**Zeitschrift:** IABSE publications = Mémoires AIPC = IVBH Abhandlungen

**Band:** 34 (1974)

Artikel: Mild steel beams: behaviour and failure under cyclic alternating loads

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**DOI:** https://doi.org/10.5169/seals-26278

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# Mild Steel Beams: Behaviour and Failure Under Cyclic Alternating Loads

Poutres en acier doux: comportement et ruine sous l'influence de charges cycliques alternatives

Flussstahl-Balken: Verhalten und Versagen unter zyklischer Wechselbelastung

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#### Introduction

The importance of studying the behaviour of structural components subjected to low-cycle fatigue conditions, as an extension of simple plastic theory, has been described by the authors in earlier publications [1–4]; a technique to predict the behaviour of structural components under alternating deflections has been formulated. The investigation, however, will be incomplete unless it is extended to cover the behaviour of components under a constant load range. As most of the studies reported on this subject are inconclusive, the present investigation is aimed at examining the behaviour and failure of mild steel beams under alternating load and deflection conditions. A brief account is presented of the fatigue machine, fixtures for pure bending, and cantilever tests.

Cyclic moment curvature models which are discrete functions of the instantaneous cyclic history, are generated from cyclic tests on beams under pure bending. These models are then employed to predict the behaviour of cantilever beams under deformation and load control and are compared with experiments.

A failure criterion for estimating the life of structural components is established by coupling strain range and number of cycles to failure from the

<sup>\*)</sup> Dr. S. Krishnasamy was associated with the University of Waterloo during the course of this investigation.

tests on beams under pure bending. The above criterion is then employed to predict the life of cantilever beams subjected to alternating tip loads and the results are compared with experiments.

# Test Equipment and Procedure

The system was augmented by special bending fixtures and transducers suitable to control either load or deformation. The testing machine and the various fixtures are fully explained elsewhere [1, 2].

For the pure bending tests described here the fixtures shown in Figs. 1a and 1b were used. The rig in Fig. 1b is a modified version of the fixture in Fig. 1a. Curvature was measured by a transducer shown in Fig. 2.

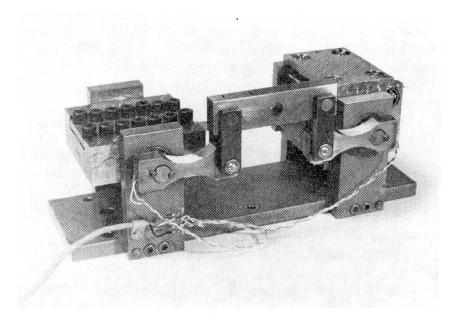


Fig. 1a.
Pure-bending fixture.

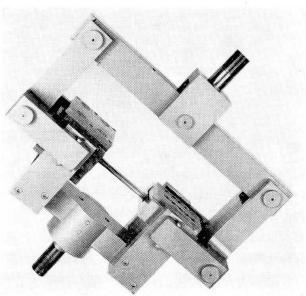


Fig. 1b.

Modified version of the fixture in Fig. 1a.

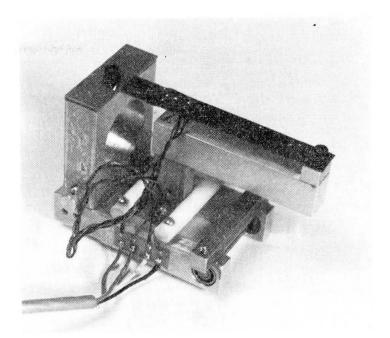


Fig. 2. Curvature meter.

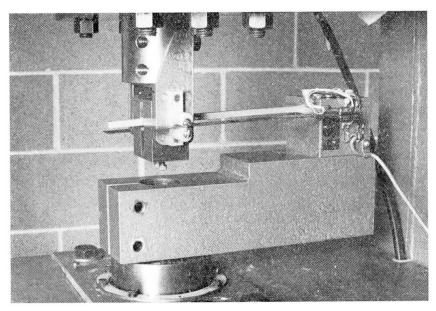


Fig. 3. Cantilever fixture.

The fixture in Fig. 3 was used to test the cantilever specimens. The load on the specimen was measured by the load cell and the measurement of the deflection was achieved by an LVDT (Linear Voltage Displacement Transducer), which is an integral part of the testing system. A strip-chart recorder and X-Y plotter were used to record the load and deformation, respectively.

# Material Properties and Test Specimens

Fully killed 1020 steel was used in making the pure bending and cantilever beam specimens. The chemical properties are tabulated below:

Si	$\mathbf{S}$	P	$\mathbf{M}\mathbf{n}$	$\mathbf{C}$	$\operatorname{Cr}$	${ m Fe}$
0.037	0.025	0.004	0.47	0.143	0.01	Remainder %

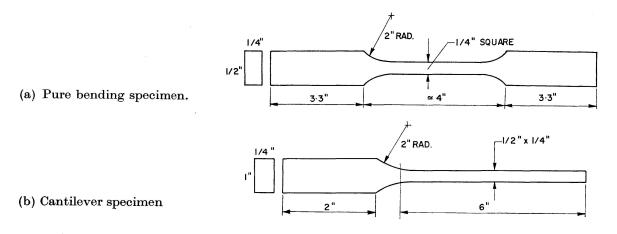
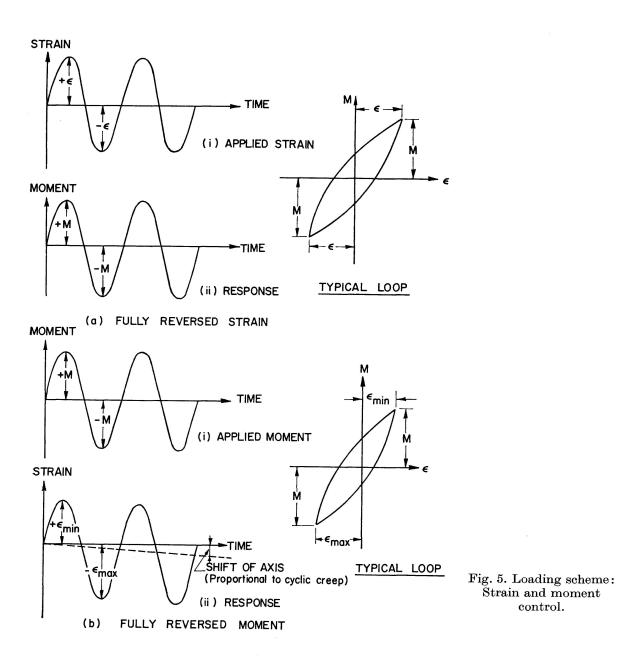


Fig. 4. Test specimens.



The pure bending and cantilever specimens used in the cyclic tests are sketched in Fig. 4. The pure bending test piece had a uniform section  $\frac{1}{4}''$  square and approximately 4'' long about the middle section; the two ends were larger in cross-section. The transition from the middle section to the end was uniform and smooth to avoid stress concentrations. The cantilever beam specimens were approximately 12'' long and  $\frac{1}{2}'' \times \frac{1}{4}''$  in cross-section. The built-in end of the beam was  $2\frac{1}{2}''$  long and  $\frac{1}{2}'' \times 1''$  in cross-section. The smooth transition between these two portions reduces stress concentration. The span of the cantilever beam is  $6\frac{1}{2}''$ .

The specimens were cut roughly to the required size. They were then soaked at 1600°F for about 30 minutes and allowed to cool in air. Great care was taken in machining to reduce the contact stresses. A finish of four microinches was attained in order to achieve a fairly correct estimate of life.

The lower yield stress of the material is found to be 33.3 ksi from static bending tests. The Elastic Modulus and the yield strain of the material from static tension tests are determined as  $29.85 \times 10^6$  psi and 0.14% respectively.

#### **Moment-Curvature Characteristics**

Fig. 5 shows the type of loading and control imposed on the test specimens. The results of pure bending tests on specimens subjected to both strain and moment control are shown in Figs. 6 and 7. The cyclic moment-curvature relations for the analysis of structural components can be formulated from these plots as explained below.

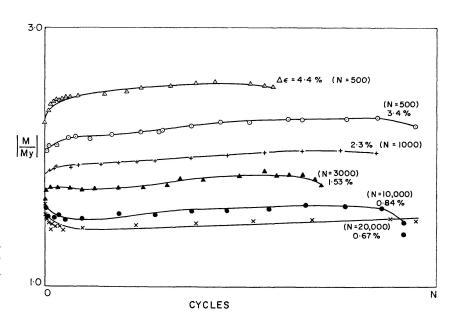
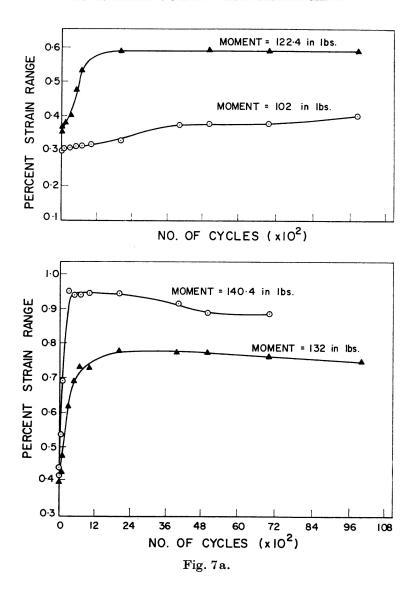
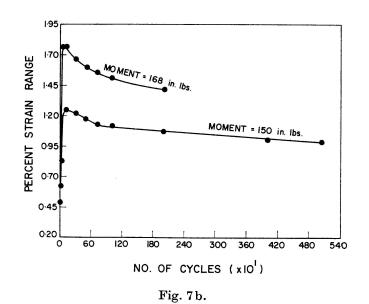


Fig. 6. Cyclic variation of moment amplitude under constant strain range.





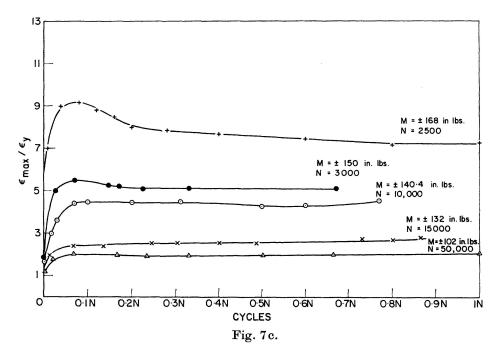


Fig. 7a-c. Cyclic variation of strain range under constant moment.

# Strain Control Tests [1]

Values of moment and curvature (or strain) are selected from the curves of Fig. 6 to construct cyclic moment-curvature characteristics. These relationships can be represented mathematically in non-dimensional form:

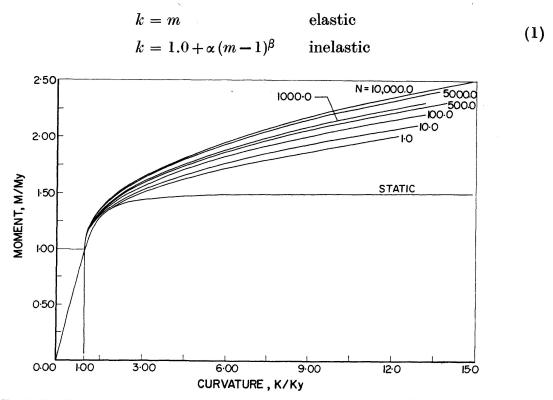


Fig. 8. Cyclic moment curvature characteristics for various cycles: Strain control.

k and m being dimensionless moment and curvature, respectively, and  $\alpha$ ,  $\beta$  geometric parameters which are functions of cyclic material behaviour. It is to be noted from Fig. 6 that the material softens up to a certain range of strain and subsequently hardens. There will be neither softening nor hardening of the material when it is strain cycled in the elastic range. Hence, it will be appropriate to assume a moment-curvature relationship which is linear up to the yield point and nonlinear beyond (Eq. (1)). The constants  $\alpha$ ,  $\beta$  for various numbers of cycles are computed by plotting the data from Fig. 6 in log-log form. The cyclic moment-curvature models generated by employing these constants are shown in Fig. 8. These moment-curvature characteristics are employed to predict analytically the cyclic response of structural components under reversed bending.

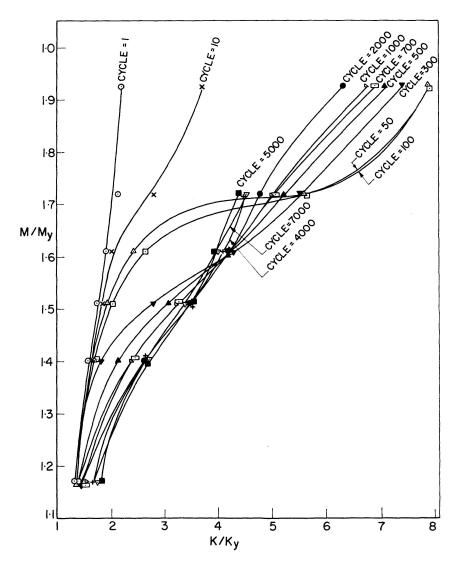


Fig. 9. Cyclic moment-curvature relations for various cycles: Moment control (curvature range vs. moment amplitude).

### Moment Control Tests [2]

The data from Fig. 7 a and b is plotted as moment vs. curvature and curves for all cycles are shown on the same graph, Fig. 9. A few comments are evident. Up to about 100 cycles, softening goes on over the entire range and thereafter there seems to be a converging point for all the curves up to about 5,000 cycles. Below the value of the moment at the point of convergence, the material softens while above this value the material hardens. At this value of moment (for cycles between 3,000 and 5,000), the material seems to be independent of cyclic history.

When the moment range is held constant, initially the material strainsoftens and then strain hardens. Eventually it settles down to a stable condition. However, unlike the case of strain control tests, the material creeps under constant moment range cycling as in Fig. 5b. Plots of  $M/M_y$  vs.  $K/K_y$ for half strain range and for maximum amplitude are shown on the same graph for each cycle (Fig. 10). For the same moment, the difference between the two curves on the abscissa shows the effect of cyclic creep.

For the reasons indicated in the case of strain cycling, the moment-curvature relationship up to the yield moment can be written as

$$k=m$$
.

Values of moment and strain ranges selected from the plots in Fig. 7a-b are plotted on a log-log scale in dimensionless form as in Fig. 11 to develop cyclic

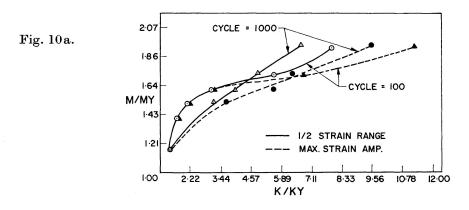
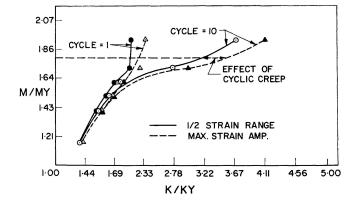


Fig. 10b.

Fig. 10a-b. Cyclic moment-curvature characteristics for various cycles (showing the effect of "cyclic creep"): Moment control.



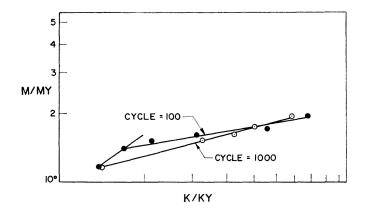


Fig. 11b.

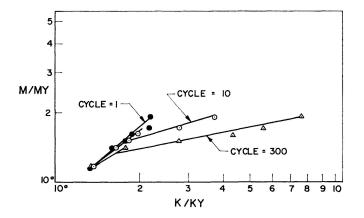


Fig. 11a.

Fig. 11a-b. Moment - curvature characteristics for various cycles: Moment control (curvature-range vs. moment amplitude on a loglog scale).

moment-curvature relationships. It can be seen that for cycles from 10 to 300 the test points fall on two straight lines. The moment-curvature relation might thus be represented beyond the yield point by a power law of the form

$$k_i = \alpha_i \, m^{\beta_i}, \tag{2}$$

where  $\alpha_i$ ,  $\beta_i$  have only one value for all cycles except from 10 to 300 and two values for cycles from 10 to 300. For purposes of computer programming it is easy to consider  $\alpha$ ,  $\beta$  as always having two values each, though these may be identical in some cases.

The relationships representing the cyclic state of a material at any stage as developed above, when applied to find structural response, would yield deflections which are equal to half the deflection range. In order to compute the creep effect the maximum deflection amplitudes are also necessary. An attempt is made to formulate a set of moment-curvature relationships which, when applied to yield response of a structure, would give maximum deflection amplitudes.

The value of  $\alpha_i$  in the new relationship will be the same as in Eq. (2), but  $\beta_i$  will be modified to include time,  $\beta_i t_i$ , reflecting the creep effect. These modified parameters can be calculated from the maximum strain amplitudes calculated earlier in the pure bending tests. The calculations for this are shown

in Appendix A. Consequently, the moment-curvature relation beyond yield can be written more generally as

$$k = \alpha_i m^{\beta_i t_i}, \tag{3}$$

where the value of i has already been discussed.

In the above expression  $\beta t$  has the following value (Appendix A):

$$\beta t = \frac{\log\left(\frac{2\epsilon_{max}}{\Delta \epsilon}\right)}{\log m} + \beta.$$

Here,  $\beta$  is a constant for each cycle and  $\beta t$  will have a value depending upon  $\epsilon_{max}$  and  $\Delta \epsilon$  which, in turn, depend on time and on the moment range applied.  $\beta t$  is thus a function of time and moment range although the variation in  $\beta t$  for each cycle is not much (see Table 1 for few cycles), since, when m increases,

Table 1

		Value of $\beta t$ fo	r
Moment	Cycle = 1	$\mathrm{Cycle} = 2$	$\mathrm{Cycle} = 10,000$
102.0 in. lbs	1.1436	3.8386	2.8579
122.4 in. lbs	1.1654	3.6535	2.7823
132.0 in. lbs	1.1319	3.6321	2.8650
140.4 in. lbs	1.1192	3.7350	
150.0 in. lbs	1.1646	3.8780	
168.0 in. lbs	1.1957	4.0644	

 $2 \epsilon_{max}/\Delta \epsilon$  also increases. For simplicity, the value of  $\beta t$  is considered constant for each cycle. For this purpose, it is seen that the highest value of  $\beta t$  for each gives better results.

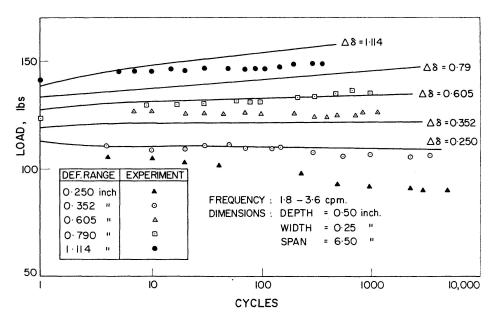


Fig. 12. Cyclic variation of load on cantilever beam under constant deflection range: Theory and experiment.

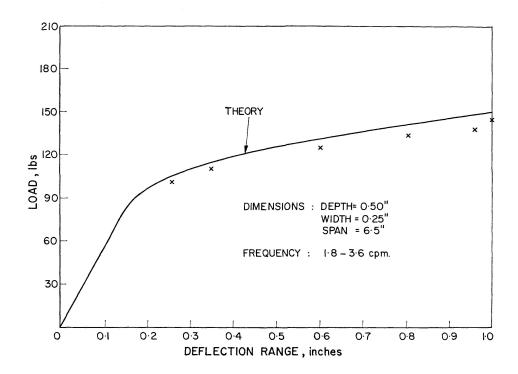


Fig. 13. Load-deflection of cantilever beam under constant deflection range: N=50.

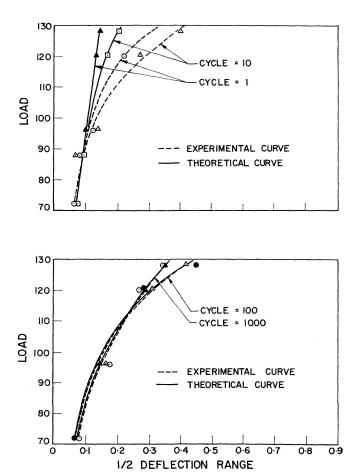


Fig. 14a. Load-deformation behaviour of cantilever beam: Experimental and theoretical results.

# **Analysis of Structural Components**

The moment-curvature relations developed above are applied to predict the cyclic response of structures subjected to repeated loading under ambient conditions of load and deformation control. An approach based on the momentarea technique has been developed and cantilever beams are analyzed theoretically and experimentally. Theoretical investigations are reported in detail elsewhere [4, 5].

# Deformation Control

The method developed (in Ref. 4) is applied to predict the variation of cyclic load on cantilever beams subjected to fully reversed deflections. The results are compared with experiments.

Tests are carried out on six cantilever beam specimens. The deflection ranges are so chosen to cover the whole strain range possible, i. e., up to  $\pm 2.5\%$ . The frequency of the loading varies from 1.8 cpm to 6 cpm, depending upon the deflection range. The computed results are plotted with the experimental

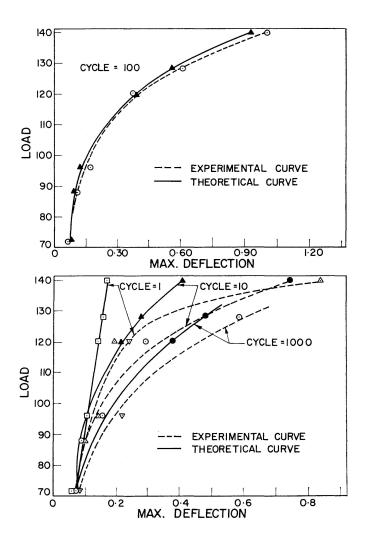


Fig. 14b. Load-deformation behaviour of cantilever beam: Experimental and theoretical results showing the "cyclic creep" effect.

results in Fig. 12. In Fig. 13, the load-deflection curve is plotted for one particular cycle along with the corresponding experimental results.

#### Load Control

Predictions are made of the variation in cyclic deflection of a cantilever beam subjected to reversed loading under load control conditions. As mentioned earlier, deflections are found both by neglecting and by including creep effects and the results are compared with experiments.

Six cantilever beam specimens were subjected to loads ranging from  $\pm$  72 lbs to  $\pm$  140 lbs. The test speed varied from 3 cpm to 30 cpm depending upon the amount of load imposed on the beams. It is noted that in the above experiments deflection amplitudes in hogging and sagging are unequal and that a continuous shift of the control axis of the beam takes place in one direction. The theoretical results are plotted and compared with experiments in Figs. 14a and 14b.

# **Failure of Structural Components**

Many investigations have been carried out in the past to examine the failure of metals under low endurance fatigue conditions. These investigations have established the existence of a relationship coupling the value of cyclic life,  $N_f$ , and true plastic strain range. In this investigation a failure criterion is established from cyclic straining tests on specimens under pure bending. This failure criterion, in conjunction with "stable" moment-strain curves derived from cyclic pure bending tests, are used to predict the life of structural components under low endurance fatigue.

# "Stable" Moment-Strain Range Relationship

As explained earlier, a metal subjected to cyclic loading "shakes" down to a "stable" state (Figs. 6 and 7), which usually extends from 25% to 75% of the life to failure. Moment-strain curves, then, can be established by plotting strain and moment amplitudes pertaining to half life. Fig. 15 shows the stable moment-strain curve for strain and moment control tests. It is to be noted that the material "shakes" down to the same "stable" state, irrespective of the control to which it is subjected. Therefore, a single moment-strain range curve in Fig. 15 is sufficient for further calculations. The experimental points can be represented by a mathematical function of the form

$$k=a_1(m)^{b_1},$$

where  $k = K/K_y$ ,  $m = M/M_y$  and  $a_1, b_1$  are geometrical constants which can be computed from Fig. 15.

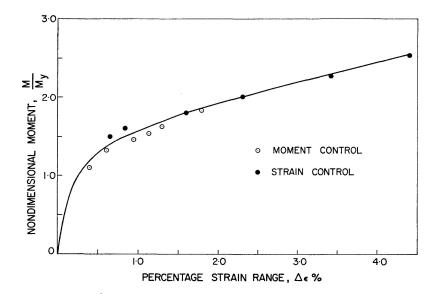


Fig. 15. Stable moment-curvature relationship: Half life values from constant moment range tests.

# "Stable" Load-Deformation Characteristic

The "stable" moment-strain range relationship developed above is now applied to predict the "stable" load-deformation relationship for a cantilever beam subjected to load and deformation control. The analytical technique has been explained earlier in this paper.

It can be seen that cantilever beams also "shake-down" to a "stable" state after an initial period of softening or hardening (Figs. 16 and 17). The theo-

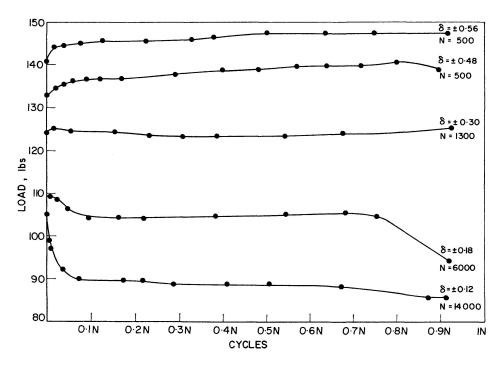


Fig. 16. Cyclic variation of load on cantilever beam (experiment): Constant deflection range.

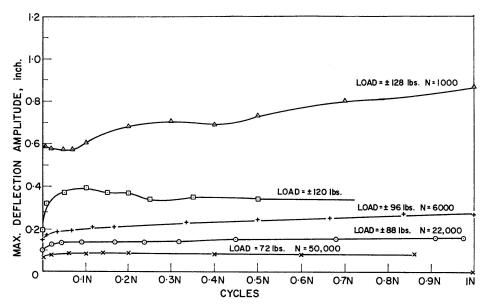


Fig. 17. Cyclic variation of maximum deflection amplitude for cantilever beam under load control.

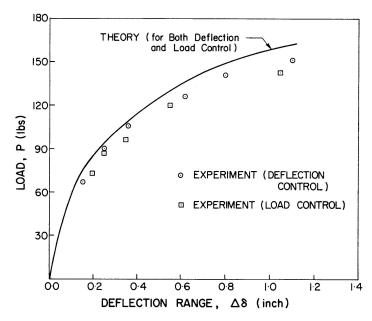


Fig. 18. "Stable" load-deflection curve for cantilever beam: Load and deformation control.

retical predictions are compared with experimental values pertaining to half life state for different beams (Fig. 18). The theory is seen to correlate reasonably well with experiments.

# Failure Criterion from Pure-bending Tests

A failure criterion is established, as shown in Fig. 19, from strain as well as moment control tests. A more defined failure criterion than the usual complete fracture of the specimen is necessary since a structural component in an actual situation may be rendered unserviceable due to excessive growth of cracks, refusal of further load, or increased deflection, prior to complete fracture.

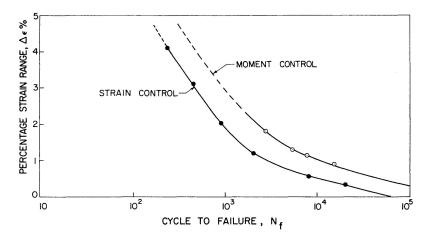


Fig. 19. Failure of beams under pure-bending: Moment and strain control.

Consequently, the failure criterion adopted is that of final reduction in bending moment in the case of strain control tests and increase in strain range when the specimens are under moment control.

Based upon the above failure criteria, the cycles to failure,  $N_f$ , for various tests were found from experiments.

### Life of Structural Components under Cyclic Loading

The failure criteria developed above can be used in predicting the life of cantilever beams subjected to deformation as well as load control. The procedure adopted is as follows:

- 1. The "stable" moment-curvature relationship (Fig. 15) developed earlier is applied to predict the "stable" load-deformation characteristic of cantilever beam; theoretical results are compared with experiments (Fig. 18).
- 2. Values of bending moments (yielding the maximum strain range in the beam) corresponding to a "stable" state are computed (a) from the theoretical load-deformation characteristic in Fig. 18 for deformation-control tests, and (b) from the imposed loads on the beams in the case of load-control tests.
- 3. The strain ranges corresponding to the "stable" state are then calculated from Fig. 15 and then are employed to predict the life of cantilever beams from Fig. 19.

Figs. 20 and 21 show theoretical and experimental results for cantilever beams under deformation as well as load control. Prediction of lives of beams are made in two ways: one from the results of strain-control tests and the other from the results of moment-control tests.

It can be seen from Figs. 20 and 21 that the failure criterion based upon moment-control tests yields an upper bound while the criterion derived from strain control tests provide a lower bound on test performance.

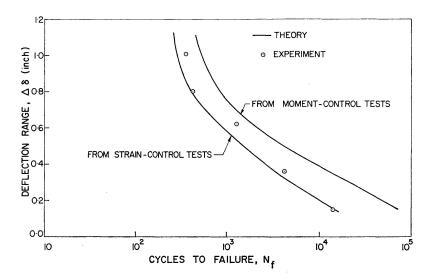


Fig. 20. Failure of cantilever beams under deflection control: Theory and experiment.

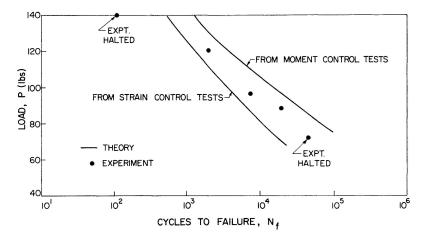


Fig. 21. Failure of cantilever beams under load control: Theory and experiment.

When a beam in service or test is subjected to deflection control, it is neither strain nor moment controlled. In fact, when the deflection range at one point is controlled, the strain range and the moment at every other point changes from cycle to cycle because of the varying strain hardening rates under different strain ranges. However, the behaviour of a beam under preset deflection ranges can be approximated to a strain controlled case without introducing significant error. Hence, the lives of beams predicted from the failure criterion derived from strain control tests are closer to experimental values than the lives predicted from the failure criterion based upon moment control tests (Fig. 20).

In a beam under load control, the moment at every point will remain constant while the strain range will change from cycle to cycle. Therefore, it is logical to expect that the endurance of beams can be very closely predicted by employing the failure criterion derived from moment controlled tests. As indicated earlier, the material creeps under constant moment (load) cycling introducing mean strain in the beam: the mean strain (or creep) is zero in the case of strain (deflection) cycling. However, it can be seen from Fig. 21 that predictions based upon the criterion derived from moment-controlled tests are non-conservative. The value of mean strain in a beam under pure bending may be more than in a cantilever beam for the same moment-range, thus causing cantilever beams to fail earlier than beams under pure bending.

#### **Discussion and Conclusions**

A method is developed for determining the behaviour and life of simple structural components subjected to a constant range of alternating deflection or load. Cyclic moment-curvature models developed from pure bending tests are applied to predict the cyclic behaviour of cantilever beams and the results are compared with experiments.

A "stable" moment-strain range relationship is established by plotting values of moment amplitude and strain range pertaining to half-life of several different beams. It has been established that, irrespective of the type of control imposed on a beam, it shakes down to the same stable state. This moment-strain range characteristic, coupled with a failure criterion developed from pure-bending tests, is employed in predicting the lives of cantilever beams subjected to cyclic alternating load or deflection.

A reasonably good correlation has been found in the prediction of cyclic behaviour of cantilever beams subjected to alternating deflections. However, in the case of beams under alternating loads, the theoretical values of cyclic deflection and experiments correlate well for all but the first 10 cycles. The correlation is not good at high loads. The discrepancy at low life and high loads may be associated with the difference in ranges under which two types of experiments were conducted:

- 1. to formulate the theory (pure bending tests), and
- 2. to compare the results (load control tests on cantilever beams).

For pure bending tests under moment control, the maximum moment amplitude applied was  $1.92\,M_y$  while for cantilever tests, the maximum moment amplitude attained was  $2.154\,M_y$ . A glance at Fig. 7 shows that strain range rises abruptly as the moment amplitude increases. One can imagine the strain range to rise to this peak value in the very first cycle at moment amplitudes greater than the largest value applied in the pure bending tests. Discontinuous yielding in mild steel can keep the strain range low at low life and at comparatively lower values of moment amplitude in the case of pure bending tests. As the value of moment amplitude increases the span of low life during which the discontinuous yield phenomena takes place decreases and there will be a limiting value of the moment amplitude when this phenomenon will

disappear at the outset, giving a full value of strain range. This seems to occur in cantilever beam tests with the highest load giving rise to greater deflections at low life and high loads.

It has already been stated that "cyclic creep" occurs in the case of beams subjected to moment (load) control. Whenever ductile materials are subjected to cycles of plastic deformation between load limits, progressive deformation, generally called "cyclic creep", invariably occurs. The presence of cyclic creep during repeated tension can be readily accepted because of the mean stress present. In symmetrical reversed loading, when the mean stress is nominally zero, the presence of cyclic creep is attributed to the tensile mean stress resulting from cyclic deformation of the specimen which causes the true maximum tensile stress to exceed the complementary compressive stress during cycles between constant load limits. If this is accepted as the sole reason for the accumulation of strain, then a small value of tensile mean stress should give rise to cyclic creep in that direction. It has been found by TILLY [6] from fully reversed (push-pull) and repeated tension load tests at room temperatures that, under the action of cyclic loads, the material is less resistant to tensile stress; the tensile stroke was always applied first in Tilly's experiments. If this argument is valid, it remains to explain why cyclic creep occurs in bending tests under load control where half the section is subjected to tensile stress alternately each half cycle and degeneration of stiffness in one direction due to tensile load is compensated by the loss of rigidity in the other direction by an equal and opposite tensile load. It appears that the direction of cyclic creep depends on the nature of the residual stresses which are set up during the first half cycle of reversed loading. It seems quite logical to assume that applied load can be superimposed upon residual stresses and the combined force effect can give rise to a mean stress which is responsible for the occurrence of cyclic creep.

The life predictions based upon failure criteria derived from strain-controlled and moment-controlled tests will yield lower and upper bounds, respectively, to the actual values.

### **Notations**

K	Curvature due to flexure.
$K_y \ k$	Curvature at yield.
$\boldsymbol{k}$	Non-dimensional curvature $K/K_y$ .
M	Moment amplitude.
$M_y$	Moment at yield.
$egin{array}{c} M_y \ m \end{array}$	Non-dimensional moment $M/M_y$ .
N	Number of cycles.
$N_{\!\scriptscriptstyle f}$	Number of cycles to failure.

 $\begin{array}{lll} \varDelta \, \epsilon & & \text{Strain range for any cycle.} \\ \epsilon_{max} & & \text{Maximum strain amplitude for any cycle.} \\ \epsilon_y & & \text{Strain at yield.} \\ \alpha_1, \alpha_2, \beta_1, \beta_2 & \text{Constants in moment-curvature relationships for half deflection range.} \\ \alpha_1 t_1, \alpha_2 t_2, & & \text{Constants in moment-curvature relationships for} \end{array}$ 

maximum deflection amplitude.

# Acknowledgments

This investigation was carried out in the Department of Civil Engineering of the University of Waterloo, with financial assistance from the National Research Council of Canada, Grant No. A-1582, and the Department of University Affairs, Ontario.

# Appendix A. Calculations for Modified Material Parameters to Yield Maximum Deflection Amplitude

 $k = \alpha m^{\beta},$  (1)  $k = \frac{K}{K_y} \text{ and } m = \frac{M}{M_y}.$   $K = \frac{\epsilon}{d},$ 

 $K_y = \frac{\epsilon_y}{d}$ 

 $\frac{K}{K_y} = \frac{\epsilon}{\epsilon_y} = \frac{2\,\epsilon}{2\,\epsilon_y} = \frac{\varDelta\,\epsilon}{2\,\epsilon_y}.$ 

Substituting in (1) we get

$$\frac{\Delta \epsilon}{2 \epsilon_y} = \alpha m^{\beta}. \tag{2}$$

Let modified relation be represented by

 $\beta_1 t_1, \beta_2 t_2$ 

 $k_{max} = \alpha m^{\beta t},$ 

where

where

 $k_{max} = \frac{K_{max}}{K_y} = \frac{\epsilon_{max}}{\epsilon_y}.$ 

Thus  $\frac{\epsilon_{max}}{\epsilon_n} = \alpha \, m^{\beta t}. \tag{3}$ 

Divide (2) by (3) 
$$\frac{\Delta \epsilon}{2 \epsilon_y} \frac{\epsilon_y}{\epsilon_{max}} = \frac{m^{\beta}}{m^{\beta}t},$$
 or 
$$m^{(\beta t - \beta)} = \frac{2 \epsilon_{max}}{\Delta \epsilon},$$
 or 
$$(\beta t - \beta) \log m = \log \left(\frac{2 \epsilon_{max}}{\Delta \epsilon}\right).$$
 Therefore, 
$$\beta t = \frac{\log \left(\frac{2 \epsilon_{max}}{\Delta \epsilon}\right)}{\log m} + \beta.$$

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#### Summary

A method is developed for predicting the life of simple structural components subjected to a constant range of cyclic alternating load or deflection. The method makes use of the fact that, irrespective of the nature of the control imposed, the components shake down to the same "stable" terminal state. The moment-strain range characteristics coupled with a failure criterion, both developed from data on beams under pure bending, is used in estimating structural life. Controlled moment and strain are also seen to bound the test performance, the former from above, the latter from below.

#### Résumé

On développe une méthode pour estimer la durée de vie d'éléments structuraux simples soumis à des charges cycliques alternatives ou à des déflexions variant entre des limites constantes. La méthode tient compte du fait que, indépendamment de la nature du contrôle imposé, les éléments atteignent à la ruine le même état final "stable". On utilise pour estimer la durée de vie de la structure les caractéristiques moment-déformation liées à un critère de ruine, tous deux étant obtenus à partir d'essais de poutres soumises à la flexion pure. On voit que le moment et la déformation contrôlés influencent l'exécution de l'essai, le premier par le haut, le dernier par le bas.

### Zusammenfassung

Es wird eine Methode zur Voraussage der Lebensdauer einfacher baulicher Komponenten entwickelt, die einer konstanten Reihe wechselnder zyklischer Belastungen oder Durchbiegungen ausgesetzt sind. Die Methode bedient sich der Tatsache, dass die Komponenten, unabhängig von der Natur der Kontrolle, sich auf den gleichen «stabilen» Endzustand einspielen. Die Moment/Deformation-Charakteristiken, zusammen mit einem Versagenskriterium, wobei beide aus Daten von Balken unter reiner Biegung entwickelt werden, benützt man um die Lebensdauer eines Bauwerkes zu schätzen. Das kontrollierte Moment und die Deformation werden auch zur Durchführung der Versuche herangezogen, das erstere von oben, das letztere von unten.

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