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Hybrid Yield-Line Finite Element Analysis

Une analyse hybride des lignes de rupture moyennant la méthode des éléments finis

Eine hybride Bruchlinien-Analyse mittels der finiten Elementenmethode

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Introduction

Plastic yield-line analysis has proved a powerful tool in the limit design of concrete slabs under lateral loading. However it is of less value in the design of steel plates because their generally more slender proportions induce membrane stresses. These increase the load necessary to cause a given deflection and in many cases the limiting load may be considerably greater than that estimated from plastic bending alone. Thus yield-line design would be wasteful of material.

Rigorous large deflection elasto-plastic analysis is time-consuming and inconvenient. CLARKSON [1] and later YOUNG [2] described simplified approaches but these only consider infinitely long plates. Empirical formulae have been proposed [3] for plates of finite aspect ratio. These usually employ the concept of a limiting lateral load associated with a maximum allowable permanent set. JAEGER [4] developed an approximate method of analysis which involves estimating the strength of an equivalent infinitely long plate whose thickness is given by a conversion factor. HOOKE [5] presented the first genuinely two-dimensional approximate analysis and published charts from which load-deflection curves may be obtained. A rigid-plastic method has been developed by JONES [8] but in this as in all previous theoretical work except Young's the plate edges are considered to be completely restrained from in-plane mouvement or pull-in. Moreover, the effect of in-plane loading has not been considered.

These deficiencies are remedied by the present paper which describes a completely new method. In this, the lateral load is estimated as the sum of a first component due to plastic bending action alone calculated by orthodox yield-line analysis, plus a second component due to the membrane stresses in each of the elements bounded by the yield-lines. These elements are assumed to remain flat so that all bending is confined to the hinges. The plane stress is assumed to be uniform in each element and virtual work or strain energy principles used to analyse the plate.

Beam with End Restraint

The principle of the method is most easily explained in terms of a simple example which is relatively trivial, a rectangular beam of unit width, depth t, length b, whose ends are clamped and restrained from inward movement. Under a uniform lateral load of p_1 per unit area, simple plastic theory shows that

$$p_1 = \frac{16\, M_p}{b^2}$$

where M_p is the full plastic moment, which may be modified by the presence of axial stress. Assuming that the two halves of the beam remain straight, the extension in each is δ^2/b, (see Fig. 1). This causes a tension of $2tE\delta^2/b^2$. By virtual work

$$\frac{1}{2} p_2 b = \frac{2tE\delta^2}{b^2} . 2 . \frac{2\delta}{b}$$

i.e.

$$p_2 = \frac{16 E t \delta^3}{b^4}$$

The essence of the author's proposal is that the limiting load is given by the sum of p_1 and p_2 where δ is some arbitrary standard of acceptable deflection. Clearly δ doesn't represent the actual deflection since elastic bending is ignored in the analysis. It does however give some indication of the amount of plastic bending. Its principal virtue, whatever value is actually used as a design limit, is that it provides a consistent criterion of performance for the gamut of design parameters. (The Perry-Robertson method of strut design adopts a similar heuristic approach in its treatment of initial imperfections). When we consider the limiting deflection in a plate, representation at one point is inadequate owing to the variety possible in the shape of the failure mechanism. A more significant parameter is the root of the mean square deflection denoted by $\bar{\delta}$. Using this parameter, p_2 for a beam becomes

$$p_2 = \frac{83\, E t \bar{\delta}^3}{b^4} \text{ since } \bar{\delta} = \delta/\sqrt{3}$$

A reasonable value for $\bar{\delta}$ might be $b/100$.

Now the plastic moment is affected by the presence of any axial stress and is given by standard plastic theory as

$$M_p = \tfrac{1}{4}\sigma_Y t^2\left[1 - \left(\frac{F_y}{\sigma_y}\right)^2\right] \text{ where } F_y = \frac{6\, E\bar{\delta}^2}{b^2}$$

When the axial stress reaches the yield point, M_p vanishes. At the same time the membrane action becomes plastic, and assuming no work-hardening, further deflection causes a reduction in the effective modulus of elasticity so that

$$E' = \sigma_Y E/F_y$$

Replacing this reduced modulus in the expression for p_2 we have

$$p_2 = 13{\cdot}8\,\sigma_Y \bar{\delta} t/b^2$$

which represents the post-yield load curve when

$$\sigma_Y < 6E(\bar{\delta}/b)^2$$

The deflected shape may not be very realistic for such an extreme condition, but the use of an RMS instead of mid-span deflection ought to improve correlation with measured data since RMS values are not so shape sensitive.

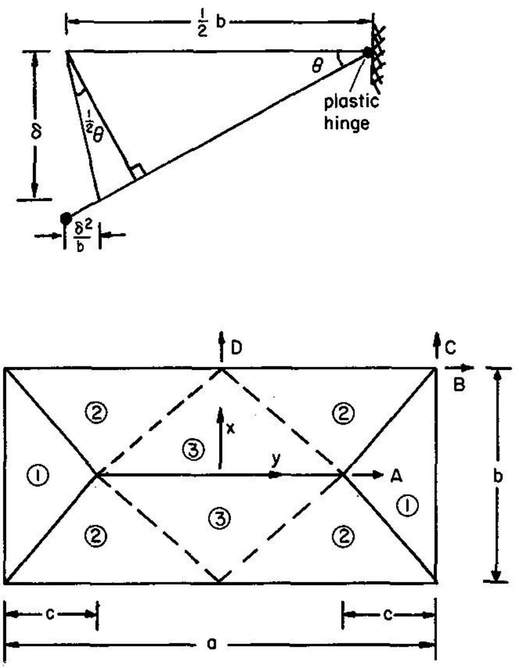

Fig. 1. Basic geometry.

The Rectangular Plate

The principle of analysis is exactly the same as for the beam but the calculation becomes more complex because membrane stresses are generated even when there is no external restraint on inward movement at the boundaries. The yield-line analysis giving p_1 uses the square Johansen yield envelope rather than the more realistic yield criteria of Tresca or von Mises. However it is considered that the loss of accuracy involved in this assumption is not sufficient to justify complicating a method, one of whose virtues is simplicity. The reader should see Ref. 6. for further discussion of the yield-line method.

The analysis of the membrane action starts by considering the finite elements formed by the pattern of hinge-lines. The full lines in Fig. 1 show a typical pitched roof mechanism. The discontinuous lines show a further division so that all elements are triangular. The in-plane displacements A, B, C and D represent the maximum number of degrees of freedom with free edges, bearing symmetry in mind. Assuming uniform strains in each element and taking into account the foreshortening due to deflection δ at the ridge line, the strains are

$$\left.\begin{aligned} \varepsilon_x &= \frac{\delta^2}{2c^2} + \frac{B-A}{c} \\ \varepsilon_y &= \frac{2C}{b} \\ \gamma_{xy} &= 0 \end{aligned}\right\} \quad \text{in element 1}$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{2B}{a} \\ \varepsilon_y &= \frac{4cD}{ab} + \frac{4C(\frac{1}{2}a-c)}{ab} + \frac{2\delta^2}{b^2} \\ y_{xy} &= \frac{4B(\frac{1}{2}a-c)}{ab} - \frac{2A}{b} + \frac{2(C-D)}{a} \end{aligned}\right\} \quad \text{in element 2}$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{A}{\frac{1}{2}a-c} \\ \varepsilon_y &= \frac{2D}{b} + \frac{2\delta^2}{b^2} \\ \gamma_{xy} &= 0 \end{aligned}\right\} \quad \text{in element 3}$$

These three equations enable the stresses to be calculated. The displacements A, B, C and D are evaluated by solving the simultaneous equations of virtual work associated with each degree of freedom. A fuller analysis is given in Appendix A. The virtual work associated with A is always zero since this point is always free. However, if there are in-plane stresses externally applied at the edges, these will contribute to the virtual work associated with B, C and D. Thus for each state of in-plane loading there will be a different solution to the four unknown displacements. In addition there are also various combinations of restraint on edge displacement. Thus if $C=D$, all edges are free to move but remain straight, reducing the number of simultaneous equations to three. If all edges are restrained against inward movement, B, C and D are all zero, a situation which might arise if the plate were heavily framed. The applied lateral load for a given value of δ can also be calculated by virtual work in terms of the displacements.

Solution for these displacements is straightforward and results have been computed for a large number of combinations of aspect ratios, boundary conditions, in-plane loading and proportions of finite elements (as defined by the parameter c in Fig. 1). These results were checked by hand calculation of some simple cases.

Yielding occurs in the membrane according to the von Mises criterion when the strain energy of distortion reaches a limit:

$$\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = \sigma_Y{}^2$$

The three elements may not yield simultaneously, and for the sake of simplicity the mean strain energy of distortion is calculated for the whole plate in terms of A, B, C, D and δ. This provides a single parameter $\bar{\sigma}$, the so-called effective stress which can be compared with σ_Y. If $\bar{\sigma} > \sigma_Y$ then general yielding is established and E must be reduced by a factor $\sigma_Y/\bar{\sigma}$ in exactly the same way as already shown in the beam example.

Now the pattern of elements which gives rise to the lowest lateral load with respect to membrane action may not correspond with the pattern of plastic hinge-lines giving rise to the lowest load associated with the yield line mechanism. Thus in order to estimate the limit load, strictly we should find the value of c for which the sum $(p_1 + p_2)$ is a minimum. This is quite feasible, but tedious to the designer who wants rapid results. Fortunately, p_1 is not very sensitive to the value of c (see Fig. 2) and for the sake of simplicity, we can estimate the limit load as the sum of the minimum loads associated with plasticity and membrane action separately, i.e.

$$(p_1)_{\min} + (p_2)_{\min}$$

Even though the corresponding values of c may differ, the error in load will not be large and the strength will in any case be underestimated. The yield-line load $(p_1)_{\min}$ is obtained from Fig. 3. High in-plane stresses will modify the effective plastic moment in each direction and it may be desirable to evaluate p_1 for an appropriately affine isotropic plate as follows. If the plastic moment per unit length

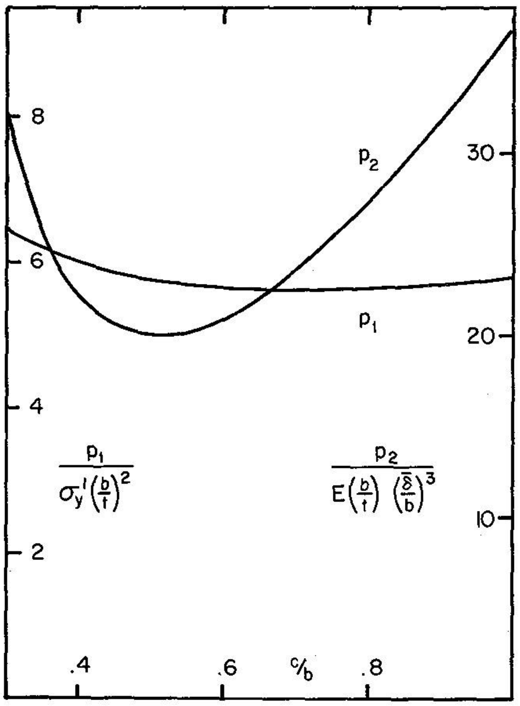

Fig. 2. Variation of p_1 and p_2 with c/b. $b/a = 0.3$, edges clamped and straight.

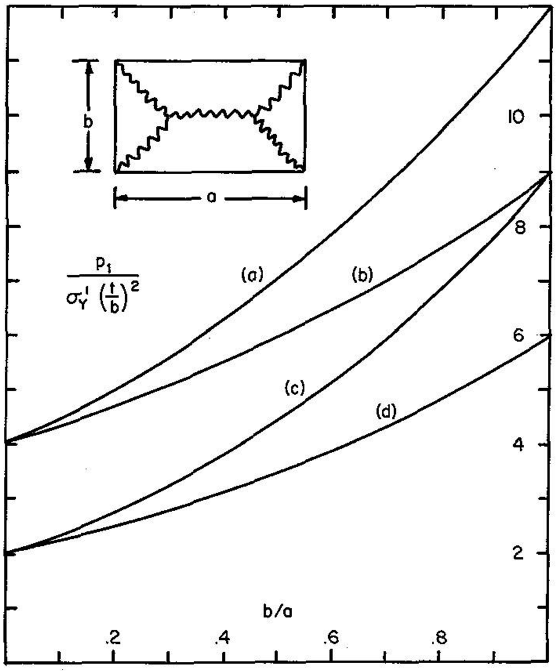

Fig. 3. p_1 given by yield-line analysis:
(a) All edges restrained.
(b) Long edges restrained, short edges straight.
(c) All edges straight.
(d) Long edges unrestrained, short edges straight.

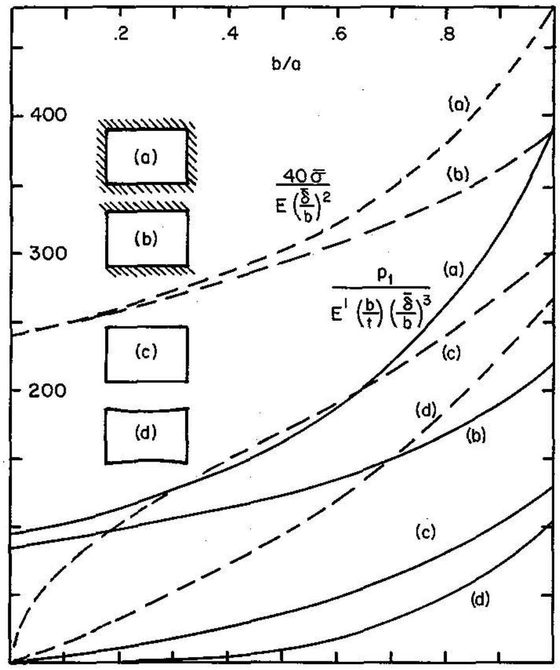

Fig. 4. p_2 (full line) and $\bar{\sigma}$ (discontinuous) due to membrane action. No in-plane loading.
(a) All edges restrained.
(b) Long edges restrained, short edges straight.
(c) All edges straight.
(d) Long edges unrestrained, short edges straight.

of plate (i.e. across the width) is M_p and that per unit width μM_p, then the socalled affine isotropic plate with M_p in both directions has a length $a/\sqrt{\mu}$. According to Johansen's Theorem, p_2 is the same for this plate as for the original one. It is shown in Appendix B that under the influence of in-plane loading and membrane action the plastic moment across the width is obtained by replacing

$$\sigma_Y \text{ by } \sigma_Y' = \frac{\sigma_Y{}^2 - \bar{\sigma}^2}{\sqrt{F_y{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}}$$

At the same time the aspect ratio b/a is factored by $\sqrt{\mu}$ i.e.

$$\sqrt[4]{\frac{F_y{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}{F_x{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}}$$

Figs 4 to 7 summarise the values of $(p_2)_{\min}$ for membrane action. These have been non-dimensionalised and Poisson's ratio taken as 0.3. It is clear that for a given value of $\bar{\delta}/b$, the limiting lateral load increases under the influence of tensile in-plane forces (negative sign convention). Conversely the lateral strength is reduced by the application of compressive in-plane loading. A negative total lateral load implies instability and in such cases the in-plane loading is excessive and either it must be reduced, or the limiting value of $\bar{\delta}/b$ allowed to increase. General yielding is determined by the value of $\bar{\sigma}$ given in Figs. 4, 5 and 8. These curves were found to depend only on the type of boundary restraint and consequently it is possible to use the family of curves in Fig. 8 for any general in-plane loading.

Examples of the use of these diagrams are presented in Appendix C.

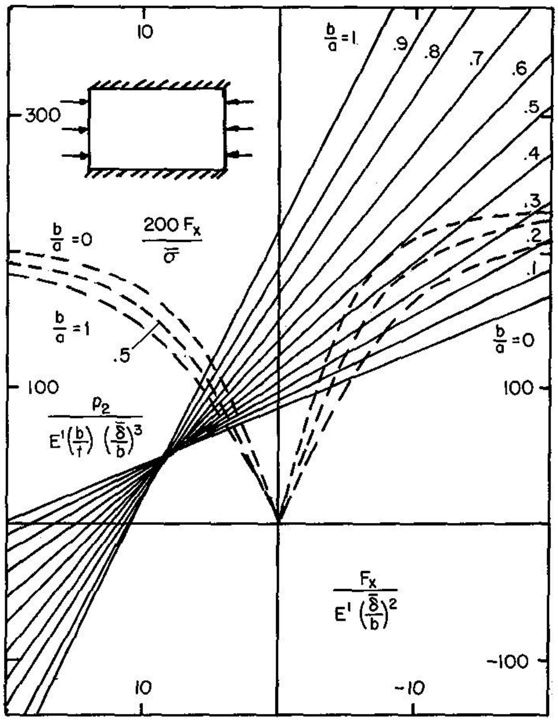

Fig. 5. p_2 (full line) and $\bar{\sigma}$ (discontinuous) with in-plane loading. Long edges restrained and short edges straight.

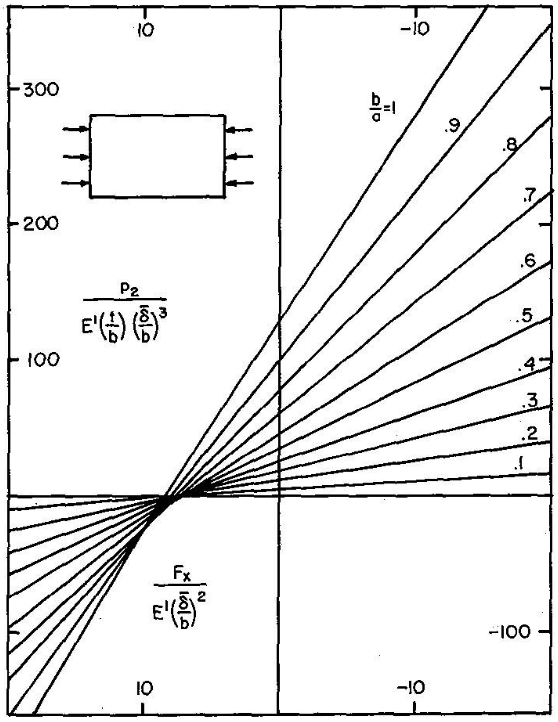

Fig. 6. (a) p_2 with all edges straight, $F_y = 0$.

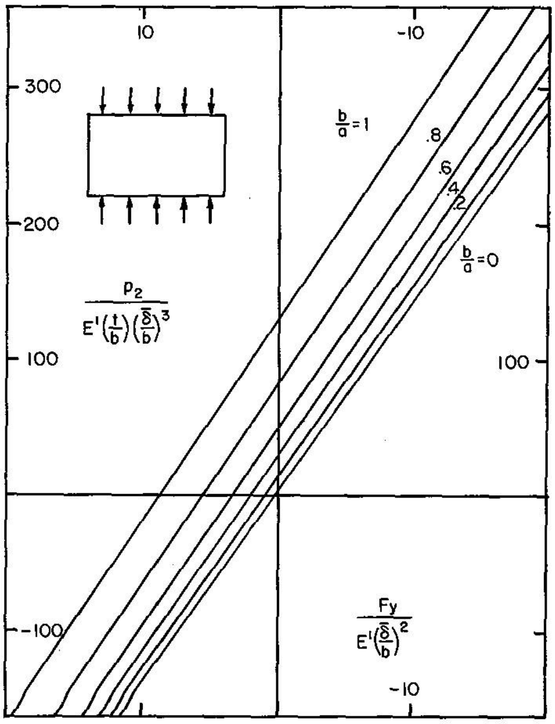

Fig. 6. (b) p_2 with all edges straight, $F_x = 0$.

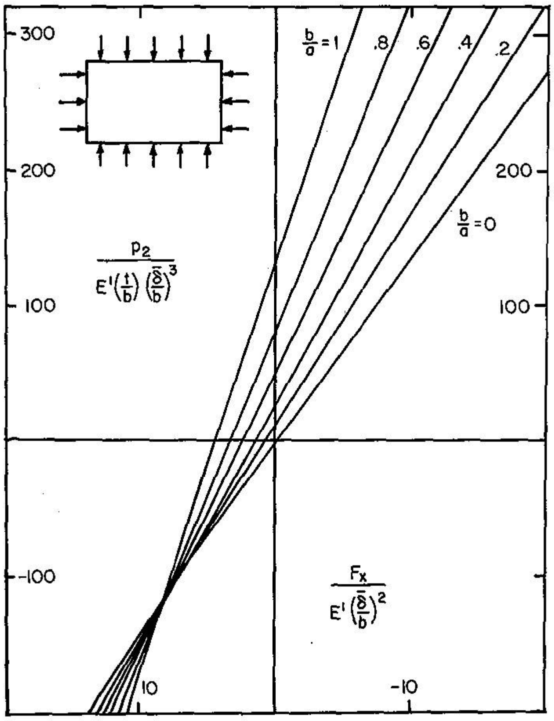

Fig. 6. (c) p_2 with all edges straight, $F_x = F_y$.

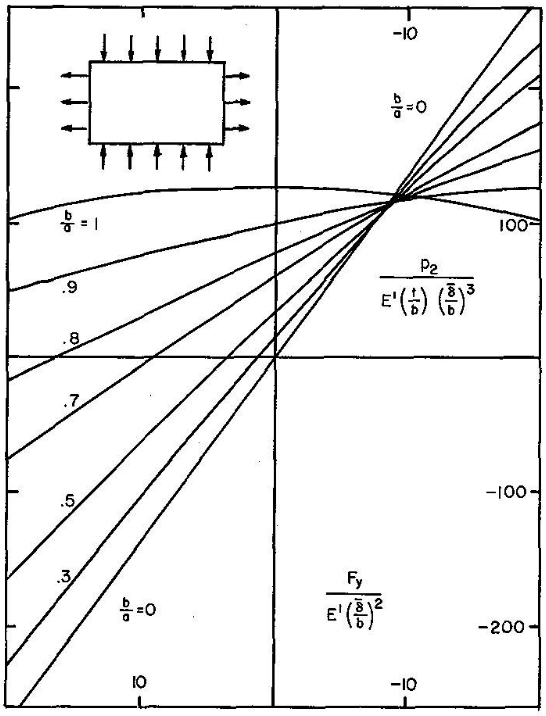

Fig. 6. (d) p_2 with all edges straight, $F_x - F_y$.

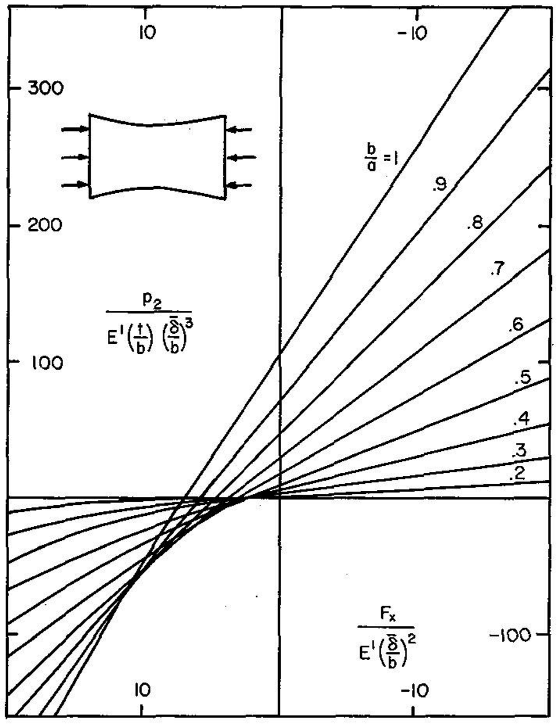

Fig. 7. (a) p_2 with short edges straight, long edges unrestrained, $F_y = 0$.

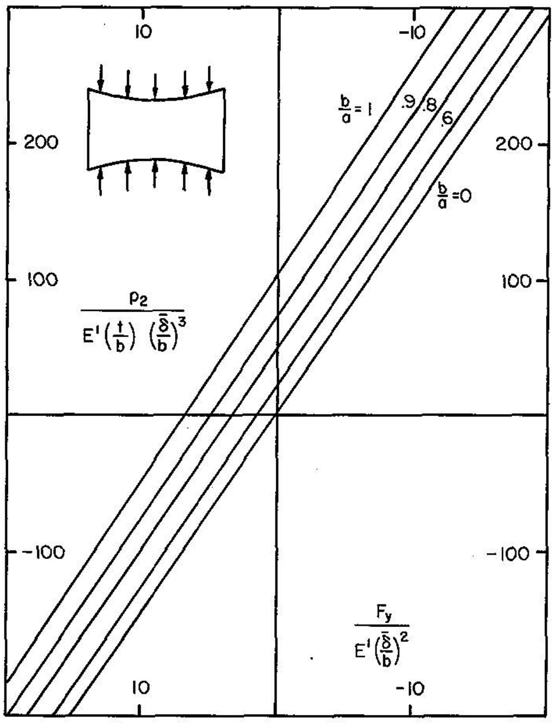

Fig. 7. (b) p_2 with short edges straight, long edges unrestrained, $F_x = 0$.

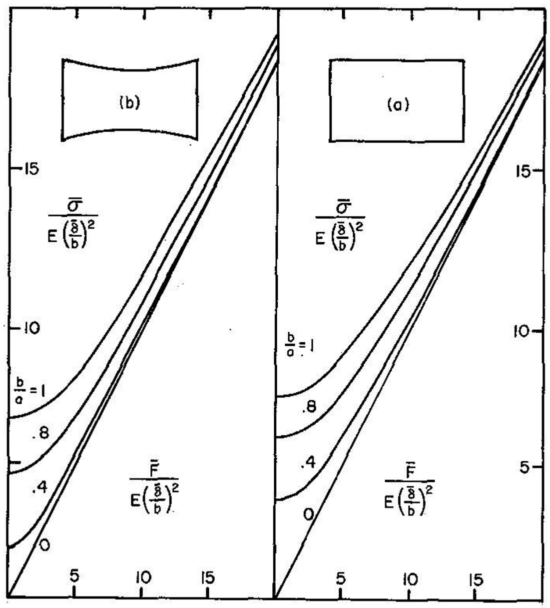

Fig. 8. $\bar{\sigma}$ for any combination of in-plane loading, short edges straight.

$$\bar{F}^2 = F_x{}^2 + F_y{}^2 - F_xF_y + 3F_{xy}^2.$$

(a) Long edges straight.
(b) Long edges unrestrained.

Effect of Initial Imperfections

The two principal imperfections in welded steel plating are initial bowing ("hungry horse" phenomenon) and residual stresses. The latter can be estimated approximately [7] and can be idealised as uniform compression. A corresponding adjustment to F_x or F_y allows for their effect on p_2.

Initial deflections are rather more difficult to quantify but if some nominal figure can be placed on the initial value of $\bar{\delta}_0$, p_2 should be reduced by the load associated with $\bar{\delta}_0$. In other words, the reduced value of p_2 is

$$p'_2 = p_2[1 - (\bar{\delta}_0/\bar{\delta})^3]$$

Initial imperfections do not of course affect p_1.

Conclusion

The method outlined in this paper enables the designer of steel plating to exploit the merits of yield-line analysis without sacrificing the reserve of strength resulting from the membrane stresses. The limiting lateral load is the sum of $(p_1)_{\min}$ and $(p_2)_{\min}$ associated with plastic bending and membrane action in relation to a nominal limiting lateral RMS deflection $\bar{\delta}$. Non-dimensional curves enable the limiting load to be rapidly estimated for rectangular plates with any symmetrical boundary conditions. The effect of membrane stress on the plastic moments, and the behaviour after the onset of general plasticity are both considered. Asymmetric boundary conditions and more complex shapes and loading can be analysed in exactly the same way, and while the computation may take slightly longer as the number of degrees of freedom increases, the method preserves its essential simplicity.

Although the effect of in-plane loading is allowed for, it is important to appreciate that this is not a buckling analysis but merely a means of estimating lateral strength, on the basis of a limiting lateral deflection. If this limiting lateral load is positive and increases with increasing deflection then buckling analysis is unnecessary since the maximum load is reached after the allowable deflection is exceeded. If the lateral load is found to decrease with increasing deflection, then design is impossible on the basis of a limiting deflection, but a more conventional buckling analysis is necessary. In-plane strength is then defined on the basis not of deflection but maximum load.

Appendix A: Finite Element Analysis of Membrane Action

The stresses in each of the three elements are calculated by Hooke's Law in terms of A, B, C, D and δ from the expressions for strain given in the main part of the paper. The virtual work associated with A is obtained by integrating the product of each stress, and corresponding unit strain obtained by putting $A=1$ and $B=C=D=0$.

$$bc\left[\frac{-\delta^2}{2c^3}-\frac{(B-A)}{c^2}-\frac{2\nu C}{bc}\right]-a(1-\nu)\left[\frac{2B(\frac{1}{2}a-c)}{ab}-\frac{A}{b}+\frac{C-D}{a}\right]$$
$$+b\left[\frac{A}{\frac{1}{2}a-c}+\frac{2\nu D}{b}+\frac{2\nu\delta^2}{b^2}\right]=0$$

In the case of B, C and D there is also work done by the stresses F_x and F_y (compressive positive) applied at the boundaries. For B, C and D respectively:

$$bc\left[\frac{\delta^2}{2c^3}+\frac{(B-A)}{c^2}+\frac{2\nu C}{bc}\right]+2b\left[\frac{B}{a}+\frac{2\nu cD}{ab}+\frac{2\nu C(\frac{1}{2}a-c)}{ab}+\frac{\nu\delta^2}{b^2}\right]$$
$$+2(1-\nu)\,(\tfrac{1}{2}a-c)\left[\frac{2B(\frac{1}{2}a-c)}{ab}-\frac{A}{b}+\frac{C-D}{a}\right]=\frac{-2F_xb(1-\nu^2)}{E}$$

$$bc\left[\frac{4C}{b^2}+\frac{\nu\delta^2}{bc^2}+\frac{2\nu(B-A)}{bc}\right]+2(\tfrac{1}{2}a-c)\left[\frac{4cD}{ab}+\frac{4C(\frac{1}{2}a-c)}{ab}+\frac{2\delta^2}{b^2}+\frac{2\nu B}{a}\right]$$
$$+b(1-\nu)\left[\frac{2B(\frac{1}{2}a-c)}{ab}-\frac{A}{b}+\frac{C-D}{a}\right]=\frac{-F_y(1-\nu^2)a}{E}$$

$$2c\left[\frac{4cD}{ab}+\frac{4C(\frac{1}{2}a-c)}{ab}+\frac{2\delta^2}{b^2}+\frac{2\nu B}{a}\right]-b(1-\nu)\left[\frac{2B(\frac{1}{2}a-c)}{ab}-\frac{A}{b}+\frac{C-D}{a}\right]$$
$$+2(\tfrac{1}{2}a-c)\left[\frac{2D}{b}+\frac{2\delta^2}{b^2}+\frac{\nu A}{\frac{1}{2}a-c}\right]=\frac{-F_y(1-\nu^2)a}{E}$$

The net work done by any shear F_{xy} applied at the boundaries is zero, assuming the displacements remain symmetrical. This is certainly so for $b/a=1$ and 0, and there is no reason to expect a significant effect for intermediate aspect ratios.

Putting $b/a=\alpha$, $c/b=\gamma$, $\nu=0.3$ and collecting terms together we obtain the four simultaneous equations:

$$\left(\frac{A}{b}\right)\left[\frac{0.7}{\alpha}+\frac{1}{\gamma}+\frac{\alpha}{0.5-\alpha\gamma}\right]-\left(\frac{B}{b}\right)\left[\frac{1}{\gamma}+\frac{0.7-1.4\alpha\gamma}{\alpha}\right]-1.3\left(\frac{C}{b}\right)+1.3\left(\frac{D}{b}\right)=$$
$$=\frac{\delta^2}{b^2}\left[\frac{1}{2\gamma^2}-0.6\right] \tag{1}$$

$$\left(\frac{A}{b}\right)\left[\frac{1}{\gamma}+\frac{1.4(0.5-\alpha\gamma)}{\alpha}\right]-\left(\frac{B}{b}\right)\left[\frac{1}{\gamma}+2\alpha+\frac{2.8(0.5-\alpha\gamma)^2}{\alpha}\right]-\left(\frac{C}{b}\right)\left[1.9-2.6\alpha\gamma\right]-$$
$$-\left(\frac{D}{b}\right)\left[2.6\alpha\gamma-0.7\right]=\frac{\delta^2}{b^2}\left[0.6+\frac{0.5}{\gamma^2}+\frac{1.82F_x}{E(\delta/b)^2}\right] \tag{2}$$

$$-1.3\left(\frac{A}{b}\right)-\left(\frac{B}{b}\right)\left[1.9-2.6\alpha\gamma\right]-\left(\frac{C}{b}\right)\left[4\gamma+0.7\alpha+\frac{8(0.5-\alpha\gamma)^2}{\alpha}\right]$$
$$-\left(\frac{D}{b}\right)\left[8\gamma(0.5-\alpha\gamma)-0.7\alpha\right]=\frac{\delta^2}{b^2}\left[\frac{0.3}{\gamma}+\frac{4(0.5-\alpha\gamma)}{\alpha}+\frac{0.9F_y}{\alpha E(\delta/b)^2}\right] \tag{3}$$

$$-1.3\left(\frac{A}{b}\right)+\left(\frac{B}{b}\right)\left[0.7-2.6\alpha\gamma\right]-\left(\frac{C}{b}\right)\left[8\gamma(0.5-\alpha\gamma)-0.7\alpha\right]$$
$$-\left(\frac{D}{b}\right)\left[8\alpha\gamma^2+0.7\alpha+\frac{4(0.5-\alpha\gamma)}{\alpha}\right]=\frac{\delta^2}{b^2}\left[\frac{2}{\alpha}+\frac{0.9F_y}{\alpha E(\delta/b)^2}\right] \tag{4}$$

These may be solved for A, B, C and D for each given value of F_x and F_y. The lateral load p_2 may then be calculated by virtual work:

$$\frac{p_2 b(1-\nu^2)(\frac{1}{2}a-\frac{1}{3}c)}{Et\delta}=\delta^2\left[\frac{b}{2c^3}+\frac{8a}{b^3}-\frac{8c}{b^3}\right]+A\left[\frac{4\nu}{b}-\frac{b}{c^2}\right]+B\left[\frac{b}{c^2}+\frac{4\nu}{b}\right]$$
$$+C\left[\frac{2\nu}{c}+\frac{8(\frac{1}{2}a-c)}{b^2}\right]+\frac{4aD}{b^2}$$

Substitution of the solutions to the simultaneous equations yields an expression for p_2 proportional to δ^3. Now the RMS deflection $\bar{\delta}$ is given by

$$ab\bar{\delta}^2=\delta^2[2bc\tfrac{1}{6}+b(a-2c)\tfrac{1}{3}]$$

Hence:

$$\frac{p_2}{E(\frac{t}{b})(\frac{\bar{\delta}}{b})^3}=\frac{17.32}{(1.5-\alpha\gamma)(1-\alpha\gamma)^{1.5}}\left\{\frac{1}{2\gamma^3}+\frac{8}{\alpha}-8\gamma+\left(\frac{A}{b}\right)\left[1.2-\frac{1}{\gamma^2}\right]+\left(\frac{B}{b}\right)\left[\frac{1}{\gamma^2}+1.2\right]\right.$$
$$\left.+\left(\frac{C}{b}\right)\left[\frac{0.6}{\gamma}+\frac{8(0.5-\alpha\gamma)}{\alpha}\right]+\frac{4}{\alpha}\left(\frac{D}{b}\right)\right\}$$

Appendix B: Effect of Membrane Stresses on Plastic Moments

Consider a hinge-line parallel to the longer side. The plastic moment per unit length is

$$\tfrac{1}{4}t^2\sigma'_Y\left[1-\left(\frac{\sigma_y}{\sigma'_Y}\right)^2\right]$$

where σ'_Y is the stress at which the material yields in the y-direction. If there are no other stresses apart from σ_y then σ'_Y is the uniaxial yield stress σ_Y. Otherwise, from von Mises

$$\sigma'^2_Y=\sigma_Y{}^2-\sigma_x{}^2+\sigma_x\sigma_y-3\tau_{xy}^2=\sigma_Y{}^2+\sigma_y{}^2-(\sigma_x^2-\sigma_x\sigma_y+\sigma_y{}^2+3\tau_{xy}^2)$$

Clearly, this expression will take a different value for each element, but for the sake of simplicity we consider mean values. Thus the term in parentheses becomes $\bar{\sigma}^2$, while the mean of σ_y is F_y. Hence

$$\sigma'_Y\doteqdot\sqrt{\sigma_Y{}^2+F_y{}^2-\bar{\sigma}^2}$$

and $$M_p\doteqdot\tfrac{1}{4}t^2\left[\frac{\sigma_Y{}^2-\bar{\sigma}^2}{\sqrt{\sigma_Y{}^2+F_y{}^2-\bar{\sigma}^2}}\right]$$

Similar expressions for the x-direction can be obtained by replacing F_y by F_x.

Appendix C: Examples of Plate Analysis

1. What load can be carried by a 4 mm thick mild steel plate 1 *m* x 0.8 m, near the middle of a lightly framed grillage? Take $\sigma_Y = 24$ kgf/mm^2 and $E = 20{,}000$ kgf/mm^2. Aspect ratio is 0.8 and all edges are straight so that from Fig. 4c

$$\bar{\sigma} = 6E\left(\frac{\bar{\delta}}{b}\right)^2$$

With a limit of 0.01 on $\bar{\delta}/b$, $\bar{\sigma} = 12$ kgf/mm^2
Since $\bar{\sigma} < \sigma_Y$ the membrane remains elastic.

$$p_2 = 80E\left(\frac{t}{b}\right)\left(\frac{\bar{\delta}}{b}\right)^3 = 8 \text{ tonnes/m}^2$$

The effective yield stress for the plastic moment

$$\sigma'_Y = \sqrt{\sigma_Y{}^2 - \bar{\sigma}^2} = 12\sqrt{3} \text{ kgf/mm}^2$$

Since all edges are clamped the yield-line load is given by Fig. 3a.

$$p_1 = 9.8\,\sigma'_Y\left(\frac{t}{b}\right)^2 = 5.1 \text{ tonnes/m}^2$$

$\therefore p = 13.1$ tonnes/m^2

If a higher limit of 0.02 were placed on $\bar{\delta}/b$, $\bar{\sigma}$ becomes 48 kgf/mm^2. The plastic moment vanishes and the membrane action becomes plastic. The modulus is reduced to $\frac{24}{48}$ E and hence $p = p_2 = 32$ tonnes/m^2.

2. The flange of a box girder is 1 m wide, 10 mm thick and framed transversely at 2.5 m intervals along its length. It is subjected to an in-plane compressive stress $F_x = 20$ kgf/mm^2 due to bending and a shear stress $F_{xy} = 15$ kgf/mm^2 due to torsion. What lateral load can be sustained if the permanent RMS deflection is not to exceed 10 mm? Take $\sigma_Y = 45$ kgf/mm^2 and $E = 20{,}000$ kgf/mm^2.

The boundary conditions are taken as unrestrained and simply supported along the sides and straight and clamped across the width.

$$\frac{\sqrt{F_x{}^2 + 3F_{xy}^2}}{E(\bar{\delta}/b)^2} = 16.4 \text{ kgf/mm}^2$$

From Fig. 8b and for $b/a = 0.4$, $\bar{\sigma} = 33$ kgf/mm^2. Thus the membrane action remains elastic.

$$\frac{F_x}{E(\bar{\delta}/b)^2} = 10$$

From Fig. 7a $p_2 = 2.3E\left(\frac{t}{b}\right)\left(\frac{\bar{\delta}}{b}\right)^3 = -4.6$ tonnes/m^2

For yield-line action $\sigma'_Y = \sqrt{\sigma_Y{}^2 - \bar{\sigma}^2} = 30.6$ kgf/mm^2

Coefficient of orthotropy $\mu = \sqrt{\dfrac{\sigma_Y{}^2 - \bar{\sigma}^2}{F_x{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}} = 0.836$

Affine aspect ratio $= b\sqrt{\mu}/a = 0.365$

From Fig. 3c $p_1 = 11.1$ tonnes/m^2

Thus the net lateral limiting load is $p = 6.5$ tonnes/m^2

Since p_2 is negative it is pertinent to ask whether this condition is stable. Repeating the calculation for $\bar{\delta}/b = 0.011$ instead of 0.01 shows that the lateral load increases to 7.0 tonnes/m^2. Thus the strength is limited by deflection, not by buckling. However, if there were no lateral load a reduced buckling strength would probably result from a different mode of deflection.

Notation

a	plate length in x-direction.
b	beam length, or plate width in y-direction, $b < a$.
t	plate thickness.
δ	central deflection.
$\bar{\delta}$	RMS deflection.
p_1	UDL per unit area due to bending action.
p_2	UDL per unit area due to membrane action.
$F_x F_y$	mean effective stress applied to respectively short and long sides, compressive positive.
$\bar{\sigma}$	effective von Mises stress.
σ_Y	yield stress.
σ'_Y	reduced yield stress in y-direction $= \dfrac{\sigma_Y{}^2 - \bar{\sigma}^2}{\sqrt{F_y{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}}$
μ	coefficient of plastic orthotropy $= \sqrt{\dfrac{F_y{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}{F_x{}^2 + \sigma_Y{}^2 - \bar{\sigma}^2}}$.
E	Young's modulus.
E'	reduced modulus $= \dfrac{\sigma_Y}{\bar{\sigma}} E$ if $\bar{\sigma} > \sigma_Y$.

Definition of boundary conditions:

a) Bending action.
 (i) simply-supported.
 (ii) clamped (i.e. moments developed).

b) Membrane action.
 (i) restrained (in-plane movement suppressed).
 (ii) straight (pull-in allowed, but edges kept straight).
 (iii) unrestrained (edges warp under uniform edge stress).

Practical Relevance

The practical value of the proposed method is that it provides a means of analysing the strength of steel plating which is almost as simple as conventional plastic yield line analysis, but by allowing for membrane stresses is not so wasteful of material. At the same time the complexities of general loading and boundary conditions are simplified so that a more rational and hence reliable design can be made. Appendix C considers two examples.

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Summary

The principal objection to Johansen yield-line analysis of steel plating is that significant membrane action is neglected. A simple technique of analysing this effect in isolation is described. Curves are obtained which can be used directly in the design of rectangular plates under uniform lateral load with various combinations of in-plane loading and boundary conditions, on the basis of a limiting lateral deflection.

Résumé

L'objection principale contre l'analyse de la ligne de rupture pour plaques en acier est due au fait que l'effet caractéristique de membrane y est négligé. Ici on présente une méthode simple pour calculer cet effet comme facteur isolé. On obtient des courbes pouvant être utilisées à projeter des plaques rectangulaires sous charge uniforme latérale, avec différentes combinaisons de charges dans le plan et conditions limites, sur la base d'une déflexion limite latérale.

Zusammenfassung

Der hauptsächliche Einwand gegen die Bruchlinien-Analyse für Stahlplatten besteht darin, dass die charakteristische Membranwirkung vernachlässigt wird. Hier wird ein einfaches Verfahren sur Berechnung dieser isoliert auftretenden Wirkung beschrieben. Man erhält Kurven, die sich direkt zum Entwurf rechteckiger Platten unter gleichförmiger seitlicher Belastung mit verschiedenen Kombinationen von Randlast und Grenzbedingungen auf Basis einer begrenzenden seitlichen Krümmung verwerten lassen.

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