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Autor: Levy, M. / Spira, E.

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Experimental Study of Masonry Walls Strengthened by Reinforced Concrete Elements

Etude expérimentale de murs en maçonnerie renforcés d'éléments en béton armé

Experimentelle Untersuchung an durch Stahlbetonelemente verstärktem Mauerwerk

M. LEVY

D.Sc., Previously Research Fellow, Struct. Dept., Building Research Station, Technion - Israel Institute of Technology, Presently Chief Struct. Engineer in "Yotam" Advanced Design Consultants Ltd. Haifa, Israel.

E. SPIRA

Professor, Fac. of Civil Engineering, Technion - Israel Institute of Technology.

Introduction

The structural behaviour of a masonry wall bordered by reinforced concrete, which is a structural element of some importance, has not been analyzed in a satisfactory manner till now. It has been under intensive investigation during the last decades. In masonry infilled frames the influence of openings on the lateral rigidity [3] and the resistance against differential support settlements [6] was studied. The great majority of the theoretical studies, which either determine displacements by the Finite-Element method [3] or the Stress Function by Finite Differences techniques [2, 4, 8] are based on the assumption of linear elasticity. But in view of early cracking of the masonry, owing to the brittleness and separation of the components, the validity of the linear elastic analysis is rather limited in range as far as the structural behaviour is concerned.

In [8] the authors report on an experimental investigation of masonry walls bordered by r.c. elements. In the present paper the findings of the above experiments are examined in the light of a theoretical linear analysis. Special attention is paid to the question of the validity range of the theoretical analysis, and procedures for predicting cracking or failure loads are suggested.

General Description of Experiments

The objective was to study the structural behaviour of masonry walls bordered by r.c. elements and to compare it with the predicted results obtained by linear elastic analysis, cf. [2, 8].

Two test series are discussed:

Series A – Dealing with uniformly loaded and simply supported walls.

Series B – Dealing with continuous walls with openings, acted upon by differential support settlements.

Series *A* comprises 8 specimens with and without edge columns, which represent 1:2 scale models of actual wall panels, the masonry being either of 7.5 cm thick "Ytong" blocks (light weight aerated cellular concrete) or 10 cm thick hollow block concrete. They were tested in 1958-1960 in the Building Research Station of the Technion - Israel Institute of Technology, by Dr. S. Rosenhoupt, as reported in [5]. Some additional conclusions flowing from these experiments are brought here.

Details of the specimens are shown in Figs. 1-3 where the failure crack patterns are indicated.

Series *B* comprises two specimens (*B*1, *B*2), representing 1:2 scale models of hollow concrete block walls on three supports with door openings. Details are shown in Figs. 5-7.

The loading was by forces corresponding to differential support settlements.

Material Properties

Concrete (age 28 days):

Series	Cube strength (kg/cm^2) measured on 12 cm cubes		Modulus of elasticity (kg/cm^2) measured on 10/10/52 cm prisms	
A	142	191	175,000	235,000
B	206	228	230,000	270,000

Note: The higher values correspond to bottom beams, the smaller ones — to top beams and posts.

Concrete hollow blocks (two holes):

Series	Dimensions	Crushing strength (kg/cm^2) related to gross area	Modulus of elasticity (kg/cm^2) measured on masonry pier
A	10/10/20	32	20,000
B	10/10/20	78	75,000

"Ytong" blocks (Series *A*): – 7.5/15/25 cm, 650 kg/m^3 specific weight; crushing strength 14.5 kg/cm^2; modulus of elasticity 6900 kg/cm^2, measured on a 7.5/61.5/50.5 cm masonry.

Mortar for joints:

Series	Crushing strength (kg/cm^2) measured on 7 cm cube	Tensile strength in flexure (kg/cm^2) measured on prisms 7/7/28 cm
A	4.5	3.4
B	89	22

Reinforcement steel: modulus of elasticity 2,090,000 kg/cm^2:

Series	Diam (mm)	Yield joint (kg/cm^2)	Ultimate strength (kg/cm^2)
A	6	4,830	6,220
	8	3,150	4,100
	12	3,140	4,240
B	6	3,129	4,450
	8	3,770	4,830
	10	4,270	5,400

Loading, measurement and results

All specimens of series *A* were loaded on the top beam by means of an AMSLER hydraulic press.

Measurements read included:

— Vertical strains across the first masonry layer (on the bottom beam) and horizontal strains in 3 vertical sections, measured by mechanical strain gauges of 0.001 mm reading accuracy.

— Deflection of the bottom beams by means of deflectometers of 0.01 mm reading accuracy.

Figs. 1, 2 give the measured results versus their predicted values for specimens A4 and A7. Dotted lines and figures in brackets are values obtained by elastic analysis.

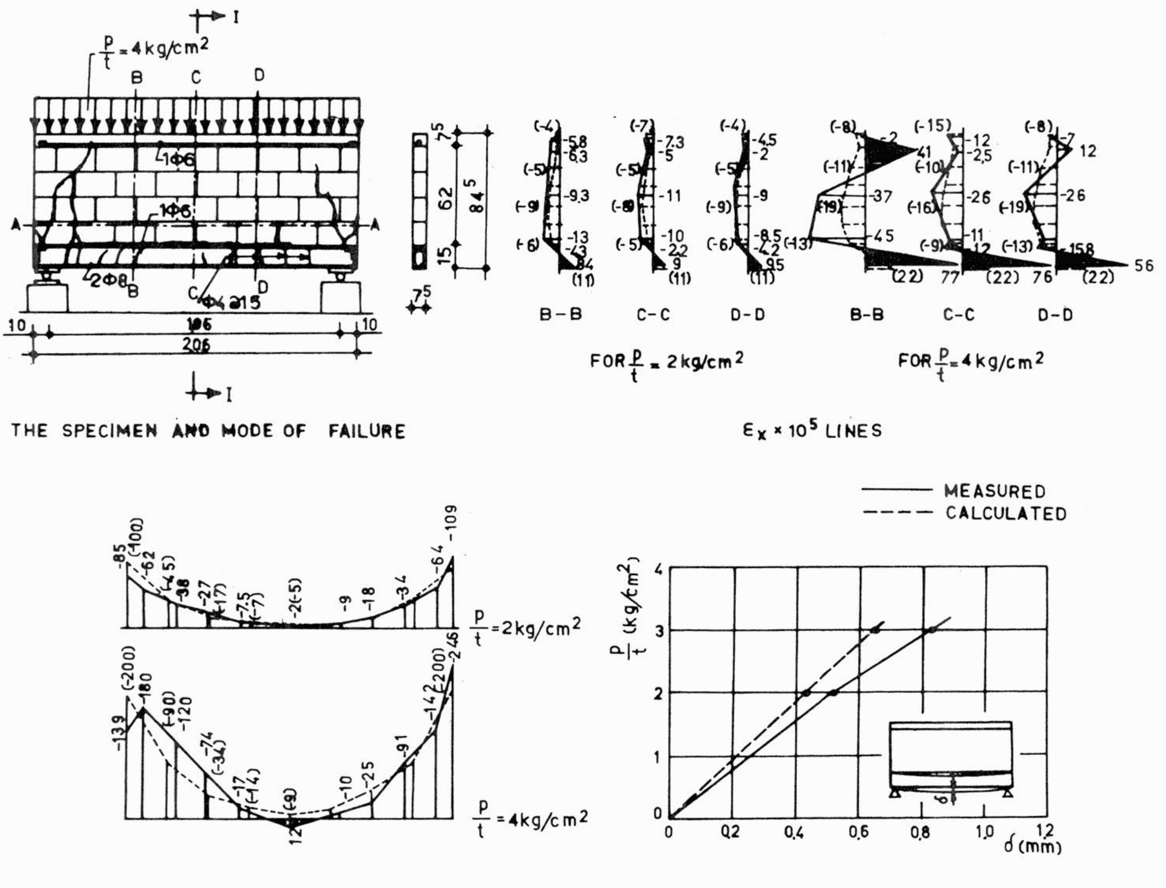

Fig. 1. Test A4.

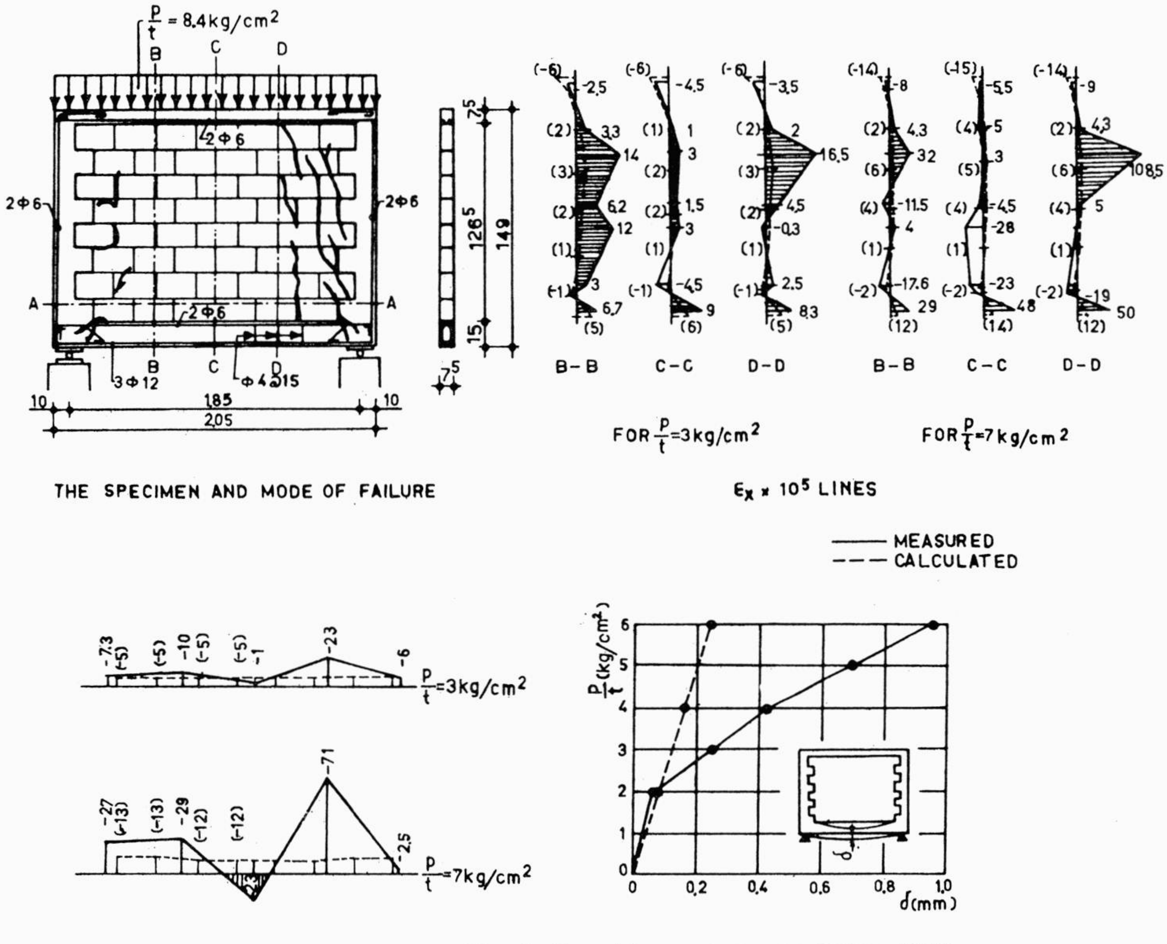

Fig. 2. Test A7.

Fig. 3 shows the crack patterns at ultimate load and in Fig. 4 the curves of mid span deflection of bottom beams.

Specimens B1 and B2 – (Figs. 5 to 7) which were erected as continuous units together with the base blocks and base columns, rest on 3 pairs of (base) columns, and were separated from them by paper layers, in order to eliminate tensile forces between them.

The loading included 2 parts:

- Uniformly distributed load at the top amounting to a total of 4.0 tons (i.e. = 1.0 t/m) supplied by 4 jacks of 17.0 tons capacity each.
- A variable concentrated load from below, supplied by a 10.0 ton jack (Fig. 6). The load (from below) was applied in two manners:
- First at the central support, while the two edge columns were anchored in the testing floor up to ultimate load ("Convex bending").
- Then the load was applied at one of the edge supports, while the other 2 columns were anchored in the testing floor ("Concave bending").

"Convex bending" — The load was applied in rates of 200 to 500 kgs until incipient failure, while at each load level the upward displacement of the point of load application was measured by 3 deflectometers. In addition, the strains in the masonry were measured, once before applying the load from below, and a second time, when it had the value of 4.5 tons.

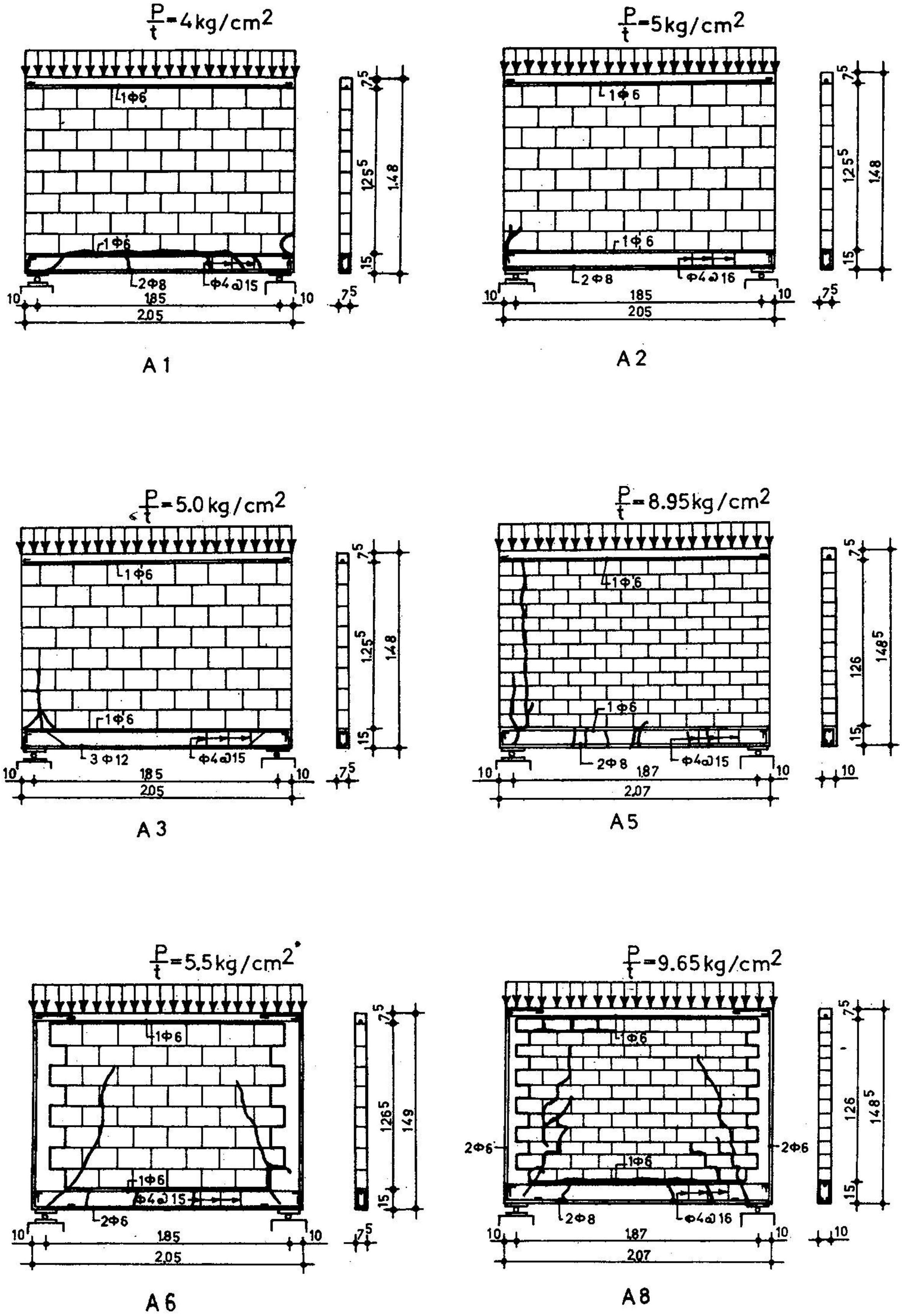

Fig. 3. Series A, Specimens and Modes of Failure.

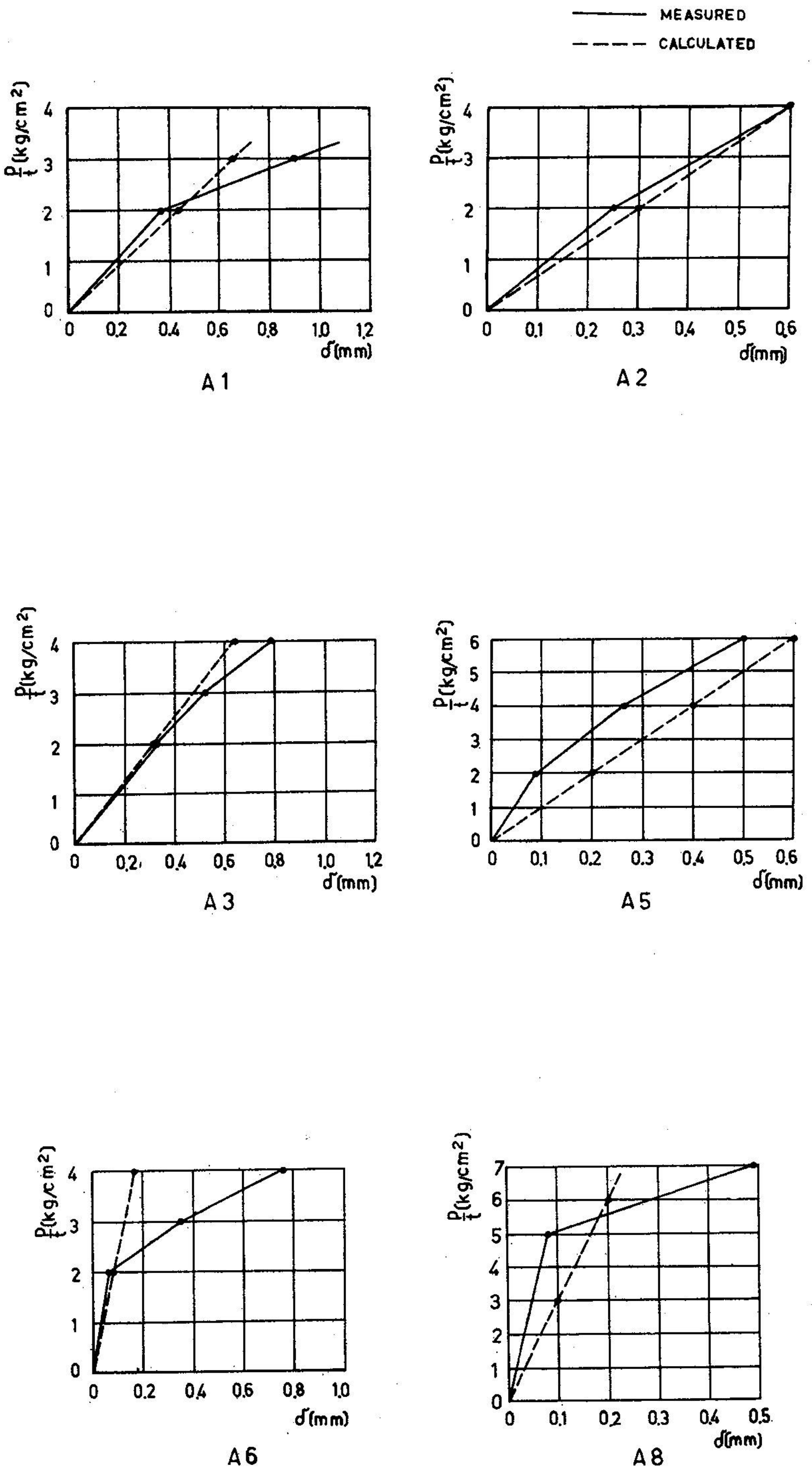

Fig. 4. Series A, Load Deflection Curves.

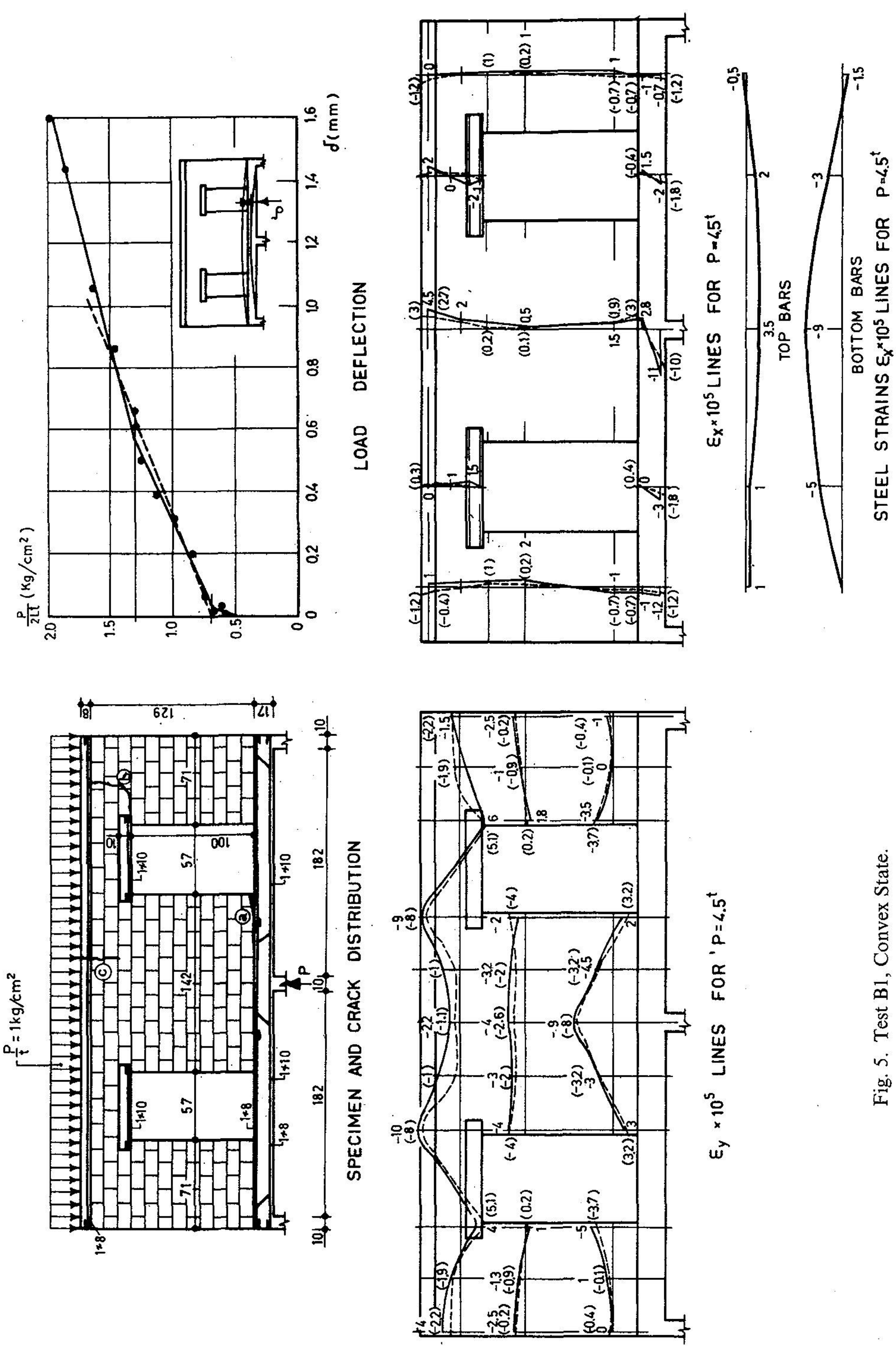

Fig. 5. Test B1, Convex State.

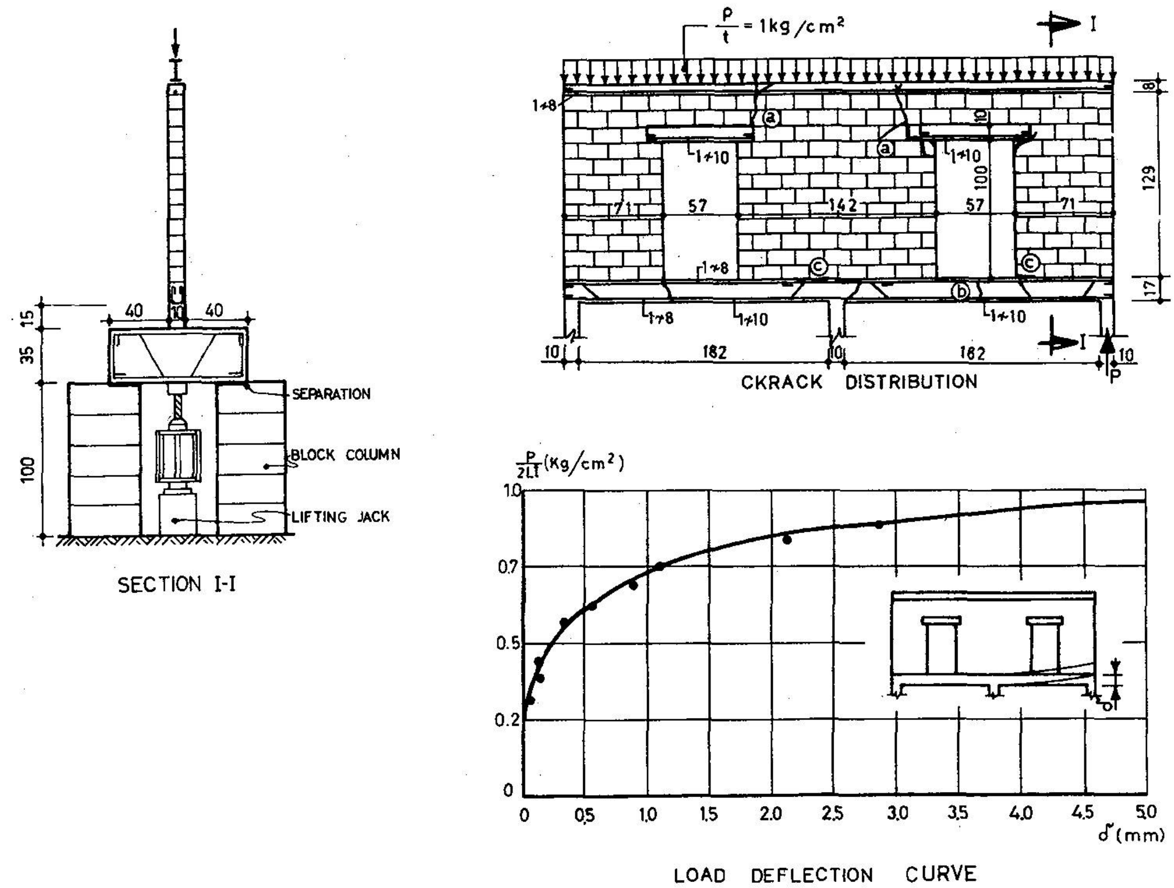

Fig. 6. Test B1, Concave State.

The reinforcement strains were measured by electric-resistance strain gauges. The crack formation was recorded at each load level.

"Concave bending" — The load was again applied in rates of 200 to 500 kgs until complete failure of the structures. The upward displacement of the point of load application was measured at each load level, and the formation of additional cracks was recorded.

Part of the test results, including $P/2Lt$ — δ graphs and cracks patterns, vertical and horizontal strains in B1 are shown in Figs. 5 to 7.

The theoretical analysis was performed for "Convex bending" only. For this purpose 2 separate load cases of the wall, considered supported at the 2 external footings only, were analyzed:

— Uniformly distributed external load.
— Concentrated upward load at the central support.

From the numerical results (omitted here), the midspan deflections and the strains in masonry were evaluated and indicated in Figs. 5, 7 by dotted lines and values in brackets.

A summary of loads causing visible cracking and failure and description of modes of cracking and failure is given in Table 1.

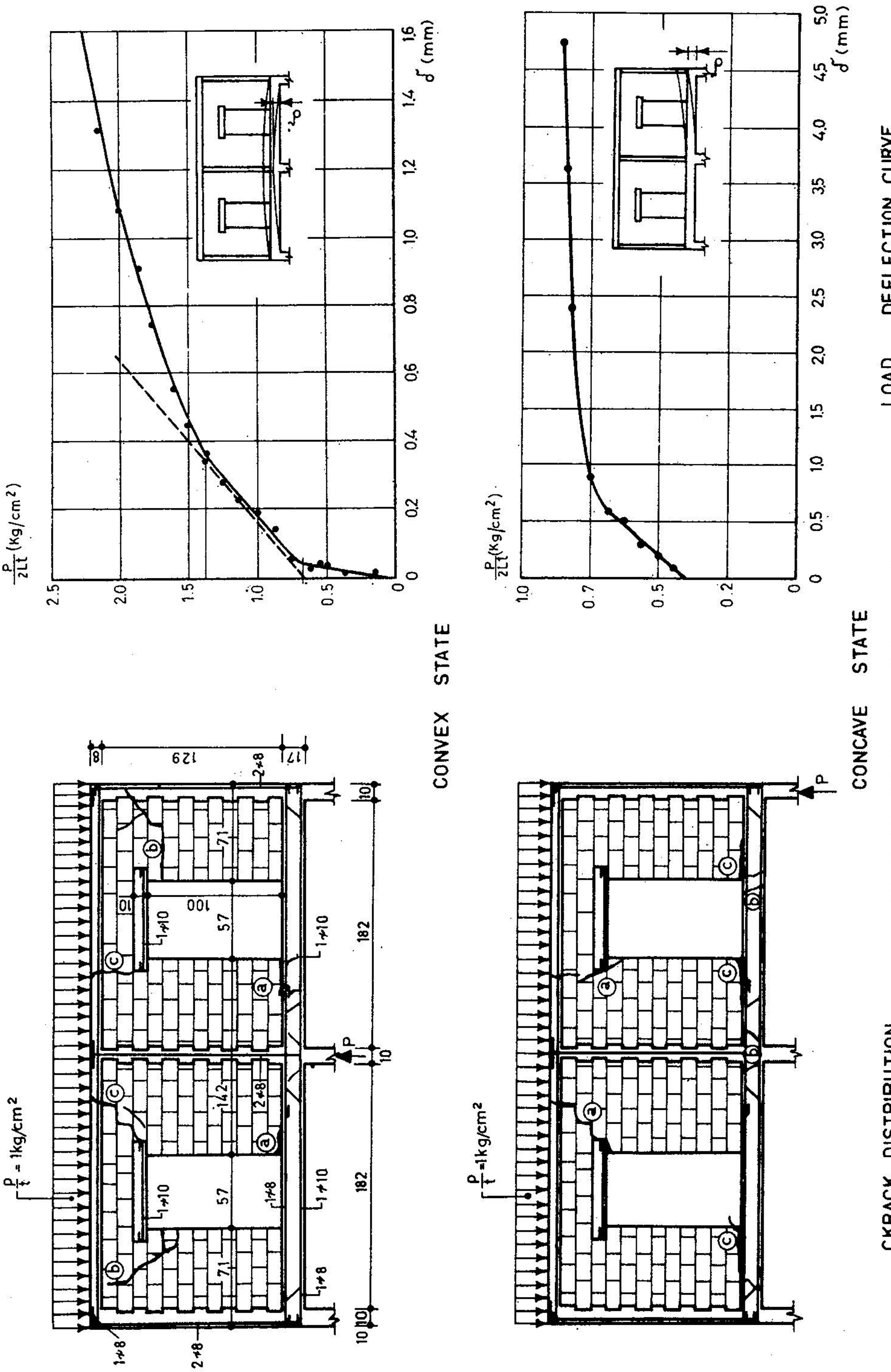

Fig. 7. Test B2, Convex and Concave State.

Table 1

Specimen, wall type	Masonry material	Cracking load (kg/cm²) (1)	Mode of cracking	Failure load (kg/cm²) (1)	Mode of failure (2)
A1, without edge columns	Ytong	3.0	Tension cracks in bottom beam	4.0	Rapidly increasing cracks and deflections of bottom beam, and crushing of outermost block above support
A2, without edge columns	Ytong	5.0	Cracks in outermost block above support	5.0	Crushing of outermost block above support
A3, without edge columns	Ytong	5.07	As in A2	5.07	As in A2
A4, without edge columns	Ytong	4.0	Tension cracks in bottom beam	4.0	Crushing of outermost block above support and diagonal cracks along the height of masonry
A5, without edge columns	Conc. hollow blocks	5.0	Tension cracks in bottom beam	8.95	Rapidly increasing cracks and deflections of bottom beam, vertical cracks along the height of masonry
A6, with edge columns	Ytong	3.0	Diagonal crack at the beampost joint	5.5	Rapidly increasing deflections and cracking of bottom beam, and diagonal cracks in beam and wall
A7, with edge columns	Ytong	4.0	As in A6	8.4	Crushing of masonry and diagonal cracks in wall near support
A8, with edge columns	Conc. hollow blocks	7.0	As in A6	9.65	As in A6
B1, without edge columns	Conc. hollow blocks	1.40	Separation of wall from bottom beam near the inner bottom corner of opening (a), cracking of wall near the top outer corner of opening (b) and tension cracks in top beam (c)	2.03	Disruption of wall continuity near the top outer corner of opening (b), and yield of top beam reinforcement (c) - fig. 5.
B2, with edge columns	Conc. hollow blocks	1.52	As in B1	2.20	As in A6 - fig. 7

[1] The cracking and the failure load is defined

for series A as $\dfrac{\text{uniformly dist. load}}{\text{wall thickness}}$ i.e. $\dfrac{p}{t}$ for series B as $\dfrac{\text{concent. load at central support}}{\text{wall length} \times \text{wall thickness}}$ i.e. $\dfrac{P}{2Lt}$

[2] The primary cause of failure indicated here represents the authors' opinion which does not necessarily conform with that stated in [5].

Discussion of Test Results, Proposed Procedures for Predicting Ultimate Loads

Series A – full masonry.

Perfect agreement between test results and those to be expected from (linear elastic) calculations was not achieved and could not be expected because of the following reasons:

a) The strength of the masonry, acting as a whole, seems to be considerably undervalued by the results of the preliminary tests. Possibly, this is due to the very low quality of the joint mortar in those preliminary test specimens. Also the crushing strength of "Ytong" blocks, stated to be 14.5 kg/cm^2, is very much lower than the minimum values known to the authors (18-24 kg/cm^2).

b) While the assumption of linear behaviour appeared to be justified for "Ytong" masonry, it did not appear to be so for concrete block masonry, even at very low load levels.

c) In infilled frames (A6, A7, A8) there is a geometrical difference between the model for analysis and the test specimens, in as much as the r.c. columns are connected with masonry by indentations.

d) In some cases difficulties were encountered in determining the primary cause of failure, because of the appearance of additional cracks at high load levels, which blurred the visibility of the primary cracks or the crushed part of the masonry.

1. Walls without edge columns

For "Ytong" masonry, for which a realistic estimate of modulus of elasticity was made, reasonable even quantitative agreement between measured and calculated results was obtained, up to cracking load.

For the majority of these specimens failure load was not much above cracking load so that the elastic analysis can be considered valid for almost the entire load range. On the other hand, for hollow block masonry, where the masonry showed non-linear behaviour throughout (therefore a realistic E_w could not be determined) the agreement between test and calculation was only a quantitative one.

The load carrying capacity of masonry walls without edge columns is determined mainly by two factors:

— The tensile strength of the bottom beam.
— The crushing strength of the masonry.

Masonry crushing is sometimes accompanied by spalling and flexural-shear failure of the bottom beam; this is because, with progressive masonry crushing an increasing part of the total shear acts on bottom beam, which is not designed for it.

The intrinsic reason for the predominance of the two above named factors can be seen in the arching effect typical of deep beams [1].

In order to demonstrate this, the principal compressive and tensile stresses in the masonry of specimen A4 were calculated and found to be $-4.6\frac{p}{t}$ and $0.35\frac{p}{t}$ respectively, which confirms the above statement. Although the appearance of (shear) cracks, due to the principal tensile stresses is probable in most cases (as in A4 and A5) the wall will continue to transmit load as long as the crushing strength of the masonry, or the tensile strength of the bottom beam, has not been exceeded.

In view of the above said, prediction of the load carrying capacity of a wall without edge columns may be reduced to the evaluation of the maximal compressive stresses in the masonry and the inner forces of the bottom beam; for this purpose linear analysis or design aids (in the form of approximation formulae or graphs based on linear analysis) may be used. Design aids of this kind were given by the authors in [2].

For illustration, approximate analysis for A4 is shown in the following:
By formula (17) of [2]

$$K_1 = 2\left(\frac{E_c I}{E_w t}\right)^{1/3} = 2\left(\frac{235{,}000 \times 2{,}494}{6{,}800 \times 7.5}\right)^{1/3} = 45 \text{ cm}$$

$$\left(\bar{a} = \frac{a}{K_1} \cong 0.22;\ \frac{2L}{K_1} = \frac{206}{45} \cong 4.6\right)$$

The maximal values of compressive stress in the wall and the bending moment of the bottom beam are obtained from the graphs of Fig. 11 in [2] as follows: (values obtained by full numerical analysis are given in brackets)

$$\sigma_{y,\,\max} = 1.995\frac{R}{K_1 t} = -1.995 \times \frac{103}{45}\frac{p}{t} = -4.56\frac{p}{t}\left(-4.51\frac{p}{t}\right)$$

$$M_{x=2.3\,K_1} = 2 \times 0.0468\, RK_1 = 0.0934 \times \frac{45}{103} pL^2 = 0.0406\, pL^2\ (0.037\, pL^2)$$

Note: Multiplication by 2 takes account of action of concentrated loads (support reactions) at both ends.

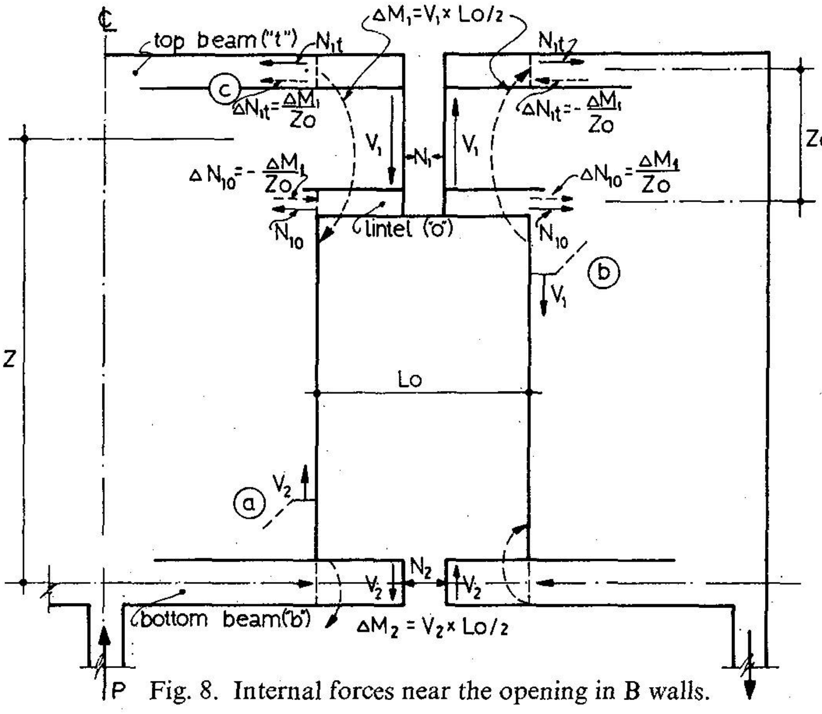

Fig. 8. Internal forces near the opening in B walls.

For the evaluation of the tensile force in the bottom beam use of this graph is not advisable because here $2L < 8K_1$; therefore expression (22) in [2] is recommended

$$\overline{N} = \frac{N}{R} = 2\left[\frac{4}{\pi^2 - 4}\left(\frac{\pi}{2} + \frac{2\frac{x}{K_1} - \pi}{2\left(\frac{x^2}{K_1^2} + 1\right)} - \text{arc tan}\frac{x}{K_1}\right) - 2\frac{\frac{a}{K_1}\frac{x^2}{K_1^2}}{\left(\frac{x^2}{K_1^2} + 1\right)^2}\right]$$

$$= 2 \times 0.682\left(1.5708 + \frac{2 \times 2.3 - 3.1416}{2(2.30^2 + 1)} - \text{arc tan } 2.3\right)$$

$$- 2 \times 2\frac{0.22 \times 2.3^2}{(2.3^2 + 1)} = 0.57\,R = 0.57\,pL(0.56\,pL)$$

Wall specimen A4 failed at $\frac{p}{t} = 4.0$ kg/cm^2 with computed a maximal compressive stress of 18 kg/cm^2.

Tensile cracks appeared on the bottom beam. The steel strain measured at this load level was $E_s = 890 \times 10^{-6}$ corresponding to a stress of 1870 kg/cm^2.

The computation yielded:

$M = 0.0406 \times 4.0 \times 7.5 \times 103^2 = 12{,}900$ kg/cm^2 (11,800 kg/cm^2)
$N = 0.57 \times 4.0 \times 7.5 \times 103 = 1{,}770$ kg (1760 kg)

These values produce, by customary r.c. section calculations, a stress of 1,950 kg/cm^2 (1,812 kg/cm^2) which is very close to the above measured value. So, even at a load level near failure load, linear theoretical analysis yields results which may be considered reliable.

To sum up the findings of these sub-series tests, most specimens failed by masonry crushing; In wall A1 cracks opened up in the weakly reinforced ($3\phi6$) bottom beam (while the calculated reinforcement stress was 3,000 kg/cm^2); in A5 the compressive stress in the masonry approached the crushing strength but the load carrying capacity was determined by the exhaustion of the tensile strength of the bottom beam. There appeared cracks in masonry which were vertical, not inclined as would be expected, probably due to weakness of the vertical mortar joints.

2. *Walls with edge columns*

This series comprises 3 specimens. For two of them, namely A6 and A7, there is good agreement of test results with the calculations, up to a load level of $\frac{p}{t} = 2.0$ kg/cm^2. Beyond this, the measured deflections exceeded the calculated values (see Figs. 2 and 4) which seems to be due to the gradual loosening of the connection of the masonry to the edge columns.

Some measure of verification of this supposition can be had from the calculated values for $\frac{p}{t} = 2.0$ kg/cm^2.

$\tau_{max} = 0.64\frac{p}{t} = 1.28$ kg/cm^2
$\sigma_1 = 0.54\frac{p}{t} = 1.08$ kg/cm^2

The value of the principal tensile stress is close to the tensile strength of "Ytong" blocks. From this load level on, the wall appears to transmit loads by a changed statical schema i.e. as if without edge columns. With the reorientation of the wall behaviour towards arch action, the strength of tension element (bottom beam) and that of the compression arches (masonry) become decisive for the carrying capacity of the wall.

In A6 the bottom was weak (reinforcement $3\phi 6$); the carrying capacity of the wall was exhausted at $\frac{p}{t} = 5.5$ kg/cm^2, due to excessive tension of the bottom beam accompanied by diagonal cracks in the wall.

In A7 the tension beam was strong (reinforcement $2\phi 6 + 3\phi 12$) and the wall failure at $\frac{p}{t} = 8.4$ kg/cm^2 was due to crushing of the masonry accompanied by shear cracks.

To complete the record, it is noted that specimen A8 (in which the lack of agreement between tests and calculations is similar to that in A5) failed due to tension of the bottom beam (as in A6) accompanied by shear cracks in the wall and separation between bottom beam and masonry.

The described three tests may warrant the following conclusions concerning the behaviour under load of walls with edge columns:

The presence of the edge columns causes a drastic reduction of the compressive stresses in the masonry (cf. [2], p. 149); consequently the schema of internal action emphasizes the "cables" (as opposed to the "arch action"). This is illustrated by the relatively large value of the principal tensile stress $-\sigma_1 = 0.54\frac{p}{t}$ in A6 and A7 — (compared with $0.35\frac{p}{t}$ in A4). But this situation can remain unchanged only until these stresses reach the tensile strength of the masonry, which occurs usually at 20%–30% of the ultimate load.

From this load stage on, a gradual shear cracking develops, invisible at first and clearly seen at the last load stages. Due to the emphasized arch action the wall carrying capacity returns to the dictated by the crushing strength of the masonry and/or by the tensile strength of the bottom beam.

A satisfactory ultimate load theory for infilled frames has not yet been proposed; so, based on the above discussed test results and those reported in [1,7] the following procedure may be suggested:

We may first evaluate the load carrying capacity of a wall without edge columns — which corresponds to the given wall in every respect (measurements, strength of masonry and edge beams) — by the procedure described in the preceding paragraph.

Then the effect of edge columns will be expressed by addition (to the above value) of the following percentages:

If the carrying capacity of the corresponding wall is dictated by the bottom beam — approximately 10% ÷ 15%.

— If it is dictated by the strength of the masonry.

— 40% ÷ 60%, depending on the depth of the edge columns. For the greater part of the usual cases the depth is approximately one twentieth of the span, for these the lower value (40%) is recommended.

The test results of A7, A8 and A6 — which correspond to A3, A5 and A1 respectively, may now be reviewed as follows:

All three specimens of this subseries had deep edge columns. A3 failed through masonry crushing the ratio of ultimate loads for A7 and A3 amounts to $\frac{8.40}{5.07} = 1.66$, A5 failed by excessive tension of the bottom beam, the ratio of ultimate loads for A8 and A5 is $\frac{9.65}{8.95} = 1.08$. The failure of A1 was due to combination of masonry crushing and bottom beam insufficiency and the ratio of ultimate loads for A6 and A1 was $\frac{5.5}{1.4} = 1.38$ which is an average of the values suggested for the two failure modes (1.10 and 1.60).

— Series B — walls with openings.

As can be seen from the description of the loading arrangement, the specimens of this series, B1 and B2, were loaded from below by a lifting jack at the middle support (Fig. 6).

Before describing the behaviour of the specimens under such loading beyond the value corresponding to the reaction force, it may be mentioned that the measured reaction (of 2,000 kg for both specimens) was in good agreement with the values calculated using the "Force Method" [2,8] and also to that obtained by the proposed approximate calculation in [2] (1,900 kg and 1,940 kg for B1 and B2 respectively).

Jacking loads beyond the value of the original support reaction caused "Convex bending" (see graphs $\frac{P}{2Lt} - \delta$) in Figs. 5, 7 and diminishing external reactions. At the moment of their becoming negative (i.e. anchoring forces) there appeared cracks (see Figs. 5, 7 and Table 1) at:

— Inner bottom corner of the opening (marked *a*).
— Top outer corner of the opening (marked *b*).
— Tension cracks at top beam (marked *c*).

The development of cracks *b* and *c* — from their appearance until failure load — was rapid, the load increase amounting to 50%. The load carrying capacity was dictated mainly by the tensile strength of top beam and the diagonal tensile strength of the outer regions of the masonry. No particular strengthening influence can be attributed to the presence of the r.c. posts in specimen B2. The agreement between the calculations and measurements taken during the experiments was satisfactory up to 70% of ultimate loads.

In the following an approximate design procedure is proposed, which supplements the qualitative indications given in [2].

The key question for walls with openings is the estimation of the integral inner forces in the vertical sections of the wall parts above and below the opening.

On the basis of the analysis of walls B and other walls with openings it may be assumed that these parts have points of Zero-curvature (inflection points), approximately at their centres; so, that the integral moment produces a couple of axial forces, obtained easily from lever arm considered as the distance between the middle lines.

Further, the shear force in each of these parts is approximately proportional to the relative equivalent stiffnesses of the top (subscript 1) and bottom (subscript 2) wall parts

$$V_i = V \frac{S_i}{\sum\limits_{j=1,2} S_j} \quad (i = 1, 2) \tag{1}$$

Where the equivalent stiffness is defined by

$$S_i = D_i + \sum_k K_{1k} \tag{2}$$

D_i = the depth of the masonry above (and sometimes below) the opening.
K_{1k} = the relative bending rigidity of the r.c. stiffening element as defined by expression (17) in [2], namely

$$K_{1k} = 2\left(\frac{E_c I_k}{E_w t}\right)^{1/3}$$

In Fig. 8 the disposition of the inner forces in the wall parts above ($j=1$) and below ($j=2$) the opening is shown. Now the tendency of cracking at *a*, *b* and *c* can easily be understood.

To ensure the load carrying capacity of the wall the vicinity of the opening should be strengthened. The authors' recommendations are as follows:

a) Vertical r.c. members at both sides of the opening should be arranged and designed for a tensile force equal to the greater of the shear forces acting in the parts above or below the opening. These elements should be well connected to edge beams.

b) The lintel and the edge beams should be designed for axial forces stemming from: 1) The integral moment — N_j and 2) The local moments due to the shearing forces at the sections in line with the vertical edges of the opening — ΔN_j (see Fig. 8). There is always a cumulative tension in edge of the lintel, therefore sufficient anchorage length should be provided.

These calculations, for designing the strengthening elements are shown in the following, for a wall whose measurements are as those of walls *B*; The loading is 1.4 t/m applied at the top (including self weight of 0.4 t/m) and uplifting force of 8.0 tons applied at the central support.

The integral moment at the centre of the opening

$$M = \left(4.00 - \frac{3.94}{2} \times 1.40\right) \times 0.95 + \frac{1.40}{2} \times 1.00^2 = 1.88 \text{ tm}$$

The lever arm is $Z = 1.28$ m

The axial forces due to the integral moment are:

$$N_1 = -N_2 = \frac{1.88}{1.28} = 1.47 \text{ t}$$

Assuming that the masonry does not transfer tensile forces the axial force N_1 — above the opening — shall be split between the lintel (subscript *o*) and the top beam (subscript *t*), proportionally to their axial stiffnesses

$$N_{1t} \cong 1.47 \times \frac{8}{8.0 + 10.0} = 0.65^t; \; N_{1o} \cong 1.47 \times \frac{10.0}{8.0 + 10.0} = 0.82^t$$

The integral shear at the middle of the opening is

$$V = 1.24 + 1.00 \times 4.40 = 2.64^t$$

The wall part above the opening is composed of the lintel (10 cm deep), top beam (8 cm deep) and a masonry strip 21 cm deep. The equivalent stiffness of this part is

$$S_1 = 2\left[\left(\frac{230{,}000 \times \frac{10.0 \times 10^3}{12}}{75{,}000 \times 10}\right)^{1/3} + \left(\frac{230{,}000 \times \frac{10.0 \times 8.0^3}{12}}{75{,}000 \times 10}\right)^{1/3}\right] + 21 =$$

$= 23 + 21 = 44$ cm.

The wall part below the opening consists in this case of the bottom beam only (with $E_w = 270{,}000$ kg/cm^2) and its equivalent stiffness is

$$S_2 = 2\left(\frac{270{,}000 \times \frac{10 \times 17^3}{12}}{75{,}000 \times 10}\right)^{1/3} \cong 23 \text{ cm}$$

The shear forces are split between the two wall parts as follows

$$V_1 = \frac{44}{44+23} V \cong 0.66\ V = 0.66 \times 2.64 = 1.74^t$$

$$V_2 = \frac{23}{44+23} V \cong 0.34\ V = 0.34 \times 2.64 = 0.90^t$$

It should be noted that the numerical analysis gave results for $B_1 - 0.66\ V$, $0.34\ V$ and for $B_2 - 0.70\ V$, $0.30\ V-$ in good agreement with the above approximate calculations.

The vertical r.c. strengthening elements on both sides of the opening shall be designed for a tensile force of 1.74^t, to counteract the vertical cracking in *a* and *b* zones.

The local moment in the wall part 1 (above the opening) at the section in line with the vertical edges of the opening is:

$$\Delta M_1 = 1.740 \times 0.57/2 \cong 0.495 \text{ tm}$$

and is resolved into a couple of axial forces in the lintel (*o*) and the top beam (*t*) (with lever arm $Z_o = 0.29_m$)

$$\Delta N_1 = \frac{0.495}{0.29} \cong 1.71^t$$

The maximal tensile forces are produced 1) in the top beam above the top inner corner of the opening (zone *c*) and 2) in the lintel's edge at the top outer corner of the opening (zone *b*). These forces are

$$N_{1t} = 0.65 + 1.71 = 2.36^t;\ N_{10} = 0.82 + 1.71 = 2.53^t.$$

The required anchorage length of the lintel (assuming allowable bond stress of 2.0 kg/cm^2) is

$$\Delta 1_o = \frac{2{,}530}{2 \times 2.0 \times 10} \cong 64 \text{ cm}$$

The above calculations, besides showing in detail the proposed approximate numerical procedure, shows the importance of the strengthening of masonry in the vicinity of the opening, wherever large shear forces are present.

A similar conclusion was also reached in [6]. There it is stated that proper stiffenings by r.c. elements or prestressing may enhance the carrying capacity of walls with openings "to give equal or even higher strength than that of solid wall".

Conclusions for Practical Application

This paper deals with load-bearing masonry walls, which are structural elements frequently used owing to their relatively low cost and other properties. The *main purpose* is to provide the tools for rational design ensuring stability and soundness without overdimensioning or exaggerating strength requirements.

Estimation of the load carrying capacity is based on the elastic-linear analysis of the plane-stress problem, by means as of a digital computer [2], formulas (1)-(16), pp. 144–149. For the purpose of engineering design tables, graphs and approximation formulas were given which permit immediate calculation of the stresses in the critical parts of the wall ([2], pp. 149–163).

Masonry is an essentially brittle material, susceptible of cracking, devoid of tensile strength. It must be, therefore, seen to it that, for the dominant mode of load transfer, the masonry is in compression while the internal tensile forces are carried by r.c. stiffening elements. In fact, loading tests of full masonry walls without strengthening posts under vertical loading confirm the above indicated distribution of the internal forces, and furthermore, analytical determination of the load carrying capacity based on the crushing strength of the masonry and the tensile strength of the bottom beam ([2], formulas 17-22) give results in good agreement with the experiments. The experiments also show that strengthening posts (of reinforced concrete) cause some increase of carrying capacity, but their effect is particularly pronounced in weak masonry. The evaluation of the load-carrying capacity of such walls with stiffening posts was based on empirical adaptation of test results (see par. 2.: "Walls with edge columns").

Openings cause a substantial reduction of the load-carrying capacity, especially if they are large and situated in the region of large shear forces. (A theoretical analysis of the plane-stress problem of the multiply connected region was given in [2], pp. 146–148.) The results of the numerical and experimental investigation indicate tensile stress concentrations near the openings, which can lead to early failure. This can, of course, be prevented by appropriate strengthening elements, but, often designers do not pay the necessary attention to this problem, and content themselves with lintels. In this paper an approximate, but sufficiently reliable, calculation is shown for the horizontal and vertical forces near the openings and for the required strength of the stiffening elements. (See formulas (1), (2), and the numerical example in the section: Series B: "Walls with openings").

In conclusion, the present paper offers tools for the rational design of typical masonry, in consideration of the state of failure. But it must be said that even under service loads shear cracking and/or separation of the masonry from the stiffening elements can not be disregarded, as it may impair the proper performance. Control of such phenomena was not yet sufficiently covered by research.

Notation

a	distance of reaction from wall corner.
D	depth of masonry strip in wall part near opening Eq. (2).
K_1	relative flexural rigidity of stiffener.
E_c, E_w	modulus of elasticity of stiffener and wall, respectively.
L	half length of wall.
M	bending moment.
N	axial force.
ΔN	additional axial force (in walls with openings).
P	concentrated load.
p	distributed load.
V	shear force.
S_i	equivalent stiffness of wall part i above or below the opening.
t	wall thickness.
ε	normal strain.
σ	normal stress.
τ	shear stress.

Subscripts

b	of bottom beam.
c	of stiffener.
o	of opening.
t	of top beam.
w	of wall.
i, j, k	of wall part above or below the opening, or its components.

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Key words

Masonry walls, load, reinforced concrete, beam, strains, deflections.

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Summary

The paper presents the results of experimental studies on masonry walls, bordered by reinforced concrete elements.

Two series of wall specimens were loaded to destruction:

a) Simply supported solid walls, acted upon by uniformly distributed vertical load.

b) Two bay walls, with opening, subjected to support elements.

The test results were examined in the light of a theoretical linear stress analysis up to cracking load. An attempt was made to interprete the behaviour of the wall specimens after cracking until failure with the purpose of proposing design procedures for evaluating the load carrying capacity of such walls.

Résumé

Cette contribution présente les résultats expérimentaux sur des murs en maçonnerie bordés d'éléments en béton armé. Deux séries de murs ont été chargés jusqu'à rupture:

a) Des murs pleins supportés simplement, sous l'effet d'une charge verticale uniforme.

b) Des murs à deux panneaux, avec ouvertures, agissant sur poutres-support.

Les résultats d'essai ont été examinés du point de vue de l'analyse admettant une répartition linéaire des contraintes jusqu'à fissuration. Les auteurs ont étudié le comportement des murs de la fissure jusqu'à la rupture, afin de proposer une méthode de dimensionnement à la rupture pour de tels murs.

Zusammenfassung

Die Arbeit berichtet über die Ergebnisse von Versuchen mit Wänden aus Mauerwerk, die durch Stahlbetonelemente eingefasst sind. Dabei wurden zwei Versuchsreihen von Mauern bis zum Bruch belastet, und zwar:

a) Einfach gelagerte volle Mauerwerkscheiben unter Einwirkung gleichmässig verteilter Last.

b) Zweifeldrige Wände mit Öffnungen und Stützträgern.

Die Versuchsergebnisse wurden verglichen mit einer Berechnung unter Annahme einer linearen Spannungsverteilung bis zur Bruchlast. Im Hinblick auf vorgeschlagene Berechnungsmethoden zur Abschätzung der Traglast solcher Wände wurde versucht, das Verhalten der Wände nach dem Reissen bis zum Versagen zu interpretieren.