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A Unified Design Method for Composite Columns

Une méthode unifiée de calcul des colonnes mixtes acier-béton

Eine vereinheitlichte Methode für den Entwurf von Verbundstützen

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Introduction

A design method for rectangular composite columns was proposed by BASU and SOMMERVILLE [1] in 1969. The method was derived on the basis of results obtained from analytical studies on numerous composite columns in uniaxial bending. Later the method was extended [2] to include concrete filled circular steel tubular columns. Design studies [3, 4] which were subsequently carried out to explore the application of the method to practical cases concluded that, although it was more comprehensive than other available methods, there were also some shortcomings.

Two principal drawbacks were related to the design of axially loaded columns. In the first instance it was found that in the case of slender composite columns, design loads given by the new method for some encased sections were less than those allowed by existing codes for the corresponding uncased sections. Furthermore, no allowance was made for the augmentation in strength of concentrically loaded concrete filled circular steel tubular columns due to the triaxial containment of the concrete.

The aim of this paper is to present a design method, for axially loaded composite columns, which overcomes the objections to Basu and Sommerville's original proposals, and to establish the validity of the approach by comparison with available test results and analytically derived data of proven accuracy. The proposed method relates the design of axially loaded composite columns to the design of bare steel columns. A later paper will deal with the design of eccentrically loaded composite columns.

Basu and Sommerville's Composite Column Curve

The strength of pin-ended axially loaded bare steel columns is related to the ratio of the length to the radius of gyration, L/r. In the case of columns made up of two or more materials with different material properties the radius of gyration has no meaning. BASU and SOMMERVILLE [1], however, defined an equivalent radius of gyration for composite columns as

$$r_e = \sqrt{(\sigma_y^* I_s + \sigma_y^* E_c I_c / E_s) / P_s} \quad (1)$$

This radius of gyration was used to obtain the column slenderness ratio (L/r_e) for subsequent use in design. It was stated that by taking $E_c = 360\ \sigma_u$ and $\sigma_y^* = 16$ tonf/in^2 (247 N/in^2) it was possible to minimise the scatter in plotting a large number of analytical results as P_a/P_s against L/r_e. In arriving at their basic buckling curve, or K_1 curve, they first found the lower bound of the narrow band of scattered points. The curve was subsequently lowered further, mainly in the intermediate slenderness range of $L/r_e = 50$-150, to ensure the safe design of certain eccentrically loaded rectangular hollow sections filled with concrete using other formulae they derived.

The excessively conservative nature of the K_1 curve so produced is illustrated by comparing available test ultimate loads [5, 6, 7, 8] with the corresponding design strength as shown in Figure 1. Nominal values of cross-sectional areas, and mean values of concrete strength, corrected for the minimum standard deviation as recommended by CEB [9] (see Appendix I), are used throughout. It may be seen that several columns, particularly those tested by Stevens, show markedly high strengths as compared with the K_1 curve. The mean value of the ratio of test ultimate loads to design strength is 1.928 with a standard deviation of 202 per cent. For a good design method, the two values should be 1.000 and 0 per cent, or as close to these values as possible.

A similar comparison is made with analytically computed results in Figure 2. The theoretical results [1] were obtained using a verified computer program [10]. The practical column cross-sections have concrete contribution parameter, α, varying from 0.12 to 0.80. The stress-strain curve for concrete was assumed to be that given in the CEB recommendations [9], and a bi-linear curve was adopted for steel. An initial lack of straightness of L/1000 in the plane of bending is assumed throughout. It will be seen that the K_1 curve lies well below the lower envelope of all the analytical results, and that the margin of conservativeness increases with slenderness ratio. The mean value of the ratio of analytical ultimate loads to design strengths is 1.567 and the standard deviation is 59 per cent.

The comparison of the design strengths with both the experimental and theoretical ultimate loads shows clearly the conservative nature of the basic composite column buckling curve proposed by Basu and Sommerville, and confirms the findings of the design studies referred to earlier. The following sections show how the anomalies between the design of composite and bare steel columns are eliminated.

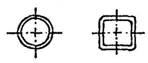
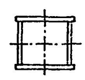
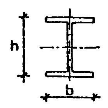
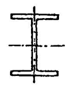
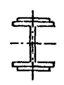
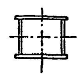
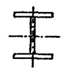
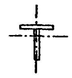
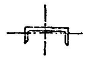
New Bare Steel Column Curves

The design of bare metal sections has traditionally been based on relationships between the column critical stress and the column slenderness ratio. Typical of these relationships is the PERRY-ROBERTSON formula [11] which incorporates an

[1] Details of the analytical and experimental results plotted in this paper may be found in Reference 28.

imperfection factor to account for any initial lack of straightness, accidental eccentricity of loading, and residual stresses. The committee drafting the new European code for steel structures have given the problem the most exhaustive treatment to date. Following numerous tests and analytical studies, BEER and SCHULZ [12] recommended three basic buckling curves (Fig. 3) which are applicable to cross-sections of different shapes. A table, reproduced here as Table 1, is provided to enable designers to select the appropriate curve for a given column cross-section. Representative residual stress distributions in the cross-section and an initial lack of straightness of L/1000 are allowed for in the derivation of the curves. Two additional curves cater for certain extreme cases.

Table 1.

SHAPE OF SECTION		CURVE
	Rolled tubes Welded tubes	a
	Welded box sections	b
	I and H rolled sections: -Buckling parallel to the web h/b > 1·2 h/b < 1·2 -Buckling parallel to flanges h/b > 1·2 h/b < 1·2	 a b b c
	I and H welded sections: -Buckling parallel to the web a) Flame cut flanges b) Rolled flanges -Buckling parallel to flanges a) Flame cut flanges b) Rolled flanges	 b b c
	I and H sections with welded flange cover plates -Buckling parallel to flanges -Buckling parallel to web	 a b
	Box sections, stress relieved by heat treatment	a
	I and H sections, stress releived by heat treatment -Buckling parallel to the web -Buckling parallel to flanges	 a b
	T-sections or half I-sections	c
	Channels	c

Parallel work carried out in connection with the new British code for steel buildings and steel bridges has resulted in four curves that approximate the European curves very closely. Both the British and European curves have small plateaux to take account of the strain hardening effects in stocky columns.

The European curves are presented as relationships between $\overline{N}$, the ratio of the column critical stress to its yield stress, and its slenderness factor $\overline{\lambda}$, the ratio between the column slenderness ratio ($\lambda = L/r$) and a critical slenderness ratio λ_e. The critical slenderness ratio is defined as that for which the column Euler stress equals the yield stress of the material of the cross-section, and is given by

$$\lambda_e = \pi \sqrt{\frac{E_s}{\sigma_y}} \tag{2}$$

The use of the slenderness factor, $\bar{\lambda}$, rather than the slenderness ratio, λ, makes the curves independent of the material properties and thus the same curves can be used to design columns with different yield strengths.

Application of Bare Steel Column Curves to Composite Column Design

Proposed Interpretation for Column Strength, $\bar{N}$

In the context of the design of bare steel sections $\bar{N}$ is defined as the ratio of critical stress σ_a to the yield stress σ_y, that is

$$\bar{N} = \frac{\sigma_a}{\sigma_y} \tag{3}$$

In the alternative interpretation now proposed, and which is applicable to bare metal sections as well as composite sections, $\bar{N}$ is defined as the ratio of the column critical load P_a to its squash load P_s, thus

$$\bar{N} = \frac{P_a}{P_s} \tag{4}$$

For a composite column the squash load may be expressed as follows:

$$P_s = \Sigma\, A_s\, \sigma_y + A_c\, \sigma_u \tag{5}$$

The summation sign is intended to include not only the main steel core but also other steel areas such as longitudinal reinforcement. The column ultimate load under concentric loading may be expressed as

$$P_a = \Sigma\, A_s\, \sigma_a + A_c\, \sigma_b \tag{6}$$

where σ_a and σ_b are the average stresses in steel and concrete respectively, associated with the ultimate thrust P_a, and are not necessarily the stresses associated with the tangent modulus load.

It is easy to see that the new interpretation is an exact equivalent of the existing one when applied to bare metal sections. That is, for $A_c = 0$,

$$\bar{N} = \frac{P_a}{P_s} = \frac{A_s\, \sigma_a}{A_s\, \sigma_y} = \frac{\sigma_a}{\sigma_y}$$

Basu and Sommerville have also adopted a similar non-dimensionalisation for the failure loads of composite columns.

Proposed Interpretation for Slenderness Factor $\bar{\lambda}$

The existing expression for slenderness factor is

$$\bar{\lambda} = \frac{\lambda}{\lambda_e} \tag{7}$$

$$\text{where } \lambda = \frac{L}{r} \qquad r = \sqrt{I/A}$$

and λ_e is as defined in Equation (2).

In the new interpretation applicable to bare metal sections as well as composite sections, the slenderness factor $\bar{\lambda}$ is defined as the ratio of the column length L to a unit critical length of the column L_c, which, in turn, is defined as the length of the column for which its Euler load equals its squash load. Thus

$$\bar{\lambda} = \frac{L}{L_c} \tag{8}$$

Also, by definition,

$$P_s = \pi^2 \frac{(\Sigma\, E_s I_s + E_c I_c)}{L_c^2} \tag{9}$$

$$\text{or } L_c = \pi \sqrt{\frac{(\Sigma\, E_s I_s + E_c I_c)}{P_s}} \tag{10}$$

For the bare metal section the proposed definition of the slenderness factor $\bar{\lambda}$ agrees exactly with the existing one, as

$$\bar{\lambda} = \frac{L}{L_c} = \frac{L}{\pi \sqrt{\dfrac{E_s I_s}{A_s \sigma_y}}} = \frac{L/\sqrt{I_s/A_s}}{\pi\sqrt{E_s/\sigma_y}} = \frac{L/r}{\lambda_e} = \frac{\lambda}{\lambda_e}$$

By adopting the proposed interpretation of slenderness factor $\bar{\lambda}$, it is no longer necessary to define an 'equivalent' radius of gyration. The column slenderness is now measured with respect to a single parameter which contains not only the geometric properties of the cross-section such as areas and moments of inertia, but also mechanical properties such as material strengths and moduli of elasticity. The merit of the new interpretation of slenderness factor thus lies in the generality of its application to bare metal sections as well as composite sections.

Formulation of Design Method for Axially Loaded Composite Columns

General

The design procedure for composite columns should now be clear in outline. Having calculated the column slenderness factor $\bar{\lambda}$ using Equations (5), (10) and (8), the designer selects the appropriate basic buckling curve applicable to the corresponding bare metal section from Table 1. A value of $\bar{N}$ is then given directly by the particular curve of Figure 3, and the ultimate column load P_a is calculated from Equations (5) and (4).

The method is applicable to composite columns of many types and all the cross-sectional shapes included in Table 1 can be adopted as the basic steel core. The particular problem of triaxial containment of concrete in concentrically loaded concrete filled circular hollow sections is discussed in a later section.

It is necessary, at this stage, to investigate which value of E_c gives the best correlation with results before recommending an appropriate expression for the initial modulus. (See also Appendix I.)

Comparison with Analytical Results

The 'exact' analytical results are now compared with design strengths obtained by the use of the corresponding European design curves. The mean value of the ratio of the analytical ultimate load to the design strengths for the three curves a, b, and c are 0.953 (standard deviation 3.88%), 0.966 (s.d. 10.0%) and 0.990 (s.d. 9.3%) respectively. The results are plotted in Figures 4-6 and a very good agreement between the theoretical values and the design curves may be observed. The correlation shown with the European curves is substantially better than that observed in the case of Basu and Sommerville's K_1 curve (cf. Fig. 2). The results shown are based on a value of E_c equal to the CEB initial modulus E_{co} as defined in Equation (21) which corresponds to the initial modulus of the stress-strain curve used in the theoretical calculations.

As an alternative to Equation (21) one may use the CP110 value given by Equation (23) to define E_{co}[1]. The CEB value of E_{co} will be larger than the CP110 value for $\sigma_u > 4500$ N/cm^2 approximately. As all the theoretical results included in this study fall within this range, the use of the CP110 value will have the effect of reducing the slenderness factor for these cases, and consequently the results will appear to be on the unsafe side. The average values for the ratio of the theoretical ultimate load to the design ultimate load for the three curves a, b, and c are 0.881 (s.d. 8.67%), 0.832 (s.d. 19.36%), and 0.845 (s.d. 14.5%).

The design values can be made to appear safer by adopting a smaller value of E_c than that previously assumed for design. Thus if E_c is taken as 0.5 of the CEB value, representing the slope of the dashed line in Figure 12, it is found that the average values of the ratio of the theoretical ultimate load to the design ultimate loads for the three curves are 1.032 (s.d. 3.85%), 1.162 (s.d. 10.25%), and 1.257 (s.d. 11.05%). Most of the results now lie above the corresponding design curves.

Key to symbols used in figures.

Fig. 1, 7, 9. Tests on Rectangular Columns.

TESTS REPORTED BY	REFERENCE	SYMBOL
STEVENS	(5)	△
JONES AND RIZK	(6)	◇
BONDALE	(7)	×
JANSS(ENCASED)	(8)	⊙
FURLONG	(20)	✸
NEOGI	(14)	⧖
KNOWLES AND PARK	(17)	✳
JANSS(SQUARE TUBES)	(21)	⊡

Fig. 10, 11. Tests on Circular Filled Tubes.

TESTS REPORTED BY	REFERENCE	SYMBOL
KLOEPPEL AND GODER	(13)	△
KNOWLES AND PARK	(17)	◇
SALANI AND SIMS	(18)	×
GARDNER AND JACOBSON	(19)	✳
FURLONG	(20)	⧖
NEOGI	(14)	✸
NEOGI,SEN	(15)	⊡
JANSS(OLD SERIES)	(21)	⊙
JANSS(NEW SERIES)	(21)	⧗

[1] For the sake of simplicity, the value of E_{co} given by Equation (21) will be referred to in this paper as the CEB value, and that given by Equation (23) as the CP110 value.

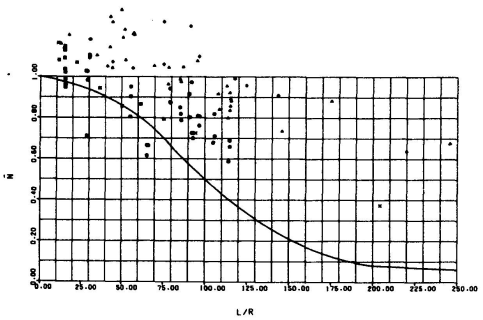


Fig. 1. Test Results for Rectangular Columns Compared with Basu and Sommerville's K_1 Curve.

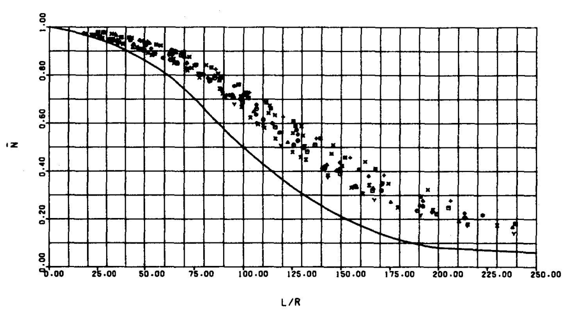


Fig. 2. Analytical Results for Rectangular Columns Compared with Basu and Sommerville's K_1 Curve.

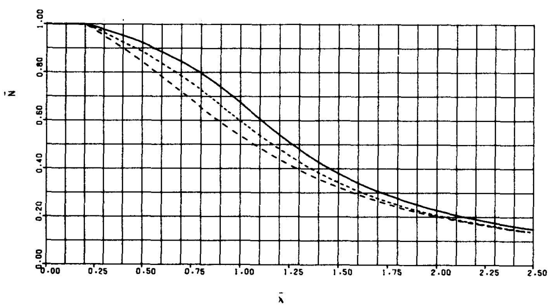


Fig. 3. European Curves a, b, and c — Top Downwards.

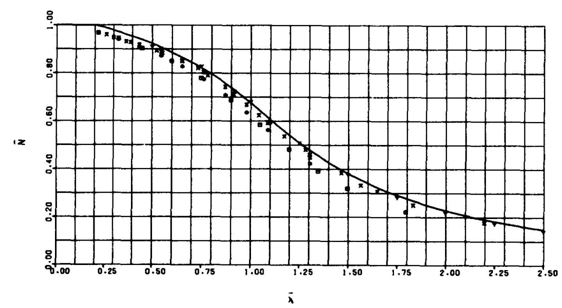


Fig. 4. Analytical Results for Rectangular Columns Compared with Curve a Using Equation (21).

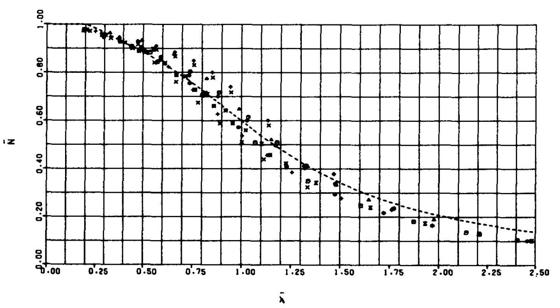


Fig. 5. Analytical Results for Rectangular Columns Compared with Curve b Using Equation (21).

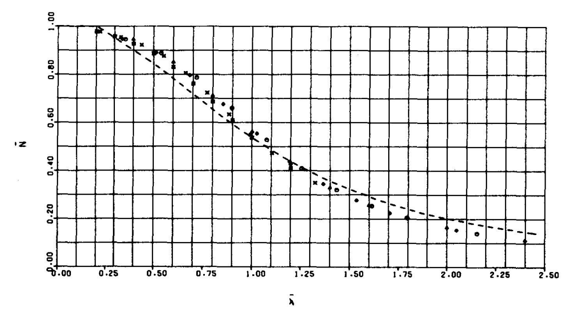


Fig. 6. Analytical Results for Rectangular Columns Compared with Curve c Using Equation (21).

It is evident that the reduction in the value of E_c makes the results appear safer. However, it may be noted that the use of E_c equal to the CEB value brings the theoretical results closest to the three design curves.

Comparison of Proposed Design Method with Test Results

The available test results are now compared with the three European design curves. The results are computed for two values of E_c, namely, the CEB value and the CP110 value. Tables 2-4, corresponding to Figures 7-9, list the comparable values for the CEB value. The average value of the ratio of test loads to the design loads relevant to the three curves a, b, and c are 1.084 (s.d. 19.4%).

1.426 (s.d. 43.6%) and 1.230 (s.d. 28.9%) respectively. The corresponding values obtained when E_c is taken as the CP110 value are respectively 1.069 (s.d. 20.7%), 1.225 (s.d. 17.2%), and 1.104 (s.d. 18.6%). It is noteworthy that in either case, the majority of test ultimate loads are safely predicted by the design curves since most of the test values appear above the design curves. In all the present comparisons with test loads, the value of γ_m is taken as 1.0. If the value of γ_m were taken in the range 1.3-1.6, as is required in real design situations, the few points lying below the design curves will shift closer to the design curves, and many more will lie above the line.

Table 2. Comparison with european curve a.

NUMBER	COLUMN	L/L_C	P_{TEST}	P_S	(4)/(5)	DESIGN $\bar{N}$	(6)/(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	RS120	.807782	47.800	65.937	.724934	.792531	.914707
2	F1	.303351	117800.000	99665.425	1.181955	.977196	1.209537
3	F2	.303351	109800.000	99665.425	1.101686	.977196	1.127395
4	F3	.305914	150000.000	137298.535	1.092510	.976581	1.118709
5	F4	.305914	152000.000	137298.535	1.107077	.976581	1.133625
6	F5	.282133	360000.000	365070.840	.986110	.982388	1.003789
7	DF3	.112362	549000.000	280003.946	1.960687	1.000000	1.960687
8	DF4	.110754	201600.000	172938.875	1.165730	1.000000	1.165730
9	KP1	.754644	80000.000	100745.378	.794081	.819078	.969482
10	KP2	.620391	86600.000	100007.293	.865937	.877048	.987332
11	KP3	.493728	95000.000	110813.849	.857294	.925195	.926608
12	KP4	.358594	104000.000	110173.293	.943967	.963552	.979675
13	KP5	.223195	113700.000	106297.506	1.069639	.995561	1.074409
14	KP6	.111598	115000.000	106297.506	1.081869	1.000000	1.081869
15	JS21	.138685	445000.000	468678.755	.949478	1.000000	.949478
16	JS22	.138841	450000.000	433802.243	1.037339	1.000000	1.037339
17	JS23	.138007	475000.000	435182.014	1.091497	1.000000	1.091497
18	JS24	.138266	450000.000	471360.489	.954683	1.000000	.954683
19	JS25	.145186	598000.000	625617.413	.955856	1.000000	.955856
20	JS26	.146039	596000.000	616392.260	.966917	1.000000	.966917
21	JS27	.146160	595000.000	581678.061	1.022903	1.000000	1.022903
22	JS28	.146493	575000.000	581678.061	.988519	1.000000	.988519
23	JS29	.150132	825000.000	724112.272	1.139326	1.000000	1.139326
24	JS30	.151217	830000.000	722871.864	1.148198	1.000000	1.148198
25	JS31	.150491	815000.000	721167.194	1.130112	1.000000	1.130112
26	JS32	.149207	830000.000	730015.357	1.136962	1.000000	1.136962
						ARITHMETIC MEAN	1.083667
						STANDARD DEVIATION	.194345

Extension of Proposed Method to Concrete Filled Circular Steel Tubes

Background to Problem

The behaviour of concrete filled circular hollow sections differs from other types of composite column in that under concentric loading such columns exhibit an enhanced strength, particularly for columns of short lengths. This is explained by the fact that the concrete core in such columns is contained triaxially thereby achieving far greater strength than the corresponding cube strength. The effects of triaxial containment tend to diminish as the column length increases, or as the end moments on the column increase. Different methods have been proposed [13, 14, 15, 16, 17] to account for the triaxial containment of concrete.

Formulation of Design Approach

Based on the results from the tests carried out at Imperial College, Sen [15] derived an expression for the ultimate load of concentrically loaded concrete filled circular hollow sections of very short length:

$$P_H = A_s \frac{\sigma_y}{\bar{\phi}} + A_c\left(\sigma_u + \frac{2t\,\delta\,\phi\,\sigma_y}{d\,\bar{\phi}}\right) \tag{11}$$

where,

σ_u = uniaxial concrete strength in member
σ_y = yield strength of steel
t = thickness of the tube
d = diameter of the tube
δ = a constant (Sen's range of values = 4 to 10)
ϕ = another constant depending upon the Poisson's ratios of steel and concrete (Sen's range of values = 0.2 to 0.5)

$$\text{and } \bar{\phi} = \sqrt{1 + \phi + \phi^2} \tag{12}$$

Since the lengths of the test columns were around 5 times their diameter, it is reasonable to assume that Equation (11) gives the squash load of such columns including triaxial effects. It follows that the augmented strength of concrete under confinement from the surrounding steel shell is given by

$$\sigma_{cL} = \sigma_u + \frac{2t}{d}\frac{\delta\phi}{\bar{\phi}}\sigma_y \tag{13}$$

and the reduced strength of steel by $\sigma_{yL} = \dfrac{\sigma_y}{\bar{\phi}}$ (14)

Table 3. Comparison with european curve b.

NUMBER	COLUMN	L/L_C	P_{TEST}	P_S	(4)/(5)	DESIGN $\bar{N}$	(6)/(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	B1	.689769	37.000	34.366	1.076654	.790627	1.361772
2	B2	1.076810	27.300	29.994	.910172	.551382	1.650710
3	B3	1.285139	28.600	32.423	.882093	.436130	2.022544
4	B4	1.631224	19.800	30.966	.639416	.297808	2.147074
5	B5	1.778363	23.080	34.123	.676379	.255509	2.647181
6	B6	2.144718	16.400	32.180	.509633	.182592	2.791096
7	B7	2.255538	15.400	35.580	.432827	.166780	2.595197
8	FA1	.148610	478.000	397.098	1.203732	1.000000	1.203732
9	FA2	.292894	450.000	407.389	1.104595	.967629	1.141548
10	FA3	.451128	421.000	389.381	1.081204	.904949	1.194769
11	FA4	.587875	426.000	404.816	1.052329	.844062	1.246743
12	FA5	.734844	424.000	404.816	1.047388	.765339	1.368529
13	S1G	.534047	240.000	199.200	1.204819	.869838	1.385107
14	S1E	.489563	281.000	232.391	1.209172	.889379	1.359568
15	S1S	.512859	258.000	212.684	1.213069	.879413	1.379407
16	S2G	.418803	290.000	235.689	1.230436	.917579	1.340959
17	S2E	.380600	380.000	286.294	1.327305	.932860	1.422834
18	S2S	.397107	310.000	260.175	1.191504	.926186	1.286462
19	S3G	.327777	364.000	287.190	1.267454	.954189	1.328305
20	S3E	.309542	380.000	331.778	1.145345	.961374	1.191363
21	S3S	.311101	423.000	327.084	1.293245	.960771	1.346050
22	FE1	.819876	440.000	451.123	.975343	.714877	1.364350
23	FE2	.785619	471.000	476.850	.987732	.735628	1.342706
24	RA1	.656405	68.000	54.523	1.247177	.808741	1.542122
25	RA2	.663310	58.000	52.773	1.099053	.805046	1.365206
26	RA3	.386355	100.000	95.993	1.041742	.930558	1.119481
27	RA*1	.847360	54.000	54.523	.990405	.697763	1.419400
28	RA**1	.518588	67.000	52.773	1.269596	.876835	1.447930
29	RA*2	.856273	50.500	52.773	.956934	.692085	1.382683
30	RA**2	.518588	56.000	52.773	1.061155	.876835	1.210210
31	RA**3	.302059	103.000	95.993	1.072995	.964217	1.112814
32	RW120	1.505184	23.600	63.151	.373705	.340682	1.096931
33	J9.1	.692323	233000.000	239511.563	.972813	.789222	1.232622
34	J9.2	.666517	258000.000	281913.760	.915173	.803346	1.139202
35	J9.3	.675550	210000.000	261666.793	.802547	.798447	1.005135
36	J10.1	.778122	235000.000	345770.894	.679641	.740108	.918300
37	J10.2	.787282	276000.000	336893.824	.819249	.734631	1.115185
38	J10.3	.788142	241000.000	338833.498	.711264	.734115	.968873
						ARITHMETIC MEAN	1.426161
						STANDARD DEVIATION	.435730

If the modified strengths of concrete and steel as defined by Equations (13) and (14) are varied to take into account the fact that the effects of triaxial containment reduce with increasing column length, it is then possible to use these values to determine the column slenderness factor using Equations (10), (8) and (5). The design ultimate loads can then be obtained from the applicable curve a and Equations (5) and (4).

For ideally straight columns with elastic plastic behaviour, failure is governed by Euler buckling for $\bar{\lambda} > 1$ and by material yield for $\bar{\lambda} < 1$. It therefore appears reasonable to postulate that for columns having $\bar{\lambda} > 1$ the triaxial effects will be negligible. For columns in the range $0 < \bar{\lambda} \leq 1$, the triaxial effects will be maximum at $\bar{\lambda} = 0$ and zero at $\bar{\lambda} = 1$.

Table 4. Comparison with european curve c

NUMBER	COLUMN	L/L_C	P_{TEST}	P_S	(4)/(5)	DESIGN $\bar{N}$	(6)/(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	A1	.110357	160.000	137.726	1.161727	1.000000	1.161727
2	A2	.399866	140.000	134.188	1.043309	.900071	1.159141
3	A3	.696097	144.000	137.726	1.045555	.721159	1.449825
4	A4	.686022	135.000	140.084	.963705	.727546	1.324598
5	A5	1.005506	131.000	137.136	.955254	.533882	1.789261
6	A6	1.268328	105.000	142.796	.735313	.408319	1.800830
7	RE1A	1.012875	134.000	145.375	.921756	.530005	1.739146
8	RE1B	1.036667	125.000	141.837	.881292	.517633	1.702542
9	RE2A	1.024417	123.000	143.606	.856510	.524003	1.634553
10	RE2B	.985105	120.000	150.091	.799513	.544694	1.467820
11	RE3A	.963089	140.000	152.639	.917197	.556701	1.647558
12	RE3B	.990755	124.000	147.359	.841483	.541700	1.553411
13	RE4A	1.016648	121.000	144.785	.835721	.528043	1.582676
14	RE4B	1.032501	127.000	142.427	.891686	.519799	1.715443
15	AE6	.364375	130.000	123.613	1.051621	.918725	1.144653
16	J1.1	.747423	219000.000	287745.243	.761090	.688550	1.105352
17	J1.2	.756456	222000.000	275414.128	.806059	.682704	1.180686
18	J1.3	.752472	213000.000	263369.544	.808750	.685293	1.180151
19	J2.1	.624064	239000.000	272874.324	.875861	.767458	1.141249
20	J2.2	.636186	222000.000	253849.175	.874535	.759579	1.151342
21	J2.3	.618793	263000.000	279216.040	.941923	.770885	1.221873
22	J3.1	.431944	268000.000	281821.203	.950958	.882731	1.077290
23	J3.2	.428982	228000.000	283230.473	.804998	.884360	.910261
24	J3.3	.438814	239000.000	264557.641	.903395	.878952	1.027809
25	J4.1	.227167	260000.000	264557.641	.982773	.986316	.996407
26	J4.2	.231092	252000.000	245532.492	1.026341	.984354	1.042654
27	J4.3	.219283	280000.000	272308.628	1.028245	.990259	1.038360
28	J5.1	.680896	240000.000	304830.464	.787323	.730826	1.077305
29	J5.2	.669674	268000.000	315047.673	.850665	.737909	1.152805
30	J5.3	.676953	252000.000	307649.004	.819115	.733320	1.116996
31	J6.1	.911229	240000.000	364235.339	.658915	.586375	1.123709
32	J6.2	.896100	220000.000	374452.548	.587524	.595440	.986706
33	J6.3	.906293	253000.000	367406.197	.688611	.589324	1.168475
34	J7.1	.733968	252000.000	360048.216	.699906	.697160	1.003939
35	J7.2	.724210	267000.000	368151.520	.725245	.703348	1.031133
36	J7.3	.732685	262000.000	360752.851	.726259	.697981	1.040513
37	J8.1	.520520	248000.000	373149.753	.664613	.831288	.799498
38	J8.2	.506191	241000.000	391822.584	.615074	.839885	.732331
39	J8.3	.507988	260000.000	390413.314	.665961	.838807	.793938
						ARITHMETIC MEAN	1.230102
						STANDARD DEVIATION	.288588

The above criterion would require the determination of $\bar{\lambda}$ twice during the design process; firstly, to determine whether the triaxial effects are to be considered at all, and secondly, having found the new concrete and steel strengths, to obtain the value of $\bar{N}$ from curve a. As the point where the triaxial effects cease to be worth considering can only be approximately defined, it is suggested that the criterion postulated above be replaced by an equivalent but simpler criterion. For most practical columns, the value of $\bar{\lambda} = 1$ corresponds to a length to diameter ratio (L/d) varying between 24 and 29. It is therefore proposed that the effects of triaxial containment be ignored for columns with L/d > 25. For columns in the rage 0 < L/d < 25, the effects of triaxial containment may be considered by making δ and ϕ linear functions of L/d. Thus

$$\delta = 0.25\,(25 - L/d) \quad 0 \le \delta \le 6.25 \tag{15}$$

$$\text{and } \phi = 0.02\,(25 - L/d) \quad 0 \le \phi \le 0.5 \tag{16}$$

These expressions correspond to $\delta = 5.0$ and $\phi = 0.4$ for $L/d = 5$, the average values for Sen's tests.

Table 5

L/D	δ	φ	$\bar{\Phi}$	$2\phi\delta/\bar{\Phi}$	$1/\bar{\Phi}$
1	6.000	0.480	1.3078	4.4043	0.7646
2	5.750	0.460	1.2929	4.0916	0.7735
3	5.500	0.440	1.2781	3.7868	0.7824
4	5.250	0.420	1.2635	3.4903	0.7915
5	5.000	0.400	1.2490	3.2026	0.8006
6	4.750	0.380	1.2347	2.9239	0.8099
7	4.500	0.360	1.2205	2.6547	0.8193
8	4.250	0.340	1.2065	2.3954	0.8289
9	4.000	0.320	1.1926	2.1465	0.8385
10	3.750	0.300	1.1790	1.9084	0.8482
11	3.500	0.280	1.1655	1.6817	0.8580
12	3.250	0.260	1.1522	1.4667	0.8679
13	3.000	0.240	1.1391	1.2641	0.8779
14	2.750	0.220	1.1262	1.0744	0.8879
15	2.500	0.200	1.1136	0.8980	0.8980
16	2.250	0.180	1.1011	0.7356	0.9082
17	2.000	0.160	1.0889	0.5878	0.9184
18	1.750	0.140	1.0768	0.4550	0.9286
19	1.500	0.120	1.0651	0.3380	0.9389
20	1.250	0.100	1.0536	0.2373	0.9492
21	1.000	0.080	1.0423	0.1535	0.9594
22	0.750	0.060	1.0313	0.0873	0.9696
23	0.500	0.040	1.0206	0.0392	0.9798
24	0.250	0.020	1.0101	0.0099	0.9900
25	0.000	0.000	1.0000	0.0000	1.0000

Table 6. Comparison with european curve a

NUMBER	COLUMN	L/L_C	P_{TEST}	P_S	(4)/(5)	DESIGN $\bar{N}$	(6)/(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	KG7	.580450	212900.000	222713.070	.955939	.892129	1.071525
2	KG8	.579317	210800.000	224202.624	.940221	.892560	1.053398
3	KG9	.578139	203900.000	219989.411	.926863	.893007	1.037912
4	KG10	.372079	228800.000	250590.256	.913081	.960180	.950947
5	KG11	.370916	226600.000	250067.658	.906155	.960471	.943448
6	KG12	.370916	232400.000	250067.658	.929348	.960471	.967597
7	KG13	.795878	199300.000	218146.834	.913605	.798561	1.144064
8	KG14	.790111	203900.000	212294.593	.960458	.801444	1.198408
9	KG15	.795972	206100.000	216377.895	.952500	.798514	1.192841
10	KG41	.388148	147500.000	129847.864	1.135945	.955081	1.188126
11	KG42	.415024	154300.000	149093.340	1.034922	.949043	1.090430
12	KG43	.391412	147500.000	126282.757	1.1680[illegible]	.955233	1.222753
13	KG44	.599577	127400.000	117540.905	1.083[illegible]8	.884957	1.224781
14	KG45	.640127	136200.000	135079.923	1.008292	.869248	1.159959
15	KG46	.604761	129400.000	114108.289	1.134011	.882991	1.284283
16	KG47	.805429	120600.000	107313.877	1.123805	.793731	1.415853
17	KG48	.858005	127200.000	122294.915	1.040109	.764557	1.360407
18	KG49	.812471	109600.000	104549.868	1.048304	.790065	1.326857
19	KG53	.409506	230000.000	370960.800	.620012	.950528	.652281
20	KG54	.415834	412400.000	385114.498	1.070850	.948825	1.128607
21	KG55	.413081	514600.000	417488.512	1.232609	.949568	1.298073
22	KG56	.412329	503300.000	419585.341	1.199232	.949771	1.262653
23	KG69	.459462	553400.000	587361.361	.942180	.936351	1.005214
24	KG70	.460985	544300.000	586729.676	.927684	.935885	.991238
25	KG71	.417362	630300.000	545077.976	1.156348	.948412	1.219245
26	KG72	.465257	659200.000	655573.288	.990424	.934518	1.059824
27	KG73	.810274	112000.000	109629.413	1.021624	.791252	1.291148
28	KG74	.813087	103300.000	101976.832	1.012975	.789733	1.282680
29	KG75	.829224	106300.000	109501.212	.970766	.781019	1.242948
30	KG76	.804498	92810.000	105809.466	.877143	.794206	1.104427
31	KG83	.357613	156300.000	160986.396	.970889	.953797	1.007359
32	KG84	.372151	167800.000	176641.064	.949949	.960162	.989363
33	KG85	.353810	188100.000	175261.875	1.073251	.962248	1.115359
34	KG86	.372343	194900.000	190325.454	1.024035	.960114	1.066577
35	KG89	.383917	224400.000	245305.204	.914779	.957182	.955700
36	KG90	.383039	228900.000	238291.520	.960168	.957410	1.002881
37	KG91	.377278	247100.000	238057.954	1.037983	.958891	1.082494
38	KG92	.373329	242500.000	235860.322	1.028151	.959868	1.071138
39	KG95	.711555	144000.000	132837.905	1.084028	.839700	1.290971
40	KG96	.738176	141500.000	144206.807	.981230	.827275	1.186098
41	KG97	.725524	156300.000	142820.850	1.094378	.833304	1.313301
42	KG98	.740292	169800.000	152383.728	1.114292	.826254	1.348607
43	KG101	.757764	176800.000	199213.530	.887490	.817519	1.085591
44	KG102	.756146	183400.000	193639.106	.947123	.818327	1.157399
45	KG103	.747697	196400.000	197167.253	.996109	.822552	1.210998
46	KG104	.739349	194500.000	188323.165	1.032799	.826712	1.249285
47	KP1	.825483	138200.000	173185.560	.797988	.783039	1.019091
48	KP2	.694170	160000.000	180255.520	.887629	.847148	1.047784
49	KP3	.563574	160800.000	191907.044	.837906	.898606	.932451
50	KP4	.427058	206500.000	210905.416	.979112	.945724	1.035304
51	KP5	.279443	223000.000	230248.698	.968518	.993039	.985228
52	KP6	.899572	50500.000	77966.666	.647713	.745757	.868531
53	KP7	.744149	66200.000	75993.333	.871129	.824326	1.056773
54	KP8	.598509	80000.000	83776.351	.954923	.885352	1.078581
55	KP9	.450040	90000.000	89554.560	1.004974	.939188	1.070046
56	KP10	.292417	110000.000	96789.916	1.136482	.979896	1.159799
57	KP11	.151401	119200.000	103786.548	1.148511	1.000000	1.148511
58	SS30F	.966078	27100.000	39283.990	.689848	.697288	.989330
59	SS42F	.[illegible]59712	72000.000	73609.768	.978131	.861218	1.135753
60	SS49F	1.945567	3540.000	10393.952	.340583	.239720	1.420754
61	SS50F	1.945567	3490.000	10393.952	.335772	.239720	1.400687
62	SS51F	1.369063	25400.000	40627.903	.625186	.442787	1.411933
63	SS52F	1.369063	24000.000	40627.903	.590727	.442787	1.334110
64	SS71F	.701792	51900.000	53809.175	.964520	.843929	1.142891
65	GJ1	.744067	184000.000	189549.623	.970722	.824367	1.177537
66	GJ2	.747001	180000.000	186713.954	.964041	.822900	1.171518
67	GJ5	.431038	260000.000	268229.036	.969321	.944609	1.026160
68	GJ6	.432851	245500.000	259655.369	.945869	.944102	1.001872
69	GJ7	.434832	213500.000	252695.967	.844889	.943547	.895439
70	GJ11	.495284	211000.000	240064.187	.878932	.924651	.950555
71	GJ12	.494984	198000.000	240319.799	.823902	.924756	.890940
72	GJ18	.806685	55000.000	52829.518	1.041085	.793091	1.312693
73	GJ20	.355343	92500.000	78645.052	1.176171	.964364	1.219633
74	GJ21	.358741	74250.000	66762.779	1.112147	.963515	1.154260
75	F1	.378681	160000.000	192722.387	.830210	.958530	.866128
76	F2	.378681	170000.000	192722.387	.882098	.958530	.920261
77	F3	.301254	141000.000	166994.717	.844338	.977699	.863597
78	F4	.301254	140000.000	166994.717	.838350	.[illegible]7699	.857472
79	F5	.301254	148000.000	166994.717	.886256	.977699	.906471
80	F6	.260158	153400.000	153316.919	1.000542	.987664	1.013039
81	F7	.258769	162200.000	169462.959	.957141	.987983	.968783
82	F8	.258769	164800.000	169452.959	.972484	.987983	.984312
83	A1	.245091	2576000.000	2092912.088	1.230821	.991080	1.241899
84	A4	.245975	2408000.000	2005576.737	1.200652	.990885	1.211696
85	A5	.221352	790700.000	879545.086	.898987	.995930	.902662
86	A6	.268337	1671000.000	1448733.953	1.153421	.985782	1.170056
87	B1	.243038	289000.000	234957.479	1.230010	.991532	1.240515
88	B1X	.238208	289000.000	225711.179	1.280397	.992558	1.289997
89	B2	.252674	293500.000	290578.187	1.010055	.989385	1.020892
90	B2X	.245438	293500.000	274284.463	1.070057	.990993	1.079783
91	DF1	.127976	663000.000	435169.233	1.523545	1.000000	1.523545
92	DF1X	.128671	663000.000	439908.447	1.507132	1.000000	1.507132
93	DF2	.129574	410000.000	306475.277	1.337791	1.000000	1.337791
94	DF2X	.131027	410000.000	313391.228	1.308269	1.000000	1.308269
95	SC1	.211474	451000.000	360220.322	1.252012	.997820	1.254747
96	SC2	.210590	502000.000	405449.081	1.238133	.997988	1.240630
97	SC3	.210809	475000.000	404765.416	1.173519	.997946	1.175934
98	SC4	.211874	392000.000	326135.663	1.201954	.997744	1.204672
99	M11	.202765	201.000	158.813	1.265643	.999475	1.266308
100	M12	.202034	224.000	181.430	1.234638	.999614	1.235115
101	M13	.203067	175.000	146.347	1.195789	.999417	1.195486
102	M14	.202013	212.000	179.901	1.178426	.999617	1.178877
103	M15	.204011	261.000	208.442	1.252146	.999238	1.253101
104	M16	.202197	253.000	212.685	1.189552	.999583	1.190048
105	M17	.204301	280.000	232.111	1.206317	.999183	1.207304
106	M18	.204918	241.000	203.305	1.185411	.999066	1.186520
107	M19	.204783	268.000	232.206	1.154149	.993091	1.155198
108	M20	.207858	294.000	250.769	1.172396	.998507	1.174149
109	M21	.212798	340.000	288.151	1.179938	.997568	1.182814
110	M22	.208399	349.000	290.886	1.199784	.998404	1.201702
111	M23	.208121	340.000	271.400	1.252762	.998457	1.254698
112	M24	.208177	361.000	299.270	1.206270	.998446	1.208147
113	1J	.760419	175000.000	213899.049	.818143	.816194	1.002387
114	2J	.589775	210500.000	240526.967	.875162	.888585	.984893
115	3J	.413698	246000.000	249101.508	.987549	.949402	1.040181
116	4J	.190069	281000.000	292278.789	.961411	1.000000	.961411
117	5J	.189455	280000.000	289316.941	.957797	1.000000	.957797
118	6J	.188506	286000.000	281570.516	1.015731	1.000000	1.015731
119	8.1J	1.684819	20600.000	47295.721	.435649	.311058	1.400541
120	8.2J	1.681357	15800.000	46745.772	.337998	.312166	1.082753
121	8.3J	1.677149	14300.000	46380.808	.308317	.313541	.983340
122	9.1J	1.110580	28500.000	48451.917	.588212	.598906	.982144
123	9.2J	1.110687	28700.000	47917.645	.598944	.598833	1.000186
124	9.3J	1.107625	29700.000	48992.565	.606214	.600915	1.008819
125	10.1J	.751509	37000.000	46118.339	.802284	.820645	.977626
126	10.2J	.753119	41500.000	46291.656	.895490	.819841	1.093493
127	10.3J	.753575	41500.000	46288.126	.896558	.819613	1.093681
128	11.1J	.587748	45300.000	51132.737	.885930	.889356	.996148
129	11.2J	.588314	45000.000	50736.387	.886937	.889141	.997522
130	11.3J	.590120	50500.000	50312.324	1.003730	.888456	1.129747
131	12.1J	.418591	53500.000	55826.457	.958327	.948080	1.010808
132	12.2J	.417377	51700.000	54344.755	.951334	.948408	1.003085
133	12.3J	.418617	54400.000	55399.701	.981955	.948073	1.035737
134	13.1J	.225721	65000.000	63255.301	1.027582	.995056	1.032689
135	13.2J	.225087	64000.000	63771.804	1.003578	.995183	1.008436
136	13.3J	.225574	68000.000	63492.336	1.070995	.995085	1.076285
137	1 JN	.174330	770000.000	738471.368	1.042694	1.000000	1.042694
138	2 JN	.176815	770000.000	774382.797	.994340	..000000	.994340
139	3 JN	.132142	785000.000	752420.777	1.043299	1.[illegible]0000	1.043299
140	4 JN	.133639	760000.000	765806.598	.992418	1.000000	.992418
141	5 JN	.137822	1000000.000	1172815.144	.852649	1.000000	.852649
142	6 JN	.138551	1000000.000	1125043.015	.888955	1.000000	.888955
143	7 JN	.188458	720000.000	675568.258	1.065612	1.000000	1.065612
144	8 JN	.187955	695000.000	681021.995	1.020525	1.000000	1.020525
145	9 JN	.187179	733000.000	658078.181	1.113849	1.000000	1.113949
146	10 JN	.304289	835000.000	850185.171	.982138	.976971	1.005289
147	11 JN	.303506	830000.000	850566.300	.975820	.977159	.998631
148	12 JN	.306246	855000.000	849236.824	1.006786	.976501	1.031014
149	13 JN	.306246	820000.000	849236.824	.965573	.976501	.988809
150	14 JN	.315802	850000.000	844605.163	1.005387	.974149	1.033093
151	15 JN	.316191	840000.000	844416.828	.994769	.974052	1.021269
						ARITHMETIC MEAN	1.109291
						STANDARD DEVIATION	.145673

Thus for columns with L/d < 25 the designer may use Equations (12)-(16) to calculate the modified values of concrete and steel strength to be used in determining the slenderness factor $\bar{\lambda}$. Once the slenderness factor is known the strength of the column can be taken from column curve a, and Equations (5) and (4). For convenience, the values of $1/\bar{\phi}$ and $2\delta\phi/\bar{\phi}$ have been listed in Table 5 for different values of L/d. It may be added that for L/d values ranging between 20 and 25, the collapse loads calculated with or without triaxial effects would not be much different. Hence, to minimise effort, the upper limit of L/d for which triaxial effects are calculated may be restricted to 20.

Comparison of Proposed Method with Test Loads

The validity of the proposed design methods for concrete filled circular tubular columns under concentric loading is now verified against all known test results [13-21]. It must be noted that the material strengths used in the following comparison are not corrected for the recommended standard deviation errors as this correction is of little significance in terms of the enhanced concrete strength. The factors $k_1 = 0.67$ for cubes and $k_2 = 0.85$ for cylinders are applied as appropriate and the value of γ_m is again assumed to be unity.

Table 6 lists the comparative test ultimate loads against the design loads obtained from European curve a and the results are also shown in Figure 10. The value of E_c used is obtained from Equation (21) (CEB value) using the uniaxial strength of concrete. The average value of the ratio of the test ultimate load to the corresponding design strength for 151 columns is 1.109 with a standard deviation of 14.7 per cent. Similar comparison based on E_c as given by Equation (23) (CP110 value) gives the average value of the ratio of test ultimate load to the relevant factored design load as 1.097 (s.d. 14.1%). It is clear that both values of E_c yield good correlation with tests.

To illustrate the effect of ignoring the triaxial effects, Figure 11 shows the comparative values with E_c equal to the CEB value. The average value of the ratio of test ultimate load to the corresponding design strength is 1.297 (s.d. 21.9%). With the CP110 value this ratio becomes 1.285 (s.d. 22.3%).

The proposed design method has thus been shown to give good correlation with a very large number of available test results on concrete filled circular tubes loaded concentrically.

Recommended Design Procedure for Concentrically Loaded Composite Columns

Summary of Design Procedure

The following design procedure covers axially loaded concrete encased steel sections and concrete filled rectangular and circular steel tubes.

1. For sections other than concrete filled circular steel tubes proceed to step 6.
2. Calculate column L/d ratio.
3. If L/d > 20 proceed to step 6.
4. Obtain values of $1/\bar{\phi}$ and $2\delta\Phi/\bar{\phi}$, from Table 5.
5. Calculate modified strength of concrete and steel using Equations (13) and (14).

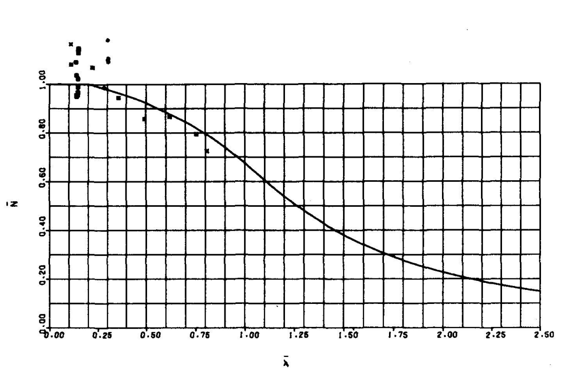


Fig. 7. Test Results for Rectangular Columns Compared with Curve a Using Equation (21).

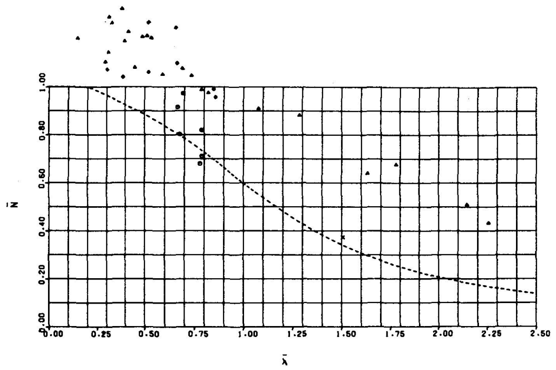


Fig. 8. Test Results for Rectangular Columns Compared with Curve b Using Equation (21).

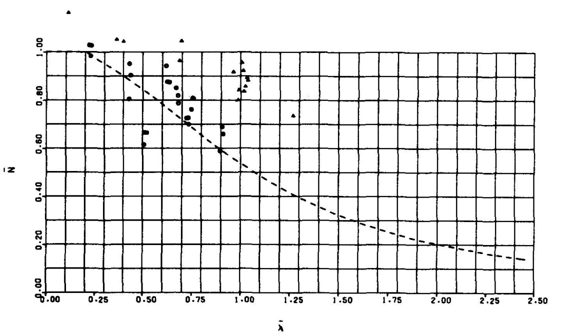


Fig. 9. Test Results for Rectangular Columns Compared with Curve c Using Equation (21).

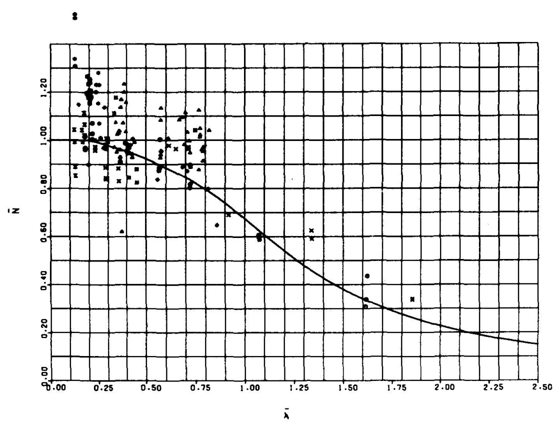


Fig. 10. Test Results for Circular Filled Tubes Compared with Curve a Using Equation (21) and Including Triaxial Effects.

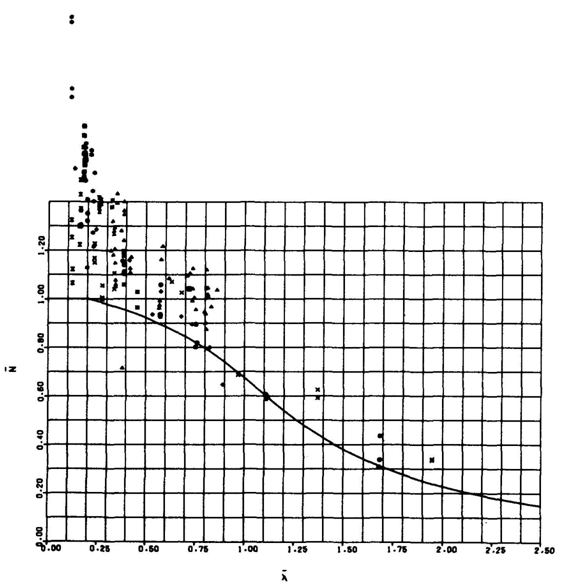


Fig. 11. Test Results for Circular Filled Tubes Compared with Curve a Using Equation (21) and Ignoring Triaxial Effects.

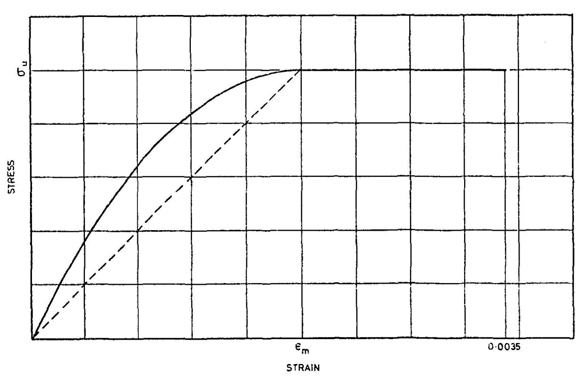


Fig. 12. Stress-strain Relationship for Concrete.

6. Calculate the column slenderness factor $\bar{\lambda}$ using Equations (10), (5) and (8) (using the modified material strengths where applicable).
7. Select appropriate European Curve from Table 1 for the bare steel section, and obtain $\bar{N}$ from Figure 3. As an alternative to Figure 3, the values of $\bar{N}$ may be calculated from Tables given in Ref. 27.
8. Calculate P_a, the ultimate load from Equations (5) and (4) (using modified material strengths where applicable).

To calculate $\bar{\lambda}$, the value of E_c should be that derived from Equation (21), i.e. the CEB value, and should be based on the uniaxial strength of concrete.

Table 7

	Length in	Theoretical* tonf	Present Design Method tonf	Basu and Sommerville tonf	Present Design Steel Core Only tonf
UCE	100	586.508	594.707	561.214	154.118
Minor Axis	300	498.225	465.499	373.619	53.879
Curve 'c'	500	321.574	319.083	153.551	22.296
RHA	100	232.281	238.802	219.644	154.191
Major Axis	300	191.266	194.853	165.078	129.256
Curve 'a'	500	110.895	115.719	75.136	82.35
IBA	100	165.141	158.910	137.994	36.391
Minor Axis	300	46.359	55.968	14.250	-
Curve 'b'	500	-	(Outside Slenderness Range)	-	-

*Excluding root areas.

Table 8

Length	L/d	Analytical Ultimate Load	Basu and Sommerville's Ultimate Load	Present Design without containment	Present Design with containment
Diameter 6.625 in		Thickness 0.176 in	σ_u = 2400 lbf/in²	σ_y = 16 tonf/in²	
in		tonf	tonf	tonf	tonf
72	10.8	83.9	82.5	86.4	99.4
144	21.6	72.2	63.6	73.6	73.6
216	32.4	50.0	37.0	52.3	52.3
Diameter 12.75 in		Thickness 0.250 in	σ_u = 7200 lbf/in²	σ_y = 23 tonf/in²	
72		685.4	667.5	695.0	855.5
144		657.1	604.6	654.7	720.8
216		592.0	499.8	594.0	606.8
288		479.4	353.0	509.7	509.7

Examples of Application of the Design Method

The design method has been used to calculate the ultimate load capacities of a concrete encased joist section, a concrete encased H universal column, and a concrete filled rectangular hollow section over a range of column lengths. The results are presented in Table 7 and are compared with the theoretically exact ultimate loads, those predicted using the method of Basu and Sommerville, and the ultimate design loads for bare steel columns alone. It will be noted that the design loads given by the new method correspond more closely with the 'exact' ultimate loads than do the values given by Basu and Sommerville's design method. It will also be seen that the latter method predicts a lower carrying capacity for the concrete filled rectangular section than is predicted for the bare steel tube. No such anomaly can arise with the new approach.

In Table 8 the design load capacities of a range of practical concrete filled tubes of varying lengths are presented. These are compared with the exactly calculated capacities ignoring triaxial containment and those obtained using the design method of Basu and Sommerville. The enhanced load-carrying capacities of short concentrically loaded columns due to triaxial effects as predicted by the new method can be seen by comparing the tabulated values.

Practical and Economic Consequences

The design of composite columns under axial loading has been made just as simple as the design of bare steel axially loaded columns. By suitably redefining the column slenderness factor, the newly developed European curves for the design of bare steel columns can be used as the basic design curves for composite columns. Thus full advantage can be taken of the contribution of concrete towards the strength of composite columns. In the case of axially loaded circular tubes filled with concrete, further economies can be made by allowing for the enhanced strength of concrete due to triaxial containment in the design method.

Conclusions

A new method of design for composite columns under concentric loading has been presented. The method unifies the design of concrete filled circular tubular sections under concentric loading with that of other types of concentrically loaded composite columns, such as encased sections and rectangular filled tubes, by calculating an augmented strength of concrete and a corresponding reduced strength of steel. The effects of triaxial containment are made to vary with the column length to diameter ratio up to a value of 25, beyond which no such effects are considered.

The design method introduces a new concept of column slenderness. The column slenderness factor is defined as the ratio of column length to a unit critical length. This unit critical length is the length for which the column Euler load equals its squash load. This definition leads to the same expression as that used in the currently proposed European design curves for bare steel columns and enables these curves to be used as the basic design curves for composite columns. It is not therefore necessary to define a fictitious radius of gyration for such sections.

The method has been compared with a large number of known experimental results on encased sections as well as rectangular and circular filled tubes. The agreement is shown to be excellent. Good correlation has also been obtained with a large number of theoretically exact results for several encased sections and filled rectangular tubes for the range of practical slenderness factors. The proposed method overcomes many of the disadvantages of a method proposed earlier by Basu and Sommerville and can be confidently recommended for use in design specifications.

Appendix

Factors Affecting the Value of Slenderness Factor $\bar{\lambda}$

The mechanical properties of steel are, in general, well defined both with respect to σ_y and E_s. Problems arise, however, in the determination of the appropriate values of σ_u and E_c for use in the expression for slenderness factor.

Design Strength of Concrete σ_u

In both the CEB recommendations [9] and the British code of practice [22] the design strength of concrete, i.e. the maximum design stress attainable by concrete in a reinforced concrete column is specified as

$$\sigma_u = \frac{k_1 \, \sigma_{cu}}{\gamma_m} = \frac{k_2 \, \sigma_{cyl}}{\gamma_m} \tag{17}$$

The recommended value of k_1 is 0.67 [22]. The factor k_2 which is used when the concrete strength is obtained from cylinder tests rather than cube tests has a recommended value of 0.85 [9]. This corresponds to the observation that the ratio of concrete cylinder strength to concrete cube strength is approximately 0.80.

The value of the characteristic concrete strength as obtained from tests is frequently taken as the mean value of the strengths of the specimen .tested. However, both the CEB recommendations [9] and the Handbook on CP110 [23] stipulate that the characteristic strength of concrete should be taken in accordance with the formula

$$\sigma_k = \sigma_m - 1.64 \text{ S} \tag{18}$$

where S is the standard deviation of test results. The CEB recommendations further stipulate that when the probabilistic distribution of test data is not known a minimum value of 300 N/cm^2 should be taken for *in situ* concrete and 200 N/cm^2 for factory cast concrete. In most practical cases, as also in laboratory tests, only a few cube or cylinder tests are carried out. Thus when comparing ultimate load calculations with test results, in the absence of sufficient experimental data, a minimum value of 200 N/cm^2 for cylinder tests, or 250 N/cm^2 for cube tests, or their equivalent related to factory cast concrete, should be used as the value of S in Equation (18) to obtain the characteristic strength of concrete.

The coefficient γ_m is the material safety factor. The design value of γ_m associated with the ultimate limit state of design recommended by CP110 is 1.5 while the CEB recommendations specify values in the range 1.3-1.6 depending upon the care and control exercised in the production of concrete. When correlating test results with ultimate load calculations, it is customary to take $\gamma_m = 1.0$, assuming that the laboratory conditions permit the production of concrete of a uniform quality.

Modulus of Elasticity of Concrete, E_c

A number of equations have been proposed to represent the concrete stress-strain relationship [24, 25, 26]. The curves recommended by CEB and CP110 have similar shapes (Fig. 12) characterised by a parabolic section up to the peak concrete stress, followed by a horizontal plateau, even though the observed stress-strain relationships do not exhibit any discernible flat plateau. Both curves have the same limiting value of strain corresponding to the crushing of concrete, namely 0.0035, but the exact shapes of the parabolas in the two curves are defined by slightly different criteria. The general equation of the parabola may be written as

$$\frac{\sigma}{\sigma_u} = \frac{\varepsilon}{\varepsilon_m}\left(2 - \frac{\varepsilon}{\varepsilon_m}\right) \tag{19}$$

The value of the initial modulus is thus given by $E_{co} = 2\,\dfrac{\sigma_u}{\varepsilon_m}$ (20)

In the CEB recommendations, the value of ε_m is fixed at 0.0020. This results in the following value, independent of units, for the initial modulus of concrete:

$$E_{co} = 1000\,\sigma_u \tag{21}$$

In CP110, on the otherhand, the value of the initial modulus of concrete is specified as

$$E_{co} = 55000\sqrt{\frac{\sigma_{cu}}{\gamma_m}} \tag{22}$$

where both E_{co} and σ_{cu} are expressed in N/cm^2. By substituting $k_1 = 0.67$ in Equation (17), Equation (22) may be rewritten as follows

$$E_{co} = 67193\sqrt{\sigma_u} \tag{23}$$

This value of E_{co} is close to the value of

$$E_{co} = 66000\sqrt{\sigma_u} \tag{24}$$

specified in the CEB recommendations for the initial modulus of concrete for cases where the stresses under working conditions do not exceed 40 per cent of the compressive strength. Thus it appears that while the CEB recommendations differentiate between the elastic moduli of concrete at origin relating to the ultimate limit state calculations and to the instantaneous loading calculations, CP110 recommends the use of a single initial modulus of elasticity.

The value of ε_m as deduced from CP110 may be expressed as follows

$$\varepsilon_m = \frac{2}{67193}\sqrt{\sigma_u} \tag{25}$$

which implies that ε_m depends upon σ_u, unlike the CEB recommendations in which ε_m is fixed at 0.0020.

Notation

- A area of cross-section
- A_c area of concrete
- A_s area of steel
- d nominal diameter of tube
- E_c modulus of elasticity of concrete
- E_{co} modulus of elasticity of concrete at origin
- E_s modulus of elasticity of steel
- I second moment of area
- I_c second moment of area for the concrete section
- I_s second moment of area for the steel section
- K_1 Basu and Sommerville's coefficient for basic column strength under axial load ($=P_a/P_s$)
- k_1 coefficient relating the bending strength of concrete in a member to its characteristic cube strength
- δ coefficient used in estimating triaxial concrete strength
- ε strain
- ε_m strain in concrete corresponding to peak stress
- λ slenderness ratio
- $\bar{\lambda}$ slenderness factor
- λ_e unit critical slenderness ratio ($= Lc/r$)
- σ stress
- σ_a average stress in steel
- σ_b average stress in concrete
- σ_{cL} augmented concrete strength under triaxial containment
- L column length
- L_c unit critical length, for which the Euler load equals the squash load for a given column cross-section
- $\bar{N}$ column strength non-dimensionalised with respect to its squash load ($= P_a/P_s$)
- P_a column strength for axial load
- P_s column squash load
- r radius of gyration
- r_e equivalent radius of gyration for composite columns
- S standard deviation of test results on concrete strength
- t tube thickness
- α concrete contribution parameter [1]
- γ_m partial safety factor for material strength
- σ_{cu} characteristic cube strength
- σ_{cyl} characteristic cylinder strength
- σ_k characteristic concrete strength
- σ_m mean value of concrete strength from tests
- σ_u design strength of concrete
- σ_y yield strength of steel
- σ_y^* reference yield strength of steel [1]
- σ_{yL} reduced longitudinal strength of steel under hoop tension
- ϕ coefficient used in estimating reduced steel strength and enhanced concrete strength
- $\bar{\phi}$ $\sqrt{1+\phi+\phi^2}$

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Summary

A new method for the design of composite columns under concentric loading is presented. The method adopts the recently developed European curves for the design of axially loaded bare metal sections as the basic design curves for composite columns. The column slenderness factor has been redefined as the ratio of column length to a unit critical length, which is defined as the length for which the column squash load equals its Euler load. The design load shows excellent agreement with over 200 analytically exact results and with over 100 experimental results.

The method has been extended to include the design of concrete filled circular tubes taking due account of triaxial containment of the concrete. Comparison with over 150 experimental results on concrete filled circular tubes shows that the unified method gives very good correlation with the experimentally obtained ultimate load.

Résumé

Les auteurs présentent une nouvelle méthode pour le calcul des colonnes mixtes soumises à des efforts centrés. Les courbes de base adoptées sont les courbes de flambement de la Convention Européenne. L'élancement intrinsèque a été défini

comme rapport de la longueur de la colonne à celle pour laquelle la charge de ruine de la colonne est égale à la charge de flambage d'Euler. La méthode montre une excellente concordance avec plus de 200 résultats analytiques et plus de 100 résultats expérimentaux.

Le procédé a été étendu à l'étude des tubes remplis de béton, en tenant particulièrement compte des sollicitations triaxiales du béton. Là aussi la comparaison avec plus de 150 résultats expérimentaux montre une très bonne concordance.

Zusammenfassung

Die Autoren behandeln eine neue Methode für den Entwurf von zentrisch beanspuchten Verbundstützen. Als grundlegende Entwurfskurven werden dabei die Knickkurven der europäischen Stalbaukonvention angenommen. Der bezogene Schlankheitsgrad wurde neu definiert als Verhältnis der Stützenlänge zur derjenigen Länge, für welche die Quetschlast der Stütze ihre Eulersche Knicklast erreicht. Die Übereinstimmung des Verfahrens mit über 200 theoretischen genauen Resultaten und über 100 Versuchsergebnissen ist ausgezeichnet.

Die Methode wurde auf betongefüllte Rohrstützen unter besonderer Berücksichtigung der dreiachsigen Betonbenaspruchung erweitert. Auch hier zeigt ein Vergleich mit über 150 entsprechenden Versuchsergebnissen eine sehr gute Übereinstimmung.