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Panel Method for Multistorey Flat Plate Structures

Calcul de structures en dalles plates à plusieurs étages

Berechnung von mehrstöckigen aus Flachdecken und Stützen bestehenden Bauwerken

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Introduction

The continuing search for economy in buildings has led to the recognition of the contribution made by flat plate floors to the overall lateral structure stiffness. Current practice is to approximate each floor by equivalent beams spanning between the columns, in the direction of the lateral load. These may be used in the analysis of an equivalent frame as mentioned in most building codes, e.g. the ACI Code 1971 [1] and the SAA Code 1480-1974 [2]. This approach produces a very inadequate representation of the three dimensional behaviour of each floor. Transverse behaviour is only approximated by adding a torsional component to the column stiffness, whilst diagonal interaction is neglected completely. CARPENTER [3] found that the moment carried over longitudinally from one column to the next by the floor is less than that predicted by the ACI equivalent beam approximation. The ACI method, when estimating the effective width of the equivalent beam, takes no account of the effects of the dimensions of the column, or the plate. In addition, the equivalent frame method is generally applicable only to the internal bents in a structure.

To overcome these problems, this paper presents an alternative method of analysis, referred to here as the "Panel Method". This method considers a structural system consisting of columns and floor slab panels, which are defined by the grid of lines through the column centres in both directions. Once the bending stiffnesses of the panels are known, the structure is analysed by the stiffness procedures used for ordinary frame structures.

The bending stiffness of regular square column supported plates has been investigated by FAULKES [4] and SMITH [5], using non-conforming finite elements. They found that appreciable moments did not carry over more than one bay away from the point of moment application. They were thus able to produce (8×8) matrices representing the moment stiffnesses of the three types of floor panels, namely the corner, edge and internal panels, as shown in Fig. 2.

In this paper the panel stiffnesses have been recalculated using compatible quadrilateral elements [6]. The scope has been expanded to include rectangular panels with an aspect ratio of 1.5:1 and also to include the vertical freedom at each node (Fig. 1). This inclusion is essential where column axial deformation effects are significant. The three freedoms at each node are thus $\left(w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial x}\right)$.

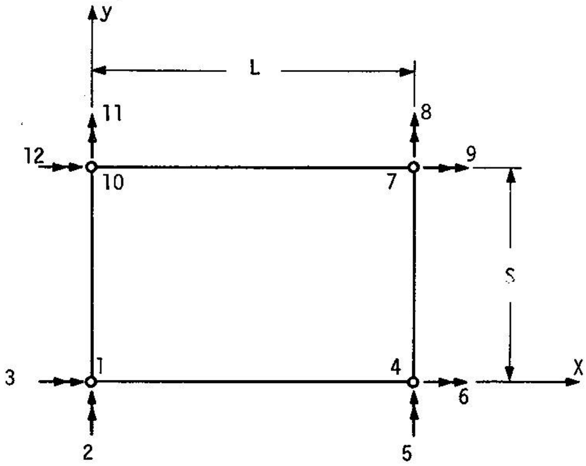
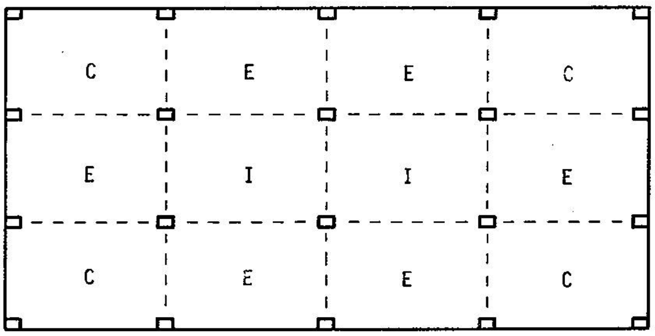


Fig. 1. 12 Degree of freedom panel.



C = Corner Panel
E = Edge Panel
I = Internal Panel

Fig. 2. Flat plate floor comprised of corner edge and internal panels.

The evaluation and the application of these stiffness matrices are described in this paper. Results have been obtained for three values of column/span (c/L), covering the normally used range. The stiffnesses of both rectangular and square panels are tested in multistorey building analyses. To provide a performance comparison, analyses of the buildings were also carried out using a complete finite element representation and the equivalent frame method.

Stiffness of Panel Element

Method of Obtaining Panel Stiffness

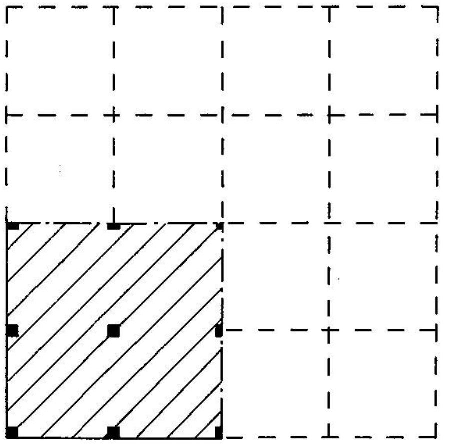
A finite element analysis was carried out for each of the panel configurations investigated. The plates considered were 4 × 4 bays. However, by utilizing symmetry only one quarter of each plate had to be analysed. Boundary conditions specified along the lines of symmetry represented either symmetry or antisymmetry.

Stiffness values at the columns were obtained by applying in turn unit displacements at each column freedom, the resultant forces at each freedom being the stiffness terms for that displacement.

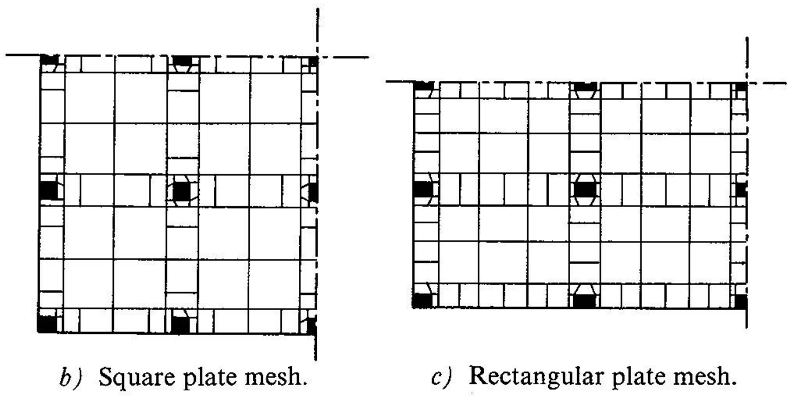
Finite Elements Used in Obtaining Panel Stiffness

Compatible quadrilateral elements, as described by KABAILA *et al.* [6], were chosen for the analysis. Their superior performance has been demonstrated by BLACK *et al.* [7]. The meshes used for the square and rectangular cases are shown in Fig. 3b and c. To provide accurate modelling in regions of high curvature, the mesh grading technique of SOMERVAILLE [8] was used extensively.

Column axes were connected to the boundaries of abutting elements by "rigid arm".



a) Quarter plate analysed.



b) Square plate mesh. *c)* Rectangular plate mesh.

Fig. 3. 4 × 4 Bay plates used in stiffness generation.

Extraction of Panel Stiffnesses

Output from the finite element program consisted of the (27×27) stiffness matrix of the quarter plate for each value of c/L and each set of boundary conditions. Suitable combination of the boundary conditions produced the stiffness which would result from an analysis of the complete plate. The (12×12) stiffnesses of the corner, edge and internal panels were extracted from these results. The numerical value of each panel stiffness coefficient had to be divided by the number of panels through which that action was transmitted.

The non-dimensionalization of each panel stiffness followed the method of tabulation used by PRZEMIENIECKI [9]. First, the whole stiffness matrix was divided by D/LS, where $D = Et^3/12(1-\mu^2)$ and L and S are the side lengths of the panel. This left factors of S and L in all rows and columns relating to the freedoms $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ respectively. The appropriate rows and columns were then divided by S or L as indicated in the tables. This procedure, on reversal, gives the stiffness of any size panel with that aspect ratio.

Tables 1 to 3 give the non-dimensionalized stiffness coefficients for corner, edge and internal square panels respectively. The three values in each square of the tables are the stiffness values for column/span ratios of 1/8, 1/16 and 1/20 respectively. Similarly, Tables 4 to 7 give the stiffness matrices for rectangular panels which have an aspect ratio of 1.5:1 and the same column/span ratio in both directions. Four tables are required in this case because the edge panels may have either their long or short sides along the free edge.

Illustrative Examples

To use the tabulated stiffnesses it is necessary only to reintroduce values of the appropriate parameters as described in the Appendix. Stiffness values for c/L ratios other than those tabulated may be obtained by interpolation. The matrices thus formed may then be used directly as input data for a three dimensional stiffness program. Assembly of the structure stiffness follows the same procedure as for other structural elements.

The performance of the panel method is demonstrated by the analysis of the two structures given below. A description of these buildings is followed by an outline of the two methods used for comparison.

Example 1: Interior Bay of a Ten Storey Building

To check the performance of the stiffness matrices of rectangular panels, one bay of the building shown in Fig. 4 was analysed by the panel method for a distributed lateral load. The structure has three bays in the transverse direction and is assumed to have many bays in the longitudinal direction. The columns have a uniform cross-section and are proportioned so that the column/span ratio (c/L) is 1/8 in each direction.

The building was also analysed using a full finite element solution and by the equivalent frame method. Displacement and column moment diagrams are given in Figs. 6, 7 and 8. The values are given in Table 8.

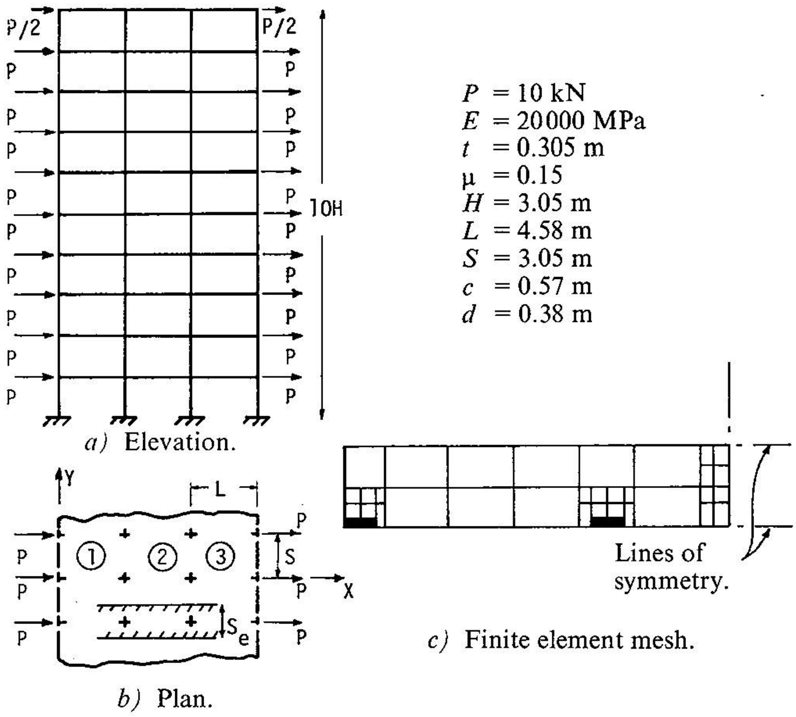


Fig. 4. Example 1: Interior bay of a 10 storey building.

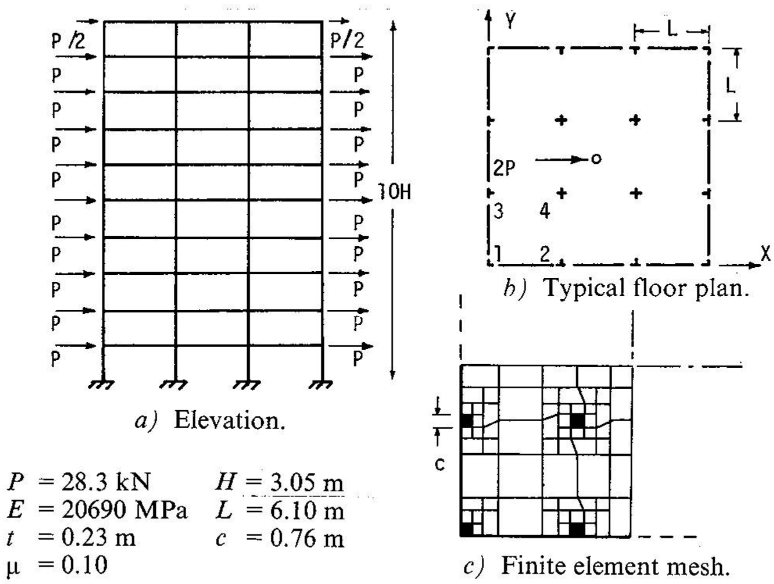


Fig. 5. Example 2: 3 × 3 bay 10 storey building.

Example 2: 3 × 3 Bay Ten Storey Building

This example demonstrates the manner in which horizontal shear loads are distributed within a structure. The building, whose details are given in Fig. 5 spans three bays in each direction. The tabulated stiffnesses for $c/L = 1/8$ and aspect ratio = 1 were used. The results (Figs. 9a, 9b and Table 9) show the horizontal deflection and column shear forces for a uniformly distributed lateral load.

The power of the method over equivalent frame methods is also demonstrated by the analysis of the structure subjected to a distributed twist load. The rotations produced by this loading are plotted in Fig. 9c. For each loading case the full finite element solution results are also given.

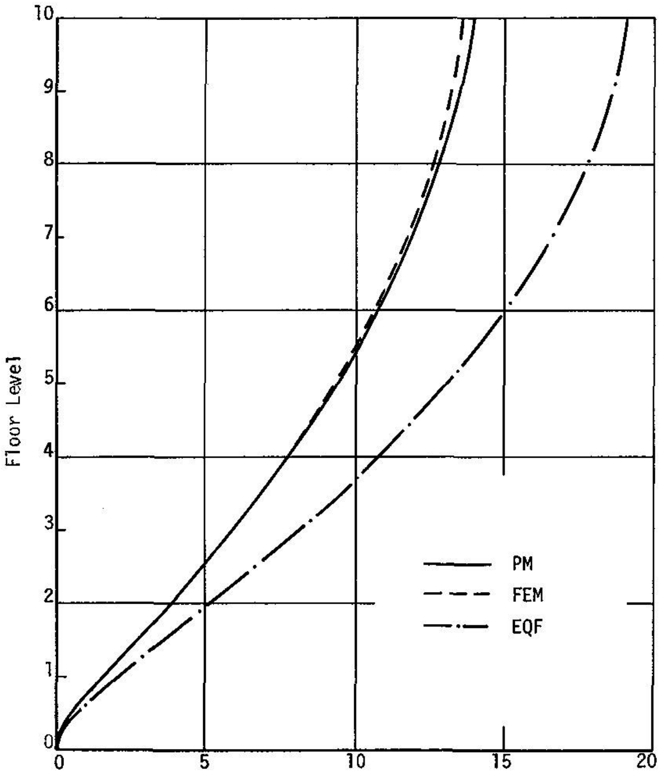


Fig. 6. Example 1 : lateral deflection (mm).

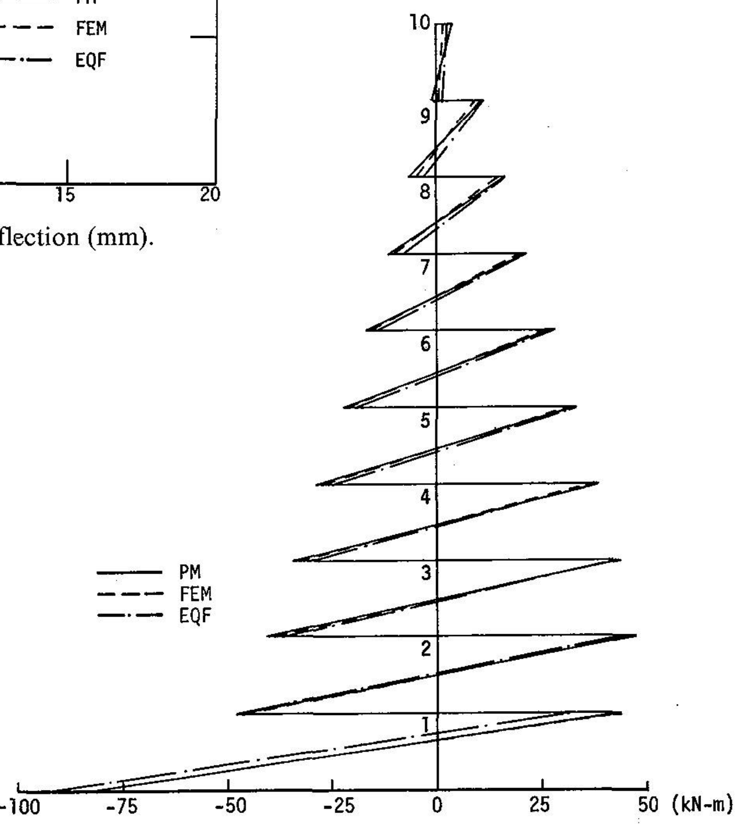


Fig. 7. Example 1 : bending moment diagram of exterior column.

Finite Element Method (FEM)

Both structures were analysed using finite element meshes to represent the floor plate. Compatible quadrilateral elements were used in the substructure program described by PULMANO *et al.* [10]. The division of the floor plates into finite elements is shown in Figs. 4c and 5c. This method provides the closest available approximation to the elastic solution and its results are used as reference values.

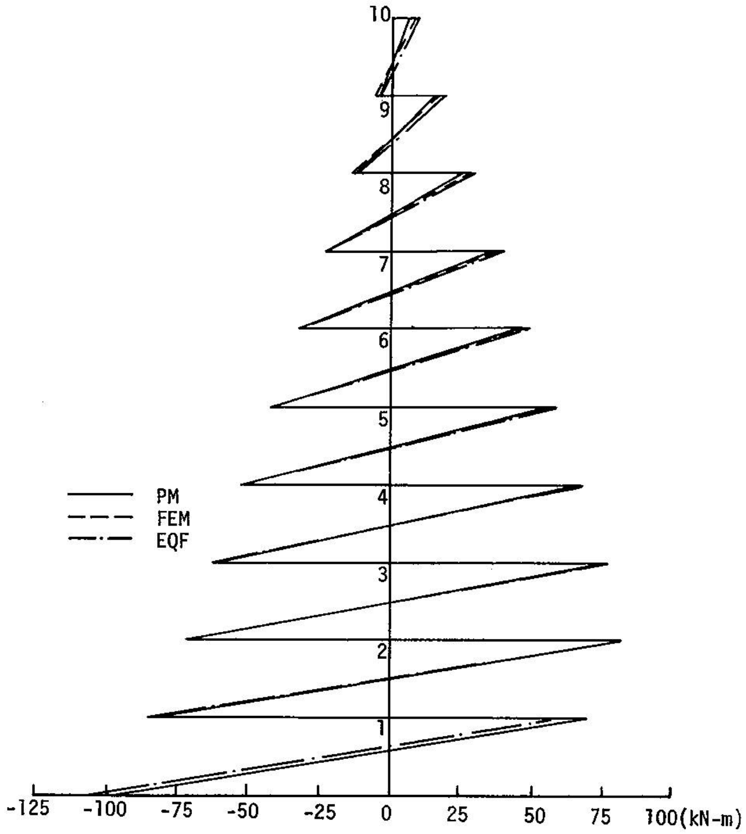
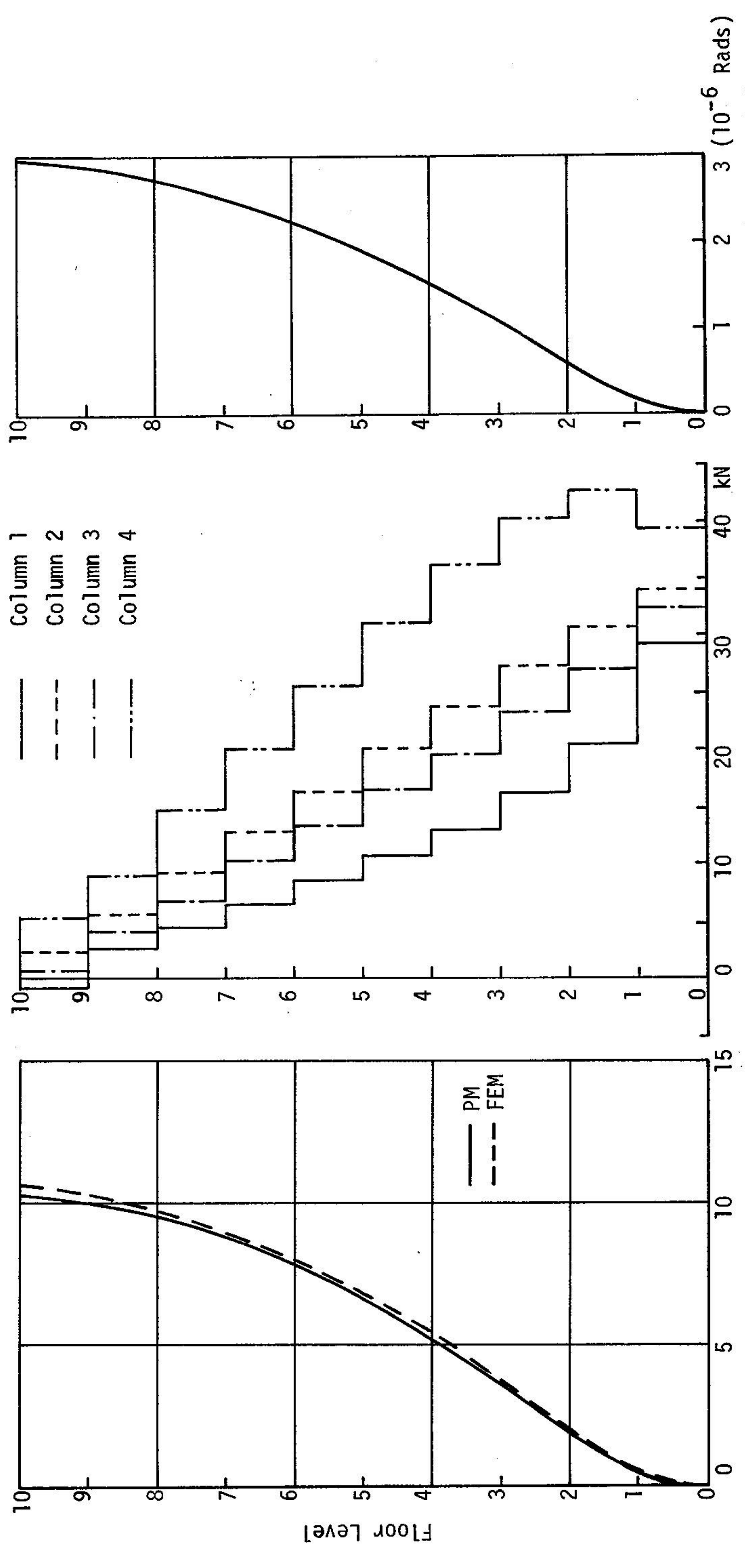


Fig. 8. Example 1 : bending moment diagram of interior column.

Equivalent Frame (EQF)

A two dimensional equivalent frame representation of the first example was analysed for the applied lateral loads. Several investigators have suggested various parameters with which to estimate a reliable value of the effective width (S_e) of the slab. QADEER and STAFFORD SMITH [11], by simultaneously rotating two columns of an internal panel using finite difference methods, produced a family of curves for effective width. In a discussion of this paper, MICHAEL [12] produced a single graph for effective width. From this graph the value of $S_e = 0.59S$ was obtained for example 1.

Note that the recommendations of both papers are only strictly valid for internal panels of an internal bay. In a structure such as that of example 2 the distribution of shear forces differs significantly from the assumed conditions thus rendering the effective width recommendations inapplicable.



a) Lateral deflection (mm).

b) Column shear forces-PM analysis.

c) Lateral twist PM analysis.

Fig. 9. Example 2: lateral deflection, column shear and lateral twist diagrams.

Discussion of Results

The lateral deflection curves for both examples show close correlation between the panel method and the finite element method. The maximum difference, which is less than 2½%, is due to the difference between the finite element meshes used in the FEM solution and those used in generating the panel method (PM) stiffnesses. From the deflection and column moment results it can be concluded that both the square and rectangular PM stiffnesses provide a close approximation to the more sophisticated methods of analysis. The reduction in the number of unknowns and hence of storage requirements and computation cost savings, is an attractive feature.

In contrast with these results, the deflection at the top of the equivalent frame is 40% greater than the FEM value. This results from the failure of the method to properly reflect all the relevant parameters. Although the deflection predictions for this case are on the safe side, there is no guarantee that this will be maintained for other column and panel configurations.

The column shear force diagram for example 2, shown in Fig. 9b indicates the way in which shear forces are distributed within a flat plate building. Although the horizontal displacement of all columns is the same at each floor level, the interior columns carry the largest shear loads. This is due to the greater slab bending stiffness at these joints causing them to attract a greater share of the load. An equivalent frame analysis could not take this transverse redistribution into account.

Excellent correlation is achieved between the FEM and PM results for the second example when it is subjected to a distributed twist load. The analysis of non-symmetric structures or non-symmetric loading is not possible using the equivalent frame method.

Applications

Until the present time, the situation relating to the lateral analysis of flat plate structures has been one of conflict and uncertainty. The ACI Committee 442 on Response of Buildings to Lateral Forces [13] noted that little research has been done to represent the stiffness of flat plates which connect columns. In fact, available test results have shown that values for effective width of less than the full slab width [11], equal to the full width [14], and greater than full width [15] are valid under different circumstances.

This situation is greatly clarified by the application of the Panel Method to the analysis of these buildings. As the method accounts for all the dimensional parameters of the plate, the resultant stiffness is more accurate, thus giving greater reliability to the results. The danger of significantly over or underestimating the plate stiffness is effectively eliminated. The economy inherent in the concept of flat plate structures may thus be expected to be enhanced.

For regular structures, the implementation of the Panel Method is straightforward. The column centre to centre spans in each direction define the size of the panels and their aspect ratios (L/S). By following the method outlined in the Appendix, the stiffness matrix of each panel may be tabulated. This data, plus the column stiffness matrices would then form the overall structure stiffness to which vertical and lateral loads may be applied. The analysis would then follow the stiffness analysis procedures of ordinary frame structures. Such an analysis would yield the moment transferred, and the movement of each column/slab joint. Any of several available mthods could then be used to design or check the details of the structural members.

Conclusions

In this paper, the proposed panel method of analysis for multistorey flat plate buildings under lateral and/or vertical loads has been presented. The method assumes a structural system which consists of columns and floor slab panels which are defined by adjacent column lines in both directions. The method has been shown to model plate behaviour with comparable accuracy to a refined finite element analysis. It achieves this with a smaller number of equations and hence with less computation cost.

The proposed structural system requires the least approximation of the actual structure whilst allowing for vertical, horizontal and twist loadings. The problems of determining the stiffness and carry over factors of conventional equivalent beams are eliminated whilst all the necessary parameters, particularly the column/span ratio, are included. Finally, the proposed method of analysis allows the moment resultants in the floor slabs to be defined more accurately, thereby enabling engineers to achieve greater economy and safety in the design of buildings with flat plate floors.

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

symmetric

1	14.80 10.05 9.23											
S	3.68 2.23 1.98	2.38 1.49 1.33										
L	3.68 2.23 1.98	0.48 0.24 0.20	2.38 1.49 1.33									
1	-8.45 -6.15 -5.78	-3.17 -2.00 -1.80	-0.46 -0.25 -0.22	14.30 10.48 9.85								
S	2.99 1.84 1.63	0.91 0.47 0.40	0.15 0.07 0.06	0.02 0.03 0.04	2.23 1.44 1.29							
L	-0.56 -0.40 -0.38	-0.21 -0.14 -0.12	-0.27 -0.12 -0.10	3.35 2.30 2.11	0.00 0.00 0.00	2.11 1.43 1.30						
1	1.22 1.19 1.19	-0.26 -0.19 -0.17	-0.26 -0.19 -0.17	-7.31 -5.72 -5.46	0.02 0.03 0.03	-2.58 -1.76 -1.61	13.70 10.82 10.38					
S	0.22 0.17 0.16	0.15 0.09 0.08	0.10 0.06 0.06	0.02 0.03 0.03	-0.18 -0.10 -0.09	0.00 0.00 0.01	0.02 0.04 0.04	1.99 1.44 1.33				
L	0.22 0.17 0.16	0.10 0.06 0.06	0.15 0.09 0.08	2.41 1.62 1.47	0.00 0.00 0.00	0.69 0.39 0.34	0.02 0.04 0.04	0.00 0.00 0.00	1.99 1.43 1.33			
1	-8.45 -6.15 -5.78	-0.46 -0.25 -0.22	-3.17 -2.00 -1.80	1.11 1.06 1.06	0.14 0.07 0.06	-0.32 -0.25 -0.23	-7.31 -5.72 -5.46	2.41 1.62 1.47	0.02 0.03 0.03	14.30 10.48 9.85		
S	-0.56 -0.40 -0.38	-0.27 -0.12 -0.10	-0.21 -0.14 -0.12	-0.32 -0.25 -0.23	0.14 0.08 0.07	-0.15 -0.10 -0.09	-2.58 -1.76 -1.61	0.69 0.39 0.34	0.00 0.00 0.01	3.35 2.30 2.11	2.11 1.43 1.30	
L	2.99 1.84 1.63	0.15 0.07 0.06	0.91 0.47 0.40	0.14 0.07 0.06	-0.06 -0.02 -0.02	0.14 0.08 0.07	0.02 0.03 0.03	0.00 0.00 0.00	-0.18 -0.10 -0.09	0.02 0.03 0.04	0.00 0.00 0.00	2.23 1.44 1.29
	1	S	L	1	S	L	1	S	L	1	S	L

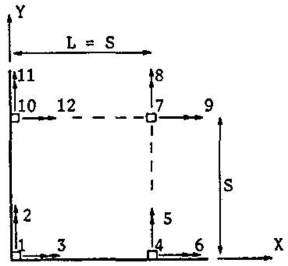
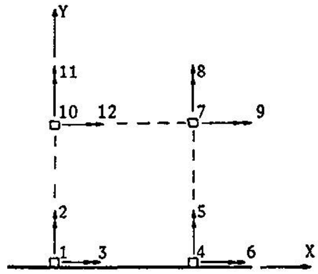


Table 1: Lower Triangular Stiffness Matrix for Square Corner Panel
[All values to be multiplied by $Et^3/12\,(1-\mu^2)\,LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

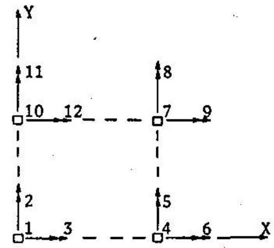


symmetric

1	14.31 10.68 10.05											
S	0.00 0.00 0.00	2.22 1.42 1.28										
L	3.35 2.32 2.13	0.00 0.00 0.00	2.11 1.43 1.30									
1	-8.92 -6.97 -6.67	-3.11 -2.01 -1.81	-0.65 -0.50 -0.48	14.31 10.68 10.05								
S	3.11 1.97 1.77	0.89 0.47 0.39	0.19 0.12 0.11	0.00 0.00 0.00	2.22 1.42 1.28							
L	-0.65 -0.51 -0.48	-0.19 -0.12 -0.11	-0.30 -0.15 -0.13	3.35 2.32 2.13	0.00 0.00 0.00	2.11 1.43 1.30						
1	0.88 0.74 0.72	-0.20 -0.14 -0.13	-0.39 -0.32 0.30	-7.20 -5.58 -5.32	0.00 0.00 0.00	-2.55 -1.73 -1.58	13.73 11.02 10.61					
S	0.30 0.27 0.26	0.13 0.08 0.07	0.14 0.09 0.08	0.00 0.00 0.00	-0.17 -0.09 -0.08	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.30				
L	0.27 0.23 0.22	0.09 0.05 0.04	0.17 0.11 0.10	2.38 1.59 1.44	0.00 0.00 0.00	0.68 0.38 0.33	0.01 0.04 0.04	0.00 0.00 0.00	1.99 1.43 1.32			
1	-7.20 -5.58 -5.32	0.00 0.00 0.00	-2.55 -1.73 -1.58	0.88 0.74 0.72	0.20 0.14 0.13	-0.39 -0.32 -0.30	-7.77 -6.53 -6.38	2.51 1.73 1.59	0.02 0.03 0.03	13.73 11.02 10.61		
S	0.00 0.00 0.00	-0.17 -0.09 -0.08	0.00 0.00 0.00	-0.30 -0.27 -0.26	0.13 0.08 0.07	-0.14 -0.09 -0.08	-2.53 -1.79 -1.65	0.67 0.38 0.33	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.30	
L	2.38 1.59 1.44	0.00 0.00 0.00	0.68 0.38 0.33	0.27 0.23 0.22	-0.09 -0.05 -0.04	0.17 0.11 0.10	0.02 0.03 0.03	0.00 0.00 0.00	-0.21 -0.13 -0.11	0.02 0.04 0.04	0.01 0.00 0.00	1.99 1.43 1.32
	1	S	L	1	S	L	1	S	L	1	S	L

Table 2: Lower Triangular Stiffness Matrix for Square Edge Panel
[All values to be multiplied by $Et^3/12(1-\mu^2)LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

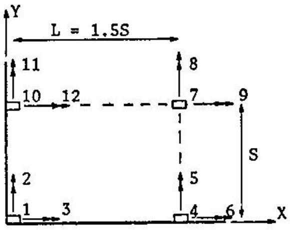


symmetric

1	13.75 11.23 10.83											
S	0.00 0.00 0.00	1.97 1.40 1.29										
L	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29									
1	-7.65 -6.38 -6.21	-2.49 -1.70 -1.56	0.00 0.00 0.00	13.75 11.23 10.83								
S	2.49 1.76 1.62	0.66 0.37 0.32	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29							
L	0.00 0.00 0.00	0.00 0.00 0.00	-0.20 -0.11 -0.10	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29						
1	0.59 0.33 0.30	-0.35 -0.33 -0.32	-0.35 -0.33 -0.32	-7.65 -6.38 -6.21	0.00 0.00 0.00	-2.49 -1.76 -1.62	13.75 11.23 10.83					
S	0.35 0.31 0.30	0.15 0.10 0.09	0.12 0.08 0.07	0.00 0.00 0.00	-0.20 -0.11 -0.10	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29				
L	0.35 0.31 0.30	0.12 0.08 0.07	0.15 0.10 0.09	2.49 1.70 1.56	0.00 0.00 0.00	0.66 0.37 0.32	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29			
1	-7.65 -6.38 -6.21	0.00 0.00 0.00	-2.49 -1.76 -1.62	0.59 0.33 0.30	0.35 0.33 0.32	-0.35 -0.33 -0.32	-7.65 -6.38 -6.21	2.49 1.70 1.56	0.00 0.00 0.00	13.75 11.23 10.83		
S	0.00 0.00 0.00	-0.20 -0.11 -0.10	0.00 0.00 0.00	-0.35 -0.31 -0.30	0.15 0.10 0.09	-0.12 -0.08 -0.07	-2.49 -1.76 -1.62	0.66 0.37 0.32	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29	
L	2.49 1.70 1.56	0.00 0.00 0.00	0.66 0.37 0.32	0.35 0.31 0.30	-0.12 -0.08 -0.07	0.15 0.10 0.09	0.00 0.00 0.00	0.00 0.00 0.00	-0.20 -0.11 -0.10	0.00 0.00 0.00	0.00 0.00 0.00	1.97 1.40 1.29
	1	S	L	1	S	L	1	S	L	1	S	L

Table 3: Lower Triangular Stiffness Matrix for Square Internal Panel
[All values to be multiplied by $Et^3/12(1-\mu^2)LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

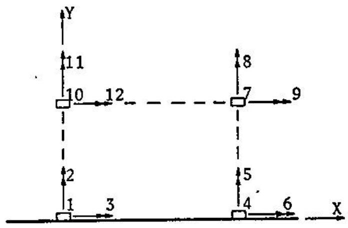


symmetric

	1	S	L	1	S	L	1	S	L	1	S	L
1	17.52 11.56 10.55											
S	3.78 2.30 2.04	3.34 2.09 1.87										
L	4.13 2.42 2.12	0.51 0.26 0.22	1.82 1.11 0.98									
1	-4.96 -3.84 -3.65	-2.47 -1.61 -1.46	-0.11 -0.06 -0.05	15.34 11.02 10.32								
S	2.38 1.53 1.37	1.04 0.56 0.48	0.07 0.04 0.04	0.01 0.02 0.02	3.02 1.97 1.78							
L	-0.16 -0.13 -0.13	-0.11 -0.08 -0.07	-0.06 -0.02 -0.02	3.40 2.22 2.02	0.00 0.00 0.00	1.51 1.00 0.90						
1	1.12 1.06 1.06	-0.24 -0.22 -0.21	-0.21 -0.12 -0.11	-12.54 -9.39 -8.91	0.01 0.02 0.02	-3.17 -2.08 -1.89	15.99 12.40 11.87					
S	0.28 0.24 0.23	0.30 0.21 0.19	0.10 0.06 0.05	0.01 0.02 0.02	-0.68 -0.37 -0.32	0.00 0.00 0.00	0.01 0.02 0.02	2.89 2.07 1.92				
L	0.11 0.08 0.08	0.07 0.04 0.04	0.04 0.02 0.02	2.95 1.87 1.69	0.00 0.00 0.00	0.57 0.30 0.26	0.03 0.05 0.05	0.00 0.00 0.00	1.47 1.01 0.93			
1	-14.94 -10.23 -9.48	-1.43 -0.79 -0.69	-4.00 -2.40 -2.13	1.02 0.95 0.96	0.19 0.14 0.13	-0.24 -0.17 -0.15	-4.10 -3.32 -3.19	1.87 1.29 1.17	0.02 0.02 0.02	18.29 13.01 12.15		
S	-1.61 -1.06 -0.97	-0.95 -0.45 -0.38	-0.36 -0.21 -0.18	-0.33 -0.29 -0.28	0.29 0.20 0.17	-0.13 -0.08 -0.07	-1.94 -1.36 -1.25	0.79 0.47 0.41	0.01 0.01 0.01	3.92 2.74 2.51	3.18 2.12 1.92	
L	3.76 2.19 1.91	0.25 0.11 0.09	0.78 0.39 0.32	0.07 0.02 0.01	-0.05 -0.02 -0.01	0.04 0.02 0.02	0.02 0.03 0.02	-0.01 -0.01 -0.01	-0.04 -0.02 -0.02	0.03 0.05 0.05	0.01 0.01 0.01	1.76 1.09 0.97

Table 4: Lower Triangular Stiffness Matrix for Rectugular Corner Panel
[All values to be multiplied by $Et^3/12\,(1-\mu^2)LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

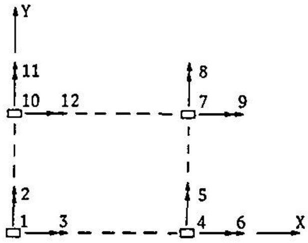


symmetric

	1	S	L	1	S	L	1	S	L	1	S	L
1	15.30 11.13 10.44											
S	0.00 0.00 0.00	3.01 1.95 1.78										
L	3.38 2.24 2.03	0.00 0.00 0.00	1.51 1.00 0.91									
1	-5.07 -4.12 -3.98	-2.43 -1.62 -1.48	-0.17 -0.16 -0.16	15.30 11.13 10.44								
S	2.44 1.60 1.45	1.02 0.55 0.47	0.10 0.07 0.07	0.00 0.00 0.00	3.01 1.95 1.77							
L	-0.16 -0.16 -0.16	-0.10 -0.07 -0.07	-0.06 -0.03 -0.03	3.38 2.24 2.03	0.00 0.00 0.00	1.51 1.00 0.91						
1	1.02 0.89 0.88	-0.21 -0.18 -0.17	-0.25 -0.18 -0.17	-12.48 -9.37 -8.88	0.00 0.00 0.00	-3.16 -2.08 -1.89	15.96 12.49 11.96					
S	0.33 0.32 0.31	0.29 0.20 0.18	0.12 0.08 0.07	0.00 0.00 0.00	-0.71 -0.35 -0.30	0.00 0.00 0.00	0.00 0.00 0.00	2.88 2.05 1.90				
L	0.11 0.10 0.09	0.06 0.04 0.04	0.05 0.03 0.03	2.92 1.90 1.71	0.00 0.00 0.00	0.57 0.31 0.26	0.03 0.05 0.05	0.00 0.00 0.00	1.46 1.02 0.94			
1	-12.48 -9.37 -8.88	0.00 0.00 0.00	-3.16 -2.08 -1.89	1.02 0.89 0.88	0.21 0.18 0.17	-0.25 -0.18 -0.17	-4.18 -3.57 -3.50	1.91 1.35 1.25	0.02 0.02 0.02	15.96 12.49 11.96		
S	0.00 0.00 0.00	-0.71 -0.35 -0.30	0.00 0.00 0.00	-0.33 -0.32 -0.31	0.29 0.20 0.18	-0.12 -0.08 -0.07	-1.91 -1.38 -1.28	0.77 0.46 0.41	0.01 0.01 0.01	0.00 0.00 0.00	2.88 2.05 1.90	
L	2.92 1.90 1.71	0.00 0.00 0.00	0.57 0.31 0.26	0.11 0.10 0.09	-0.06 -0.04 -0.04	0.05 0.03 0.03	0.02 0.02 0.04	-0.01 -0.01 -0.01	-0.04 -0.02 -0.02	0.03 0.05 0.05	0.00 0.00 0.00	1.46 1.02 0.94

Table 5: Lower Triangular Stiffness Matrix for Rectugular Edge Pannel, Free Edge on Long Side
[All values to be multiplied by $Et^3/12\,(1-\mu^2)LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20

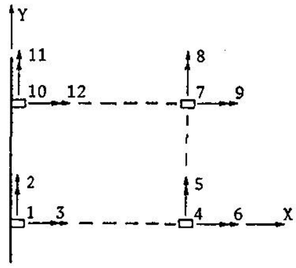


symmetric

	1	S	L	1	S	L	1	S	L	1	S	L
1	16.21 13.01 12.53											
S	0.00 0.00 0.00	2.88 2.04 1.89										
L	0.00 0.00 0.00	0.00 0.00 0.00	1.53 1.07 0.91									
1	-3.97 -3.29 -3.21	-1.85 -1.28 -1.18	0.00 0.00 0.00	16.21 13.01 12.53								
S	1.84 1.31 1.21	0.75 0.44 0.38	0.00 0.00 0.00	0.00 0.00 0.00	2.88 2.04 1.89							
L	0.00 0.00 0.00	0.00 0.00 0.00	-0.03 -0.02 -0.02	0.00 0.00 0.00	0.00 0.00 0.00	1.53 1.07 0.91						
1	0.24 0.03 0.00	-0.52 -0.48 -0.47	-0.21 -0.18 -0.17	-13.82 -11.32 -10.96	0.00 0.00 0.00	-3.11 -2.12 -1.94	16.21 13.01 12.53					
S	0.53 0.48 0.47	0.36 0.26 0.24	0.10 0.06 0.06	0.00 0.00 0.00	-0.77 -0.44 -0.39	0.00 0.00 0.00	0.00 0.00 0.00	2.88 2.04 1.89				
L	0.20 0.17 0.16	0.09 0.06 0.06	0.04 0.02 0.02	3.04 2.02 1.86	0.00 0.00 0.00	0.55 0.30 0.25	0.00 0.00 0.00	0.00 0.00 0.00	1.53 1.07 0.91			
1	-13.82 -11.32 -10.96	0.00 0.00 0.00	-3.11 -2.12 -1.94	0.24 0.03 0.00	0.52 0.48 0.47	-0.21 -0.18 -0.17	-3.97 -3.29 -3.21	1.85 1.28 1.18	0.00 0.00 0.00	16.21 13.01 12.53		
S	0.00 0.00 0.00	-0.77 -0.44 -0.39	0.00 0.00 0.00	-0.53 -0.48 -0.47	0.36 0.26 0.24	-0.10 -0.06 -0.06	-1.84 -1.31 -1.21	0.75 0.44 0.38	0.00 0.00 0.00	0.00 0.00 0.00	2.88 2.04 1.89	
L	3.04 2.02 1.86	0.00 0.00 0.00	0.55 0.30 0.25	0.20 0.17 0.16	-0.09 -0.06 -0.06	0.04 0.02 0.02	0.00 0.00 0.00	0.00 0.00 0.00	-0.03 -0.02 -0.02	0.00 0.00 0.00	0.00 0.00 0.00	1.53 1.07 0.91

Table 6: Lower Triangular Stiffness Matrix for Rectugular Internal Panel
[All values to be multiplied by $Et^3/12\,(1-\mu^2)LS$]

In each square, Row 1: c/L = 1/8
Row 2: c/L = 1/16
Row 3: c/L = 1/20



symmetrical

	1	S	L	1	S	L	1	S	L	1	S	L
1	18.52 13.49 12.65											
S	3.98 2.80 2.58	3.19 2.13 1.93										
L	0.00 0.00 0.00	0.00 0.00 0.00	1.73 1.07 0.95									
1	-3.88 -3.04 -2.91	-1.80 -1.22 -1.11	0.00 0.00 0.00	16.29 12.91 12.40								
S	1.87 1.29 1.18	0.76 0.44 0.39	0.00 0.00 0.00	0.01 0.02 0.02	2.90 2.08 1.92							
L	0.00 0.00 0.00	0.00 0.00 0.00	-0.03 -0.02 -0.02	0.00 0.00 0.00	0.00 0.00 0.00	1.43 0.99 0.91						
1	0.31 0.17 0.17	-0.49 -0.42 -0.40	-0.33 -0.13 -0.10	-13.90 -11.34 -11.01	0.01 0.02 0.02	-3.13 -2.10 -1.92	16.29 12.91 12.40					
S	0.54 0.48 0.46	0.37 0.26 0.24	0.08 0.03 0.02	0.01 0.02 0.02	-0.79 -0.47 -0.41	0.00 0.00 0.00	0.01 0.02 0.02	2.90 2.08 1.92				
L	-0.07 -0.08 -0.08	0.02 0.00 -0.01	0.03 0.02 0.02	3.06 2.03 1.86	0.00 0.00 0.00	0.55 0.29 0.25	0.00 0.00 0.00	0.00 0.00 0.00	1.43 0.99 0.91			
1	-16.28 -12.13 -11.51	-1.94 -1.38 -1.28	-3.93 -2.41 -2.14	0.31 0.17 0.17	0.49 0.42 0.40	-0.33 -0.13 -0.10	-3.88 -3.04 -2.91	1.80 1.22 1.11	0.00 0.00 0.00	18.52 13.49 12.65		
S	-1.96 -1.39 -1.30	-1.06 -0.55 -0.46	-0.32 -0.18 -0.16	-0.54 -0.48 -0.46	0.37 0.26 0.24	-0.08 -0.03 -0.02	-1.87 -1.29 -1.18	0.76 0.44 0.39	0.00 0.00 0.00	3.98 2.80 2.58	3.19 2.13 1.93	
L	3.88 2.34 2.09	0.31 0.17 0.15	0.77 0.38 0.32	-0.07 -0.08 -0.08	-0.02 0.00 0.01	0.03 0.02 0.02	0.00 0.00 0.00	0.00 0.00 0.00	-0.03 -0.02 -0.01	0.00 0.00 0.00	0.00 0.00 0.00	1.73 1.07 0.95

Table 7: Lower Triangular Stiffness Matrix for Rectangular Edge Panel, Free Edge on Short Side
[All values to be multiplied by $Et^3/12\,(1-\mu^2)LS$]

FLOOR	LATERAL DEFLECTION (mm)			COLUMN END MOMENTS (kN-m)					
				EXTERIOR COLUMN			INTERIOR COLUMN		
LEVEL	FEM	PM	EQF	FEM	PM	EQF	FEM	PM	EQF
10	13.63	13.96	19.12	1.72 -0.82	4.11 0.96	2.72 -1.73	8.55 5.80	6.13 3.82	9.56 4.68
9	13.24	13.52	18.63	9.39 4.72	10.73 6.22	11.02 3.24	17.35 14.26	15.84 12.73	18.74 12.72
8	12.63	12.85	17.81	14.79 10.22	16.24 11.74	16.31 8.46	27.22 23.97	25.57 22.46	28.94 22.48
7	11.76	11.94	16.62	20.36 15.86	21.71 17.31	21.77 13.92	36.81 33.65	35.23 32.24	38.68 32.30
6	10.65	10.78	15.05	25.90 21.56	27.14 22.92	27.13 19.42	46.37 43.34	44.88 42.05	48.43 42.17
5	9.29	9.37	13.10	31.47 27.35	32.56 28.57	32.50 25.04	55.85 52.97	54.49 51.85	58.08 52.02
4	7.68	7.74	10.79	37.09 33.22	38.01 34.28	37.80 30.78	64.00 62.59	64.02 61.65	67.54 61.94
3	5.84	5.88	8.12	42.76 39.73	43.48 40.59	42.78 37.70	74.16 71.94	73.18 71.25	76.14 71.93
2	3.79	3.81	5.13	47.01 47.02	47.48 47.59	44.60 47.88	81.69 83.36	80.98 82.97	80.95 85.59
1	1.61	1.61	2.03	42.87 82.92	43.04 83.10	31.60 93.63	68.20 95.57	67.87 95.52	57.62 106.62

Table 8, Example 1: Lateral Deflections and Column End Moments

FLOOR	LATERAL DEFLECTION (mm)		COLUMN END MOMENTS (kN-m)								ROTATION DUE TO APPLIED TWIST (10^{-6} RADS)	
			COLUMN 1		COLUMN 2		COLUMN 3		COLUMN 4			
LEVEL	FEM	PM	FEM	PM	FEM	PM	FEM	PM	FEM	PM	FEM	PM
10	10.65	10.42	5.37 -8.40	5.43 -8.41	10.88 -4.22	10.26 -4.71	7.70 -6.06	8.67 -5.85	16.38 0.34	16.11 0.12	2.98	2.99
9	10.31	10.11	16.27 -8.59	16.32 -8.64	20.20 -3.48	19.98 -3.90	18.41 -5.74	18.92 -5.32	25.08 2.61	25.06 2.54	2.88	2.89
8	9.80	9.63	20.75 -7.40	20.84 -7.50	27.62 0.15	27.33 -0.35	24.65 -2.96	25.35 -2.52	35.87 9.27	35.87 9.12	2.74	2.75
7	9.07	8.93	24.50 -4.80	24.60 -4.95	33.50 5.05	33.22 4.49	29.95 1.25	30.62 1.64	44.60 17.19	44.70 17.04	2.53	2.54
6	8.09	7.99	27.00 -1.24	27.15 -1.44	38.41 10.97	38.17 10.34	34.05 6.43	34.84 6.78	52.60 26.17	52.80 25.99	2.26	2.27
5	6.88	6.81	28.40 3.84	28.59 3.56	42.00 18.17	41.81 17.47	37.05 13.00	37.82 13.27	59.00 36.02	59.38 35.95	1.92	1.93
4	5.45	5.40	27.64 11.87	27.90 11.52	43.10 27.85	42.98 27.10	37.60 22.20	38.43 22.41	62.60 48.00	63.00 47.78	1.52	1.52
3	3.85	3.82	22.43 26.58	22.75 26.20	38.80 42.80	38.76 42.12	33.15 37.22	33.93 37.32	59.50 63.50	60.00 63.29	1.07	1.07
2	2.19	2.17	7.16 55.20	7.50 54.80	22.55 69.80	22.60 69.17	17.30 64.60	18.03 64.81	42.10 88.20	42.59 88.06	0.60	0.61
1	0.72	0.71	-29.68 118.70	-29.35 118.50	-20.47 123.30	-20.29 123.00	-23.55 121.70	23.06 121.61	-8.95 129.10	-8.44 128.92	0.20	0.20

Table 9, Example 2: Lateral Deflections, Column End Moments and Lateral Twist

Appendix

Data Preparation

To illustrate the procedure for evaluating a panel stiffness matrix, a typical panel, shown in Fig. 10b will be examined. In this example $c/L = 1/8$ in each direction and the aspect ratio $(L:S)$ is 9m:6m. Stiffness values for the panel are therefore taken from the top rows (since $c/L = 1/8$) of Table 7.

Using as data $E = 25000$ *MPa*, $t = 0.2$m and $\mu = 0.15$ the value of $Et^3/12(1-\mu^2)LS$ is 316 *kN/m*.

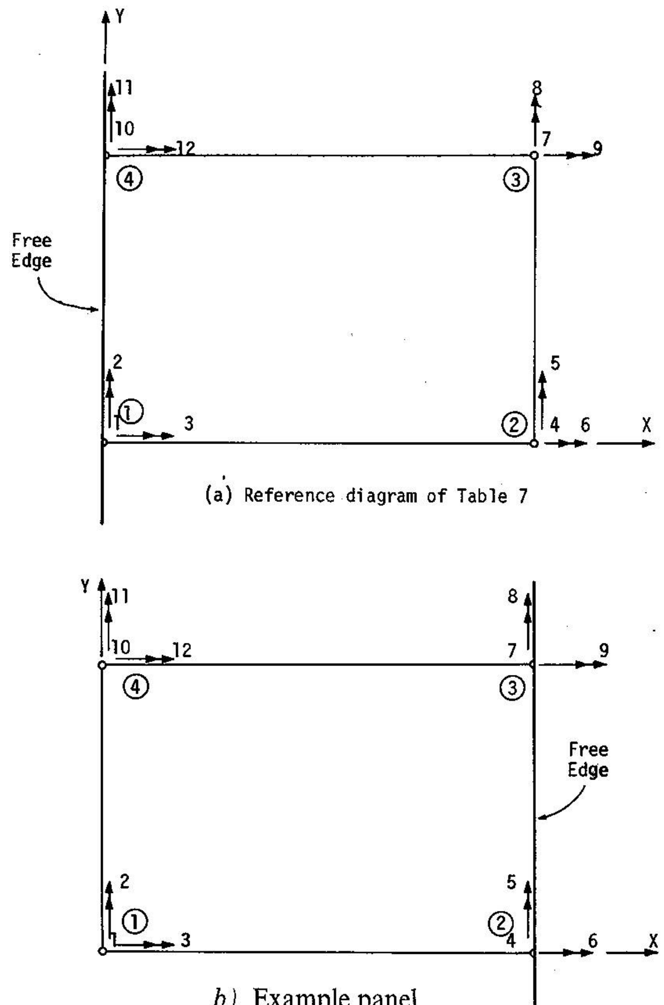
To establish the value of K(8,3), for instance, we note that the values of row 8 are multiplied by *L*, and the values of column 3 are multiplied by *S*.

$K(8,3)$ would therefore be $(316 \times 10^3) \times 9 \times 6 \times 0.084 = 1433$ *kN/m*.

In like manner, the full matrix may be evaluated.

As this panel is oriented differently from the reference diagram of Table 7, the stiffness matrix must be rotated. This is performed by a congruent transformation which has the form:

$$\overline{K} = RKR^T$$



b) Example panel.

Fig. 10. Panel rotation to suit boundary conditions.

where $\overline{K}$ is the resultant stiffness matrix.

Matrix R contains four submatrices r, which perform the axis rotation of the three freedoms at each node.

In terms of generalised forces, the matrix r forms the relation:

$$\begin{bmatrix} \overline{P}_1 \\ \overline{P}_2 \\ \overline{P}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}}_{r} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

where C_{ij} are direction cosines.

For the example panel, the rotation is 180°, therefore:

$$r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The moment sign convention used is that positive rotations cause a positive slope in the direction of the axis.

The final step is to move the nodes to their new positions. In this example, node 1 is interchanged with node 3, which is achieved by placing submatrix r in $R(3,1)$. Likewise the other nodes are interchanged as indicated in the matrix:

$$R = \begin{bmatrix} 0 & 0 & r & 0 \\ 0 & 0 & 0 & r \\ r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix}$$

Notations

The following symbols are used in this paper:

C_{ij}	direction cosines.
c	column thickness in x direction.
D	flexural rigidity of plate.
d	column thickness in y direction.
E	Young's Modulus.
EQF	equivalent frame method.
FEM	finite element method.
K	panel stiffness matrix.
$\overline{K}$	transformed panel stiffness matrix.
L	column centre to centre span in x direction.
P	lateral point load.
P_i	generalized nodal forces.
$\overline{P}_i$	transformed forces.
PM	panel method.
R	rotation transformation matrix.
r	transformation submatrix.
S	column centre to centre span in y direction.

S_e effective width in y direction.
t plate theckness.
w nodal deflection orthogonal to plate.
μ Poisson's Ratio.

Acknowledgement

The work reported in this paper forms part of a research project on the three dimensional analysis of multistorey flat plate buildings and is supported in part by the Australian Research Grants Commission. This support is gratefully acknowledged.

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Summary

Presented herein are the stiffness matrices of flat plate floor panels in non-dimensional form for use as input data for a three dimensional frame analysis computer program. The tabulated stiffness values take into account the parameters involving the size of the floor panel and the cross sectional dimensions of the supporting columns.

Two multistorey buildings are analysed to demonstrate the application of the method and its advantages over alternative methods.

Résumé

Les auteurs étudient les matrices de rigidité de dalles plates — dans une présentation sans dimension — en tant que données pour un programme de calcul à l'ordinateur de cadres tridimensionnels. Les valeurs de ridigité indiquées en forme de tableaux tiennent compte de paramètres relatifs aux dimensions des dalles ainsi qu'aux sections des colonnes.

Deux bâtiments à plusieurs étages sont calculés et montrent l'application de la méthode ainsi que ses avantages par rapport à d'autres procédés.

Zusammenfassung

In der vorliegeden Arbeit werden die Steifigkeitsmatrizen für die Felder von Flachdecken in dimensionsloser Form zum Gebrauch als Eingabedaten für ein dreidimensionales Computerprogramm dargestellt. Die tabellarisch angegebenen Steifigkeitswerte berücksichtigen die Abmessungen der Deckenplatten sowie die Querschnittsabmessungen der Stützen.

Zwei mehrstöckige Bauten werden berechnet, um die Anwendung der Methode und ihre Vorzüge gegenüber anderen Verfahren zu belegen.