Theory of dynamic analysis of box girder bridges

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Theory of Dynamic Analysis of Box Girder Bridges

Théorie d'une analyse dynamique de ponts à poutres en caisson

Theorie einer dynamischen Berechnung von Kastenträgerbrücken

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Introduction

In almost all the studies on bridge vibration subjected to moving loads (known to bridge engineers as impact loads), the bridge is idealized as a beam. A review of the literature and extensive list of references are omitted herein as it can be found in a recent (Oct. 1970) article [11]. While the aforementioned idealization is reasonable for computing bending moments and shears for the bridge as a whole, it is not possible to evaluate dynamic effects on stresses in various elements of the bridge. For instance, in the box girder bridges, such as those shown in Figure 1, dynamic effects on membrane forces, transverse moments and shears in plate elements cannot be evaluated by the beam method. Although the total bending moment due to dynamic effect is found, the distribution of longitudinal stresses in a cross section can be obtained approximately by the beam theory only when the load is symmetrical to the longitudinal centerline.

The purpose of this study is to make a more exact analysis of the dynamic effects on the deflections, stresses, and moments in all plate elements of box girder bridges. In order to reduce the problem to one consisting of finite number of degrees of freedom, the distributed masses of the plate elements were replaced by equivalent concentrated masses applying at the joints. (The edge where the plate elements joint together is referred to as a joint.) Since both inertia forces due to vertical and rotational displacements of the plate elements are considered, consistent masses [1] concentrated along the joints are used. Moreover, since taking each plate between two joints in a bridge cross section as an element would not provide enough accuracy of both frequency (eigenvalue) and mode shape (eigenvector), each plate element is further divided into several longitudinal plate strips and new joints are introduced along the edges joining the plate strips (see Fig. 2).

With the structure divided into many plate elements and each element involving a number of consistent masses, the matrix size become fairly large. In order to simplify the problem, only simply supported box girder bridges with end diaphragms but no intermediate diaphragms will be considered. Also instead of the elaborate idealization of a truck the live load is simplified as consisting of four identical wheel loads represented by spring supported masses with the force in the spring distributed over a rectangular area (see Fig. 3). Unsprung masses and internal damping of the truck as well as structural damping of the bridge are neglected since their effects are relatively small [4, 11]. Although without the above simplifications, no great complication in formulation and programming results. The simplifications were introduced in order to save computer time and storage.

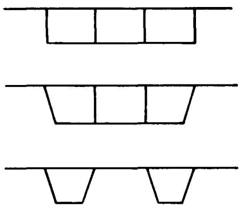

Fig. 1. Typical Box Girder Bridge Cross Section.

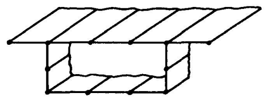

Fig. 2. Plate Element in a Box Girder Bridge.

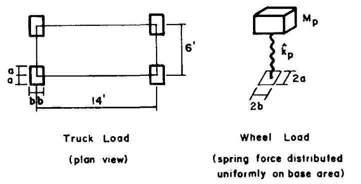

Fig. 3. Simplified Live Load.

The static relationships between displacements and internal forces are essentially those developed for folded plate analysis by GOLDBERG and LEVE [5] and others [2, 10]. Since these relationships and definition of symbols are virtually the same as given in reference [2] they will be omitted in the text. The main assumptions of the folded plate analysis are (a) plate elements are perfectly elastic, rectangular in shape and rigidly jointed along the longitudinal edges, (b) the transverse end of the plates are framed into diaphragms which are infinitely stiff in their own plane but perfectly flexible normal to their own plane and (c) no interaction exists between bending resisting forces (moments and shears) and membrane forces.

The approach of replacing the applied distributed forces by joint forces based on virtual work concept is believed to be novel. The expansion of distributed forces into a double Fourier series using normal mode functions is based on the work of IYENGAR and JAGADISH [7]. The matrix formulation of the equation of motion is based on a classical method [6].

A computer program was written in Fortran to be used in Univac 1108. Verification of the accuracy of the program were made by comparing the static and dynamic results obtained by the proposed method with those obtained by existing methods or with examples taken from published articles. Static and dynamic analysis were also performed for an example box girder bridge. However, since the inclusion of the above results will result in a paper too lengthy for publication, the results will be presented in a separate paper.

Equations for the dynamic Analysis of Box Girder Bridges

Equation of Motion of the Structure

With masses concentrated at the joints, the equation of motion of the structure neglecting damping may be written in the following form

$$[\Phi_{mn}]^t [\overline{M}_n] [\Phi_{mn}] \ddot{\Psi}_{mn} + [\Phi_{mn}]^t [\overline{K}_n] [\Phi_{mn}] \Psi_{mn} = [\Phi_{mn}]^t [Q_n] \tag{1}$$

$$\text{or } \ddot{\Psi}_{mn} + \omega_{mn}^2 \Psi_{mn} = [\overline{M}^*]^{-1} [\Phi_{mn}]^t [Q_n] \tag{2}$$

in which

Ψ_{mn} = time function for m-th mode n-th Fourier component.
$\ddot{\Psi}_{mn}$ = double dots denotes second derivative of Ψ_{mn} with respect to time (t).
$[\Phi_{mn}]$ = matrix (vector) of m-th mode for n-th Fourier component.
$[\Phi_{mn}]^t$ = superscript t denotes transpose of matrix $[\Phi_{mn}]$.
ω_{mn} = natural frequency of the m-th mode n-th Fourier component.
$[Q_n]$ = applied joint force matrix (vector) of n-th Fourier component to be defined later.
$[\overline{M}_n]$ = assembled consistent mass matrix of n-th Fourier component to be defined later.
$[\overline{M}^*] = [\Phi_{mn}]^t [\overline{M}_n] [\Phi_{mn}]$ (3)
$[\overline{M}^*]^{-1}$ = inverse of $[\overline{M}^*]$.
$[\overline{K}_n]$ = assembled stiffness matrix of n-th Fourier component $[\overline{K}_n]$ is an assembly of the stiffness matrices $[\overline{K}_n]_r$.
$[\overline{K}_n]_r = [R]_r^t [K_n]_r [R]_r$. (4)

in which

$[K_n]$ = local stiffness matrix of n-th Fourier component.
$[R]$ = rotation matrix relating local and global displacements.
$[R]^t$ = transpose of $[R]$ and suscript r denotes r-th plate element.

Frequency Equation

The m-th natural mode Φ_{mn} and frequency ω_{mn} of the n-th Fourier component are given respectively by the eigenvector and eigenvalue of the following equation

$$(\omega_{mn}^2 [\overline{M}_n] - [\overline{K}_n]) [\Phi_{mn}] = 0 \tag{5}$$

Applied Joint Forces

If a moving force is applied along a joint line, Eq. (1) can be solved by representing the force as a Fourier series in the space variable. However, if a distributed force is applied on a portion of a plate element, it must be converted to forces applied on the edges of the element such that the edge forces will cause the same dynamic effect as the distributed force.

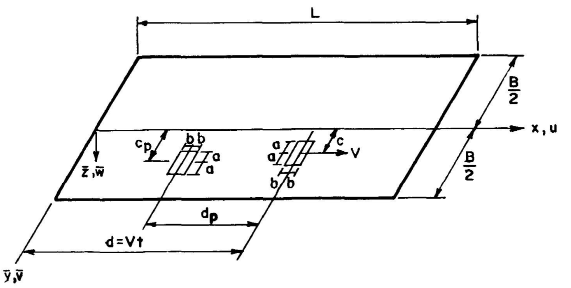

Fig. 4. Coordinates of the Wheel Loads.

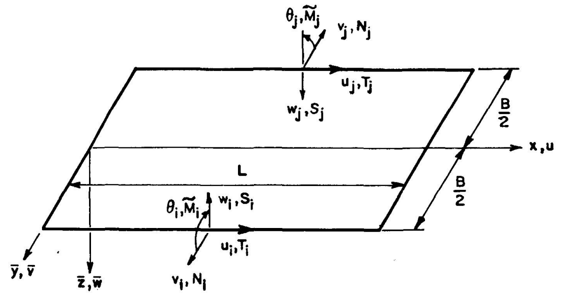

Fig. 5. Displacement Component in Local coordinate System.

As shown in Figure 4 the force $\overline{Q}$ is distributed on an area $2a \times 2b$, the center of which is at a distance c from the longitudinal centerline and a distance

$d = Vt$ from $x = 0$ in which V is the velocity and t is time. Let the force function $\hat{Q}(x, y, t)$ be expressed in the following form (6):

$$\hat{Q}(x, \bar{y}, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Omega(t)\, \hat{\Phi}_{mn}(\bar{y}) \sin \frac{n\pi x}{L} \tag{6}$$

$$\hat{Q} = \frac{\overline{Q}}{4ab} \text{ for } d - b < x < d + b \text{ and } c - a < \bar{y} < c + a \tag{7a}$$

$$= 0 \text{ elsewhere} \tag{7b}$$

in which Ω is a time function defined below and $\hat{\Phi}(y)$ is a plate deflection function due to the mode displacement Φ_{mn} along the longitudinal edges. (Expression of the deflection function will be given later in the section on consistent mass matrix). $\hat{Q}$ is also taken as equal to $\dfrac{\overline{Q}}{4ab}$ when the load enters the bridge with $d > 0$ and $b > d > -b$.

Multiplying Eq. (6) by $\hat{\Phi}_{m'n'}(\bar{y}) \sin \dfrac{n'\pi x}{L}\, dx\, d\bar{y}$ and integrating, then using the orthogonality property of $\hat{\Phi}_{mn}$ and the sine function, one obtains

$$\Omega(t) = \frac{\displaystyle\int_{c-a}^{c+a} \int_{d-b}^{d+b} \frac{\overline{Q}}{4ab} \hat{\Phi}_{mn}(\bar{y}) \sin \frac{n\pi x}{L}\, dx\, d\bar{y}}{\displaystyle\int_{-B/2}^{B/2} \int_{0}^{L} \hat{\Phi}^2_{mn}(\bar{y}) \sin^2 \frac{n\pi x}{L}\, dx\, d\bar{y}}$$
$$= \frac{4}{n\pi}\left(\frac{\overline{Q}}{4ab}\right) \frac{\displaystyle\int_{c-a}^{c+a} \hat{\Phi}(\bar{y})\, d\bar{y}}{\displaystyle\int_{-B/2}^{B/2} \hat{\Phi}^2(\bar{y})\, d\bar{y}} \sin \frac{n\pi b}{L} \sin \frac{n\pi Vt}{L} \tag{8}$$

The term $[\Phi_{mn}]^t [Q_n]$ in Eq. (1) may be considered as the virtual work of the force Q_n due to virtual displacement Φ_{mn}. In order that the edge forces will cause the same dynamic effect as the distributed force, the virtual work due to the edge forces must be equal to that due to the distributed force. Thus

$$[\Phi_{mn}]^t [Q_n] = \int_{-B/2}^{B/2} \hat{\Phi}^t_{mn}(y)\, \Omega(t)\, \hat{\Phi}(y)\, d\bar{y} = \Omega(t) \int_{-B/2}^{B/2} \hat{\Phi}^2_{mn}(\bar{y})\, d\bar{y}$$
$$= \frac{4}{n\pi} \frac{\overline{Q}}{4ab} \left(\int_{c-a}^{c+a} \hat{\Phi}_{mn}(\bar{y})\, d\bar{y} \right) \sin \frac{n\pi b}{L} \sin \frac{n\pi Vt}{L} \tag{9}$$

If there are several forces $\overline{Q}_p$ applied at $y = c_p$ and $x = Vt - d_p$ (see Fig. 4) then

$$[\Phi_{mn}]^t [Q_n] = \sum_p \frac{Q_p}{n\pi ab} \left(\int_{c_p-a}^{c_p+a} \hat{\Phi}_{mn}(\bar{y})\, d\bar{y} \right) \sin \frac{n\pi b}{L} \sin \frac{n\pi (Vt - d_p)}{L} \tag{10}$$

Equations of Motion of the Wheel Loads

As stated previously, a wheel load is considered as consisting of a mass $\hat{M}_p$ supported by a spring with spring constant $= \hat{k}_p$. The deflection of the mass $\hat{M}_p$ from its equilibrium position is ζ^p. For the mode component mn, the plate deflection at $\bar{y} = c_p$ and $x = Vt - d_p$ is given by $\Psi_{mn}(t)\,\hat{\Phi}_{mn}(c_p) \sin \frac{n(Vt - d_p)}{L}$. Then

$$\overline{Q}_p = \hat{M}_p g + \hat{k}_p \left[\zeta^p - \sum_{mn}\sum \Psi_{mn}(t)\,\hat{\Phi}_{mn}(c_p) \sin \frac{n\pi(Vt - d_p)}{L} \right] \tag{11a}$$

$$\overline{Q}_p = 0 \text{ when } Vt - d_p \leq 0 \text{ and } Vt - d_p \geq L \tag{11b}$$

$$\text{and } \hat{M}_p \ddot{\zeta}^p + \hat{k}_p \left[\zeta^p - \sum_{mn}\sum \Psi_{mn}(t)\,\hat{\Phi}_{mn}(c_p) \sin \frac{n\pi(Vt - d_p)}{L} \right] = 0 \tag{12}$$

in which g is the gravitational acceleration.

Final Equations

From Eqs. (2) and (10)

$$\ddot{\Psi}_{mn} + \omega_{mn}^2 \Psi_{mn} = [\overline{M}^*]^{-1} \sum_p \frac{\overline{Q}_p}{n\pi ab} \left(\int_{c_p - a}^{c_p + a} \hat{\Phi}_{mn}(\bar{y})\, d\bar{y} \right) \sin \frac{n\pi b}{L} \sin \frac{n\pi(Vt - d_p)}{L} \tag{13}$$

in which $\overline{Q}_p$ is given by Eq. (11). If there are four wheel loads, the final equations consist of the above equation plus four equations of Eq. (12) with $p = 1, 2, 3, 4$.

Consistent Mass Matrix

The matrix $[\overline{M}]$ is an assembled mass matrix of $[\overline{M}]$ for all plate elements. For plate r

$$[\overline{M}]_r = [R]_r^t [M]_r [R]_r \tag{14}$$

in which $[M]_r$ is the consistent mass matrix. The element $M_{\alpha\beta}^{ij}$ of $[M]_r$ is given by (1)

$$M_{\alpha\beta}^{ij} = \int_{-B/2}^{-B/2} \rho \bar{t} \eta_{\bar{\alpha} i} \eta_{\bar{\beta} j}\, d\bar{y} \tag{15}$$

in which ρ is the density, $\bar{t}$ is the thickness of the plate, $\eta_{\bar{\alpha} i}$ is a displacement function due to unit displacement of $\bar{\alpha} = (u, \bar{v}, \bar{w}, \text{ or } \theta)$ at edge i with other displacements equal to zero, and $\eta_{\bar{\beta} j}$ is defined similar to $\eta_{\bar{\alpha} i}$. The mass $M_{\alpha\beta}^{ij}$ is corresponding to the stiffness $K_{\gamma\beta}^{ij}$ in which $\bar{\gamma} = (T, N, S \text{ or } M)$ is corresponding to $\bar{\alpha} = (u, v, w \text{ or } \theta)$ with T corresponding to u, etc. The mass distribution in the x direction is a Fourier cosine series for the u displacement and Fourier sine series for the displacement v, w and θ.

The local displacement functions $\bar{w}_n$ u_n and $\bar{v}_n$ and the elements for the consistent mass matrix $[M]_r$ with subscript r omitted are given in Reference 8.

Method of Solution and Final Results

Method of Obtaining Eigenvalues and Eigenvectors

The success of the mode superposition method depends on the accuracy of the eigenvalues and eigenvectors. In the present study, the eigenvalues and eigenvectors are obtained by means of a method given in Reference [12]. Briefly, the method is as follows, for an eigenvalue problem of the following standard form;

$$[\tilde{A}]\{\Phi\} = \tilde{\lambda}\{\Phi\} \tag{16}$$

The largest eigenvalue, $\tilde{\lambda}_1$ and its corresponding eigenvector, Φ_1, are obtained by the usual method of iteration except that the pivot of the trial vector is taken in the same row as that of $[\tilde{A}]$ containing the largest absolute element. The eigenvector is normalized so that the sum of the squares of its components equal unity.

In the process of iterating for Φ_1, a vector Φ_1^* is obtained at a few iterations before reaching the final Φ_1. The vector Φ_1^* contains components of many modes ϕ_1, ϕ_2 ..., etc. Let $\phi_2' = [\tilde{A} - \tilde{\lambda}_1\ I]\phi_1^*$ in which I is the identity matrix. Then in vector ϕ_2', the ϕ_1 component is removed and the ϕ_2 component will predominate. Using ϕ_2' as the starting trial vector, and using Eq. (16) for iteration, one obtains the eigenvalue $\tilde{\lambda}_2$ and its corresponding eigenvector ϕ_2.

Let ϕ_2^* be the vector obtained at a few iterations before reaching the final ϕ_2. The components of ϕ_1 and ϕ_2 in ϕ_2^* are removed by taking $\phi_3' = [\tilde{A} - \tilde{\lambda}_1\ I]\phi_2^*$ and $\phi_3'' = [\tilde{A} - \tilde{\lambda}_2\ I]\phi_3'$. The vector ϕ_3'' is used as the starting trial vector in Eq. (16) for iteration to obtain $\tilde{\lambda}_3$ and ϕ_3. After a number of iterations, a vector $\bar{\phi}_3$ is obtained which may contain errors due to components of ϕ_1 and ϕ_2. These components are removed by taking $\bar{\phi}_3' = [\tilde{A} - \tilde{\lambda}_1\ I]\bar{\phi}_3$ and $\bar{\phi}_3'' = [\tilde{A} - \tilde{\lambda}_2\ I]\bar{\phi}_3'$. The vector $\bar{\phi}_3''$ is again substituted back into Eq. (16) for iteration to obtain $\tilde{\lambda}_3$ and ϕ_3.

Procedures similar to the one described above and used for obtaining other eigenvalues and eigenvectors. In order to save both time and storage, only single precision was used in the computation. As a result, a certain loss of accuracy becomes inevitable. Some indication of the accuracy of the results will be given in the numerical examples.

Reduction of Degrees of Freedom for Dynamic Analysis

In order to save computer time and storage, the sizes of the matrices used in the dynamic analysis were reduced by limiting the degrees of freedom taken into consideration. Since the loading is in vertical direction and may be unsymmetrical to the bridge centerline, vibrations of the bridge are considered to cause inertia forces only in the directions of global vertical displacement (Δ_z) and rotation about longitudinal axis (Δ_θ). Since there are no longitudinal and lateral oscillations, inertia forces in these global directions ($\ddot{\Delta}_x$, $\ddot{\Delta}_y$) will be assumed negligible. In Eqs. (1, 2 and 5), a reduced stiffness matrix as shown in references (6, 9) relating forces and displacements in the Δ_z and Δ_θ directions will be used instead of $[\bar{K}_n]$ and a mass matrix consisting only of the masses causing inertia forces in these directions will be used instead of $[\overline{M}_n]$. The reduced stiffness matrix $[\overline{K}]^*$ is obtained in the following manner. Let

$$\Delta_1 = \begin{bmatrix} \Delta_z \\ \Delta_\theta \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} \tag{17a, b}$$

$$\text{From} \quad \left[\begin{array}{c} \overline{F}_1 \\ \hline \overline{F}_2 \end{array}\right] = \left[\begin{array}{c|c} \overline{K}_{11} & \overline{K}_{12} \\ \hline \overline{K}_{21} & \overline{K}_{22} \end{array}\right] \left[\begin{array}{c} \Delta_1 \\ \hline \Delta_2 \end{array}\right] \tag{18}$$

with $\overline{F}_2 = 0$ it can be shown that $\quad [\overline{K}]^* = [\overline{K}_{11}] - [\overline{K}_{12}][\overline{K}_{22}]^{-1}[\overline{K}_{21}] \quad (19)$

Solution of the Dynamic Equations

The dynamic equations 11, 12 and 13 were solved simultaneously for ζ^p_{mn} and Ψ_{mn} by the 5th order ADAM'S method [3] using a Runge Kutta starting procedure.

Displacements, Forces and Stresses

Upon solution of the dynamic equations, the resulting displacements and forces must be obtained. The matrix (vector) of coefficients of the n-th Fourier component of edge displacements in global coordinates is given by

$$[\Delta_n] = \sum_m \Psi_{mn} [\Phi_{mn}] \tag{20}$$

The matrix $[\Delta_n]$ is the assembled matrix of $[\Delta_n]_r$

$$[\Delta_n]_r = [\Delta_{zi}\ \Delta_{yi}\ \Delta_{xi}\ \Delta_{\theta i}\ \Delta_{zj}\ \Delta_{yj}\ \Delta_{xj}\ \Delta_{\theta j}]_r^t \tag{21}$$

in which Δ_z Δ_y Δ_x and Δ_θ are displacements in global z, y, x and θ (rotation about x) directions respectively and i and j denote longitudinal jointing edges of plate element r.

The matrix of coefficients of n-th Fourier components of local displacements $[\delta_n]_r$ is given by

$$[\delta_n]_r = [\theta_{ni}\ \theta_{nj}\ w_{ni}\ w_{nj}\ u_{ni}\ u_{nj}\ v_{ni}\ v_{nj}]^t = [R]_r [\Delta_n]_r \tag{22}$$

in which u, v, w, θ are displacements in the local x, y, z and θ (rotation about x) directions respectively (see Fig. 5 for positive directions of θ_{ni}, w_{nj}, etc.) and $[R]$ is the rotation matrix defined previously.

The matrix of coefficients of n-th Fourier components of local edge forces acting on plate element r is given by

$$[F_n]_r = [\tilde{M}_{ni}\ \tilde{M}_{nj}\ S_{ni}\ S_{nj}\ T_{ni}\ T_{nj}\ N_{ni}\ N_{nj}]_r^t = [K_n]_r [\delta_n]_r = [K_n]_r [R]_r [\Delta_n]_r \tag{23}$$

in which $[K_n]$ = local stiffness matrix as defined previously, $\tilde{M}$ = transverse bending moment per unit length, S = transverse shear per unit length, T = membrane shearing stress resultant, N = membrane normal stress resultant and forces are acting along edges i and j as indicated by subscripts. The positive directions of $[\tilde{M}_{ni}, \tilde{M}_{nj}, S_{ni}, S_{nj}, T_{ni}, T_{nj}, N_{ni}, N_{nj}]$ are corresponding to the positive directions of $[\theta_{ni}, \theta_{nj}, w_{ni}, w_{nj}, u_{ni}, u_{nj}, v_{ni}, v_{nj}]$ respectively.

The final edge displacements and forces for plate r are given by

$$[\delta]_r = \sum_{n=1}^{\infty} [\tilde{s}_n\ \tilde{s}_n\ \tilde{s}_n\ \tilde{s}_n\ \tilde{c}_n\ \tilde{c}_n\ \tilde{s}_n\ \tilde{s}_n]\,[\delta_n]_r \tag{24}$$

$$[F]_r = \sum_{n=1}^{\infty} [\tilde{s}_n\ \tilde{s}_n\ \tilde{s}_n\ \tilde{s}_n\ \tilde{c}_n\ \tilde{c}_n\ \tilde{s}_n\ \tilde{s}_n]\,[F_n]_r \tag{25}$$

$$\text{in which } \tilde{s}_n = \sin\frac{n\pi x}{L},\ \tilde{c}_n = -\cos\frac{n\pi x}{L} \tag{26a, b}$$

The longitudinal stress at edge i in plate r is given by

$$(\sigma_{xi})_r = E\sum_{n=1}^{\infty}\frac{n\pi}{L}(u_{ni})_r \sin\frac{n\pi x}{L} + \frac{v}{t}\sum_{n=1}^{\infty}(N_{ni})_r \sin\frac{n\pi x}{L} \tag{27}$$

in which E = modulus of elasticity and v = Poisson's ratio.

The interior moments in any plate strip are given by

$$\tilde{M}_{xI} = \sum_{n=1}^{\infty} D\left(\frac{n\pi}{L}\right)^2 (1-v)\,(\tilde{M}_{xI})_n \sin\frac{n\pi x}{L} \tag{28a}$$

$$\tilde{M}_{yI} = \sum_{n=1}^{\infty} D\left(\frac{n\pi}{L}\right)^2 (1-v)\,(\tilde{M}_{yI})_n \sin\frac{n\pi x}{L} \tag{28b}$$

The coefficients of $(\tilde{M}_{xI})_n$ and $(\tilde{M}_{yI})_n$ are given in Reference 8.

Static Solution and Impact Factor

As indicated in Ref. [7], the static solution may be simply obtained by the following considerations: (a) In Eq. (11), consider only the term $\hat{M}_p g$ (omit the dynamic force in the spring). (b) In Eq. (1), consider only the effect of stiffness matrix (omit the term depending on time derivative of Ψ). Letting the solution of Ψ thus obtained $= \overline{\Psi}$, Eq. (13) becomes

$$\omega_m^2\,\overline{\Psi}_{mn}(t) = [\overline{M}^*]^{-1}\sum_p \frac{\hat{M}_p g}{n\pi ab}\left(\int_{c_p-a}^{c_p+a}\hat{\phi}_{mn}(\bar{y})\,d\bar{y}\right)\sin\frac{n\pi b}{L}\sin\frac{n\pi(Vt-d_p)}{L} \tag{29a}$$

$$\hat{M}_p = 0 \text{ when } Vt - d_p \le 0 \text{ and } Vt - d_p \ge L \tag{29b}$$

Since $\overline{\Psi}_{mn}(t)$ depends only on $Vt = d$, $\Psi_{mn}(t)$ is a function of d. Thus the static solution of $[\Delta_n]$ denoted by $[\overline{\Delta}_n]$ can be obtained by replacing Ψ_{mn} with $\overline{\Psi}_{mn}$ in Eq. (27)

$$[\overline{\Delta}_n] = \sum_m \overline{\Psi}_{mn}\,[\Phi_{mn}] \tag{30}$$

with $[\overline{\Delta}_n]$ known, the responses (deflections, forces, moments, and stress) may be obtained by the same equations as given in the previous section.

The amplification factors Λ_d and Λ_s for dynamic responses Γ_d and static response Γ_s respectively are defined by

$$\Lambda_d = \frac{\Gamma_d}{\Gamma_{sm}} \qquad \Lambda_s = \frac{\Gamma_s}{\Gamma_{sm}} \tag{31a, b}$$

is which Γ_{sm} is the maximum static response. The impact factor I, is defined as

$$I = \max|\Lambda_d| - 1 \tag{32}$$

It should be noted that in the evaluation of Λ_d, Λ_s and I, the values of Γ_d, Γ_s and Γ_{sm} should be taken for the response at the point under consideration, for example, the deflection at a specific plate joint at midspan, the moment at center of top plate at midspan, etc.

Conclusions

Theoretical formulation for the exact analysis of dynamic effects due to moving load on deflections, stresses and moments in all plate elements in simply supported box girder bridges is presented. Each plate element is further divided into several longitudinal plate strips and consistent masses are applied along the lines joining the strips. The truck is simplified into 4 identical wheel loads represented by spring supported masses with the force in the spring distributed over a rectangular area. Unsprung masses and internal damping of the truck as well as structural damping of the bridge are neglected. The spring force is expressed in the form of double Fourier series and the distributed force is replaced by forces along the joint lines based on the principle of virtual work. The general method is essentially based on mode superpositions and the method chosen for obtaining the frequencies and mode shapes are given. Governing matrix equations, displacement functions, consistent mass matrix and expressions for plate moments at interior points are presented.

Although numerical studies will be presented in a companion paper, two general observations may be made herein.

1. In order to obtain accurate results, many plate strips must be taken and many modes must be included. However, the higher the number of modes included, the smaller the time increment (which is proportional to the period of the highest mode) required in a dynamic analysis. As the computer tie is inversely proportional to the time increment, it becomes impractical to include large number of modes.
2. Since the proposed method is based on mode superposition, it is essential that not only the eigenvalues (frequencies) but also the eigenvectors (mode shapes) be determined efficiently and accurately. Many methods are available to determine accurate eigenvalues but such is not the case for eigenvectors. It is beyond the scope of this study to make an exhaustive evaluation of all available methods. However, it is hoped that more powerful methods will be developed in the near future.

As pointed out previously, practical application of the theory will be presented in a companion paper mainly for the reason of shortening the paper to reasonable length. By knowing more precisely the dynamic effects of moving loads on box girder bridges, the theorectical formulation presented herein would undoubtedly make a meaningful contribution to the safety and economy on the design of such bridges.

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Summary

Theoretical formulation for elastic analysis of dynamic effects due to moving loads on deflections, stresses and moments in all plate elements in simply supported box girder bridges is presented. The truck is simplified into 4 identical wheel loads represented by spring supported masses with the force in the spring distributed over a rectangular area. Governing equations, displacement functions, consistent mass matrix and expressions for plate moments are given. Practical application of the theory will be given in another paper to demonstrate its contribution to the safety and economy of box girder bridge design.

Résumé

L'article présente une théorie pour l'analyse élastique des effets dynamiques des charges mobiles sur les déflexions, tensions et moments dans tous les éléments de ponts à poutres en caisson sur appuis simples. Le véhicule est simplifié en

4 charges de roue qui sont représentées par des masses supportées par ressorts et dont la force du ressort est distribuée sur une surface rectangulaire. On établit les équations principales, les fonctions de déplacement, la matrice des masses et les moments des plaques. L'application pratique de la théorie sera présentée dans l'article suivant montrant aussi la contribution à la sécurité et à l'économie d'un pont à poutres en caisson.

Zusammenfassung

Es wird eine theoretische Formulierung der elastischen Berechnung von Durchbiegungen, Spannungen und Momenten in allen Plattenelementen einfach gelagerten Kastenträgerbrücken unter dynamischen Einflüssen bewegter Lasten vorgelegt. Die Belastung wird durch 4 gleiche Radlasten mit elastisch aufgelagerter Masse dargestellt, wobei die Federkraft über eine rechteckige Fläche verteilt wird. Es werden bestimmende Gleichungen, Verschiebungsfunktionen, die folgerichtige Steifigkeitsmatrix und Ausdrücke für die Plattenmomente angegeben. Die praktische Anwendung der Theorie im Hinblick auf Sicherheit und Wirtschaftlichkeit der Kastenträgerbrücken wird im anderen Beitrag dargelegt.

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