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# A Unified Design Method for Composite Columns

Une méthode unifiée de calcul des colonnes mixtes acier-béton

Eine vereinheitlichte Methode für den Entwurf von Verbundstützen

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#### Introduction

A design method for rectangular composite columns was proposed by BASU and SOMMERVILLE [1] in 1969. The method was derived on the basis of results obtained from analytical studies on numerous composite columns in uniaxial bending. Later the method was extended [2] to include concrete filled circular steel tubular columns. Design studies [3, 4] which were subsequently carried out to explore the application of the method to practical cases concluded that, although it was more comprehensive than other available methods, there were also some short-comings.

Two principal drawbacks were related to the design of axially loaded columns. In the first instance it was found that in the case of slender composite columns, design loads given by the new method for some encased sections were less than those allowed by existing codes for the corresponding uncased sections. Furthermore, no allowance was made for the augmentation in strength of concentrically loaded concrete filled circular steel tubular columns due to the triaxial containment of the concrete.

The aim of this paper is to present a design method, for axially loaded composite columns, which overcomes the objections to Basu and Sommerville's original proposals, and to establish the validity of the approach by comparison with available test results and analytically derived data of proven accuracy. The proposed method relates the design of axially loaded composite columns to the design of bare steel columns. A later paper will deal with the design of eccentrically loaded composite columns.

## **Basu and Sommerville's Composite Column Curve**

The strength of pin-ended axially loaded bare steel columns is related to the ratio of the length to the radius of gyration, L/r. In the case of columns made up of two or more materials with different material properties the radius of gyration

has no meaning. BASU and SOMMERVILLE [1], however, defined an equivalent radius of gyration for composite columns as

$$r_{e} = \sqrt{\left(\sigma_{y}^{*} I_{s} + \sigma_{y}^{*} E_{c} I_{c} / E_{s}\right) / P_{s}}$$
(1)

This radius of gyration was used to obtain the column slenderness ratio  $(L/r_e)$  for subsequent use in design. It was stated that by taking  $E_c = 360 \sigma_u$  and  $\sigma_y^* = 16 \text{ tonf/in}^2 (247 \text{ N/in}^2)$  it was possible to minimise the scatter in plotting a large number of analytical results as  $P_a/P_s$  against  $L/r_e$ . In arriving at their basic buckling curve, or  $K_1$  curve, they first found the lower bound of the narrow band of scattered points. The curve was subsequently lowered further, mainly in the intermediate slenderness range of  $L/r_e = 50-150$ , to ensure the safe design of certain eccentrically loaded rectangular hollow sections filled with concrete using other formulae they derived.

The excessively conservative nature of the  $K_1$  curve so produced is illustrated by comparing available test ultimate loads [5, 6, 7, 8] with the corresponding design strength as shown in Figure 1. Nominal values of cross-sectional areas, and mean values of concrete strength, corrected for the minimum standard deviation as recommended by CEB [9] (see Appendix I), are used throughout. It may be seen that several columns, particularly those tested by Stevens, show markedly high strengths as compared with the  $K_1$  curve. The mean value of the ratio of test ultimate loads to design strength is 1.928 with a standard deviation of 202 per cent. For a good design method, the two values should be 1.000 and 0 per cent, or as close to these values as possible.

A similar comparison is made with analytically computed results in Figure 2. The theoretical results <sup>1</sup> were obtained using a verified computer program [10]. The practical column cross-sections have concrete contribution parameter,  $\alpha$ , varying from 0.12 to 0.80. The stress-strain curve for concrete was assumed to be that given in the CEB recommendations [9], and a bi-linear curve was adopted for steel. An initial lack of straightness of L/1000 in the plane of bending is assumed throughout. It will be seen that the K<sub>1</sub> curve lies well below the lower envelope of all the analytical results, and that the margin of conservativeness increases with slenderness ratio. The mean value of the ratio of analytical ultimate loads to design strengths is 1.567 and the standard deviation is 59 per cent.

The comparison of the design strengths with both the experimental and theoretical ultimate loads shows clearly the conservative nature of the basic composite column buckling curve proposed by Basu and Sommerville, and confirms the findings of the design studies referred to earlier. The following sections show how the anomalies between the design of composite and bare steel columns are eliminated.

## **New Bare Steel Column Curves**

The design of bare metal sections has traditionally been based on relationships between the column critical stress and the column slenderness ratio. Typical of these relationships is the PERRY-ROBERTSON formula [11] which incorporates an

<sup>&</sup>lt;sup>1</sup> Details of the analytical and experimental results plotted in this paper may be found in Reference 28.

imperfection factor to account for any initial lack of straightness, accidental eccentricity of loading, and residual stresses. The committee drafting the new European code for steel structures have given the problem the most exhaustive treatment to date. Following numerous tests and analytical studies, BEER and SCHULZ [12] recommended three basic buckling curves (Fig. 3) which are applicable to cross-sections of different shapes. A table, reproduced here as Table 1, is provided to enable designers to select the appropriate curve for a given column cross-section. Representative residual stress distributions in the cross-section and an initial lack of straightness of L/1000 are allowed for in the derivation of the curves. Two additional curves cater for certain extreme cases.

SHAPE	CURVE	
•	Rolled tubes Welded tubes	a
	Welded box sections	Ь
h	I and H rolled sections: -Buckling parallel to the web h/b > 1:2 h/b < 1:2 -Buckling parallel to flanges h/b > 1:2 h/b < 1:2	a b c
	I and H welded sections: -Buckling parallel to the web a) Flame cut flanges b) Rolled flanges -Buckling parallel to flanges a) Flame cut flanges b) Rolled flanges	b b c
	I and H sections with welded flange cover plates -Buckling parallel to flanges -Buckling parallel to web	a b
ŦŦŦ	Box sections, stress relieved by heat treatment	a
	I and H sections, stress releived by heat treatment -Buckling parallel to the web -Buckling parallel to flanges	a b
	T-sections or half I-sections	c
	Channels	с

Parallel work carried out in connection with the new British code for steel buildings and steel bridges has resulted in four curves that approximate the European curves very closely. Both the British and European curves have small plateaux to take account of the strain hardening effects in stocky columns.

The European curves are presented as relationships between  $\overline{N}$ , the ratio of the column critical stress to its yield stress, and its slenderness factor  $\overline{\lambda}$ , the ratio

Table 1.

between the column slenderness ratio ( $\lambda = L/r$ ) and a critical slenderness ratio  $\lambda_e$ . The critical slenderness ratio is defined as that for which the column Euler stress equals the yield stress of the material of the cross-section, and is given by

$$\lambda_{\rm e} = \pi \sqrt{\frac{\rm E_s}{\sigma_y}} \tag{2}$$

The use of the slenderness factor,  $\overline{\lambda}$ , rather than the slenderness ratio,  $\lambda$ , makes the curves independent of the material properties and thus the same curves can be used to design columns with different yield strengths.

#### Application of Bare Steel Column Curves to Composite Column Design

# Proposed Interpretation for Column Strength, $\overline{N}$

In the context of the design of bare steel sections  $\overline{N}$  is defined as the ratio of critical stress  $\sigma_a$  to the yield stress  $\sigma_v$ , that is

$$\overline{N} = \frac{\sigma_a}{\sigma_v} \tag{3}$$

In the alternative interpretation now proposed, and which is applicable to bare metal sections as well as composite sections,  $\overline{N}$  is defined as the ratio of the column critical load  $P_a$  to its squash load  $P_s$ , thus

$$\overline{N} = \frac{P_a}{P_s} \tag{4}$$

For a composite column the squash load may be expressed as follows:

$$P_{s} = \Sigma A_{s} \sigma_{y} + A_{c} \sigma_{u}$$
<sup>(5)</sup>

The summation sign is intended to include not only the main steel core but also other steel areas such as longitudinal reinforcement. The column ultimate load under concentric loading may be expressed as

$$P_a = \Sigma A_s \sigma_a + A_c \sigma_b \tag{6}$$

where  $\sigma_a$  and  $\sigma_b$  are the average stresses in steel and concrete respectively, associated with the ultimate thrust  $P_a$ , and are not necessarily the stresses associated with the tangent modulus load.

It is easy to see that the new interpretation is an exact equivalent of the existing one when applied to bare metal sections. That is, for  $A_c = 0$ ,

$$\overline{N} = \frac{P_a}{P_s} = \frac{A_s \sigma_a}{A_s \sigma_y} = \frac{\sigma_a}{\sigma_y}$$

Basu and Sommerville have also adopted a similar non-dimensionalisation for the failure loads of composite columns.

## Proposed Interpretation for Slenderness Factor $\lambda$

The existing expression for slenderness factor is

W

$$\overline{\lambda} = \frac{\lambda}{\lambda_{e}}$$
(7)
here  $\lambda = \frac{L}{r}$   $r = \sqrt{I/A}$ 

and  $\lambda_e$  is as defined in Equation (2).

In the new interpretation applicable to bare metal sections as well as composite sections, the slenderness factor  $\overline{\lambda}$  is defined as the ratio of the column length L to a unit critical length of the column L<sub>c</sub>, which, in turn, is defined as the length of the column for which its Euler load equals its squash load. Thus

$$\overline{\lambda} = \frac{L}{L_c} \tag{8}$$

Also, by definition,

$$P_{s} = \pi^{2} \frac{\left(\Sigma E_{s}I_{s} + E_{c}I_{c}\right)}{L_{c}^{2}}$$
(9)

or 
$$L_c = \pi \sqrt{\frac{(\Sigma E_s I_s + E_c I_c)}{P_s}}$$
 (10)

For the bare metal section the proposed definition of the slenderness factor  $\overline{\lambda}$  agrees exactly with the existing one, as

$$\overline{\lambda} = \frac{L}{L_{e}} = \frac{L}{\pi \sqrt{\frac{E_{s}I_{s}}{A_{s}\sigma_{y}}}} = \frac{L/\sqrt{I_{s}/A_{s}}}{\pi \sqrt{E_{s}/\sigma_{y}}} = \frac{L/r}{\lambda_{e}} = \frac{\lambda}{\lambda_{e}}$$

By adopting the proposed interpretation of slenderness factor  $\overline{\lambda}$ , it is no longer necessary to define an 'equivalent' radius of gyration. The column slenderness is now measured with respect to a single parameter which contains not only the geometric properties of the cross-section such as areas and moments of inertia, but also mechanical properties such as material strengths and moduli of elasticity. The merit of the new interpretation of slenderness factor thus lies in the generality of its application to bare metal sections as well as composite sections.

### Formulation of Design Method for Axially Loaded Composite Columns

## General

The design procedure for composite columns should now be clear in outline. Having calculated the column slenderness factor  $\overline{\lambda}$  using Equations (5), (10) and (8), the designer selects the appropriate basic buckling curve applicable to the corresponding bare metal section from Table 1. A value of  $\overline{N}$  is then given directly by the particular curve of Figure 3, and the ultimate column load P<sub>a</sub> is calculated from Equations (5) and (4). The method is applicable to composite columns of many types and all the crosssectional shapes included in Table 1 can be adopted as the basic steel core. The particular problem of triaxial containment of concrete in concentrically loaded concrete filled circular hollow sections is discussed in a later section.

It is necessary, at this stage, to investigate which value of  $E_c$  gives the best correlation with results before recommending an appropriate expression for the initial modulus. (See also Appendix I.)

### Comparison with Analytical Results

The 'exact' analytical results are now compared with design strengths obtained by the use of the corresponding European design curves. The mean value of the ratio of the analytical ultimate load to the design strengths for the three curves a, b, and c are 0.953 (standard deviation 3.88%), 0.966 (s.d. 10.0%) and 0.990 (s.d. 9.3%) respectively. The results are plotted in Figures 4-6 and a very good agreement between the theoretical values and the design curves may be observed. The correlation shown with the European curves is substantially better than that observed in the case of Basu and Sommerville's  $K_1$  curve (cf. Fig. 2). The results shown are based on a value of  $E_c$  equal to the CEB initial modulus  $E_{co}$  as defined in Equation (21) which corresponds to the initial modulus of the stress-strain curve used in the theoretical calculations.

As an alternative to Equation (21) one may use the CP110 value given by Equation (23) to define  $E_{co}^{1}$ . The CEB value of  $E_{co}$  will be larger than the CP110 value for  $\sigma_{u} > 4500 \text{ N/cm}^{2}$  approximately. As all the theoretical results included in this study fall within this range, the use of the CP110 value will have the effect of reducing the slenderness factor for these cases, and consequently the results will appear to be on the unsafe side. The average values for the ratio of the theoretical ultimate load to the design ultimate load for the three curves a, b, and c are 0.881 (s.d. 8.67%), 0.832 (s.d. 19.36%), and 0.845 (s.d. 14.5%).

The design values can be made to appear safer by adopting a smaller value of  $E_c$  than that previously assumed for design. Thus if  $E_c$  is taken as 0.5 of the CEB value, representing the slope of the dashed line in Figure 12, it is found that the average values of the ratio of the theoretical ultimate load to the design ultimate loads for the three curves are 1.032 (s.d. 3.85%), 1.162 (s.d. 10.25%), and 1.257 (s.d. 11.05%). Most of the results now lie above the corresponding design curves.

Key to symbols used in figures.



Fig. 10, 11. Tests on Circular Filled Tubes.

TESTS REPORTED BY	REFERENCE	SYMBOL
STEVENS JONES AND RIZK	(5) (6) (7)	<b>A</b> <b>O</b>
JANSS (ENCASED)	(8) (20)	0
KNOWLES AND PARK JANSS(SQUARE TUBES)		) E

TESTS REPORTED BY	REFERENCE	SYMBOL
KLOEPPEL AND GODER KNOWLES AND PARK SALANI AND SIMS	(13) (17) (18)	40 X
GARDNER HND JHCOBSON FURLONG NEOGI	(19) (20) (14) (15)	**
JANSS(OLD SERIES) JANSS(NEW SERIES)	(21) (21)	N. N

<sup>1</sup> For the sake of simplicity, the value of  $E_{co}$  given by Equation (21) will be referred to in this paper as the CEB value, and that given by Equation (23) as the CP110 value.



Fig. 1. Test Results for Rectangular Columns Compared with Basu and Sommerville's K<sub>1</sub> Curve.



Fig. 3. European Curves a, b, and c — Top Downwards.



Fig. 5. Analytical Results for Rectangular Columns Compared with Curve b Using Equation (21).



Fig. 2. Analytical Results for Rectangular Columns Compared with Basu and Sommerville's K<sub>1</sub> Curve.



Fig. 4. Analytical Results for Rectangular Columns Compared with Curve a Using Equation (21).



Fig. 6. Analytical Results for Rectangular Columns Compared with Curve c Using Equation (21).

It is evident that the reduction in the value of  $E_c$  makes the results appear safer. However, it may be noted that the use of  $E_c$  equal to the CEB value brings the theoretical results closest to the three design curves.

#### Comparison of Proposed Design Method with Test Results

The available test results are now compared with the three European design curves. The results are computed for two values of  $E_c$ , namely, the CEB value and the CP110 value. Tables 2-4, corresponding to Figures 7-9, list the comparable values for the CEB value. The average value of the ratio of test loads to the design loads relevant to the three curves a, b, and c are 1.084 (s.d. 19.4%).

1.426 (s.d. 43.6%) and 1.230 (s.d. 28.9%) respectively. The corresponding values obtained when  $E_c$  is taken as the CP110 value are respectively 1.069 (s.d. 20.7%), 1.225 (s.d. 17.2%), and 1.104 (s.d. 18.6%). It is noteworthy that in either case, the majority of test ultimate loads are safely predicted by the design curves since most of the test values appear above the design curves. In all the present comparisons with test loads, the value of  $\gamma_m$  is taken as 1.0. If the value of  $\gamma_m$  were taken in the range 1.3-1.6, as is required in real design situations, the few points lying below the design curves will shift closer to the design curves, and many more will lie above the line.

NUMBER	COLUMN	۲۸ C	P TEST	P S	(4) / (5)	DESIGN N	(6) / (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1 2 3 4 5 5 6 7 8 9 10 11 12 13 14 15 15 16 17 18	RS120 F1 F2 F3 DF3 DF4 KP1 KP2 KP4 KP5 KP6 JS21 JS22 JS23 JS24 JS25	.807762 .303351 .305914 .305914 .305914 .282133 .112362 .110754 .493728 .358594 .223195 .111598 .1386851 .1386841 .138007 .138266 .145186	47.800 117800.000 150000.000 152000.000 360000.000 201600.000 86500.000 95000.000 113700.000 113700.000 115000.000 455000.000 455000.000	65.937 99665.425 99665.425 137298.535 355070.840 280003.946 172938.875 100745.378 10007.293 110613.849 110173.293 1106297.506 106297.506 486678.755 433802.243 435182.014 471360.489 625617.413	.724934 1.181955 1.101686 1.092510 1.107077 .986110 1.960687 1.165730 .794081 .865937 .857294 .943967 1.069639 1.061869 .949478 1.037339 1.091497 .954683 .955856	.792531 .977196 .976581 .976581 .982388 1.000000 1.000000 .819078 .877048 .925195 .963552 .995561 1.000000 1.000000 1.000000 1.000000	.914707 1.209537 1.127395 1.118709 1.13625 1.003789 1.960687 1.165730 .963482 .987332 .926608 .979675 1.074409 1.081869 .949478 1.037339 1.091497 .954683 .954683
19 20 21 22 23 24 25 26	JS25 JS26 JS27 JS28 JS29 JS30 JS31 JS32	.145160 .146160 .146493 .150132 .151217 .150491 .149207	59600.000 595000.000 575000.000 825000.000 830000.000 815000.000 830000.000	616392.260 581678.061 581678.061 724112.272 722871.864 721167.194 730015.357	.956917 .966917 1.022903 .988519 1.139326 1.148198 1.130112 1.136962 ARITHM	1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 ETIC MEAN	.955555 .965917 1.022903 .988519 1.139326 1.139326 1.148196 1.130112 1.136962

Table 2. Comparison with european curve a.

**Extension of Proposed Method to Concrete Filled Circular Steel Tubes** 

## Background to Problem

The behaviour of concrete filled circular hollow sections differs from other types of composite column in that under concentric loading such columns exhibit an enhanced strength, particularly for columns of short lengths. This is explained by the fact that the concrete core in such columns is contained triaxially thereby achieving far greater strength than the corresponding cube strength. The effects of triaxial containment tend to diminish as the column length increases, or as the end moments on the column increase. Different methods have been proposed [13, 14, 15, 16, 17] to account for the triaxial containment of concrete.

# Formulation of Design Approach

Based on the results from the tests carried out at Imperial College, Sen [15] derived an expression for the ultimate load of concentrically loaded concrete filled circular hollow sections of very short length:

$$P_{\rm H} = A_{\rm s} \, \frac{\sigma_{\rm y}}{\bar{\Phi}} + A_{\rm c} \left( \sigma_{\rm u} + \frac{2t \, \delta \, \phi \, \sigma_{\rm y}}{d \, \bar{\Phi}} \right) \tag{11}$$

where,

 $\sigma_{u}$  = uniaxial concrete strength in member

- $\sigma_y$  = yield strength of steel
- t = thickness of the tube
- d = diameter of the tube
- $\delta$  = a constant (Sen's range of values = 4 to 10)
- $\phi$  = another constant depending upon the Poisson's ratios of steel and concrete (Sen's range of values = 0.2 to 0.5)

and 
$$\bar{\phi} = \sqrt{1 + \phi + \phi^2}$$
 (12)

Since the lengths of the test columns were around 5 times their diameter, it is reasonable to assume that Equation (11) gives the squash load of such columns including triaxial effects. It follows that the augmented strength of concrete under confinement from the surrounding steel shell is given by

$$\sigma_{cL} = \sigma_u + \frac{2t}{d} \frac{\delta \phi}{\bar{\phi}} \sigma_y$$
(13)

and the reduced strength of steel by  $\sigma_{yL} = \frac{\sigma_y}{\overline{\Phi}}$ 

		LA	P	Р		_	
NUMBER	COLUMN	C	TEST	5	(4)/(5)	DESIGN N	(6)/(7)
ώ	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	B1	.689769	37.000	34.366	1.076654	.790627	1.361772
2	B2	1.076810	27.300	29.994	.910172	.551382	1.650710
3	B3	1.285139	28.600	32.423	.882093	.436130	2.022544
4	B4	1.631224	19.800	30.966	.639416	.297808	2.147074
5	85	1.778363	23.080	34.123	.676379	.255509	2.647181
6	B6	2.144718	16.400	32.180	.509633	.182592	2.791096
7	B7	2.255538	15.400	35.580	.432827	.166780	2.595197
8	FA1	.148610	478.000	397.098	1.203732	1.000000	1.203732
9	FA2	.292894	450.000	407.389	1.104595	.967629	1.141548
10	FA3	.451128	421.000	389.381	1.081204	.904949	1.194769
11	FA4	.587875	426.000	404.816	1.052329	.844062	1.246743
12	FA5	.734844	424.000	<i>4</i> 04.816	1.047388	.765339	1.368529
- 13	SIG	.534047	240.000	199.200	1.204819	.869838	1.385107
14	SIE	.489563	281.000	232.391	1.209172	.889379	1.359568
15	515	.512859	258.000	212.684	1.213069	.8/9413	1.3/940/
16	526	.418803	290.000	235.689	1.230436	.91/5/9	1.340959
17	SZE	.380600	380.000	286.294	1.327305	.932860	1.422834
18	525	.39/10/	310.000	260.175	1.191504	.926186	1.286462
19	536	.32////	364.000	287.190	1.26/454	.954189	1.328305
20	535	.309542	380.000	331.778	1.145345	.961374	1.191363
21	535	.311101	423.000	327.084	1.293245	.960771	1.346050
22	FE1	.819876	440.000	451.123	19/5343	./148//	1.364350
23	FE2	. /85019	4/1.000	4/6.850	.987732	./35628	1.342706
24	RHI	.090409	66.000	54.523	1.24/1//	.000741	1.942122
22	RH2	.003310	100,000	52.773	1.099053	.805046	1.305200
20	DOT1	. 300399	E4 000	99.993	1.041/42	.930990	1.119401
21	DOWN	510500	57.000	57.523	1.350505	.09//03	1.419400
20	POX2	910900	50 500	52.773	1.209590	.0/0035	1 202602
20	DOUCE	618688	50.500	52.773	1 061155	.032005	1.302003
31	DOX'3	302050	103.000	05 003	1.001199	.070035	1 112010
32	PLII20	1 505184	23 600	53.553	373705	340683	1.005031
32	10 1	602323	233000 000	230511 553	072913	.370002	1.090931
34	19.2	666517	258000.000	291013 760	015173	903346	1 130303
35	10 3	675550	210000.000	201913.700	902547	709447	1.135202
36	110 1	778122	235000.000	345770 804	670641	740108	019300
37	110 2	787282	276000.000	336803 824	810240	734631	1 115195
38	J10.3	.788142	241000.000	338833.498	.711264	.734115	.968873
						FTIC MEAN	1 426161
	2				STANDARD	DEVIATION	.435730

Table 3. Comparison with european curve b.

(14)

If the modified strengths of concrete and steel as defined by Equations (13) and (14) are varied to take into account the fact that the effects of triaxial containment reduce with increasing column length, it is then possible to use these values to determine the column slenderness factor using Equations (10), (8) and (5). The design ultimate loads can then be obtained from the applicable curve a and Equations (5) and (4).

For ideally straight columns with elastic plastic behaviour, failure is governed by Euler buckling for  $\overline{\lambda} > 1$  and by material yield for  $\overline{\lambda} < 1$ . It therefore appears reasonable to postulate that for columns having  $\overline{\lambda} > 1$  the triaxial effects will be negligible. For columns in the range  $0 < \overline{\lambda} \le 1$ , the triaxial effects will be maximum at  $\overline{\lambda} = 0$  and zero at  $\overline{\lambda} = 1$ .

		LA	Р	P		-	
NUMBER	COLUMN	C	TEST	5	(4) / (5)	DESIGN N	(6)/(7)
(D)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	A1	.110357	160.000	137.726	1.161727	1.000000	1.161727
2	A2	.399866	140.000	134.188	1.043309	.900071	1.159141
3	A3	.696097	144.000	137.726	1.045555	.721159	1.449825
4	A4 .	.686022	135.000	140.084	.963705	.727546	1.324598
5	A5	1.005506	131.000	137.136	.955254	.533882	1.789261
6	A6	1.268328	105.000	142.796	.735313	.408319	1.800830
7	RE1A	1.012875	134.000	145.375	.921756	.530005	1.739146
8	RE1B	1.036667	125.000	141.837	.881292	.517633	1.702542
9	RE2A	1.024417	123.000	143.606	.856510	.524003	1.634553
10	RE2B	.985105	120.000	150.091	.799513	.544694	1.467820
11	RE3A	.963089	140.000	152.639	.917197	556701	1.647558
12	RE3B	.990755	124.000	147.359	.841483	.541700	1.553411
13	RE-1A	1.016648	121.000	144.785	.835721	.528043	1.582676
14	RE4B	1.032501	127.000	142.427	.891686	.519799	1.715443
15	AE6	.364375	130.000	123.613	1.051621	.918725	1.144653
16	J1.1	.747423	219000.000	287745.243	.761090	.688550	1.105352
17	J1.2	.756456	222000.000	275414.128	.806059	.682704	1.180686
18	J1.3	.752472	213000.000	263369.544	.808750	.685293	1.180151
19	J2.1	.624064	239000.000	272874.324	.875861	.767458	1.141249
20	J2.2	.636186	222000.000	253849.175	.874535	.759579	1.151342
21	J2.3	.618793	263000.000	279216.040	.941923	.770885	1.221873
22	J3.1	.431944	268000.000	281821.203	.950958	.882731	1.077290
23	J3.2	.428982	228000.000	283230.473	.804998	.884360	.910261
24	J3.3	.438814	239000.000	264557.641	.903395	.878952	1.027809
25	J4.1	.227167	260000.000	264557.641	.982773	.986316	.996407
26	J4.2	.231092	252000.000	245532.492	1.026341	.984354	1.042654
27	J4.3	.219283	280000.000	272308.628	1.028245	.990259	1.038360
28	J5.1	.680896	240000.000	304830.464	.787323	.730826	1.077305
29	J5.2	.669674	268000.000	315047.673	.850665	.737909	1,152805
30	J5.3	.676953	252000.000	307649.004	.819115	.733320	1,116996
31	J6.1	.911229	240000.000	364235.339	.658915	.586375	1,123709
32	J6.2	.896100	220000.000	374452.548	.587524	.595440	.986706
33	J6.3	.906293	253000.000	367406.197	.688611	.589324	1,168475
34	J7.1	.733968	252000.000	360048.216	.699906	.697160	1.003939
35	J7.2	.724210	267000.000	368151.520	.725245	703348	1.031133
36	J7.3	.732685	262000.000	360752.851	.726259	697981	1.040513
37	J8.1	.520520	248000.000	373149.753	.664613	.831288	.799498
38	J8.2	.506191	241000.000	391822.584	.615074	839885	.732331
39	J8.3	.507988	260000.000	390413.314	.665961	.838807	.793938
					ARITH	TETIC MEAN	1.230102
			· · · · · · · · · · · · · · · · · · ·		STANDARD	DEVIATION	.288588

Table 4. Comparison with european curve c

The above criterion would require the determination of  $\overline{\lambda}$  twice during the design process; firstly, to determine whether the triaxial effects are to be considered at all, and secondly, having found the new concrete and steel strengths, to obtain the value of  $\overline{N}$  from curve a. As the point where the triaxial effects cease to be worth considering can only be approximately defined, it is suggested that the criterion postulated above be replaced by an equivalent but simpler criterion. For most practical columns, the value of  $\overline{\lambda} = 1$  corresponds to a length to diameter ratio (L/d) varying between 24 and 29. It is therefore proposed that the effects of triaxial containment be ignored for columns with L/d > 25. For columns in the rage 0 < L/d < 25, the effects of triaxial containment may be considered by making  $\delta$  and  $\phi$  linear functions of L/d. Thus

$$\delta = 0.25 (25 - L/d) \quad 0 \le \delta \le 6.25 \tag{15}$$

and 
$$\phi = 0.02 (25 - L/d)$$
  $0 \le \phi \le 0.5$  (16)

These expressions correspond to  $\delta = 5.0$  and  $\phi = 0.4$  for L/d = 5, the average values for Sen's tests.

		Ţ	able 5		
L/D	5	Φ	 •	205/0	1/0
1234567890 11234567890 111134567890 1222345	$\begin{array}{c} 6.000\\ 5.750\\ 5.250\\ 5.250\\ 5.250\\ 4.750\\ 4.500\\ 4.500\\ 4.500\\ 3.500\\ 3.500\\ 3.250\\ 3.250\\ 2.250\\ 2.250\\ 2.250\\ 2.250\\ 2.250\\ 1.500\\ 1.500\\ 1.500\\ 0.550\\ 0.250\\ 0.$	$\begin{array}{c} 0.480\\ 0.460\\ 0.420\\ 0.420\\ 0.380\\ 0.380\\ 0.340\\ 0.320\\ 0.220\\ 0.220\\ 0.220\\ 0.220\\ 0.180\\ 0.220\\ 0.180\\ 0.220\\ 0.180\\ 0.200\\ 0.140\\ 0.100\\ 0.100\\ 0.100\\ 0.000\\ 0.000\\ \end{array}$	$\begin{array}{c} 1.3078\\ 1.2929\\ 1.2782\\ 1.2635\\ 1.2635\\ 1.23490\\ 1.2347\\ 1.2205\\ 1.2946\\ 1.17926\\ 1.17926\\ 1.1522\\ 1.1391\\ 1.1262\\ 1.1391\\ 1.0536\\ 1.0053\\ 1.0536\\ 1.0566\\ 1.05$	$\begin{array}{c} 4.4043\\ 4.0916\\ 3.7868\\ 3.2026\\ 2.9239\\ 2.6544\\ 2.3955\\ 4.6817\\ 1.68617\\ 1.2641\\ 1.6817\\ 1.2641\\ 1.6744\\ 0.8980\\ 0.7356\\ 0.5878\\ 0.4550\\ 0.3380\\ 0.5378\\ 0.1535\\ 0.0392\\ 0.0099\\ 0.0000 \end{array}$	0.7646 0.7735 0.7925 0.8006 0.8099 0.8193 0.8289 0.83860 0.86679 0.88890 0.88890 0.88890 0.88890 0.89082 0.99082 0.99286 0.99286 0.9492 0.95940 0.96954 0.96954 0.96954 0.96900 0.96900 0.96900 0.96900 0.99900 0.99900 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.99000 0.90000 0.90000 0.90000 0.90000 0.90000 0.90000 0.90000 0.90000 0.90000 0.900000000

Table 6. Comparison with european curve a

		- LA	P	P		-				L٨	Р	P		-	
NUMBE	R COLUMN	C	TEST	5	(4)/(5)	DESIGN N	(6) / (7)	NUMBER	COLUMN	C	TEST	5	(4)/(5)	DESIGN N	(6)/(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	KG7 KG9	.580450	212900.000	222713.070	.955939	.892129	1.071525	78	F4	.301254	140000.000	166994.717	.838350	.7,7699	.857472
3	KG9	.578139	203500.000	219989.411	.926863	.693007	1.037912	80	FG	.260158	153400.000	153316.919	1.000542	.987664	1.013039
4	KG10	.372079	229800.000	250580.256	.913081	.950180	.950947	61	F7	.258769	162200.000	169462.959	.957141	.987983	.968783
5	KG11	.370916	226600.000	250067.658	.906155	.950471	.943448	82	FB	.258769	164800.000	169452.959	.972484	.987983	.984312
7	KG13	.795878	199300.000	218146.834	.929340	. 798561	1,144054	84	84	.245975	2408000.000	2005576.737	1.200652	.990885	1.211695
8	KG14	.790111	203900.000	212294.593	.960458	.801444	1.198408	85	A5	.221352	790700.000	879545.086	.898987	.995930	.902662
9	KG15	.795972	206100.000	216377.895	.952500	.798514	1.192841	86	A6	.258337	1671000.000	1448733.953	1.153421	.985782	1.170056
10	KG41	.388148	14/500.000	129547.864	1.135945	.955081	1.188125	89	BIX	238208	289000.000	225711.179	1.280397	.992558	1.289997
12	KG43	.391412	147500.000	126282.757	1.168011	.955233	1.222753	89	82	.252674	293500.000	290578.187	1.010055	.989385	1.020392
13	KG44	.599577	127400.000	117540.905	1.0835.8	.884957	1.224781	90	B2X	.245438	293500.000	274284.463	1.070057	.990993	1.079783
14	KG45	.640127	135200.000	135079.923	1.008292	.969248	1.159959	92	OFIX	.12/9/6	663000.000	439908.447	1.507132	1.000000	1.507132
15	KG47	.805429	120500.000	107313.877	1.123805	.793731	1.415853	93	DF2	.129574	410000.000	306475.277	1.337791	1.000000	1.337791
17	KG49	.858005	127200.000	122294.915	1.040109	.764557	1.360407	94	DF2X	.131027	410000.000	313391.228	1.308269	1,000000	1.308269
18	KG49	.812471	109600.000	104549.868	1.048304	.790065	1.326857	95	SC1	.211474	451000.000	360220.322	1.252012	.997820	1.254747
19	KG53	.409506	230000.000	370960.800	.620012	.950528	.652281	97	SC3	.210809	475000.000	404765.416	1.173519	.997946	1.175934
21	KG55	.413081	514660.000	417488.512	1.232609	.949558	1.298073	96	SC4	.211874	392000.000	326135.663	1.201954	.997744	1.204672
22	KG56	.412329	503300.000	415585.341	1.199232	.949771	1.262653	.99	M11	.202765	201.000	158.813	1.265643	.999475	1.256308
23	KG69	.459462	553400.000	587361.361	.942180	.936351	1.005214	100	M13	202034	175.000	145.347	1.234030	.999614	1.195485
24	KG70	417752	544300.000	545/29.5/5	.927684	.935885	.991238	102	M14	.202013	212.000	179.901	1.178426	.999617	1.178877
26	KG72	.465257	659200.000	655573.288	.990424	.934518	1.059924	103	M15	.204011	261.000	208.442	1.252146	.999538	1.253101
27	KG73	.810274	112000.600	109629.413	1.021624	.791252	1.291148	104	M16	.202197	253.000	212.685	1.189552	.999583	1.190048
28	KG74	.813087	103300.000	101976.832	1.012975	.789733	1.282680	106	MIB	.204918	241.000	203.305	1.185411	.999066	1.186520
29	KG75	874498	105300.000	105509 455	877143	794205	1 104427	107	M19	.204783	268.000	232.205	1.154149	.993091	1.155198
31	KG93	.357613	156300.000	160386.396	.970889	.953797	1.007359	108	M20	.207858	294.000	250.769	1.172396	.998507	1.174149
32	KG84	.372151	167800.000	176641.064	.949949	.960162	.989353	110	MD2	208399	349.000	290.885	1.199784	.998404	1.201702
33	KG85	.353810	188109.000	175251.875	1.073251	.962248	1.115359	111	M23	.208121	340.000	271.400	1.252762	.998457	1.254698
35	KG83	.383917	224400.000	245305.204	.914779	.957182	.955700	112	M24	.208177	361.000	299.270	1.206270	.998446	1.208147
36	K.GSO	.383039	228900.000	238291.520	.960168	.957410	1.002831	113	11	.760419	210500.000	213899.049	.818143	.816194	1.002387
37	KG91	.377278	247100.000	238057.954	1.037983	.958891	1.082494	115	31	.413698	245000.000	249101.508	.987549	.949402	1.040181
38	KG92	.373329	242500.000	235850.322	1.023151	.959868	1.071138	116	4J	.190069	281000.000	292278.789	.961411	1.000000	.981411
40	KG96	.738176	141500.000	144205.807	.981230	.827275	1.185098	117	5J	.189455	280000.000	289316.941	.957797	1.000000	.967797
41	KG97	.725524	156300.000	142820.850	1.004378	.833304	1.313301	119	8.11	1.684619	20500.000	47295.721	.435649	.311058	1.400541
42	KG98	.740292	169800.000	152383.728	1.114292	.826254	1.348607	120	8.2J	1.681357	15800.000	46745.772	.337998	.312166	1.082753
43	KG101	755145	175800.000	199213.930	.687490	.81/519	1.085591	121	8.3J	1.677149	14300.000	46380.808	.309317	.313541	.983340
45	KG103	.747697	196400.000	197167.253	.996109	.822552	1.210398	122	9.11	1.110580	28500.000	48451.917	.588212	.598905	1 000186
<del>4</del> 6	KG104	.739349	194500.000	188323.165	1.032799	.825712	1.249285	124	9.3.	1.107625	29700.000	48992.565	.606214	.600915	1.006819
47	KP1	.825483	138200.000	1/3185.560	.797988	.783039	1.019091	125	10.1J	.751509	37000.000	46118.339	.802284	.820645	.977626
49	KP3	.563574	160300.000	191907.044	.837905	.898605	.932451	126	10.2J	.753119	41500.000	45291.656	.895490	.919841	1.093493
50	KP4	.427058	206500.000	210905.416	.979112	.945724	1.035304	128	10.33	587748	41500.000	51132.737	.885930	.889356	.996148
51	KP5	.279443	223000.000	230248.598	.958518	.993039	.985228	129	11.2J	.588314	45000.000	50736.387	.886937	.889141	.997522
52	207	7441-29	50500.000	75993 333	871129	824325	.868531	130	11.3J	.590120	50500.000	50312.324	1.003730	.898456	1.129747
54	KP8	.598509	80000.000	83776.351	.954923	.885352	1.078581	131	12.1J	.418591	53500.000	54344.755	.951334	.948408	1.003085
55	KP9	.450040	9000.000	89554.560	1.004974	.939188	1.070046	133	12.3J	.418617	54400.000	55399.701	.981955	.9-9073	1.035737
56	KP10	.292417	110000.000	90789.916	1.136482	.979895	1.159799	134	13.1J	.225721	65000.000	63255.301	1.027582	.995056	1.032589
57	5530F	.955078	27100.000	39283,990	.589848	697288	1.148511	135	13.2J	.225087	64000.000	63771.804	1.003578	.995183	1.008436
59	5542F	. 359712	72000.000	73509.768	.978131	.661218	1,135753	137	13.33 1 IN	174330	770000.000	738471.358	1.042694	1.000000	1.042594
60	55-99F	1.945567	3540.000	10393.952	.340583	.239720	1.420754	133	2 JN	.176815	770000.000	774392.797	.994340	000000	.994340
61	SS50F	1.945567	3490.000	10393.952	.335772	.239720	1.400687	139	3 JN	.132142	785000.000	752420.777	1.043299	1.030000	1.043299
62	SS51F	1.369063	25400.000	40527.903	.625186	.442787	1.411933	140	4 JN	.133639	760000.000	1172815 144	852549	1.000000	.852549
64	5571F	.701792	51900.000	53809.175	.964520	.843929	1.142891	142	6 JN	.135551	1000000.000	1125043.015	.886355	1.000000	.823955
65	GJ1	.744057	184000.000	189549.623	.970722	.824357	1.177537	143	7 JN	.183453	720000.000	675558.258	1.055612	1.000000	1.065612
65	6.15	431039	180000.000	258229.035	.954041	.022900	1.171518	144	NL B	.187955	695000.000	658078 191	1.020525	1.000000	1.113949
68	GJ6	.432851	245500.000	259655.369	.945869	.944102	1.001872	146	9 JN 10 JN	.10/1/9	835000.000	850185.171	982138	.976971	1.005289
69	GJ7	.434832	213500.000	252695.967	.844689	.943547	.895439	147	11 JN	.303506	830000.000	850566.300	.975820	.977159	.998531
70	6,112	495284	211000.000	240064.187	.878932	.924651	.950555	148	12 JN	.306246	855000.000	849235.824	1.006786	.976501	1.031014
72	GJ18	.806685	55000.000	52829.518	1.041085	.793091	1.312693	150	13 JN	.305246	820000.000	844605.163	1.005387	.974149	1.033093
73	GJ20	.355343	92500.000	78545.052	1.176171	.964364	1.219633	151	15 JN	.315191	840000.000	844416.828	.994769	.974052	1.021269
74	GJ21 F1	358741	74250.000	65762.779	1.112147	.963515	1.154260								
75	F2	.378681	170000.000	192722.387	.882098	.958530	.920261						ARITH	ETIC MEAN	1.109291
77	F3	.301254	141000.000	155994.717	.844338	.277699	.863597						STANDARD	DEVIATION	.145673

#### K.S. VIRDI and P.J. DOWLING

Thus for columns with L/d < 25 the designer may use Equations (12)-(16) to calculate the modified values of concrete and steel strength to be used in determining the slenderness factor  $\overline{\lambda}$ . Once the slenderness factor is known the strength of the column can be taken from column curve a, and Equations (5) and (4). For convenience, the values of  $1/\overline{\varphi}$  and  $2\delta\varphi/\overline{\varphi}$  have been listed in Table 5 for different values of L/d. It may be added that for L/d values ranging between 20 and 25, the collapse loads calculated with or without triaxial effects would not be much different. Hence, to minimise effort, the upper limit of L/d for which triaxial effects are calculated may be restricted to 20.

## Comparison of Proposed Method with Test Loads

The validity of the proposed design methods for concrete filled circular tubular columns under concentric loading is now verified against all known test results [13-21]. It must be noted that the material strengths used in the following comparison are not corrected for the recommended standard deviation errors as this correction is of little significance in terms of the enhanced concrete strength. The factors  $k_1 = 0.67$  for cubes and  $k_2 = 0.85$  for cylinders are applied as appropriate and the value of  $\gamma_m$  is again assumed to be unity.

Table 6 lists the comparative test ultimate loads against the design loads obtained from European curve a and the results are also shown in Figure 10. The value of  $E_c$  used is obtained from Equation (21) (CEB value) using the uniaxial strength of concrete. The average value of the ratio of the test ultimate load to the corresponding design strength for 151 columns is 1.109 with a standard deviation of 14.7 per cent. Similar comparison based on  $E_c$  as given by Equation (23) (CP110 value) gives the average value of the ratio of test ultimate load to the relevant factored design load as 1.097 (s.d. 14.1%). It is clear that both values of  $E_c$  yield good correlation with tests.

To illustrate the effect of ignoring the triaxial effects, Figure 11 shows the comparative values with  $E_c$  equal to the CEB value. The average value of the ratio of test ultimate load to the corresponding design strength is 1.297 (s.d. 21.9%). With the CP110 value this ratio becomes 1.285 (s.d. 22.3%).

The proposed design method has thus been shown to give good correlation with a very large number of available test results on concrete filled circular tubes loaded concentrically.

#### **Recommended Design Procedure for Concentrically Loaded Composite Columns**

#### Summary of Design Procedure

The following design procedure covers axially loaded concrete encased steel sections and concrete filled rectangular and circular steel tubes.

- 1. For sections other than concrete filled circular steel tubes proceed to step 6.
- 2. Calculate column L/d ratio.
- 3. If L/d > 20 proceed to step 6.
- 4. Obtain values of  $1/\phi$  and  $2\delta\Phi/\phi$ , from Table 5.
- 5. Calculate modified strength of concrete and steel using Equations (13) and (14).



Fig. 7. Test Results for Rectangular Columns Compared with Curve a Using Equation (21).



Fig. 8. Test Results for Rectangular Columns Compared with Curve b Using Equation (21).



Fig. 9. Test Results for Rectangular Columns Compared with Curve c Using Equation (21).

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Fig. 11. Test Results for Circular Filled Tubes Compared with Curve a Using Equation (21) and Ignoring Triaxial Effects.



Fig. 12. Stress-strain Relationship for Concrete.

- 6. Calculate the column slenderness factor  $\overline{\lambda}$  using Equations (10), (5) and (8) (using the modified material strengths where applicable).
- 7. Select appropriate European Curve from Table 1 for the bare steel section, and obtain  $\overline{N}$  from Figure 3. As an alternative to Figure 3, the values of  $\overline{N}$ may be calculated from Tables given in Ref. 27.
- 8. Calculate  $P_a$ , the ultimate load from Equations (5) and (4) (using modified material strengths where applicable).

To calculate  $\overline{\lambda}$ , the value of  $E_c$  should be that derived from Equation (21), i.e. the CEB value, and should be based on the uniaxial strength of concrete.

	Length in	Theoretical <sup>*</sup> tonf	Present Design Method tonf	Basu and Sommerville tonf	Present Design Steel Core Only tonf
UCE	100	586.508	594.707	561.214	154.118
Minor Axis	300	498.225	465.499	373.619	53.879
Curve 'c'	500	321.574	319.083	153.551	22.296
RHA	100	232.281	238.802	219.644	154.191
Major Axis	300	191.266	194.853	165.078	129.256
Curve 'a'	500	110.895	115.719	75.136	82.35
IBA	100	165.141	158.910	137.994	36.391
Minor Axis	300	46.359	55.968	14.250	-
Curve 'b'	500	-	(Outside Slenderness Range)	-	-

Table 7

\* Excluding root areas.

Table 8

Diamete	er 6.62	in Thickness	0.176 in $\sigma_{\rm u} = 2400 \ {\rm lb}$	$f/in^2 \sigma_y = 2$	l6 tonf/in <sup>2</sup>		
		Analytical	Basy and Companyillale	Present	t Design		
Length L/d	Ultimate Load	Ultimate Load	without containment	with containment			
in		tonf tonf		tonf	tonf		
72	10.8	83.9	82.5	86.4	99.4		
144	21.6	72.2	63.6	73.6	73.6		
216	32.4	50.0	37.0	52.3	52.3		
Diamete	r 12.75	in Thickness	0.250 in $\sigma_u = 7200$ 1b	$f/in^2 \sigma_y = 2$	23 tonf/in <sup>2</sup>		
72	685.4		667.5	695.0	855.5		
144		657.1	604.6		720.8		
216		592.0	592.0 499.8		606.8		
288		479.4	353.0	509.7	509.7		

## Examples of Application of the Design Method

The design method has been used to calculate the ultimate load capacities of a concrete encased joist section, a concrete encased H universal column, and a concrete filled rectangular hollow section over a range of column lengths. The results are presented in Table 7 and are compared with the theoretically exact ultimate loads, those predicted using the method of Basu and Sommerville, and the

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ultimate design loads for bare steel columns alone. It will be noted that the design loads given by the new method correspond more closely with the 'exact' ultimate loads than do the values given by Basu and Sommerville's design method. It will also be seen that the latter method predicts a lower carrying capacity for the concrete filled rectangular section than is predicted for the bare steel tube. No such anomaly can arise with the new approach.

In Table 8 the design load capacities of a range of practical concrete filled tubes of varying lengths are presented. These are compared with the exactly calculated capacities ignoring triaxial containment and those obtained using the design method of Basu and Sommerville. The enhanced load-carrying capacities of short concentrically loaded columns due to triaxial effects as predicted by the new method can be seen by comparing the tabulated values.

#### **Practical and Economic Consequences**

The design of composite columns under axial loading has been made just as simple as the design of bare steel axially loaded columns. By suitably redefining the column slenderness factor, the newly developed European curves for the design of bare steel columns can be used as the basic design curves for composite columns. Thus full advantage can be taken of the contribution of concrete towards the strength of composite columns. In the case of axially loaded circular tubes filled with concrete, further economies can be made by allowing for the enhanced strength of concrete due to triaxial containment in the design method.

## Conclusions

A new method of design for composite columns under concentric loading has been presented. The method unifies the design of concrete filled circular tubular sections under concentric loading with that of other types of concentrically loaded composite columns, such as encased sections and rectangular filled tubes, by calculating an augmented strength of concrete and a corresponding reduced strength of steel. The effects of triaxial containment are made to vary with the column length to diameter ratio up to a value of 25, beyond which no such effects are considered.

The design method introduces a new concept of column slenderness. The column slenderness factor is defined as the ratio of column length to a unit critical length. This unit critical length is the length for which the column Euler load equals its squash load. This definition leads to the same expression as that used in the currently proposed European design curves for bare steel columns and enables these curves to be used as the basic design curves for composite columns. It is not therefore necessary to define a fictitious radius of gyration for such sections.

The method has been compared with a large number of known experimental results on encased sections as well as rectangular and circular filled tubes. The agreement is shown to be excellent. Good correlation has also been obtained with a large number of theoretically exact results for several encased sections and filled rectangular tubes for the range of practical slenderness factors. The proposed method overcomes many of the disadvantages of a method proposed earlier by Basu and Sommerville and can be confidently recommended for use in design specifications.

## Appendix

## Factors Affecting the Value of Slenderness Factor $\overline{\lambda}$

The mechanical properties of steel are, in general, well defined both with respect to  $\sigma_y$  and  $E_s$ . Problems arise, however, in the determination of the appropriate values of  $\sigma_u$  and  $E_c$  for use in the expression for slenderness factor.

# Design Strength of Concrete $\sigma_u$

In both the CEB recommendations [9] and the British code of practice [22] the design strength of concrete, i.e. the maximum design stress attainable by concrete in a reinforced concrete column is specified as

$$\sigma_{\rm u} = \frac{k_1 \, \sigma_{\rm cu}}{\gamma_{\rm m}} = \frac{k_2 \, \sigma_{\rm cyl}}{\gamma_{\rm m}} \tag{17}$$

The recommended value of  $k_1$  is 0.67 [22]. The factor  $k_2$  which is used when the concrete strength is obtained from cylinder tests rather than cube tests has a recommended value of 0.85 [9]. This corresponds to the observation that the ratio of concrete cylinder strength to concrete cube strength is approximately 0.80.

The value of the characteristic concrete strength as obtained from tests is frequently taken as the mean value of the strengths of the specimen tested. However, both the CEB recommendations [9] and the Handbook on CP110 [23] stipulate that the characteristic strength of concrete should be taken in accordance with the formula

$$\sigma_{\mathbf{k}} = \sigma_{\mathbf{m}} - 1.64 \, \mathrm{S} \tag{18}$$

where S is the standard deviation of test results. The CEB recommendations further stipulate that when the probabilistic distribution of test data is not known a minimum value of  $300 \text{ N/cm}^2$  should be taken for *in situ* concrete and  $200 \text{ N/cm}^2$  for factory cast concrete. In most practical cases, as also in laboratory tests, only a few cube or cylinder tests are carried out. Thus when comparing ultimate load calculations with test results, in the absence of sufficient experimental data, a minimum value of  $200 \text{ N/cm}^2$  for cylinder tests, or  $250 \text{ N/cm}^2$  for cube tests, or their equivalent related to factory cast concrete, should be used as the value of S in Equation (18) to obtain the characteristic strength of concrete.

The coefficient  $\gamma_m$  is the material safety factor. The design value of  $\gamma_m$  associated with the ultimate limit state of design recommended by CP110 is 1.5 while the CEB recommendations specify values in the range 1.3-1.6 depending upon the care and control exercised in the production of concrete. When correlating test results with ultimate load calculations, it is customary to take  $\gamma_m = 1.0$ , assuming that the laboratory conditions permit the production of concrete of a uniform quality.

#### A UNIFIED DESIGN METHOD FOR COMPOSITE COLUMNS

### Modulus of Elasticity of Concrete, $E_c$

A number of equations have been proposed to represent the concrete stressstrain relationship [24, 25, 26]. The curves recommended by CEB and CP110 have similar shapes (Fig. 12) characterised by a parabolic section up to the peak concrete stress, followed by a horizontal plateau, even though the observed stress-strain relationships do not exhibit any discernible flat plateau. Both curves have the same limiting value of strain corresponding to the crushing of concrete, namely 0.0035, but the exact shapes of the parabolas in the two curves are defined by slightly different criteria. The general equation of the parabola may be written as

$$\frac{\sigma}{\sigma_{u}} = \frac{\varepsilon}{\varepsilon_{m}} \left( 2 - \frac{\varepsilon}{\varepsilon_{m}} \right)$$
(19)

The value of the initial modulus is thus given by  $E_{co} = 2 \frac{\sigma_u}{\varepsilon_m}$  (20)

In the CEB recommendations, the value of  $\varepsilon_m$  is fixed at 0.0020. This results in the following value, independent of units, for the initial modulus of concrete:

$$E_{co} = 1000 \sigma_u \tag{21}$$

In CP110, on the otherhand, the value of the initial modulus of concrete is specified as

$$E_{co} = 55000 \sqrt{\frac{\sigma_{cu}}{\gamma_m}}$$
(22)

where both  $E_{co}$  and  $\sigma_{cu}$  are expressed in N/cm<sup>2</sup>. By substituting  $k_1 = 0.67$  in Equation (17), Equation (22) may be rewritten as follows

$$E_{co} = 67193 \sqrt{\sigma_u} \tag{23}$$

This value of  $E_{co}$  is close to the value of

$$\mathbf{E}_{co} = 66000 \sqrt{\sigma_{u}} \tag{24}$$

specified in the CEB recommendations for the initial modulus of concrete for cases where the stresses under working conditions do not exceed 40 per cent of the compressive strength. Thus it appears that while the CEB recommendations differentiate between the elastic moduli of concrete at origin relating to the ultimate limit state calculations and to the instantaneous loading calculations, CP110 recommends the use of a single initial modulus of elasticity.

The value of  $\varepsilon_m$  as deduced from CP110 may be expressed as follows

$$\varepsilon_{\rm m} = \frac{2}{67193} \sqrt{\sigma_{\rm u}} \tag{25}$$

which implies that  $\varepsilon_m$  depends upon  $\sigma_u$ , unlike the CEB recommendations in which  $\varepsilon_m$  is fixed at 0.0020.

# Notation

- Α area of cross-section
- area of concrete Ac
- area of steel A,
- nominal diameter of tube d
- Ec modulus of elasticity of concrete
- Eco modulus of elasticity of concrete at origin
- modulus of elasticity of steel E<sub>s</sub>
- Ι second moment of area
- I<sub>c</sub> second moment of area for the concrete section
- second moment of area for the steel section I,
- K<sub>1</sub> Basu and Sommerville's coefficient for basic column strength under axial load  $(=P_a/P_s)$
- coefficient relating the bending strength of k1 concrete in a member to its characteristic cube strength
- δ coefficient used in estimating triaxial concrete strength
- strain 3
- strain in concrete corresponding to peak εm stress
- λ slenderness ratio
- λ slenderness factor
- λε unit critical slenderness ratio (= Lc/r)
- σ stress
- $\sigma_{a}$ average stress in steel
- average stress in concrete  $\sigma_{b}$
- augmented concrete strength under triaxial  $\sigma_{cL}$ containment

- L column length
- unit critical length, for which the Euler load  $\mathbf{L}_{\mathbf{c}}$ equals the squash load for a given column cross-section
- N column strength non-dimensionalised with respect to its squash load ( $= P_a/P_s$ )
- Pa column strength for axial load
- $\mathbf{P_s}$ column squash load
- radius of gyration r
- equivalent radius of gyration for composite r<sub>e</sub> columns
- S standard deviation of test results on concrete strength
- t tube thickness
- concrete contribution parameter [1] α
- Υm partial safety factor for material strength
- characteristic cube strength  $\sigma_{cu}$
- characteristic cylinder strength  $\sigma_{cyl}$
- characteristic concrete strength  $\sigma_{\mathbf{k}}$
- mean value of concrete strength from tests  $\sigma_{m}$
- design strength of concrete  $\sigma_u$
- yield strength of steel
- σ<sub>y</sub> σ<sub>y</sub>\* reference yield strength of steel [1]
- reduced longitudinal strength of steel under  $\sigma_{yL}$ hoop tension
- φ coefficient used in estimating reduced steel strength and enhanced concrete strength

 $\sqrt{1+\phi+\phi^2}$  $\overline{\Phi}$ 

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#### Summary

A new method for the design of composite columns under concentric loading is presented. The method adopts the recently developed European curves for the design of axially loaded bare metal sections as the basic design curves for composite columns. The column slenderness factor has been redefined as the ratio of column length to a unit critical length, which is defined as the length for which the column squash load equals its Euler load. The design load shows excellent agreement with over 200 analytically exact results and with over 100 experimental results.

The method has been extended to include the design of concrete filled circular tubes taking due account of triaxial containment of the concrete. Comparison with over 150 experimental results on concrete filled circular tubes shows that the unified method gives very good correlation with the experimentally obtained ultimate load.

## Résumé

Les auteurs présentent une nouvelle méthode pour le calcul des colonnes mixtes soumises à des efforts centrés. Les courbes de base adoptées sont les courbes de flambement de la Convention Européenne. L'élancement intrinsèque a été défini comme rapport de la longueur de la colonne à celle pour laquelle la charge de ruine de la colonne est égale à la charge de flambage d'Euler. La méthode montre une excellente concordance avec plus de 200 résultats analytiques et plus de 100 résultats expérimentaux.

Le procédé a été étendu à l'étude des tubes remplis de béton, en tenant particulièrement compte des sollicitations triaxiales du béton. Là aussi la comparaison avec plus de 150 résultats expérimentaux montre une très bonne concordance.

# Zusammenfassung

Die Autoren behandeln eine neue Methode für den Entwurf von zentrisch beanspuchten Verbundstützen. Als grundlegende Entwurfskurven werden dabei die Knickkurven der europäischen Stalbaukonvention angenommen. Der bezogene Schlankheitsgrad wurde neu definiert als Verhältnis der Stützenlänge zur derjenigen Länge, für welche die Quetschlast der Stütze ihre Eulersche Knicklast erreicht. Die Übereinstimmung des Verfahrens mit über 200 theoretischen genauen Resultaten und über 100 Versuchsergebnissen ist ausgezeichnet.

Die Methode wurde auf betongefüllte Rohrstützen unter besonderer Berücksichtigung der dreiachsigen Betonbenaspruchung erweitert. Auch hier zeigt ein Vergleich mit über 150 entsprechenden Versuchsergebnissen eine sehr gute Übereinstimmung.