau)d\tau\end{aligned} \tag{2.25}$$

We note again that the theory presented here is linear in stress but nonlinear in temperature and aging, i.e., the total strain in Eq. (2.25) depends linearly on history of stress, but temperature history and aging introduce time-dependent weighting effects. Further, this weighting may be different for delayed elastic strain as apposed to flow (in general $\phi \neq \psi$ differ in Eq. (2.25)). Accordingly, although Eq. (2.25) represents a form of superposition of strains arising from various creep phenomena, it is more general than the conventional linear viscoelastic form of convolution integral, Eq. (2.14). However, in this paper we will not pursue further this generalized form of superposition; rather we shall show how Eq. (2.25) can be reduced to convolution forms which, in the absence of adequate experimental data to the contrary, appear to be capable of modeling thermomechanical creep as well as being suited to numerical computation.

Before turning to the "effective time" concept it should be mentioned that the proposed creep model is also suited for describing transient creep phenomena such as transitional thermal creep [32] for primary increases of temperature. In this case the flow term in Eq. (2.25) can be readily expanded into an isothermal component and a component which is active only if the temperature T exceeds the highest value of the previous temperature history T^* (analogous to the yield condition in plasticity).

3. An "Effective Time" Characterization of Creep

The notion of a nonlinear transformation of time to incorporate the effects of temperature and aging has been employed by a number of researchers utilizing linear viscoelastic models of concrete [2], [12], [13], [6], [34]. The constitutive equation (2.25) developed in the previous section furnishes a unified view of various approaches, as well as a means of parameter identification and computation. Introducing the constitutive postulate of an "effective time", we proceed first by setting

$$\phi[T(t), t] = \psi[T(t), t] \tag{3.1}$$

and recalling that

$$\xi(t) = \int_0^t \phi[T(s), s] ds \tag{2.23}$$

is the "effective time". This postulate is equivalent to the statement that delayed elasticity and flow are affected by temperature and age in the same manner. We note that Eq. (2.23) is assumed to be invertible such that

$$t = h(\xi) \tag{3.2}$$

We adopt the notation

$$\xi' \equiv \xi(\tau) = \int_0^\tau \phi[T(s), s] ds \tag{3.3}$$

or

$$\tau = h(\xi')$$

We note further that Eq. (3.3) is equivalent to

$$d\xi' \equiv d\xi(\tau) = \phi[T(\tau), \tau] d\tau \tag{3.4}$$

Finally, we adopt the notation, using Eqs. (3.2) and (3.3)

$$f(t) = f[h(\xi)] = \hat{f}(\xi) \tag{3.5}$$

or

$$f(\tau) = f[h(\xi')] = \hat{f}(\xi') \tag{3.6}$$

We can now write the basic equation (2.25) in the form

$$\hat{\gamma}(\xi) = \frac{\hat{\sigma}(\xi)}{\hat{E}(\xi)} + \int_0^T \alpha(T')\, dT' + \int_0^\xi \left[B_1^2 + \sum_{\alpha=2}^n B_\alpha^2 e^{-c_\alpha(\xi - \xi')} \right] \hat{\sigma}(\xi') d\xi' \tag{3.7}$$

where

$$\hat{E}(\xi) = E[\hat{T}(\xi), h(\xi)] \tag{3.8}$$

is the time- and age-dependent instantaneous elastic modulus. It is convenient to introduce the reduced strain (total strain less thermal strain) as the basis for defining the thermomechanical creep function:

$$\gamma'(t) = \gamma(t) - \int_0^T \alpha(T')dT' \tag{3.9}$$

Then, if

$$\xi_0 = \int_0^{t_0} \phi[T(s), s]ds, \ \xi \geq \xi' \geq \xi_0 \tag{3.10}$$

is the effective time corresponding to the time of application of a unit stress the reduced creep function can be expressed by combining Eqs. (3.9) and (3.7)

$$\begin{aligned} J(\xi, \xi_0) &= \hat{\gamma}'(\xi)\,|_{\hat{\sigma} = H(\xi - \xi_0)} \\ &= \frac{1}{\hat{E}(\xi)} + \int_{\xi_0}^{\xi} \left[B_1{}^2 + \sum_{\alpha} B_\alpha^2 e^{-c_\alpha(\xi - \xi')} \right] d\xi' \end{aligned} \tag{3.11}$$

which after integration yields

$$J(\xi, \xi_0) = \frac{1}{\hat{E}(\xi)} + B_1{}^2(\xi - \xi_0) + \sum_{\alpha = 2}^{n} \frac{B_\alpha^2}{C_\alpha}\left[1 - e^{-c_\alpha(\xi - \xi_0)}\right] \tag{3.12}$$

Equation (3.12) expresses the total strain, reduced by thermal expansion, produced at effective time ξ by a unit stress applied at effective time ξ_0. It follows from Eqs. (3.7) and (3.12) that the reduced strain can be expressed in the convolution form

$$\hat{\gamma}'(\xi, \xi_0) = \frac{\hat{\sigma}(\xi)}{\hat{E}(\xi)} - \int_{\xi_0}^{\xi} \hat{\sigma}(\xi')\frac{\partial J(\xi - \xi')}{\partial \xi'} d\xi' \tag{3.13}$$

Since (3.13) is a convolution integral and the creep function (3.12) possesses a socalled "degenerate" structure, it is well-adapted to numerical computation, a matter to be reviewed in Section 4. However, before such implementation is possible, the structure of the time transformation must be examined, along with parameterization of the function and identification of parameters from experimental data. A discussion of these matters follows.

3.1 Structure of the Temperature and Aging Shift Function

The function $\phi[T(\tau), \tau]$ appearing in (2.21) was introduced to reflect the effects of temperature history and age on the evolution of the internal variables which in turn control delayed elasticity and flow in the model. On physical grounds it appears plausible, following BRESLER-MUKADDAM [12], to express $\phi[T(\tau), \tau]$ as a product of two functions

$$\phi[T(\tau), \tau] = \varphi[T(\tau)]\, \psi[T(\tau), \tau] \tag{3.14}$$

in which φ is the usual temperature shift function employed in the treatment of thermorheologically simple, non-aging viscoelastic materials and ψ is an aging function which reflects the effect of temperature on aging as well. We thus retain the hypothesis of [12] that ϕ cannot be expressed as a product of a function of temperature and a function of age, since the latter is influenced by temperature. If T_0, τ_0 respectively denote a constant reference temperature and reference age of loading, we require the following properties in Eq. (3.14):

At the reference temperature T_0, $\varphi(T_0) = 1$, thus

$$\phi[T_0, \tau] = \psi(T_0, \tau) \tag{3.15}$$

At the reference temperature and reference age T_0, τ_0

$$\phi(T_0, \tau_0) = 1 \text{ thus } \psi(T_0, \tau_0) = 1 \tag{3.16}$$

Equation (3.15) expresses the condition that for an aging viscoelastic model at constant temperature, no time shift for temperature is required. The condition expressed by Eq. (3.16) is that of an isothermal, non-aging model, where according to Eq. (2.23), $\xi = t$ and the creep law (3.13) reduces to Eq. (2.14).

While the temperature shift function φ has been studied in some detail and can be obtained by conducting isothermal creep tests at various temperatures [14], the aging function ψ is less well-understood. Generalizing the proposal introduced in [12], we set

$$\psi[T(\tau), \tau] = \psi(g) \tag{3.17}$$

where the "co-age" g is calculated from

$$g(\tau, \tau_0) = \tau_0 + \int_{\tau_0}^{\tau} \Lambda[T(s)]ds \tag{3.20}$$

in which the function Λ describes the accelerative effect of temperature on aging, with the property that

$$\Lambda(T_0) = 1 \tag{3.21}$$

in order that at the reference temperature T_0, $g = \tau$, or in other words no temperature-induced distortion of aging occurs. In addition to conditions (3.15) and (3.16), we note that at the reference age Eq. (3.20) gives

$$g(\tau_0, \tau_0) = \tau_0$$

which along with Eq. (3.15) requires that

$$\psi(T, \tau_0) = \psi(\tau_0) = 1 \tag{3.22}$$

This implies that for creep tests initiated at the reference age τ_0, all time shift effects are temperature induced.

3.2 Identification of Internal State Variable Parameters

The identification of instantaneous elastic, delayed elastic and permanent flow parameters involves straightforward curve-fitting manipulations if age and temperature effects remain negligible. The separation of the three deformation components requires in this case the evaluation of E, $\eta_F(t)$ and $\eta_{DE}(t-\tau)$ as indicated in Fig. 1. The delayed elastic deformations approach asymptotically the value $\eta_{DE}(\infty)$ corresponding to the retardation spectrum of the exponential expansion in Eq. (2.10). The underlying experimental program simply involves a single creep and unloading test. The irrecoverable creep deformations due to permanent flow follow directly from the construction in Fig. 1 after deducting instantaneous elastic and delayed elastic components from total deformations.

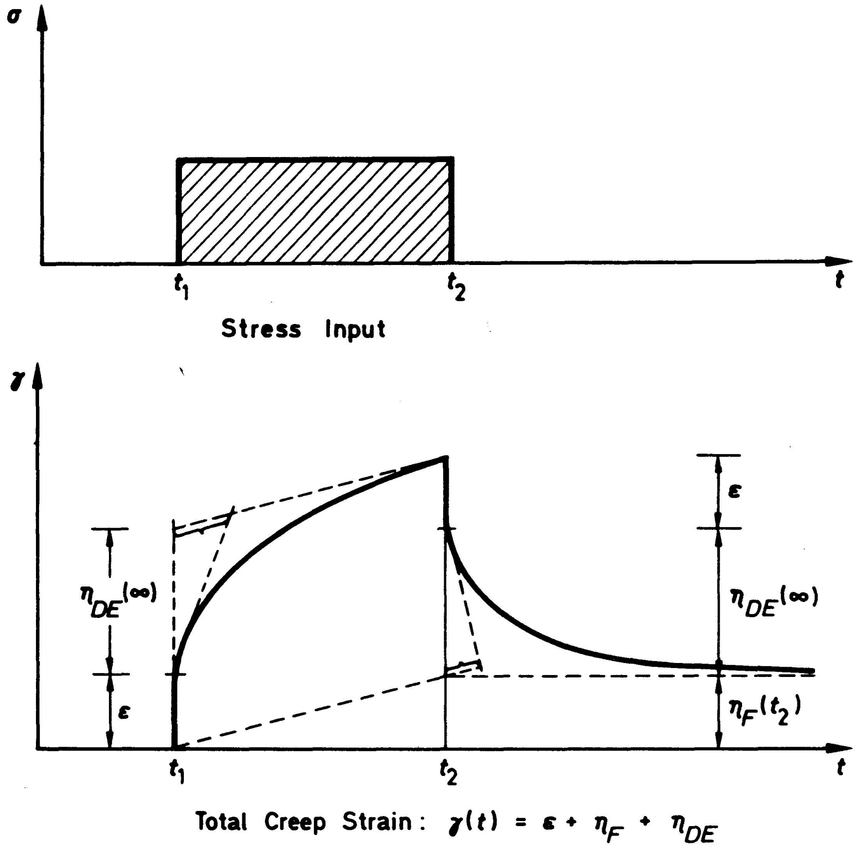

Fig. 1. Elastic, Flow and Delayed Elastic Deformations (Non-Aging Constitutive Model).

For aging behaviour and non-isothermal conditions the identification problem becomes much more complex. Basically, we are confronted with the problem of separating recoverable and irrecoverable creep deformation, Fig. 2, from load-unload tests at different times of observations. This distinction is rather simple if one component is time invariant. To this end the creep studies in [15] are very useful indicating that the magnitude of delayed elastic action is considerably smaller than that of permanent flow. Moreover, the limiting value of recoverable creep strain appears to be insensitive to age and temperature effects. Work reported in [15] suggests that it may be expressed as fraction of the instantaneous elastic response

$$\eta_{DE}(\infty) = 0.4\varepsilon \tag{3.23}$$

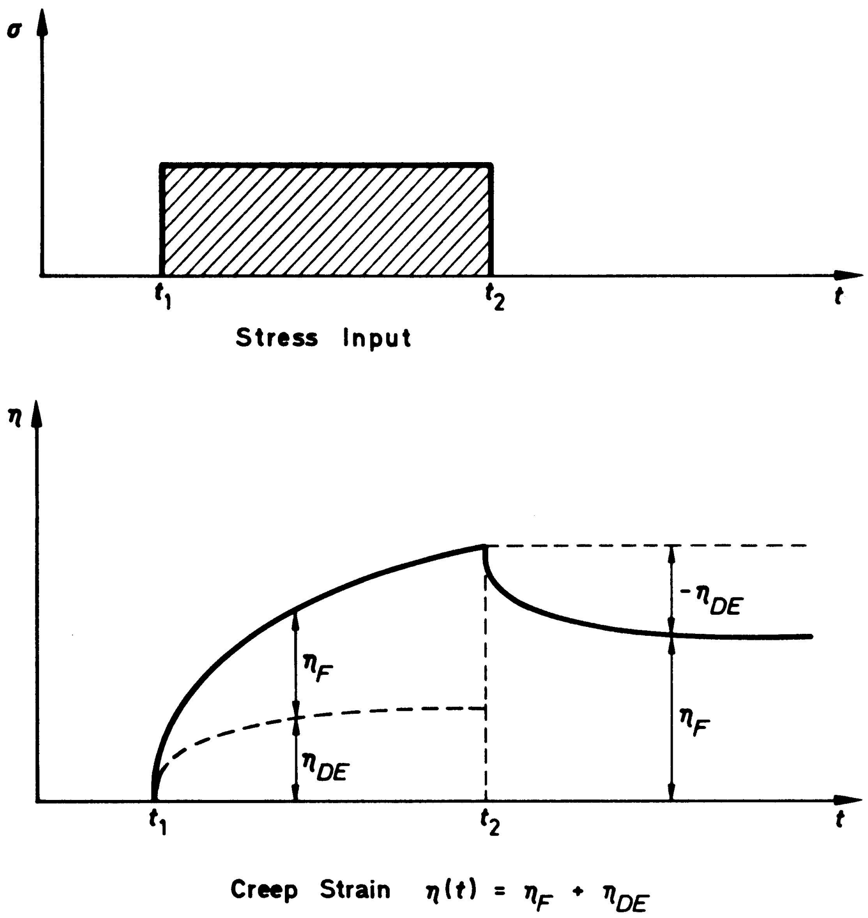

Fig. 2. Recoverable-Irrecoverable Creep Deformations.

Fig. 3 illustrates the creep deformations for two stress pulse inputs separated by a period sufficiently long for recovery. The rate of permanent flow decreases with age, while the delayed elastic component approaches the same limiting value independently of loading age. For this type of aging model the underlying experimental program involves a reference creep test and a series of parallel tests with unloading at different ages. The irrecoverable creep deformations determine then the aging of the permanent flow component. Explicit expressions for permanent flow and delayed elasticity are developed in the CEB-recommendations [16] for isothermal conditions.

In the effective time approach nonlinear transformation of the time scale compensates for both age and temperature effects. Assuming that the same transformation applies to all inelastic deformation components, Eq. (3.1), the non-aging viscoelastic model is recovered by replacing real time with effective time, for which the mapping is defined in Eq. (2.23). Fig. 4 illustrates the creep deformations versus effective time for two stress pulse inputs separated by a period sufficiently large for recovery. In transformed time the material responds exactly like a non-aging viscoelastic solid shown in Fig. 1. The structure of the corresponding age and temperature shift functions was discussed in Section 3.1 considering the interaction of both phenomena. Explicit parameter values are given for concrete data in [12] and [34], both of which are restricted to isothermal conditions at different temperatures.

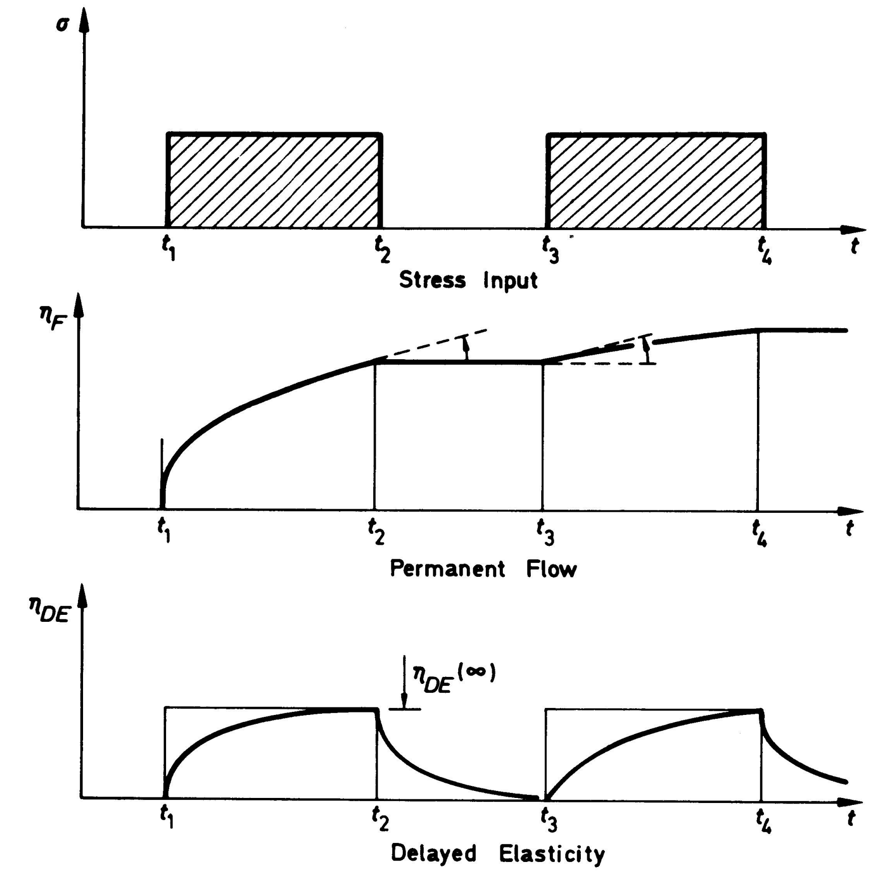

Fig. 3. Aging of Creep Components.

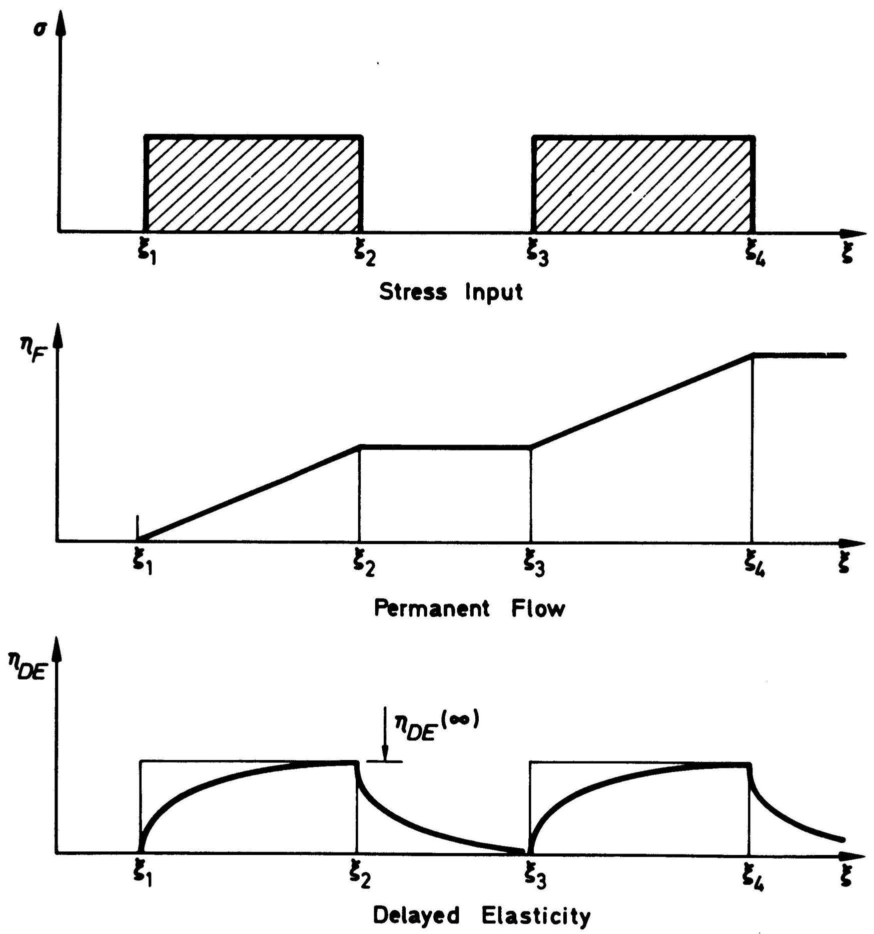

Fig. 4. Creep Components versus Effective Time.

4. Concrete Creep Philosophies

Creep analysis for a given time interval requires the integration of stress and temperature histories within the whole structure. Therefore, it must be anticipated that the elastic solution effort is magnified by the number of previous time steps, whereby all past solutions have to be retained because of material memory. Clearly both computing effort and storage requirements may overtax the capacity of third generation computers. For this reason two approaches have been pursued to reduce the computational task. One is based upon the notion of "finite material memory" in which a solution at any instant of time involves only a "small" number of past solutions. The other is based upon the concept of "degenerate constitutive models" in which the numerical effort for each time step corresponds to that of a single elastic solution and no storage of history is required. As indicated, the internal variable formulation provides a unified concept for these degenerate models while still maintaining ample flexibility to model the dominant phenomena of concrete creep.

In concrete literature three methods are subsumed as special cases of the internal variable formulation above: the flow model, also known as the "rate of creep method" [3], the delayed elastic model based on a homothetic product expansion corresponding to a generalization of ARUTYUNYAN'S method [1] and the additive combination of flow and delayed elastic models, a special version of which is known as "rate of flow method"[17]. In the following, each creep philosophy is discussed in the light of the internal variable theory examining the underlying creep mechanism, treatment of history effect, identification problem and preferred domain of application in reference to prominent examples found in the literature.

4.1 Aging Flow Model

Following the "viscous flow hypothesis" for cement paste, the creep response behaviour of concrete is in this case made up of aging flow only; thus the corresponding internal variable formulation involves only instantaneous elastic and permanent flow components. The reduced strain is according to Eq. (2.25) simply

$$\gamma'(t) = \varepsilon + \eta_F = \frac{\sigma(t)}{E[T(t), t]} + B_1{}^2 \int_0^t \psi[T(\tau), \tau]\sigma(\tau)d\tau \tag{4.1}$$

The associated creep function is in analogy to Eq. (3.12)

$$J(t, \tau) = \frac{1}{E[T(t), t]} + f(t) - f(\tau) \tag{4.2}$$

where the flow term $f(t) - f(\tau)$ corresponds to the effective time expression (2.23)

$$f(t) - f(\tau) = B_1{}^2 \int_\tau^t \psi\ [T(s), s]ds \tag{4.3}$$

In principle, there is no restriction on the "viscosity" $\psi(T, \tau)$ except that for physical reasons it must be a continuous monotonically decreasing function. For example, power laws, exponential functions and hyperbolic or logarithmic expressions are often employed for curve fitting $f(t) - f(\tau)$. Comparison of the governing integral law (4.1) with the convolution expression (3.13) indicates that $\psi(T, \tau)$ simply defines the creep rate as function of age and temperature.

For time variable stress histories the flow model (4.1) infers "superposition" according to the time-hardening concept shown in Fig. 5 for a unit stress pulse input. Clearly, the flow mechanism introduces irrecoverable strains upon unloading, for which the origin of the creep function remains fixed.

$$\begin{aligned} \eta_F(t, \tau) &= f(t) - f(t_1), t_1 < t < t_2 \\ \eta_F(t, \tau) &= f(t_2) - f(t_1), t_2 < t \end{aligned} \tag{4.4}$$

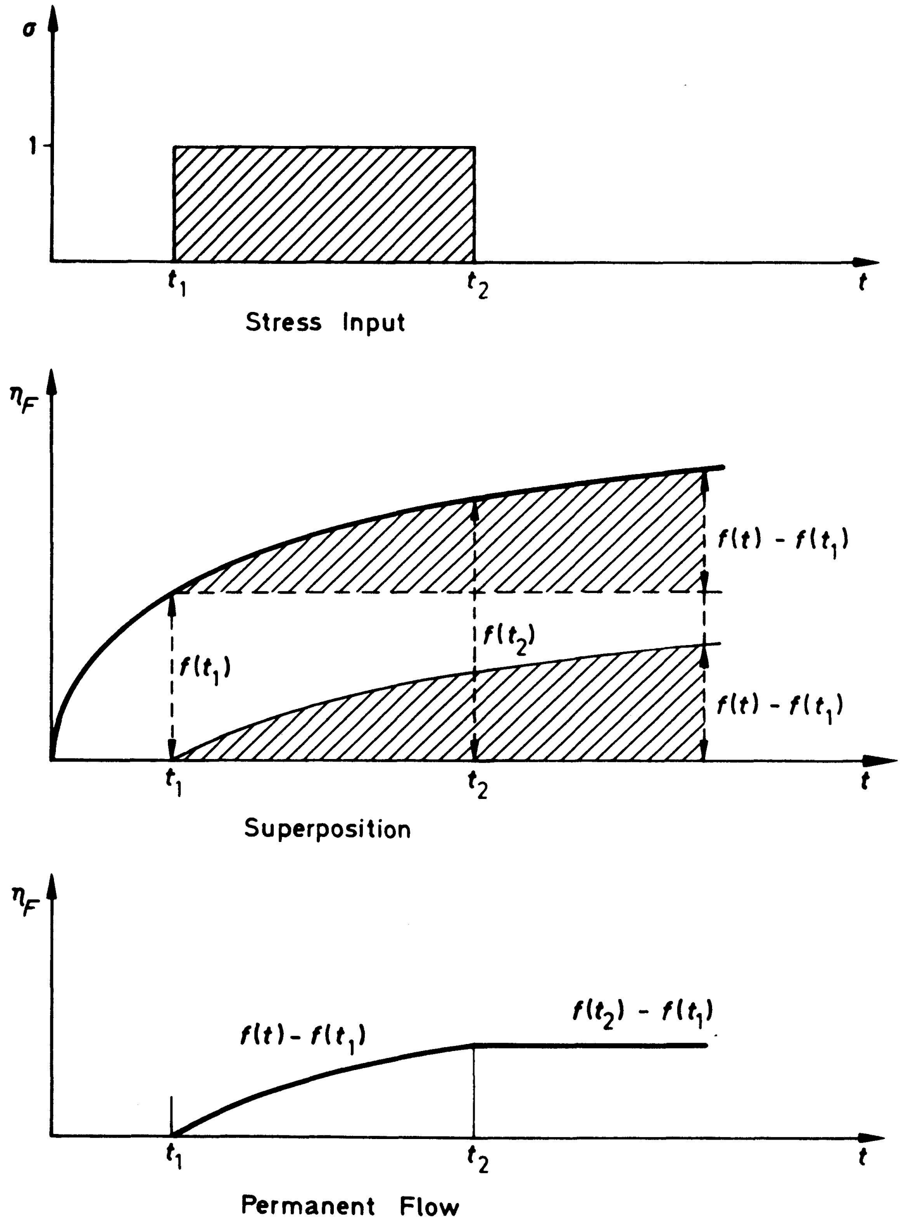

Fig. 5. Flow Model (Time Hardening Concept).

The lack of creep recovery is the most serious defect of the flow model aside from underestimating creep in mature concrete. Therefore, the rate of creep method should be limited to creep predictions of young concrete subjected to input histories which are predominantly monotonic.

The identification problem in this case reduces to the parametrization of the time function or the corresponding rate expression for which only a single creep test is needed to establish the required input data. The corresponding numerical algorithm follows the concept of degenerate models; it involves single step incrementation without storage of history effects.

In spite of the serious shortcomings noted, the flow model has been very popular mainly because of identification and computational convenience. In concrete literature the rate of creep method has been promoted by many investigators, here we mention only the names of DISCHINGER [18], ROSS [3] and ENGLAND [19]. The last reference also contains a comprehensive discussion of numerical implementation and the proper selection of appropriate time steps.

4.2 Aging Delayed Elastic Model

Following the "seepage hypothesis" in cement gel and restraining postulates of the cement paste the creep response behaviour is in this case an aging delayed elastic mechanism described, e.g., by a homothetic product law. One function specifies the time variation since loading while the other accounts for the influence of loading age. For degenerate models the time function is expanded into a series of exponentials which are "complete" in the sense of the Weierstrass' approximation theorem. The corresponding internal variable formulation thus involves aging instantaneous and aging delayed elastic components; the reduced strain is according to Eq. (2.19)

$$\gamma'(t) = \varepsilon + \eta_{DE} = \frac{\sigma(t)}{E[T(t), t]} + \sum_{\alpha=2}^{n} B_\alpha^2 \int_0^t \phi_\alpha[T(\tau), \tau] e^{-\lambda_\alpha(t-\tau)} \sigma(\tau) d\tau \tag{4.5}$$

where we assume that $B_\alpha = \text{const.}$ and $\lambda_\alpha = \text{const.}$ in contrast to the effective time model.

$$\lambda_\alpha = \frac{C_\alpha(t)}{G_\alpha(t)} = \text{const. for } \alpha > 1 \tag{4.6}$$

whence

$$\xi_\alpha(t) = \lambda_\alpha \cdot t$$

This form of aging in the evolution equation (2.8) is necessary to reproduce a creep formulation similar to that of ARUTYUNYAN [1] in which $n = 2$.

Note that Eq. (4.5) is not of convolution form. The associated creep function is in this case

$$J(t, \tau) = \frac{1}{E[T(t), t]} + \sum_{\alpha=2}^{n} B_\alpha^2 \int_\tau^t \phi[T(s), s] e^{-\lambda_\alpha(t-s)} ds \tag{4.7}$$

which can be integrated explicitly only for special classes of aging functions. The kernel of the integral law represents the creep rate of the homothetic product law which is widely used in concrete literature to describe aging of concrete creep. In principle, there is no restriction on aging of the creep rate expression $\phi(\tau, T)$, except that for physical reasons the function must be continuous and monotonically decreasing. However, special care has to be exercised in order to avoid contradictions such as indicated by HAAS [26] for the old DIN 1045 norm.

For arbitrary stress histories the aging delayed elastic model (4.5) infers "superposition" along the line of the Mc Henry principle as shown in Fig. 6 for a unit stress

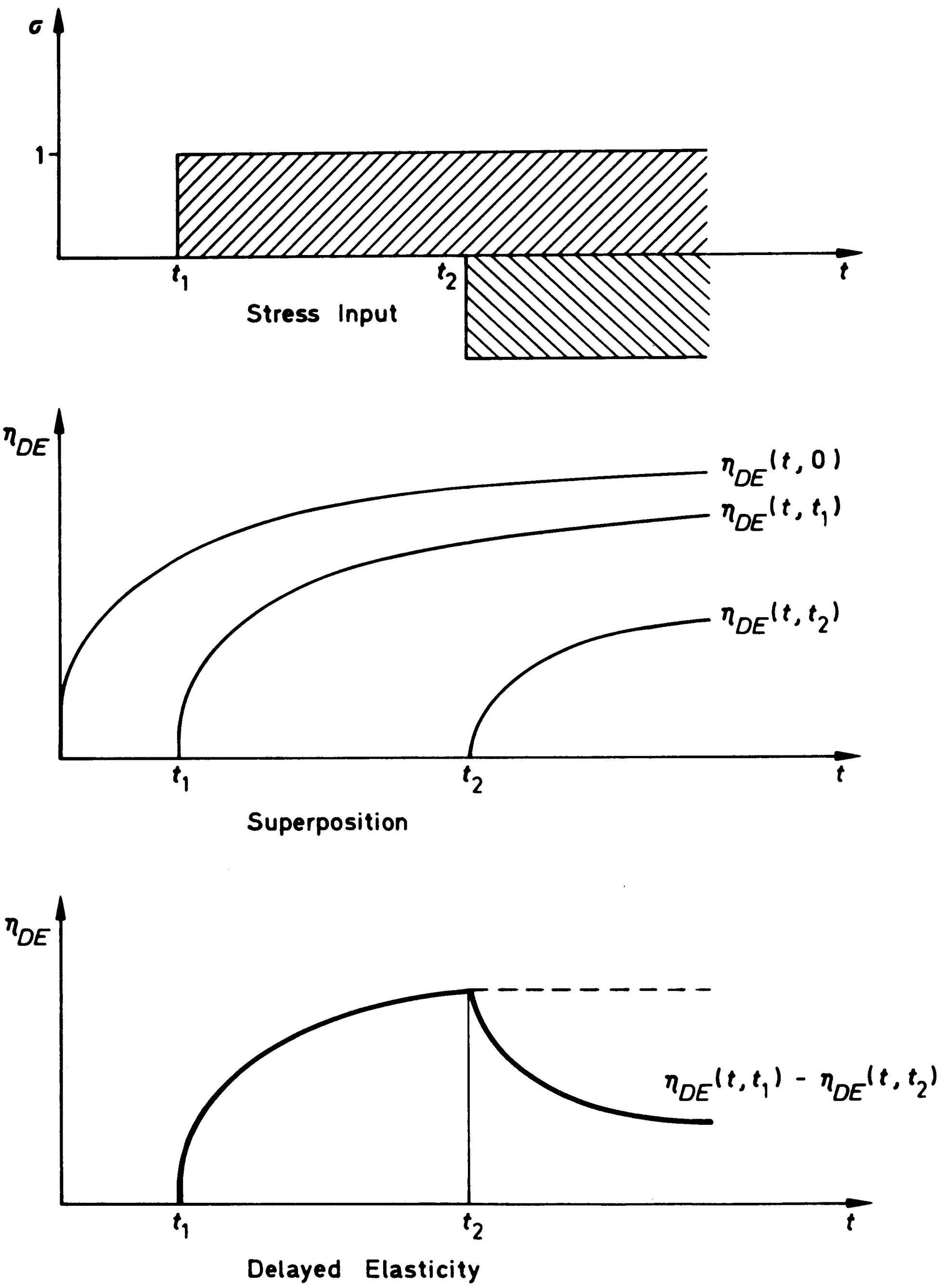

Fig. 6. Delayed Elastic Model (Mc Henry Superposition).

pulse input. Clearly, the delayed elastic mechanism reproduces creep recovery upon unloading during which the origin of the creep function is shifted along the time axis

$$\eta_{DE}(t, t_1) = \sum_{\alpha=2}^{n} B_\alpha^2 \int_{t_1}^{t} \phi[T(t_1), t_1] e^{-\lambda_\alpha(t - t_1)} ds, \; t > t_1 \tag{4.8}$$

$$\eta_{DE}(t, t_2) = \sum_{\alpha=2}^{n} B_\alpha^2 \int_{t_2}^{t_1} \phi[T(t_2), t_2] e^{-\lambda_\alpha(t - t_2)} ds, \; t > t_2$$

Note that the aging function $\phi(T, \tau)$ is responsible for an irrecoverable component upon unloading. Because of the parameter identification from creep tests the aging delayed elastic model tends to overestimate creep recovery effects in mature concrete while the long term response under constant load is sometimes underestimated due to boundedness of the exponential basis.

From the standpoint of parametrization both time and aging function have to be identified, requiring now an extensive test program. In principle a series of creep tests must be initiated at different loading ages to determine the variation of the time function and its dependence on the loading age. The associated age function should meet two criteria, on one hand it should model the decrease of creep capacity with increasing loading age, on the other hand it should reproduce irrecoverable creep upon unloading. Clearly, both conditions are difficult to satisfy because of nonlinear effects, thus the range of application should be oriented towards the underlying identification program.

The delayed elastic model has been applied by many investigators ever since the homothetic product law was published by Arutyunyan [1]. The computational advantages of the exponential series expansion for describing the time variation after loading needs no further commentary; here we refer only to the degenerate constitutive structure. A comprehensive comparison of flow and delayed elastic superposition was presented by Trost [4] for creep and relaxation problems. In the same reference an algebraic approximation of the integral law was developed to improve the effective modulus method. Numerical implementation was discussed in detail by Zienkiewicz and Watson [20] who developed a single step incremental algorithm for constant time steps with no storage of stress and deformation histories. This numerical technique was extended by Bazant and Wu to variable time steps [21]; in the latter reference attention was devoted to the identification of concrete creep data. In a recent publication by the same authors an activation energy concept was introduced to include temperature effects in an equivalent relaxation formulation [5].

An alternative approach for non-aging delayed elastic models was adopted by Taylor *et al.* [22] along the line of thermorheologically simple materials. Using the effective time concept for simulating temperature effects the problem was reduced to the solution of the linear convolution integral equation (3.13) in the kernel of which $B_1 = 0$. To this end a variable time step algorithm was presented and illustrated with a typical example. For concrete a similar delayed elastic model was proposed by Mukaddam and Bresler [12], who introduced an additional time shift factor to account for age as well as temperature effects. The resulting problem of identification of the age and temperature compensated time expression (3.14) has been discussed in detail in Section 3.1.

4.3 Combined Flow-Delayed Elastic Model

In this case the creep response behaviour is described by both viscous flow as well as delayed elastic mechanisms. The additive combination of these two models corresponds exactly to the internal variable formulation of Section 2.2 in which the reduced strain is made up of instantaneous elastic, permanent flow and delayed elastic contributions. In its most general form Eq. (2.19) furnishes for the reduced strain

$$\begin{aligned}\gamma'(t) = \varepsilon + \eta_F + \eta_{DE} &= \frac{\sigma(t)}{E[T(t), t]} \\ &+ B_1{}^2 \int_0^t \psi[T(\tau), \tau]\sigma(\tau)d\tau \\ &+ \sum_{\alpha=2}^{n} B_\alpha^2 \int_0^t \phi_\alpha[T(\tau), \tau] e^{-[\xi_\alpha(t) - \xi_\alpha(\tau)]}\sigma(\tau)d\tau\end{aligned} \tag{4.9}$$

where it is assumed that

$$B_1 = \text{const. and } B_\alpha = \text{const.}$$

and

$$\frac{1}{G_\alpha(\tau)} = \phi_\alpha[T(\tau), \tau] \tag{4.10}$$

The associated creep function is now

$$J(t, \tau) = \frac{1}{E[T(t), t]} + f(t) - f(\tau) + \sum_{\alpha=2}^{n} B_\alpha^2 \int_\tau^t \phi_\alpha[T(s), s] e^{-[\xi_\alpha(t) - \xi_\alpha(s)]} ds \tag{4.11}$$

in which the delayed elastic component can be integrated explicitely but for special classes of aging functions, e.g. using the effective time concept.

For arbitrary stress histories superposition of the combined model implies response behaviour as indicated in Fig. 3. The ensuing numerical algorithm clearly exhibits degenerate structure.

The identification of the internal variable parameters was discussed in Sections 3.1 and 3.2 from the standpoint of effective time. The combined model provides the largest flexibility for describing concrete creep phenomena under arbitrary stress and temperature histories. The model is clearly suited for applications with distinct load-unload regimes; it is this case that the separation into recoverable and irrecoverable deformations is predicted best because the same type of test data is utilized for parameter identification.

Using rheological arguments the combined model was introduced by FREUDENTHAL and ROLL [23] and by HANSEN [24]. Hansen employed an aging Burgers body made up of a Maxwell unit in series with a Kelvin solid. The corresponding viscous flow and delayed elastic action were both related to distinct physical phenomena in the concrete microstructure, i.e. viscous flow of cement paste and consolidation due to seepage from the cement gel or the restraining action of the cement paste. In contrast Freudenthal and Roll proposed a generalized Burgers body with an aging, non-linear Maxwell fluid in series with three Kelvin solids, two of them being non-linear with stress, thus applicable to load regimes up to 65 percent of the 28-day compressive strength. In the last decade there have been numerous papers on more refined rheological models, all of them being based on slightly different arrangements of Maxwell and Kelvin units. In principle they correspond to the formation of higher order differential equations from an equivalent set of first order ones, while in fact the first order law of evolution is most convenient from a computational viewpoint.

Recently, the combined model was rediscovered primarily to overcome the shortcomings of the rate of creep method. The so-called "rate of flow" method of ILLSTON [17] was newly interpreted by RÜSCH *et al.* [15] and later incorporated in the bulletins of the CEB [16]. In all three cases, aging is restricted to viscous flow while delayed elasticity is modeled by non-aging functions. As a matter of fact, Rüsch *et al.* consider delayed elastic response as instantaneous for all practical purposes, assuming that the retardation spectrum of the exponential expansion is zero.

The effective time concept was introduced in the combined model by JORDAAN [25], who developed an age-compensating shift function which applies to both delayed elasticity as well as permanent flow. The resulting convolution integral (3.13) is solved by Laplace transform techniques, a method which is traditionally used for analytical solutions of linear viscoelastic problems, but is of limited use for direct computational methods based on numerical analysis.

4.4 Summary

It was shown how the proposed internal variable approach encompasses three major creep philosophies of concrete literature. It degenerates (i) to the aging flow method of Dischinger, (ii) to the aging delayed elastic method along the line of McHenry and Arutyunyan and (iii) to a combination of these two models. One particular version of this combination is readily identified with the rate of flow method by Illston in which the delayed elastic components are non-aging or even instantaneous as proposed by Rüsch. Another special case is the effective time formulation of age and temperature effects resulting in a convolution integral equation for arbitrary stress histories as shown by Jordaan which can be readily solved by analysis techniques used in linear viscoelasticity.

All these particular forms of the internal variable model lend themselves to economical computational algorithms reducing the solution effort to single step incrementation without storage of history effects because they belong to the class of degenerate constitutive models.

5. Examples

Predictions of different creep philosophies are illustrated with two examples in which uniaxial concrete specimens are subjected to load-unload cycles at different ages and temperatures. In the first problem creep results for three code recommendations for mass concrete are compared; in the second example, creep laws for the Wylfa concrete are studied and compared with test data presented by BROWNE and BLUNDELL [14].

5.1 Code Requirements for Mass Concrete

Creep predictions of the CEB-Committee (Comité Européen du béton), the DIN 1045 norm (German reinforced concrete code) and the ACI Committee 209 (American Concrete Institute) are examined for two load cases, a unit stress pulse in the time interval 28-365 days and one at 365-702 days. The step loads are applied in this case at constant reference temperature $T_0 = 20°\,C$. Parameter values of typical mass concrete are used for identification of the governing internal state variables.

(*a*) *New CEB-Formulation*

The CEB creep law [16] is made up of elastic, permanent flow and delayed elastic components analogous to the revision of the German prestressed concrete code DIN 4227. The unit creep function is in this case

$$J(t,\tau) = \frac{1}{E_0}\{1 + \varphi_F(\infty)[f(t) - f(\tau)] + \varphi_{DE}(\infty)[1 - e^{-\beta(t-\tau)}] * 10^{-6} \tag{5.1}$$

For mass concrete the parameter values are

$$\begin{aligned} \varphi_F(\infty) &= 1.46 \\ \varphi_{DE}(\infty) &= 0.40 \\ \beta &= 0.02 \end{aligned} \tag{5.2}$$

The aging of the flow component is described by a power law

$$f(\tau) = \left(\frac{\tau}{\tau + a}\right)^{1/3} \text{ with } a = 2800 \tag{5.3}$$

In contrast, the delayed elastic behaviour is non-aging, i.e. $\varphi_{DE}(\infty)$ and β are constants.

For variable stress histories the CEB norm specifies superposition according to the rate of flow method for which the internal variable formulation is given by Eq. (4.9). The parameter values are obtained by differentiation of the unit creep function.

Instantaneous Elasticity (Non-Aging):

$$E_0 = 400000 \text{ kp/cm}^2 \text{ (assumed)} \tag{5.4}$$

Permanent Flow (Aging): $\alpha = 1$

$$B_1^2 = \frac{\varphi_F(\infty)}{E_0};\ \psi(\tau) = \frac{a}{3}\tau^{-2/3}(\tau + a)^{-4/3} \tag{5.5}$$

Delayed Elasticity (Non-Aging): $\alpha > 1,\ n = 2$

$$\begin{aligned} B_2^2 &= \frac{\varphi_{DE}(\infty)}{E_0}\beta;\ \phi_2(\tau) = 1 \\ &\qquad \xi_2(\tau) = \beta \cdot \tau \end{aligned} \tag{5.6}$$

(*b*) *DIN 1045 Code*

The DIN 1045 creep law is made up of instantaneous elasticity and creep in form of a homothetic product law. The unit creep function is in this case

$$J(t, \tau) = \frac{1}{E_0}[1 + f(\tau)g(t - \tau)] * 10^{-6} \tag{5.7}$$

where the time and aging functions are specified in form of graphs. HAAS [26] recently proposed a parameterization of the two functions along the line of Arutyunyan's aging delayed elastic model [1]. The time function is described by an exponential series expansion

$$g(t - \tau) = \sum_{\alpha=2}^{3} g_\alpha[1 - e^{-\beta_\alpha(t-\tau)}] \tag{5.8}$$

while the aging function is given by a hyperbolic law

$$f(\tau) = \varphi_0(a + \frac{b}{c + \tau}) \tag{5.9}$$

For mass concrete and loading ages $\tau > 28$ days the parameters are according to Haas [26]

$$\begin{aligned} g_2 &= 0.04131 & \varphi_0 &= 1.5 \\ g_3 &= 0.95869 & a &= 0.4 \\ \beta_2 &= 1/100 & b &= 324.25 \\ \beta_3 &= 1/1526.32 & c &= 561.75 \end{aligned} \tag{5.10}$$

For variable stress histories the DIN 1045 code is incomplete, i.e. there is no explicit specification for superposition. However, the interpretation of Haas clearly belongs to the class of delayed elastic models for which the internal variable formulation is given in Eq. (4.5). The associated parameter values are readily obtained for loading age τ

Instantaneous Elasticity (Non-Aging):

$$E_0 = 400000 \text{ kp/cm}^2 \text{ (assumed)} \tag{5.11}$$

Delayed Elasticity (Aging): $\alpha > 1, n = 3$

$$\begin{gathered} B_\alpha^2 = \varphi_0 g_\alpha / E; \ \lambda_\alpha = \beta_\alpha \\ \phi_\alpha(\tau) = (a + \frac{b}{c + \tau})/\beta_\alpha \end{gathered} \tag{5.12}$$

(*c*) *ACI Committee 209*

Similar to the DIN 1045 specification, the ACI creep law is made up of instantaneous elasticity and creep in the form of a homothetic product law. The unit creep function is analogous to Eq. (5.7)

$$J(t, \tau) = \frac{1}{E(\tau)}[1 + cf(\tau)g(t - \tau)] * 10^{-6} \tag{5.13}$$

In this case the time function is described by a hyperbolic expansion

$$g(t-\tau) = \frac{(t-\tau)^{\beta}}{10+(t-\tau)^{\beta}} \text{ where } \beta = 0.6 \tag{5.14}$$

and the aging function by a power law

$$f(\tau) = 1.25/\tau^{\delta} \text{ where } \delta = 0.118 \tag{5.15}$$

The constant c is for mass concrete

$$c = c_T * c_H * c_{\infty} = 0.7 * 0.67 * 2.35 \tag{5.16}$$

For variable stress histories the ACI recommendation is again incomplete since no explicit statement is made with regard to superposition. For simplicity we assume here that the ACI creep law is based on viscous flow, of which the internal variable formulation is given in Eq. (4.1). The associated parameter values are obtained by differentiation of the unit creep function with constant origin at $\tau = 28$.

Instantaneous Elasticity (Non-Aging):

$$E_0 = 400000 \text{ kp/cm}^2 \text{ (assumed)} \tag{5.17}$$

Permanent Flow (Aging):

$$\alpha = 1;\ B_1{}^2 = \frac{cf(\tau)}{E(\tau)}$$

$$\psi(t-\tau) = \frac{dg(t-\tau)}{dt} = 10\frac{\beta(t-\tau)^{\beta-1}}{[10+(t-\tau)^{\beta}]^2} \tag{5.18}$$

(d) Comparison of Results

In Fig. 7 the creep response behaviour of the three models is shown for loading ages of 28 days and 365 days. The elastic component is omitted in the illustration to emphasize the difference of the individual code predictions. Both results are plotted for the time one day after loading, $t - \tau + 1$, to simplify comparison of age effects.

In the constant load regime it is obvious that there are substantial deviations among the time variations of the three code recommendations for both load cases. The code specifications could be adjusted to furnish the same values for long term creep since all models are using bounded creep functions.

In the unloading regime the deviations of the three creep models are even more pronounced. The CEB specification exhibits large recovery effects because of the non-aging delayed elastic component. In contrast the flow interpretation of the ACI code does not allow for recovery, underscoring the serious shortcomings of the rate of creep method for unloading. The DIN 1045 norm predicts very small recovery effects because of limited creep capacity and negligible aging.

In conclusion, the appreciable differences among the code predictions underline a need for additional creep test data and selection of an appropriate creep model if accurate creep results are to be assured for specific concretes.

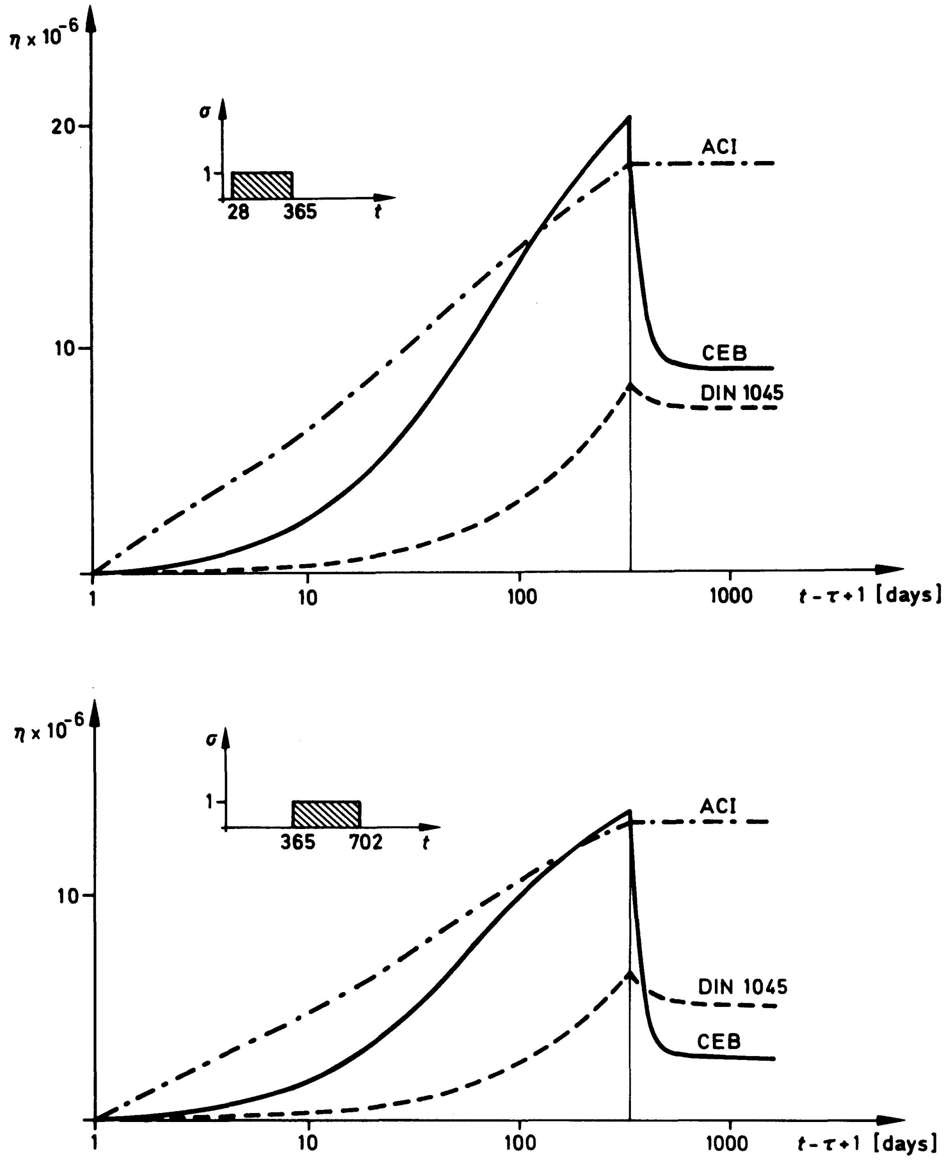

Fig. 7. Code Predictions for Unit Stress Pulse (Time Variation and Aging of Mass Concrete).

5.2 Creep Models for Wylfa Concrete

In the second example total creep deformations of the Wylfa reactor concrete, for which several creep models have been proposed in concrete literature, are examined. In particular, the formulations of Bazant-Wu [21], Hansson [28] and Bresler-Mukaddam [12] are applied and their results compared with underlying experimental data of Browne and Blundell [14]. Again the problem involves a uniaxial concrete specimen which is subjected to stress pulse inputs at 28-365 days and 365-702 days for assessing age effects at constant reference temperature $T = 20°$ C. In addition at 365 days another stress pulse is applied at $T = 65°$C to study temperature effects under isothermal conditions, the results of which could be utilized for non-isothermal conditions using superposition as long as transitional thermal creep remains negligible.

(*a*) *Bazant-Wu*

In this model the creep law is made up of instantaneous elasticity and creep in the form of a homothetic product law with exponential basis. The following unit creep function was developed by BAZANT-WU [21] for the reference temperature $T = 20°\,C$

$$J(t,\tau) = \frac{1}{E(\tau)} + \sum_{\alpha=2}^{\infty} \frac{1}{E^*(\tau)}\left[1 - e^{-\beta_\alpha(t-\tau)}\right] * 10^{-6}/\text{lbs/in}^2 \qquad (5.19)$$

which was extended by the same authors in a more recent publication [5] to include temperature effects within a relaxation formulation. In Eq. (5.19) instantaneous elasticity is assumed to age according to the law

$$\frac{1}{E(\tau)} = 0.20 + 0.45/\tau^{0.5} \qquad (5.20)$$

while aging of the delayed elastic component is modeled by the time variable coefficient $E^*(\tau)$ in the exponential expansion

$$\frac{1}{E^*(\tau)} = 0.0075 + 0.233/\tau^{0.33} \qquad (5.21)$$

There is no age effect in the exponents themselves, i.e. the retardation spectrum is sampled in decades of real time

$$\beta_\alpha = \frac{10^{-\alpha+2}}{5.63}, \alpha > 1 \qquad (5.22)$$

For variable stress histories the internal variable formulation of the corresponding delayed elastic model is given in Eq. (4.5). The governing parameter values are readily derived for loading age τ

Instantaneous Elasticity (Aging):

$$\frac{1}{E(t)} = \frac{1}{E(\tau)} + \int_\tau^t \frac{\partial}{\partial s}\left[\frac{1}{E(s)}\right] ds \qquad (5.23)$$

Aging of the elastic component gives rise to a contribution equivalent to permanent flow.

Delayed Elasticity (Aging): $\alpha = 1, n = 10$

$$B_\alpha^{\,2} = 1; \lambda_\alpha = \beta_\alpha$$

$$\phi_\alpha(\tau) = (0.0075 + 0.233\tau^{-.33})/\beta_\alpha \qquad (5.24)$$

(*b*) *Hansson*

In this model the creep law is made up of a single creep expression in the form of a power law suggested in [14]. HANSSON [28] developed explicit parameter values for temperatures between 20°–70° C, however, no aging is considered and no distinction is made for individual creep components. The following unit creep function was proposed

$$log[J(T, t-\tau)] = f(T, t-\tau)[a + bT + c\,log(t-\tau+1)] * 10^{-6}/\mathrm{kp/cm^2} \quad (5.25)$$

where $(t-\tau)$ denotes time since loading. Immediately upon loading the response is governed by

$$f(T, t-\tau) = 1 - \frac{T}{25} 10^{-(t-\tau+1)} \quad (5.26)$$

Note that Eq. (5.25) describes a linear relation in a double-logarithmic plot, the parameters for which are

$$\begin{aligned} a &= 0.46 \\ b &= 0.0045 \\ c &= 0.12 \end{aligned} \quad (5.27)$$

For variable stress histories the Hansson model is again incomplete since no specification is made with regard to superposition. In this context we assume here for convenience that the creep law is based on a flow model for which the internal variable formulation is given in Eq. (4.1). The governing parameter values derive from the corresponding creep rate expression with constant origin at $\tau = 28$.

Instantaneous Elasticity (Non-Aging):

$$\frac{1}{E(T, t)} = 10^{f(T, t-\tau)(a+bT)} \quad (5.28)$$

Permanent Flow (Aging): $\alpha = 1, B_1{}^2 = 1$

$$\psi(T, t-\tau) = c(t-\tau+1)^{cf(T, t-\tau)}\left[\frac{f(T, t-\tau)}{t-\tau+1} + ln(t-\tau+11)(1-f(T, t-\tau))\right] \quad (5.29)$$

(c) *Bresler-Mukaddam*

In this model the effective time concept was applied to account for age and temperature effects by equivalent transformations of time scale. The creep law is composed of instantaneous and delayed elastic contributions in the form of exponential series expansion. In particular, the following expression was proposed by BRESLER-MUKADDAM [12]

$$J(T,t,\tau) = \frac{1}{E(T,\tau)} + \sum_{\alpha=1}^{6} g_\alpha e^{-\beta_\alpha \phi(T,\tau)(t-\tau)} * 10^{-6}/lbs/in^2 \quad (5.30)$$

Instantaneous elasticity depends on both age and temperature, i.e.

$$\begin{aligned} \frac{1}{E(T,\tau)} &= 210\,p[T(\tau)] + 1600/\tau \\ p[T(\tau)] &= \left[-.086\left(\frac{T}{68}\right)^3 + .352\left(\frac{T}{68}\right)^2 - .019\left(\frac{T}{68}\right) + .753\right] \end{aligned} \quad (5.31)$$

The individual parameters of the exponential series expansion are

α	β_α	g_α
1	0	919.01
2	.2	− 46.27
3	.02	− 95.08
4	.002	− 156.32
5	.0002	− 197.60
6	.00002	− 370.22

(5.32)

Age and temperature compensated time is defined by a shift function in which temperature effects on aging are considered

$$\phi(T,\tau) = \varphi[T(\tau)]\,\psi[T(\tau),\tau] \tag{5.33}$$

The temperature shift is

$$\varphi[T(\tau)] = -3.55\left(\frac{T}{68}\right)^3 + 21.57\left(\frac{T}{68}\right)^2 - 33.83\left(\frac{T}{68}\right) + 16.79 \tag{5.34}$$

The age shift depends on temperature as well as age via a new variable, the co-age $g[T(\tau),\tau]$. For loading ages $\tau > 28.3$ days

$$\psi[T(\tau),\tau] = .056 + .686\,e^{-.01\,g[T(\tau),\tau]}$$

and

$$g[T(\tau),\tau] = 28 + (\tau - 28)\,e^{-.06(T-68)} \tag{5.35}$$

while at reference age $\tau_0 = 28$ days the aging function $\psi[T(\tau_0),\tau_0] = 1$.

For variable stress histories the corresponding internal variable formulation follows directly from the effective time model Eq. (3.12). The governing parameter values derive from the corresponding creep rate expression yielding for

Instantaneous Elasticity (Aging):

$$\frac{1}{E(T,t)} = \frac{1}{E(T,\tau)} + \int_\tau^t \frac{\partial}{\partial s}\left[\frac{1}{E(T,s)}\right] ds \tag{5.36}$$

Aging of the elastic component gives rise to a contribution equivalent to permanent flow. Moreover, the first term $\alpha = 1$ has a zero exponent, Eq. (5.32), which yields an additional instantaneous deformation component

$$\frac{1}{E'} = g_1 + \sum_{\alpha=2}^{6} g_\alpha = 53.52 \tag{5.37}$$

Delayed Elasticity (Aging): $\alpha > 1, n = 6$

$$B_\alpha{}^2 = -\beta_\alpha g_\alpha;\ c_\alpha = \beta_\alpha \tag{5.38}$$

$$\xi' = \int_0^\tau \phi[T(s),s]\,ds$$

(d) *Comparison of Results*

The resulting creep deformations of the three models are shown in Fig. 8 for the stress pulse 28-365 days. A comparison with the test data of BROWNE and BLUNDELL [14] indicates relatively satisfactory agreement during loading, especially if the large scatter of test data is considered (average values are shown). Basically, the results only confirm the curve-fitting technique, since the same test data were used by all authors. In contrast, there is no experimental evidence on unloading and the recovery predictions deviate by considerable margins. In particular, the flow interpretation of the creep law of Hansson does not allow for any recovery, a shortcoming of the rate of creep method discussed before. The delayed elastic model of Bazant-Wu exhibits non-monotonic recovery effects, a phenomenon rather suspect on physical grounds.

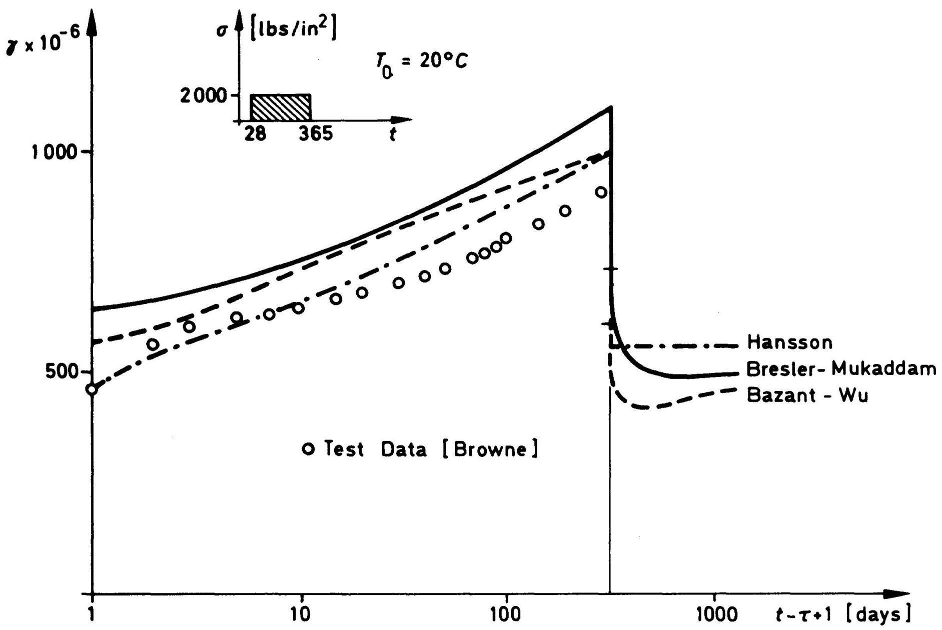

Fig. 8. Creep Predictions for Wylfa Concrete. (Loading at Reference Age and Temperature).

In Fig. 9 creep results for the same models are shown for the stress pulse 365–702 days at $T = 20°$C and $T = 65°$C. A comparison of both figures illustrates the effects of temperature on creep capacity and creep rate as predicted by the models of Hansson and Bresler-Mukaddam. In spite of the poor agreement at $T = 20°$C there is relatively close fit at $T = 65°$C except for unloading. The creep law of Hansson exhibits no aging, thus it yields identical values for the loading ages 28 days and 365 days as shown in Figs. 8 and 9.

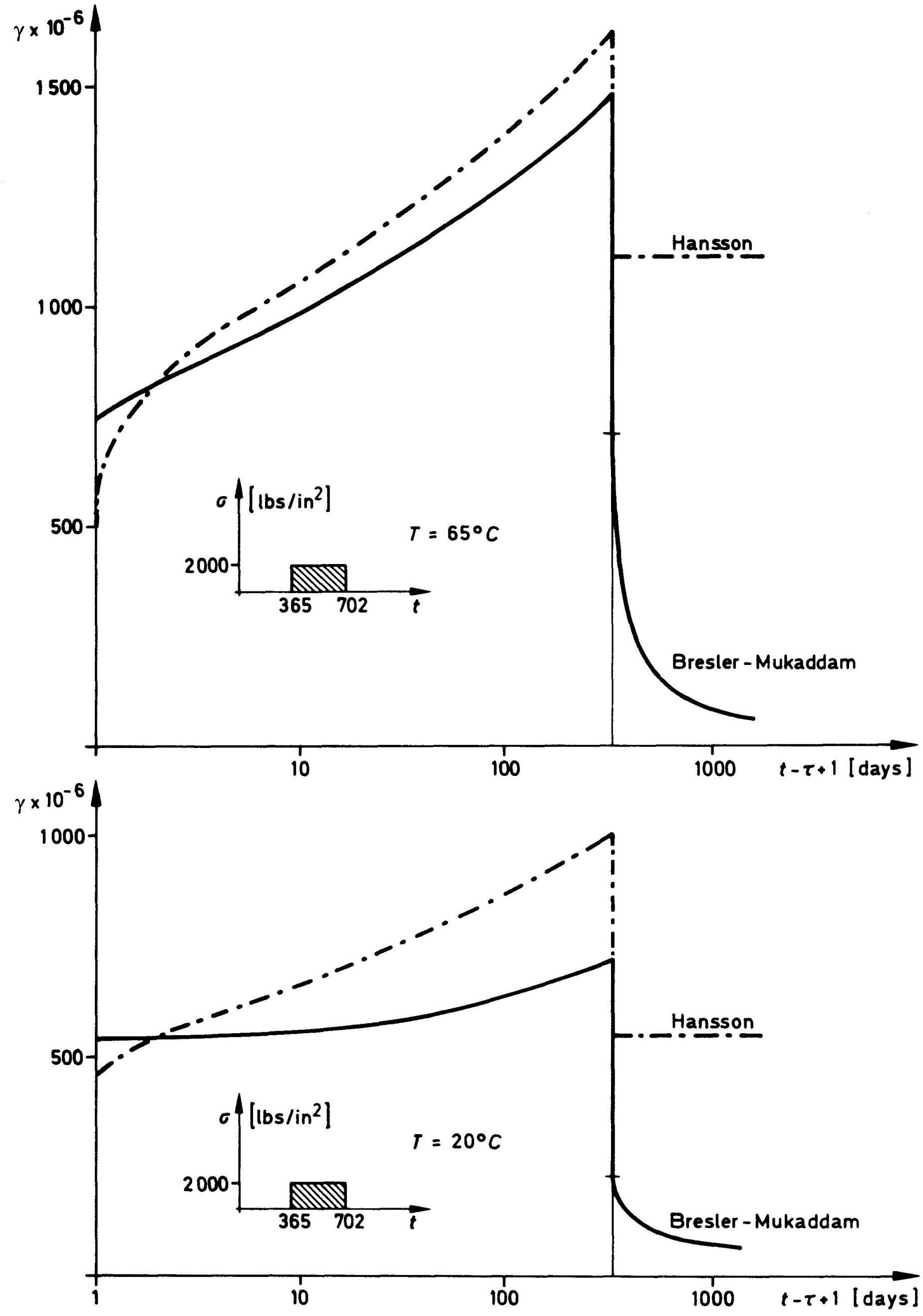

Fig. 9. Creep Predictions for Wylfa Concrete (Loading at Higher Age and Temperature).

In conclusion, there is reasonably satisfactory agreement of the different Wylfa concrete creep models for constant load regimes. However, appreciable differences arise for variable load histories, in particular during unloading. Likewise, the lack of experimental data for tests incorporating both variable stress and temperature makes it impossible to select a preferred model for thermomechanical creep at this time.

6. Safety and Economy

Creep and relaxation control the long term performance of concrete structures which is strongly affected by environmental conditions such as temperature and moisture.

During the life of reinforced and prestressed concrete structures creep deformations may exceed the instantaneous deformations by a factor of 2 to 5 and thus endanger serviceability of the structure. Moreover, creep deformations may lead to considerable prestress losses and thus reduce the safety of the structure due to excess tension under normal service conditions. On the other side, stress relaxation may be responsible for stress reversals, e.g. introducing tensile stresses because of unloading (heating-cooling cycles).

Therefore, time dependent analysis of complex concrete structures has become mandatory in the engineering decision process. Since transient creep prediction under constant and variable load histories requires integration of stress and temperature histories at "each" point of the structure we must anticipate that the elastic solution effort is magnified considerably.

The internal state variable concept provides on one side a unified theoretical frame for different concrete creep philosophies with ample flexibility for modeling time dependent phenomena. On the other side it furnishes a very economical numerical algorithm which reduces the computational effort for advancing the solution by a single time step to the solution of one additional load case. In this way creep analysis of complex concrete structures requires relatively little additional effort beyond time independent elastic solutions in spite of tracing the entire response history for constant as well as time variable loads.

7. Concluding Remarks

The principle objective of the paper has been to provide a unifying analytical structure for studying creep of concrete subjected to uniaxial stress (loading and unloading) with simultaneous changes in temperature. This has led to the general thermomechanical creep equation (2.25). In this formulation, rate and history effects are reflected by internal variables, the evolution of which is modeled by first-order differential equations expressing "growth laws" which determine the amount of creep. The total creep deformation is composed of instantaneous and delayed elastic parts as well as permanent flow. Introduction of the concept of an "effective time" permits representation of the creep law as a convolution integral, equation (3.13), whose corresponding kernel is the creep function given by equation (3.12). The "degenerate" structure leads to a numerical solution scheme which is well-adapted to automatic computation involving single step incrementation with no storage of history. The model suggests test programs from which internal variable parameters can be evaluated from measurements of recoverable and permanent creep deformation as functions of time, loading age and temperature.

As special cases of the theory presented, models currently in use appear as special cases. These include the flow model (rate of creep method), the delayed elastic model (homothetic product expansion with an exponential basis) and the combined model (rate of flow method). Values of the parameters of these models in terms of the internal variable format are given in two illustrative examples. The comparisons illustrated in Figures 7, 8, 9, indicate substantial variations in predicted creep deformation for a stress pulse of constant magnitude. Of particular concern in Fig. 7 is the lack of agreement for predicted maximum creep stress and the failure to model creep recovery. Similarly, the predictions for effects of age and temperature of loading shown in Figs. 8 and 9, while relatively better, are at best inconclusive, since experimental data on creep recovery as well as data for variable temperature history (e.g. thermal cycling) are not available.

Accordingly, while the model proposed in equation (3.13) appears qualitatively to have the necessary flexibility, at present there is insufficient data to select appropriate values of the required model parameters, particularly as far as transient temperature and moisture conditions are concerned for which additional phenomena have been reported (e.g. transitional thermal creep, drying creep).

Finally, we wish to note again that formal generalization of the proposed model in a form suitable for three-dimensional stress analysis is straight-forward, although the concomitant identification problem is made substantially more complicated. In the same vein incorporation of moisture effects and of nonlinear dependence of creep strain on stress level is possible at the expense of a more complicated mathematical structure of the equations of evolution (2.8) for internal variables.

Notation

B_1, B_α	coefficient of driving force in equation of evolution.
C_1, C_α	growth coefficient of internal variable.
D	coefficient of thermal expansion.
E	instantaneous elastic modulus.
F	thermal property.
f	function; aging function.
G_1, G_α	growth coefficient of internal variable.
g	co-age; time function since loading.
J_0, J_1, J_α	creep compliance, instantaneous, flow, delayed coefficients.
q_α, q_β	internal variables.
T	temperature.
t	time of observation.
α	coefficient of thermal expansion.
β_α	exponential coefficient.
γ	total strain.
ε	elastic strain.
η_F, η_{DE}	inelastic strain, flow, delayed components.
λ	temperature shift of co-age.
λ_α	exponential coefficient.
ϕ	temperature-age function of delayed component.
φ	temperature shift.
X	complementary energy.

ψ	temperature-age function of flow component.
ψ	aging interaction with temperature.
ξ_0, ξ_α	effective time.
σ	stress.
τ	age at time of loading.

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Summary

The objective of this paper is to develop a unified constitutive model for concrete creep with the following features: creep strain is a linear functional of stress history; besides instantaneous elastic response the model predicts partial recovery upon removal of stress due to delayed elastic action which approaches in the limit the irrecoverable part of deformation corresponding to irreversible flow. Each creep component is affected by both temperature and aging. The internal state variable model unifies the constitutive structure of energy dissipation during creep. The ensueing first order differential law of evolution lends itself to an economical computational algorithm which reduces the solution effort to single step incrementation without storage of history effects.

Résumé

L'objectif de cet article est d'établir un modèle pour le fluage du béton tenant compte des caractéristiques suivantes: l'extension due au fluage est une fonction linéaire de l'histoire des contraintes; la réponse élastique instantanée est accompagnée d'une déformation, laquelle revient partiellement au délai de l'action élastique, et tend asymptotiquement vers la déformation limite correspondant à un écoulement permanent. Chaque composante du fluage est affectée par la température et le vieillissement. La théorie des variables d'état interne fournit la structure générale des équations constitutives de base par une loi d'évolution différentielle du premier ordre. Elle conduit à un algorithme économique de calcul qui ramène l'effort de travail pour la solution numérique à une simple incrémentation, sans avoir à garder en mémoire les effets dus à l'histoire antérieure.

Zusammenfassung

Ziel der vorliegenden Veröffentlichung ist, die Rechenverfahren zum Betonkriechen durch ein Stoffmodell mit folgenden Eigenschaften zu vereinheitlichen: Kriechdehnungen sind ein lineares Funktional der Spannungsgeschichte. Neben momentan elastischem Verhalten entstehen elastische Nachwirkungen, die sich bei Entlasten in Form von Kriecherholen asymptotisch dem irreversiblen Anteil der Verformungen annähern. Alle Verformungskomponenten sind Funktionen von Temperatur und Alter bei Belastung. Das Modell der inneren Zustandsvariablen beschreibt Energiedissipation infolge Kriechens in Form von Differentialgleichungen erster Ordnung. Das resultierende numerische Verfahren bietet den Vorteil, daß sich der Aufwand des allgemeinen Superpositionsverfahrens auf den eines einfachen Inkrementierungsverfahrens, ohne Speicherung der Geschichte, reduziert.