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Moment-Curvature-Time-Relations for Reinforced Concrete Beams

Relations entre moment, flexion et temps sur les poutres en béton armé

Beziehung zwischen Moment, Biegung und Zeit an Stahlbetonbalken

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1. Introduction

Analytic studies have shown that the time-varying behaviour of a reinforced concrete section subjected to sustained bending moment is nonlinear in nature, even when the applied moment is small and the concrete itself is acting as a linear material [6, 8]. Any general method of structural analysis for concrete structures under varying sustained loading must take into account this creep-induced non-linear behaviour, as well as the non-linearities inherent in the instantaneous response to short-term loading [3].

As pointed out by FERRY BORGES [3], the development of accurate techniques of non-linear analysis is of importance, not simply for use in the design of special structures, but also, and perhaps more important, to provide a means of calibrating and evaluating simplified calculation procedures.

In the case of short-term, non-linear response to "instantaneous" loadings, consideration must be given to overall unloading of the structure during an unloading cycle and also to unloading of localized regions during a loading cycle. In order to treat such effects properly, an incremental analysis is required in which a sequence of loading stages is considered.

For the more general case of long-term response to time-varying load history, the non-linear incremental analysis becomes, in effect, a simulation of structural behaviour, whereby the state of the structural system, as represented by stresses and strains in selected sections and by deformations and deflections at selected points, is evaluated for a sequence of load levels and time instants. Simulation of structural behaviour clearly becomes a practical possibility only with the use of a digital computer.

Before the computer simulation of the general time-varying behaviour of a reinforced concrete frame structure can be undertaken, methods must be developed for generating moment-curvature-time relations for flexural members and moment-thrust-curvature-time relations for columns.

In the present paper a method is outlined for computing the curvature history of a reinforced concrete section which is subjected to any prescribed moment history. The method is an extension of a previously reported procedure for computing the biaxial moment-thrust-curvature relations of a column section under short-term loading [9]. The analysis has been generalized to take into account the effects of creep and shrinkage in the concrete. In order to simplify the presentation, the method is here described for the special case of a rectangular section in pure uniaxial bending. However, extension to a cross section of irregular shape under time-varying thrust and skew bending follows directly from the treatment of short-term loading in Ref. [9].

2. General Description of Method

The time-varying moment applied to the cross section is approximated by a finite sequence of moment values $M(1), M(2), \ldots M(n), \ldots M(N)$, being taken to act during a sequence of time intervals $\Delta t(1), \Delta t(2), \ldots \Delta t(n), \ldots \Delta t(N)$. The time intervals $\Delta t(n)$ are small enough to ensure that the moment increments

$$\Delta M(n) = M(n) - M(n-1) \tag{1}$$

are also small in magnitude. See Fig. 1. If the moment history contains a large instantaneous change in moment, for example as at time zero when the initial loading is applied, it is necessary to use a sequence of moment increments, in order to preserve the incremental nature of the analysis.

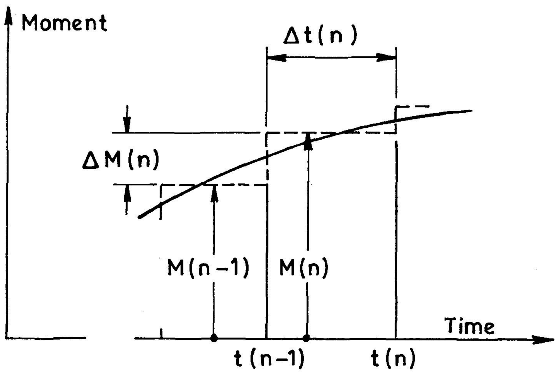

Fig. 1. Discretization of moment history.

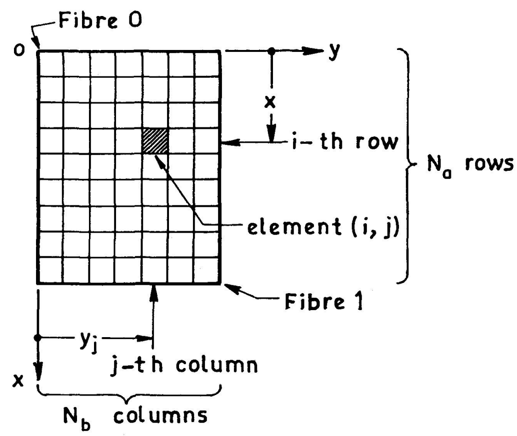

Fig. 2. Partitioning of concrete section.

No restriction is placed on the values $M(n)$. Should a moment be stipulated which is greater than the carrying capacity of the section at any particular time, then failure is recorded and the computation is terminated.

To simplify equilibrium calculations, the section is replaced by a finite number of elemental areas of concrete and steel. The N_s steel elements are actual reinforcing bars in the section or, when more convenient, groups of bars. The procedure of partitioning the concrete section has been described previously [9], and is illustrated in Fig. 2. For the special case of uniaxial bending it is appropriate to take the number of element columns, N_b, as one. However, the two-way partitioning in Fig. 2 allows an immediate extension of the analysis to treat non-rectangular sections and skew bending.

The total strain in a typical concrete element in the i-th row and j-th column is assumed to be composed of an instantaneous, a creep and a shrinkage component. The time-varying stress history of each element is also discretized by assuming that changes in stress level occur only at the time instants $t(n)$ which define the end of the time intervals. The stress $\sigma_{ij}(n)$ acting during time interval $\Delta t(n)$ is thus taken to be constant.

Computations are carried out in cycles; each cycle corresponds to the passage of a time interval. Time interval $\Delta t(n)$ begins at time $t(n-1)$ *after* the moment increment $\Delta M(n)$ has been applied. The state of stress and strain in each concrete and steel element at $t(n-1)$ is known from the previous computation cycle. All concrete elements are allowed to creep and shrink freely during $\Delta t(n)$. Resulting increments in creep and shrinkage strains in all concrete elements are calculated using an appropriate constitutive relation.

As a result of the assumed free straining of the concrete elements, strain incompatibilities develop throughout the section between adjacent concrete elements and also between steel elements and the surrounding concrete. Furthermore, at the end of the time increment, i.e. just before the instant $t(n)$, the moment increment $\Delta M(n+1)$ is applied to the section. The computation cycle is therefore completed by adjusting instantaneous strains and associated stresses in the elemental areas so that, at time $t(n)$,

a) compatibility of deformations over the entire section is re-established, and

b) all equilibrium requirements are fulfilled.

The stresses and strains thus obtained provide the initial conditions required for the next cycle of computations.

This calculation procedure is fairly standard, having been used in creep studies of both metal and concrete structures. The particular method used here resembles the "creep method" proposed by BRESLER [1]. However, a rather general formulation has been attempted, which is potentially applicable to sections of arbitrary shape subjected to arbitrarily varying moment histories ranging from zero up to the section carrying capacity.

3. Stress and Strain in Plain Concrete

A non-dimensionalized stress,

$$S = \frac{\sigma}{\sigma_u} \tag{2}$$

and a normalized strain,

$$E = \frac{\epsilon}{\epsilon_c'} \tag{3}$$

are introduced to represent stresses and strains in the concrete elements. The reference stress σ_u is the strength of the concrete in the member at a specified time, for example at 28 days after casting, and ϵ_c' is the instantaneous strain associated with σ_u. Compressive stresses and strains are taken as positive.

The total concrete strain, composed of instantaneous, creep and shrinkage components, is

$$\epsilon = \epsilon^i + \epsilon^c + \epsilon^s \tag{4a}$$

or in normalized form,

$$E = E^i + E^c + E^s. \tag{4}$$

The following equations are here used to represent the relation between instantaneous strain and stress for monotonicylly increasing strain [9].

$$E^i < 0: \qquad S = 0. \tag{5a}$$

$$0 \leqq E^i \leqq 1.0: \qquad S = \gamma_1 E^i + (3 - 2\gamma_1) E^{i\,2} + (\gamma_1 - 2) E^{i\,3}. \tag{5b}$$

$$1.0 \leqq E^i \leqq \gamma_2: \qquad S = 1 - \frac{1 - 2E^i + E^{i\,2}}{1 - 2\gamma_2 + \gamma_2^2}. \tag{5c}$$

$$E^i > \gamma_2: \qquad S = 0. \tag{5d}$$

The parameters γ_1 and γ_2 define the shape of the loading and unloading portions of the curve, respectively. See Fig. 3a. The initial slope, γ_1, is fixed by the initial elastic modulus of concrete E_c,

$$\gamma_1 = \frac{E_c \epsilon_c'}{\sigma_u}. \tag{6}$$

This parameter is also used to define the stress-strain relation for non-monotonic changes in instantaneous strain. The term $\max(E^i)$ is introduced to denote the maximum instantaneous strain which has occurred in the particular concrete element during all previous loading stages and time instants. If the current value E^i is larger than $\max(E^i)$ then Eqs. (5) apply; otherwise, the stress is determined as follows,

$$0 < E^i \leqq \max(E^i): \qquad S = \max(S) - \gamma_1 [\max(E^i) - E^i]. \tag{7}$$
$$\geqq 0$$

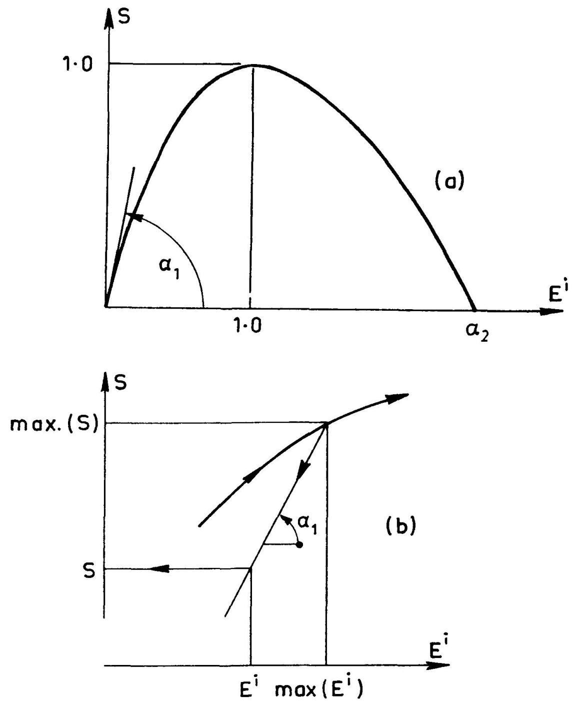

Fig. 3. Relation between stress and instantaneous strain.

Eq. (7) gives non-zero values of stress for $\max(E^i) < \gamma_2$. The term $\max(S)$ is the maximum recorded stress and is related to $\max(E^i)$ by Eqs. (5). See Fig. 3b.

The stress S corresponding to a current strain E^i and a given pre-history of strain is defined jointly by Eqs. (5) and (7), which can be expressed as

$$S = FC\{E^i, \max(E^i)\}. \tag{8a}$$

This computation is carried out by means of an elementary sub-program. The presence of $\max(E^i)$ in Eq. (8a) emphasizes the incremental, step-wise nature of the analysis. For the calculation of stress in the (i, j)th concrete element for the n-th computation cycle, Eq. (8a) can be written more precisely as

$$S_{ij}(n) = FC\{E^i_{ij}(n), \max[E^i_{ij}(m)]\}, \quad 1 \leq m < n \tag{8}$$

where the indicator m, $m < n$, refers to previous cycles.

From Eq. (7) it can be seen that the instantaneous strain is composed of a linear-elastic, recoverable component E^e, and a non-linear, non recoverable plastic component E^p,

$$E^i = E^e + E^p. \tag{9}$$

Considering briefly the stress-strain relation for the reinforcing steel, we note that total steel strain consists of an elastic and a plastic component. The stress in the k-th steel element for the n-th computation cycle can be expressed in a form similar to Eq. (8),

$$S_k(n) = F\,S\{E_k(n), \max_{1 \leqq m < n} [E_k(n)]\}. \tag{10}$$

The yield stress σ_y and the yield strain ϵ_y provide convenient reference values for the definition of non-dimensional stress S_k and normalized strain E_k.

Creep strains in the concrete are accounted for by means of a constitutive relation which takes into account non-linear effects at high stress, as well as the characteristic properties of partial ageing and partial recovery. Details of the particular model of concrete creep here used are given in Ref. [10].

The creep strain at time t, $\epsilon^c(t)$, is assumed to be made up of three components:
$\epsilon^d(t)$ is linear, ageing and non-recoverable; $\epsilon^v(t)$ is linear, non-ageing and recoverable; and $\epsilon^n(t)$ is non-linear. The rate-of-change of creep strain is thus

$$\dot{\epsilon}^c(t) = \dot{\epsilon}^d(t) + \dot{\epsilon}^v(t) + \dot{\epsilon}^n(t). \tag{11}$$

An expression for $\dot{\epsilon}^d(t)$ follows from the Dischinger creep theory (2),

$$\dot{\epsilon}^d(t) = \epsilon^e(t)\,\dot{\phi}^d(t), \tag{12}$$

in which $\epsilon^e(t)$ is the linear-elastic component of the instantaneous strain and $\dot{\phi}^d(t)$ is a creep function, yet to be defined (see Eq. (17)), but which is similar in all major aspects to the Dischinger creep function. It will be noted that $\dot{\epsilon}^d(t)$ is directly related to stress level through $\epsilon^e(t)$.

The linear viscoelastic creep rate can be expressed as (4)

$$\dot{\epsilon}^v(t) = [\epsilon^e(t)\,\phi^v_* - \phi^v(t)]\frac{1}{T_v}, \tag{13}$$

in which ϕ^v_* and T_v are constant material parameters. Depending on the relative magnitudes of the two bracketed quantities in Eq. (13), $\dot{\epsilon}^v(t)$ may be positive (creep) or negative (creep recovery).

Both $\dot{\epsilon}^d(t)$ and $\dot{\epsilon}^v(t)$ are linear with respect to stress. The additional non-linear component is assumed to be non-zero only when the instantaneous stress $\sigma(t)$ exceeds some threshold value σ_c. A creep test conducted at a constant stress σ which is less than σ_c yields a record of experimental values for the total linear component,

$$\sigma \leqq \sigma_c: \qquad \epsilon^c(t) = \epsilon^d(t) + \epsilon^v(t). \tag{14}$$

The creep functions $\phi(t)$ and $\phi^d(t)$ are defined in terms of the experimentally obtained values of $\epsilon^c(t)$ in Eq. (14) as follows,

$$\phi(t) = \frac{\epsilon^c(t)}{\epsilon^e}, \tag{15}$$

$$\phi_* = \phi(\infty) = \frac{\epsilon^c(\infty)}{\epsilon^e}, \tag{16}$$

$$\phi^d(t) = \frac{\epsilon^c(t) - \epsilon^v(t)}{\epsilon^e}, \tag{17}$$

$$\phi_*^d = \phi^d(\infty) = \frac{\epsilon^c(\infty) - \epsilon^v(\infty)}{\epsilon^e}. \tag{18}$$

The parameter ϕ_*^v is defined as

$$\phi_*^v = \frac{\epsilon^v(\infty)}{\epsilon^e}, \tag{19}$$

so that the following relations apply,

$$\phi_*^d = \alpha_d \phi_*, \qquad \alpha_d \leqq 1.0, \tag{20}$$

$$\phi_*^v = \alpha_v \phi_*, \qquad \alpha_v \leqq 1.0, \tag{21}$$

$$\alpha_d + \alpha_v = 1.0. \tag{22}$$

For the creep test at constant stress, the assumed viscoelastic component is

$$\epsilon^v(t) = \epsilon^e \phi_*^v (1 - e^{-t/T_v}). \tag{23}$$

If numerical values can be given to ϕ_*^v and T_v, then the creep function $\phi^d(t)$ can be evaluated from the experimentally determined $\phi(t)$,

$$\phi^d(t) = \phi(t) - \phi_*^v (1 - e^{-t/T_v}). \tag{24}$$

We have yet to consider non-linear creep at stresses in excess of σ_c. It is convenient to relate $\dot{\epsilon}^n(t)$ back to the total linear component by introducing a stress-dependent multiplying factor, $G(\sigma)$,

$$\dot{\epsilon}^n(t) = [\dot{\epsilon}^d(t) + \dot{\epsilon}^v(t)]\, G(\sigma). \tag{25}$$

Setting $H(\sigma) = 1 + G(\sigma)$, we obtain for the total creep rate

$$\dot{\epsilon}^c(t) = [\dot{\epsilon}^d(t) + \dot{\epsilon}^v(t)]\, H(\sigma). \tag{26}$$

The multiplier $G(\sigma)$ is taken to be a power function of stress. A convenient and appropriate non-dimensional expression for $H(\sigma)$ is then

$$\sigma \leqq \sigma_c: \qquad H(\sigma) = 1.0, \tag{27a}$$

$$\sigma_c < \sigma \leqq \sigma_u: \qquad H(\sigma) = 1.0 + \alpha_m \left[\frac{\sigma - \sigma_c}{\sigma_u - \sigma_c}\right]^{\alpha_n}. \tag{27b}$$

The material parameters α_d, α_m, α_n and T_v, together with the creep function $\phi(t)$, have to be evaluated from test data, Suggested values of the parameters are [9],

$$\alpha_d \approx 0.3, \qquad \alpha_v \approx 0.7,$$
$$30 \leqq T_v \leqq 60 \quad \text{(For time } t \text{ measured in days)},$$
$$10 \leqq \alpha_m \leqq 20,$$
$$3 \leqq \alpha_n \leqq 4.$$

A typical creep function $\phi(t)$, obtained from test data (7), is shown in Fig. 4. For convenience the function

$$f(t) = \frac{\phi(t)}{\phi_*} \leqq 1.0 \tag{28}$$

has been plotted.

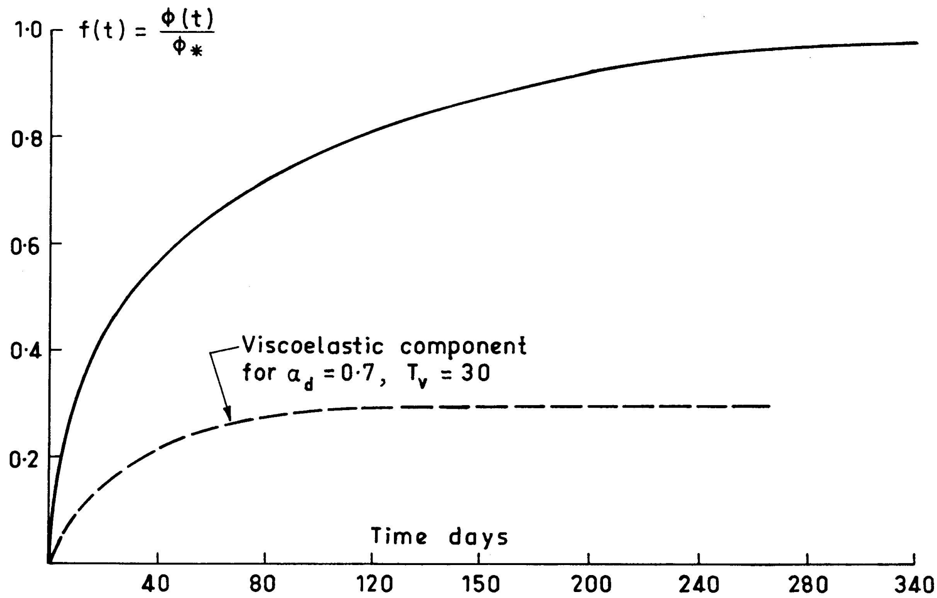

Fig. 4. Creep function used in calculations.

The above creep equations can be expressed in difference form using normalized strains. For the time interval $\Delta t(n)$ the increment in creep strain in the $(i-j)$th concrete element, which is subjected to stress $S_{ij}(n-1)$, is

$$\Delta E^c_{ij}(n) = [\Delta E^d_{ij}(n) + \Delta E^v_{ij}(n)]\, H[S_{ij}(n-1)], \tag{29}$$

where

$$\Delta E^d_{ij}(n) = E^e_{ij}(n-1)\, \Delta \phi^d(n), \tag{30}$$

$$\Delta \phi^d(n) = \phi[t(n)] - \phi[t(n-1)], \tag{31}$$

$$\Delta E^v_{ij}(n) = [E^e_{ij}(n-1)\, \phi^v_* - E^v_{ij}(n-1)]\, \frac{\Delta t(n)}{T_v}, \tag{32}$$

$$S_{ij}(n-1) \leqq S_c: \qquad H[S_{ij}(n-1)] = 1.0, \tag{33a}$$

$$S_c \leqq S_{ij}(n-1) \leqq 1.0: \qquad H[S_{ij}(n-1)] = 1.0 + \alpha_m \left[\frac{S_{ij}(n-1) - S_c}{1 - S_c}\right]^{\alpha_n}. \tag{33b}$$

Shrinkage strains, assumed to occur independently of stress level, can be expressed as

$$\epsilon^s(t) = \epsilon^s_*\, g(t), \tag{34}$$

in which ϵ^s_* is the end value and $g(t)$ is an experimentally obtained pure-time function which increases monotonically from zero to unity. Eq. (34) is rewritten as

$$\Delta E_{ij}^{s}(n) = E_{*}^{s} \Delta g(n), \tag{35}$$

$$\Delta g(n) = g[t(n)] - g[t(n-1)]. \tag{36}$$

4. Computation Cycle

To simplify the computation cycle, it is convenient to choose the sequence of time intervals $\Delta t(n)$ in such a manner that all increments $\Delta \phi(n)$ in the creep function are of the same magnitude. Thus for a total of N time steps,

$$\Delta \phi(n) = \Delta \phi = \frac{\phi_*}{N}, \tag{37}$$

$$\phi(n) = n \Delta \phi. \tag{38}$$

The appropriate time instants $t(n)$ are obtained from an experimentally obtained creep curve, such as Fig. 4.

The associated values $\Delta \phi^d(n)$ vary in magnitude, but can be obtained from Eq. (24). Corresponding increments in the shrinkage function $g(t)$, i.e. $\Delta g(n)$ in Eq. (36), are obtained from experimental curves. Eqs. (29) and (35) can thus be used to determine the strain increments $\Delta E_{ij}^{c}(n)$ and $E_{ij}^{s}(n)$ in each concrete element during time interval $\Delta t(n)$.

At the end of the time interval $\Delta t(n)$ the total creep and shrinkage strains in each element are obtained as

$$E_{ij}^{c}(n) = E_{ij}^{c}(n-1) + \Delta E_{ij}^{c}(n), \tag{39}$$

$$E_{ij}^{s}(n) = E_{ij}^{s}(n-1) + \Delta E_{ij}^{s}(n). \tag{40}$$

It is assumed that total concrete strains at time $t(n)$ are linearly distributed over the section, and also that the strain in each steel element is equal to that in the surrounding concrete. In the case of uniaxial bending, the strain distribution in the section is completely defined by the upper and lower extreme fibre strains $E_0(n)$ and $E_1(n)$. If these values are known or assumed, the strain in the (i,j)th concrete element is given as

$$E_{ij}(n) = E_0(n) - [E_0(n) - E_1(n)]\,\alpha_i, \tag{41}$$

in which α_i defines the depth of the element below the top fibre.

$$\alpha_i = \frac{x_i}{a} = \frac{i+0.5}{N_a}. \tag{42}$$

The instantaneous strain component, obtained by subtracting the creep and shrinkage components, is

$$E_{ij}^{i}(n) = E_{ij}(n) - E_{ij}^{c}(n) - E_{ij}^{s}(n) \tag{43}$$

and the associated stress, $S_{ij}(n)$, is given by Eq. (8).

The total strain in the k-th steel element is also obtained from the extreme fibre strains,

$$E_k(n) = \frac{1}{\eta}\{E_0(n) - [E_0(n) - E_1(n)]\,\alpha_k\} \tag{44}$$

and this allows the stress $S_k(n)$ in this element to be obtained by means of Eq. (10). The depth of the k-th steel element is given by α_k,

$$\alpha_k = \frac{x_k}{a} \tag{45}$$

and η is the ratio of reference strains.

$$\eta = \frac{\epsilon_y}{\epsilon_c'}. \tag{46}$$

With all stresses known, the total force in the section can be obtained by summing the elemental forces acting in all elemental areas. Defining the non-dimensional force term,

$$\bar{P} = \frac{P}{\sigma_u\, b\, a} \tag{47}$$

one obtains for the resultant force in the section,

$$\bar{P} = \frac{1}{N_s}\sum_i\sum_j S_{ij}(n) + \mu\, m \frac{1}{N_s}\sum_k S_k(n) - \mu\frac{1}{N_s}\sum_k S_k^c(n). \tag{48}$$

Here, μ is the ratio of steel and concrete areas,

$$\mu = \frac{A_s}{b\,a}, \tag{49}$$

m is the ratio of reference stresses,

$$m = \frac{\sigma_y}{\sigma_u} \tag{50}$$

and N_c is the total number of concrete elements,

$$N_c = N_b N_a. \tag{51}$$

The third summation term in Eq. (48) has been introduced to account for the concrete area which has been replaced by the steel elements. The term $S_k^c(n)$ is the concrete stress at the level of the k-th steel element.

The resultant moment acting in the section is found by taking moments of the elemental forces about the horizontal y axis shown in Fig. 2. With

$$\bar{M} = \frac{M}{\sigma_u\, b\, a^2}, \tag{52}$$

this gives

$$\bar{M} = \frac{1}{N_c}\sum_i\sum_j S_{ij}(n)\,\alpha_i + \mu\, m\frac{1}{N_s}\sum_k S_k(n)\,\alpha_k - \mu\frac{1}{N_s}\sum_k S_k^c(n)\,\alpha_k. \tag{53}$$

At any time instant $t(n)$, the correct values of $E_0(n)$ and $E_1(n)$ are such that the following equilibrium requirements are fulfilled:

a) The force P is zero;

b) The moment M is equal to the prescribed moment, $M(n+1)$,

$$M(n+1) = M(n) + \varDelta M(n+1). \tag{54}$$

For practical calculation purposes, strain values are accepted if they fulfil the equilibrium requirements to within specified tolerances t_p and t_M, viz

$$|\bar{P}| \leqq t_p, \tag{55}$$

$$|\bar{M} - \bar{M}(n+1)| \leqq t_M. \tag{56}$$

A nested search technique has been developed, which consists of an outer procedure to find $E_0(n)$ and an inner procedure to find $E_1(n)$. The inner procedure, called SEEKE 1, establishes a value of $E_1(n)$ which, together with any value of $E_0(n)$ prescribed by the other procedure, satisfies Eq. (55). Basically, SEEKE 1 consists of two phases. In the first phase, bounds on E_1 are found, E_1^U and E_1^L, for which the corresponding value of $\bar{P}$ are positive and negative, respectively. These bounds define a search region within which the required value lies. In the second phase, a halving process is used, whereby the mid point of the search region,

$$E_1^M = \tfrac{1}{2}[E_1^U + E_1^L]$$

is tested. If this value satisfies Eq. (55) the search is terminated; otherwise, the search region is halved and the process continues. Figs. 5 and 6 show schematically the two phases of SEEKE 1.

The outer search procedure, SEEKE 0, establishes a value of $E_0(n)$ which fulfills Eq. (56). However, for every trial value of E_0, SEEKE 0 calls on SEEKE 1 to carry out a subsidiary search for an E_1 value so that Eq. (55) is always fulfilled. The structure of SEEKE 0 is quite similar to that of SEEKE 1. Upper and lower bounds are first established, E_0^U and E_0^L, such that the computed $\bar{M}$ is greater than and less than $\bar{M}(n+1)$, respectively. The halving procedure then brings convergence to a set of values which together satisfy Eqs. (55) and (56).

Although the nested search procedure is certainly non optimal, it is extremely simple in structure and has proved to be surprisingly efficient and reliable, not only for the case uniaxial bending but also, in an extended form, for calculations involving biaxial bending and compression.

With the strain distribution determined, it is a simple matter to obtain curvature. A reference curvature is introduced,

$$K_{ref} = \frac{\epsilon_c'}{a}$$

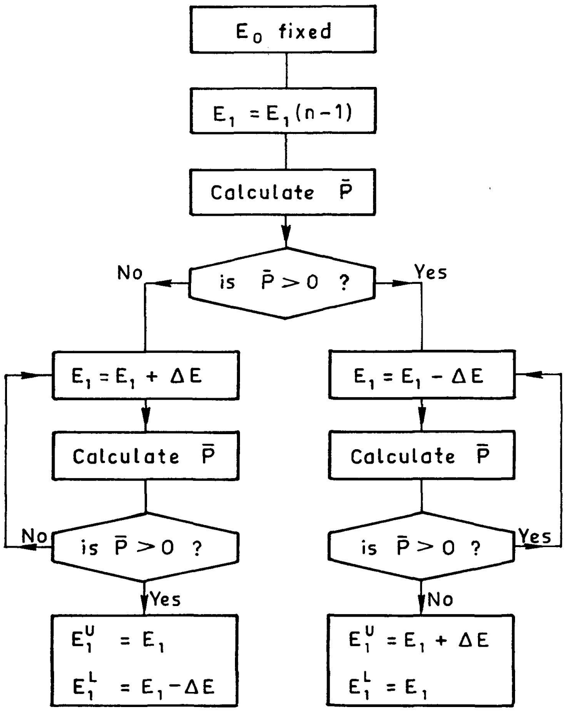

Fig. 5. Seeke 1. Bounds on E_1.

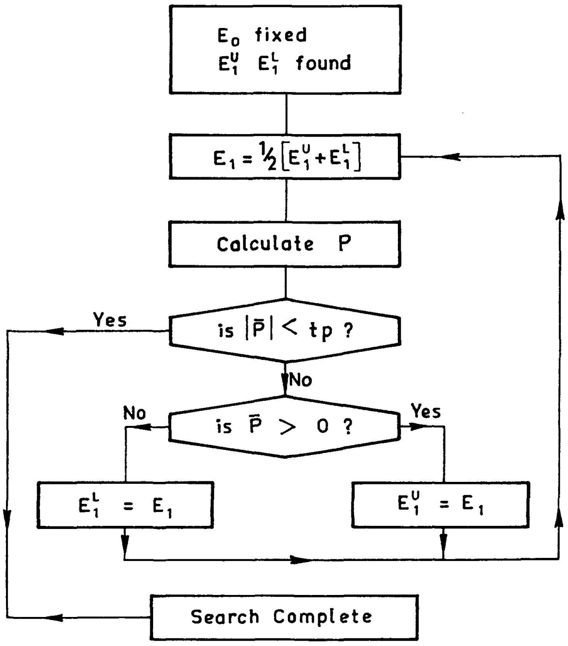

Fig. 6. Seeke 1. Halving procedure.

and the non-dimensional curvature $\bar{K}$ is calculated simply as

$$\bar{K} = \frac{K}{K_{ref}} = E_0(n) - E_1(n).$$

The computation cycle described above is applicable provided the moment increments $\Delta M(n)$ are reasonably small. It remains to take care of instantaneous loadings and unloadings, including the first loading at time zero when the initial conditions are established for the first computational cycle.

When a significant jump in moment occurs, the extreme compressive fibre strain E_0 is incremented repeatedly by a small value ΔE. Thus, for the initial loading, a sequence of strains ΔE, $2\Delta E, \ldots$ is considered. An appropriate value of E_1 is in each case determined, by means of SEEKE 1, such that Eq. (55) is fulfilled, and the corresponding moment $\bar{M}$ is calculated by means of Eq. (53). The strain E_0 is thus incremented until $\bar{M}$ becomes larger than the prescribed value. When this occurs, upper and lower bounds on E_0 have been established and the halving process in SEEKE 0 can be used to fulfill Eq. (56).

In the case of a sudden unloading, a similar calculation procedure is adopted, whereby E_0 is decremented incrementally.

5. Numerical Results

In order to test the adequacy of the computation procedure prior to its use in the analysis of time varying structural behaviour, calculations were made for a variety of different cross sections, material parameter values and loading histories. The calculations provided a means for checking the reliability of the search procedures and also gave information on the sensitivity of numerical results to variations in the values of the parameters which are used to define the deformation characteristics (including creep and shrinkage) of the concrete.

Results of several typical calculations are presented in Figs. 8 to 15. Except where otherwise noted, calculations were made for the section details given in Fig. 7 and for the parameter values given in Table 1. The curve used for the creep function $\phi(t)$ is that shown in Fig. 4. The calculations were made with an IBM 360/50 installation. Execution time for a complete variable moment history, followed by an incremental loading to failure, was usually in the order of 30 seconds.

A characteristic of the computed results is the slightly non-smooth nature of almost all resulting curves. This can be seen in Fig. 11 for stress variation

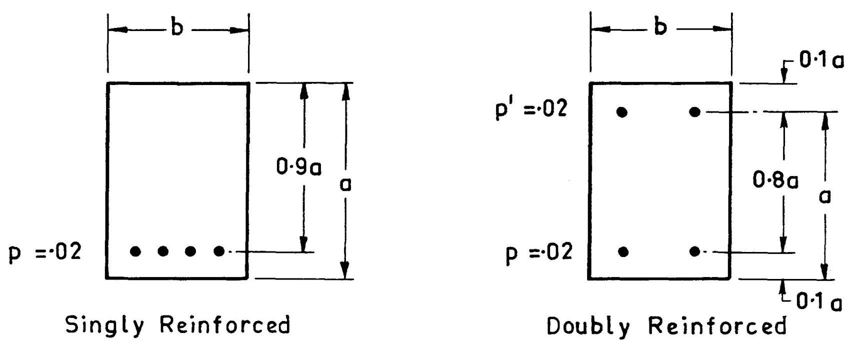

Fig. 7. Cross sections for numerical calculations.

Table 1. Values for Numerical Computations

Stress-strain Relation:	γ_1	$= 2.0$
	γ_2	$= 3.0$
Creep Law:	α_d	$= 1.0,\ 0.7,\ 0.5$
	α_m	$= 20$
	α_n	$= 4$
	T_v	$= 30,\ 60$
	σ_c	$= 0.4\ \sigma_u$
	ϕ_*	$= 3.0$
	E_*^s	$= 0,\ 0.25$
Section Details:	μ	$= 0.02,\ 0.04$
	p	$= 0.02$
	p^1	$= 0,\ 0.02$
Calculation Details:	N_a	$= 20$
	N_b	$= 1$
	N_s	$= 4$
	t_P	$= 0.01$
	t_M	$= 0.01$

over the cross-section, in Fig. 10 for variation of stress with time, and in Fig. 9 for time variation of curvature. The non-smoothness stems of course from the finite computation procedure. Smoothing of the results can nevertheless be achieved, at the expense of additional computation time, by using a finer partitioning grid for the section, smaller time intervals, and, most important of all, finer tolerances t_p and t_M.

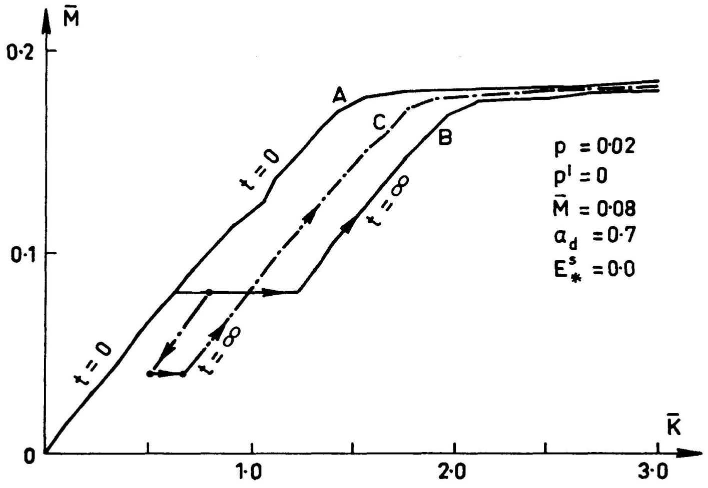

Fig. 8. Curvature histories for three loading cases.

In Fig. 8 curvature histories are shown for three different moment histories applied to a singly reinforced section. Curve A shows the moment-curvature relation for short-term, monotonically increasing loading. Curve B shows moment versus curvature for the case where the moment is raised instantaneously to the value $\bar{M}=0.08$ (about 40 percent of the ultimate moment) and held there indefinitely, with a final loading to failure at time infinity. In the third case, represented by curve C, the moment is raised instantaneously to $\bar{M}=0.08$ and held there for 7 days, when it is reduced to $\bar{M}=0.04$ and held there indefinitely, with a final loading to failure at time infinity. The curves are terminated arbitrarily at a curvature value of $\bar{K}=3.0$. It is seen that the effet of prior history of loading on moment capacity, and also on the latter portion of the moment-curvature relation, is almost imperceptible.

Fig. 9 shows the increase in curvature with time for a singly reinforced and a doubly reinforced section, each subjected to a moment of $\bar{M}=0.08$ (about 40 percent of ultimate moment). As is to be expected, the compression reinforcement plays a decisive role in restricting the development of curvature. Fig. 9 also shows that concrete shrinkage plays a significant part in the time-increase in curvature in both the singly reinforced and the doubly reinforced section.

It is significant that the curvature in the doubly reinforced section for non-zero shrinkage reaches a maximum value and then remains constant over the final few time intervals. The reason for this can be seen in Fig. 10, where the attenuation of maximum concrete compressive stress in the section is plotted.

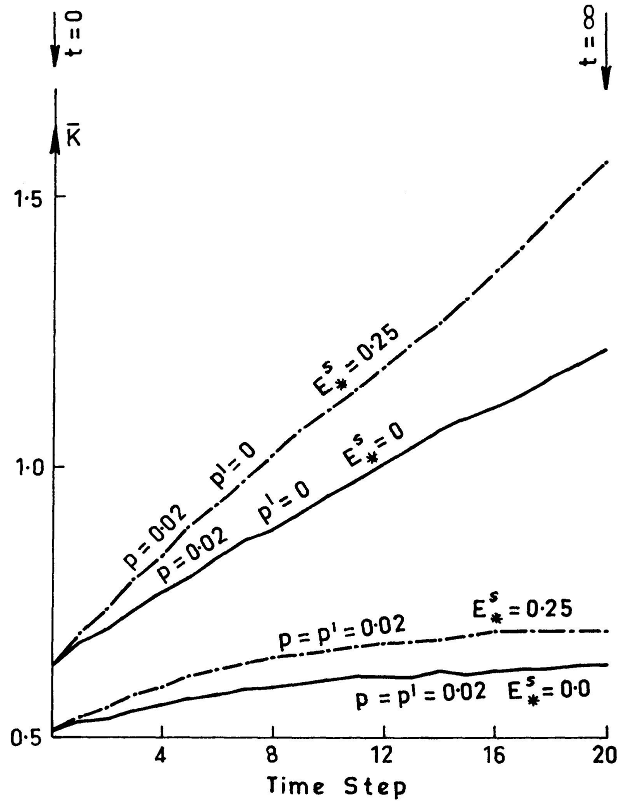

Fig. 9. Curvature histories.

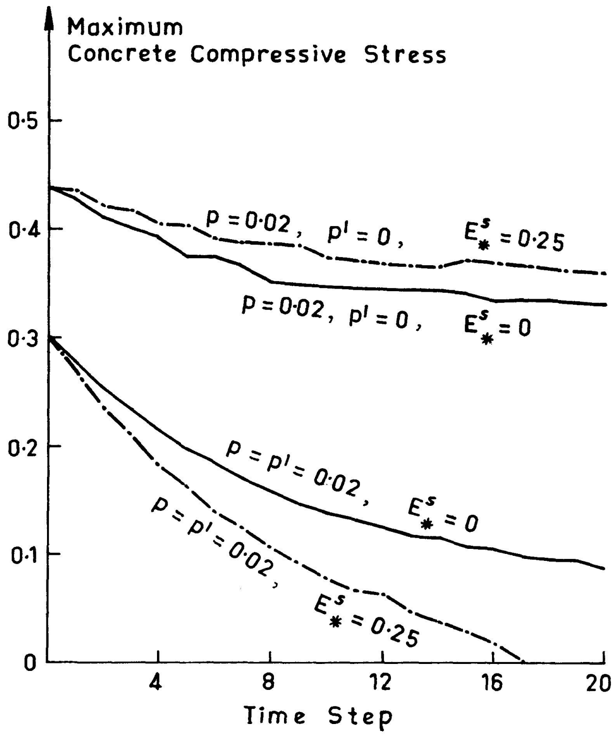

Fig. 10. Attenuation of max. concrete compressive stress.

Due to the restraint provided by the compression reinforcement, coupled with both creep and shrinkage of the concrete, the concrete compressive stresses have completely disappeared and the applied moment is being resisted entirely by a compressive steel force and a tensile steel force. In fact, tensile stresses would develop in the upper fibres of this section. The development of tensile stress fields in the "compression" zone of doubly reinforced beams has been reported previously [6]. Fig. 10 also shows that the concrete compressive stress cannot disappear completely without the presence of concrete shrinkage.

Although shrinkage strains result in an overall reduction in concrete compressive stress in the doubly reinforced section, they result in slightly higher stresses, relative to the no-shrinkage case, in the singly reinforced section. This is explained by the redistribution of stresses in the section with time, as shown in Fig. 11 for the singly reinforced section and in Fig. 12 for the doubly reinforced section. The neutral axes of stress and strain coincide for times

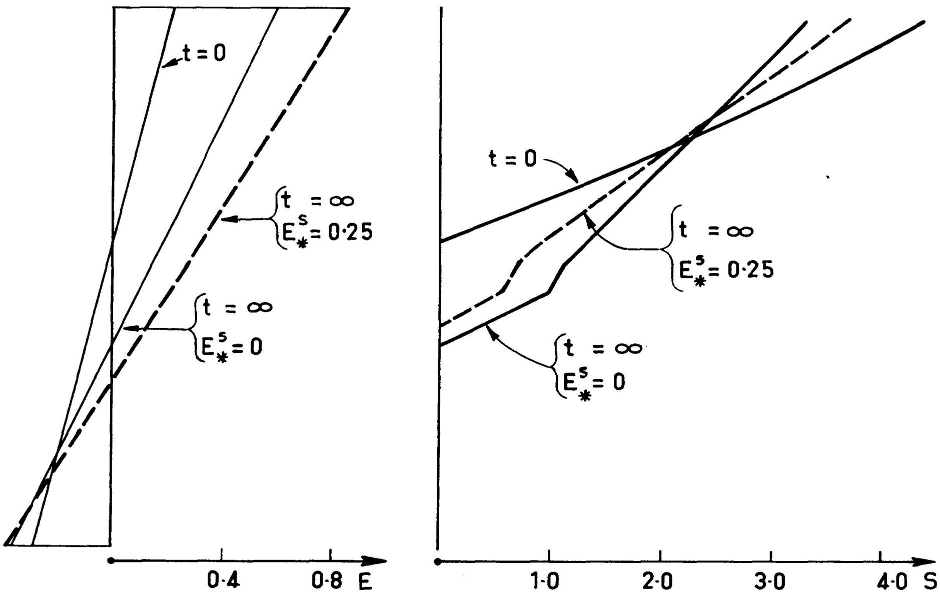

Fig. 11. Stresses and strains in concrete. Singly reinforced section.

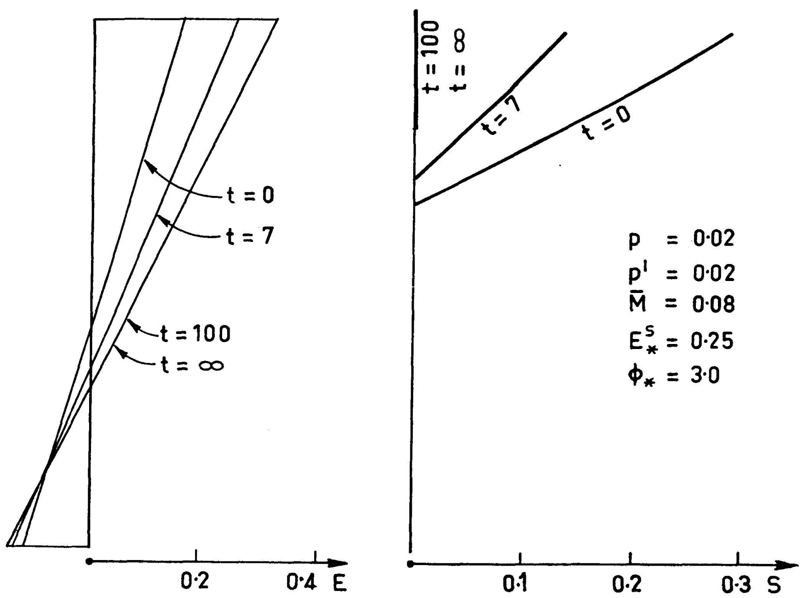

Fig. 12. Stresses and strains in concrete. Doubly reinforced section.

$t > t_0$ only when shrinkage strains are zero. For non-zero shrinkage the stress axis rises above the strain axis, and the effective reduction in the area of the compressive stress block requires in the singly reinforced section some increase in stress over that calculated for zero shrinkage.

Shrinkage strains in the doubly reinforced section also force the neutral axis of stress to rise above that of strain. See Fig. 12. However, the reduction of total concrete compressive stress, due to unloading onto the compression steel, is the over-riding effect here.

A question of prime importance in any analysis of time-varying structural behaviour is the sensitivity of the results to the type of creep law assumed and, further, to the numerical values used for the parameters of the creep law.

It is clear tht ϕ_* and E_*^s are measures of total creep and shrinkage and that they must have an important influence on final deformations in the section. It is also clear that the shape of the creep and shrinkage curves, i.e. $f(t)$ and $g(t)$, will determine, to a large extent, the rate of increase of curvature in the case of constant sustained loading.

The importance of secondary parameters such as α_d, α_m, α_n and T_v is not so obvious. If concrete stress does not change greatly with time, for example as in a singly reinforced section under constant sustained moment, it is reasonable to expect that creep calculations will not depend greatly on α_d and T_v, since these parameters define the ageing and recovery properties of the concrete. Situations most sensitive to variations in these parameters are likely to be those in which large changes occur in the stress level with time.

Fig. 13 shows the attenuation of concrete compressive stress in a doubly reinforced section calculated for values of α_d ranging from 0.5 to 1.0. The value $\alpha_d = 1.0$ corresponds, in the linear range, to Dischinger creep. For the lower value of $\alpha_d = 0.5$, the time constant T_v must be increased, in order to allow some non-recoverable creep to occur during the first few days of loading,

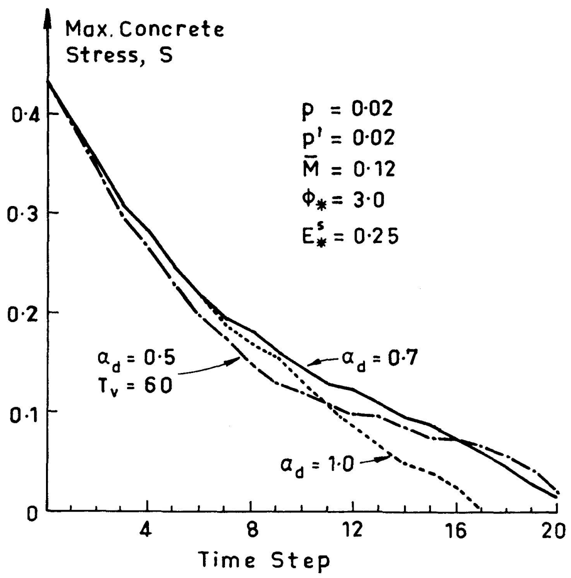

Fig. 13. Effect of parameter α_d-stresses.

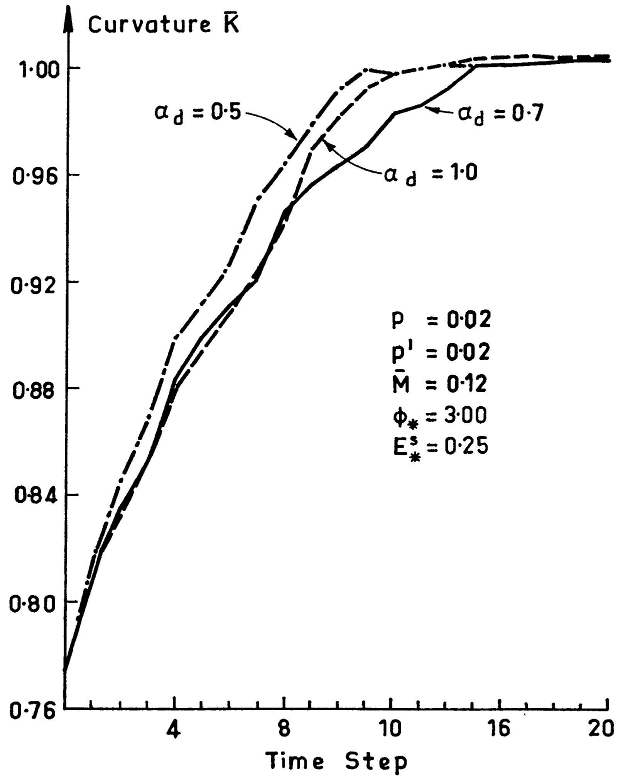

Fig. 14. Effect of parameter α_d-curvature.

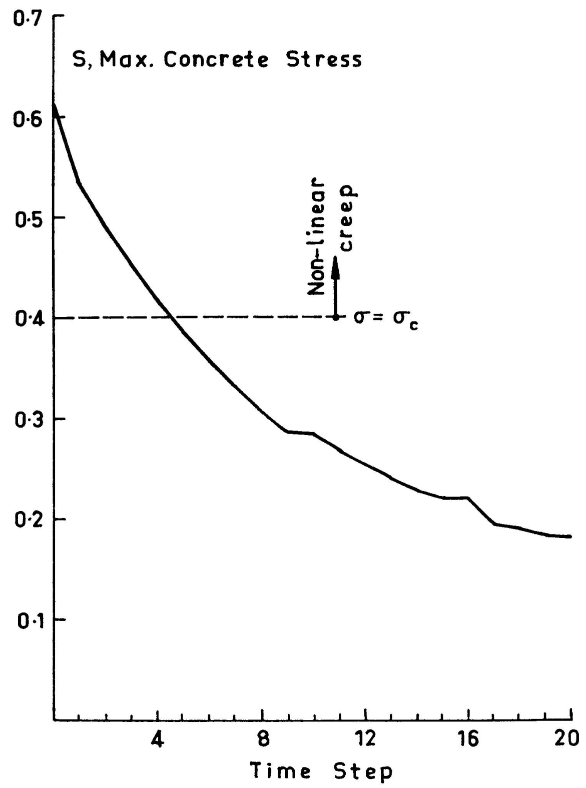

Fig. 15. Non linear creep.

i. e. to ensure that $\epsilon^c(t) > \epsilon^v(t)$. For these calculations T_v was increased from 30 to 60. Creep behaviour shown in Fig. 13 is essentially linear, since the extreme concrete stress exceeds the threshold level of $S_c = 0.4$ only during the first time interval, i. e. during portion of the first day of loading. Results of the calculations shown in Fig. 13 are encouraging, in that changes in α_d have not resulted in very significant changes in the stress history. Corresponding curvature histories are shown in Fig. 14.

The very strong tendency for concrete to unload onto surrounding steel reinforcement lessens the importance of non-linear creep in the case of pure flexure. In Fig. 15a sustained overload moment of $\bar{M}=0.18$ (nearly 90 percent of ultimate moment) produced an initial concrete stress of 0.53. This has attenuated to the threshold level of 0.4 in five time intervals, so that, even at high overload, non-linear creep has occurred only during the first week of loading. Non-linear creep increases the rate of transfer of stress from concrete to steel and, to this extent, tends to be self destroying. Non-linear creep may nevertheless become of prime importance in sections of slender columns. Here, increase in curvature with time results in large increases in deflection and in the geometric non-linearities. This can, in turn, bring about creep buckling.

6. Concluding Remarks

The procedure described in this paper can be used to calculate the curvature history of a reinforced concrete section for any prescribed moment history. This allows the time-varying behaviour of a statically determinate member to be analysed for any variable load history [5]. The procedure also provides a basis for the analysis of time-varying behaviour of indeterminate members, and is at present being used for this purpose.

The procedure can be extended without difficulty to apply to members of irregular section subjected to skew bending.

Although one specific constitutive relation for concrete has been used throughout the present study, changes both in the instantaneous stress-strain relation and in the creep law can be made without affecting in any way the calculation procedure. It is only necessary that the creep law be expressed in difference formulation.

Possible variations with time in concrete material properties such as concrete strength and stiffness have not been considered. Although such variations can be accounted for without difficulty, they do not usually have a significant effect on structural behaviour. Indeed, some of the effects included in the present analysis have proved to be of minor importance. This was, of course, not unexpected: a prime motivation in the development of such a complex procedure is to provide a standard of comparison for simplified methods of analysis [11].

7. List of Symbols

a	=	depth of section
A_s	=	total area of steel in section
b	=	width of section
E_c	=	initial elastic modulus of concrete
$E_{ij}(n)$	=	$\frac{\epsilon_{ij}(n)}{\epsilon_c'}$; normalized concrete strain in (i, j)th element at time $t(n)$
$E_{ij}^c(n)$	=	creep component of $E_{ij}(n)$
$E_{ij}^d(n)$	=	ageing component of $E_{ij}^c(n)$
$E_{ij}^e(n)$	=	linear elastic component of $E_{ij}^i(n)$
$E_{ij}^i(n)$	=	$E_{ij}^e(n) + E_{ij}^p(n)$; instantaneous component of $E_{ij}(n)$
$E_{ij}^p(n)$	=	plastic component of $E_{ij}^i(n)$
$E_{ij}^v(n)$	=	viscoelastic component of $E_{ij}^c(n)$
$E_{ij}^s(n)$	=	shrinkage component of $E_{ij}(n)$
$E_k(n)$	=	$\frac{\epsilon_k(n)}{\epsilon_y}$; normalized strain in k-th steel element at time $t(n)$
$E_0(n)$	=	normalized concrete strain in extreme compression fibre
$E_1(n)$	=	normalized concrete strain in extreme tension fibre
$f(t)$	=	pure time function defining the shape of the creep function
$g(t)$	=	pure time function defining the shape of the shrinkage function
$G(\sigma)$	=	non-linear, power function of stress
$H(\sigma)$	=	$1 + G(\sigma)$; non-linear multiplying factor
K	=	curvature
K_{ref}	=	$\frac{\epsilon_c'}{a}$; reference curvature
$\bar{K}$	=	$\frac{K}{K_{ref}}$; non dimensionalized curvature
m	=	$\frac{\sigma_y}{\sigma_u}$
$\max(E^i)$	=	maximum value of instantaneous strain occurring in previous loading history
$\max(S)$	=	maximum value of S occurring in previous loading history
$\bar{M}$	=	$\frac{M}{\sigma_u b a^2}$; non-dimensionalized moment
$M(n)$	=	moment prescribed for time increment $\Delta t(n)$
N_a	=	number of rows of concrete elemental areas
N_b	=	number of columns of elemental concrete areas
N_c	=	$N_a N_b$; total number of elemental concrete areas in rectangular section
p	=	proportion of tension steel
p'	=	proportion of compression steel
$\bar{P}$	=	$\frac{P}{\sigma_u b a}$; resultant longitudinal force acting in section, non-dimensionalized

S = $\frac{\sigma}{\sigma_u}$; non-dimensionalized concrete stress

S_c = $\frac{\sigma_c}{\sigma_u}$; threshold stress level at which creep becomes non-linear

$S_{ij}(n)$ = non dimensional stress in (i, j)th concrete element during time interval $\Delta t(n)$

$S_k(n)$ = non-dimensional stress in k-th steel element during time interval $\Delta t(n)$

$S_k^c(n)$ = equivalent stress (non-dimensional) in concrete at k-th steel element

t_p = tolerance on computed longitudinal force in section

t_M = tolerance on computed moment in section

$t(n)$ = time instant

T_v = time parameter of viscoelastic component of creep

x_i = x component of (i, j)th concrete element

x_k = x component of k-th steel element

α_i = $\frac{x_i}{a}$

α_k = $\frac{x_k}{a}$

$\left.\begin{matrix}\alpha_d, \alpha_m \\ \alpha_n, \alpha_v\end{matrix}\right\}$ = parameters in creep law

γ_1, γ_2 = parameters defining shape of instantaneous stress-strain relation for concrete

$\epsilon(t)$ = $\epsilon^i(t) + \epsilon^c(t) + \epsilon^s(t)$; total strain in concrete at time t

ϵ_c' = instantaneous strain corresponding to σ_u

ϵ_y = steel yield strain

$\epsilon^c(t)$ = $\epsilon^d(t) + \epsilon^v(t) + \epsilon^n(t)$; creep strain

$\epsilon^d(t)$ = ageing component of $\epsilon^c(t)$

$\epsilon^e(t)$ = linear elastic component of $\epsilon^i(t)$

$\epsilon^i(t)$ = $\epsilon^e(t) + \epsilon^p(t)$; instantaneous component of $\epsilon(t)$

$\epsilon^n(t)$ = non-linear component of $\epsilon^c(t)$

$\epsilon^p(t)$ = plastic component of $\epsilon^i(t)$

$\epsilon^s(t)$ = shrinkage component of $\epsilon(t)$

$\epsilon^v(t)$ = viscoelastic component of $\epsilon^c(t)$

η = $\frac{\epsilon_y}{\epsilon_c'}$

μ = $\frac{A_s}{b\,a}$; total steel proportion

$\sigma_{ij}(n)$ = stress in (i, j)th concrete element during time interval $\Delta t(n)$

σ_u = strength of concrete in member

σ_y = steel yield stress

$\phi(t)$ = creep function

ϕ_* = $\phi_*^d + \phi_*^v$; end value of $\phi(t)$

$\phi^d(t)$ = creep function (Dischinger component of $\phi(t)$)

ϕ^d_*	=	end value of $\phi^d(t)$
ϕ^v_*	=	concrete creep parameter
ΔE	=	increment in normalized strain
ΔM	=	increment in moment
$\Delta t(n)$	=	time interval

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Summary

A method is described for determining the non-linear, time-varying response of a reinforced concrete beam section to any prescribed moment history. The cross section is partitioned into a finite number of elemental areas of steel and concrete, and a non-linear constitutive relation is used to represent the behaviour of each concrete element.

An incremental, step-wise numerical computation procedure allows curvature and stresses and strains to be evaluated for moment histories which may include loading and unloading cycles as well as periods of sustained loading at the service load level and at any overload level up to the carrying capacity of the section.

Résumé

On décrit une méthode pour la détermination de la réaction non-linéaire et dépendant du temps de la section d'une poutre en béton armé à des changements prescrits du moment. La section est divisée en un nombre fini d'éléments en acier et en béton. On se sert d'une relation non-linéaire constitutive pour représenter le comportement de chaque élément en béton.

Une procédure graduelle supplémentaire par ordinateur permet à évaluer flexion, tension et déformation pour le changement du moment, comprenant des cycles de charge et de décharge ainsi que des périodes de charges continues au niveau de service et à n'importe quel niveau de surcharge jusqu'à la capacité de charge de la section.

Zusammenfassung

Es wird eine Methode zur Bestimmung der nichtlinearen, zeitabhängigen Reaktion eines Stahlbetonquerschnitts auf ein vorgeschriebenes veränderliches Moment beschrieben. Der Querschnitt ist in eine finite Anzahl von Elementflächen aus Stahl und Beton unterteilt, und eine nichtlineare konstitutive Beziehung dient zur Darstellung des Verhaltens jedes Betonelements.

Ein inkrementales, stufenweises Computerverfahren gestattet Biegung, Spannung und Deformation für ein veränderliches Moment auszuwerten, welches Belastungs und Entlastungszyklen sowie Perioden dauernder Betriebslast einschliesst, und bei irgendwelchem Überlastniveau bis zur Belastbarkeit des Querschnitts.

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