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Buckling of Welded Beams and Girders

Flambement de poutrelles soudées

Kippen geschweisster Balken und Träger

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1. Notation

a	distance from point on cross section to shear centre.
B_y	minor flexural rigidity.
b_f	width of flanges.
C_T	torsional rigidity.
C_w	warping rigidity.
c	width of assumed rectangular stress block.
d	depth of section between flanges.
$\overline{K} = \int_A \sigma a^2 dA$	property of cross section.
k	effective length factor.
L	span.
L_y	span for which $M_{cr} = M_y$.
M_{cr}	critical moment.
M_p	fully plastic moment.
M_y	moment at which section starts to yield, the effects of residual stresses being neglected.
M_{yr}	moment at which section starts to yield, the effects of residual stresses being included.
r_y	minor radius of gyration.
$s_w = \frac{2c}{d}$	proportion of web containing tensile residual stresses.
t_f	thickness of flanges.
t_w	thickness of web.

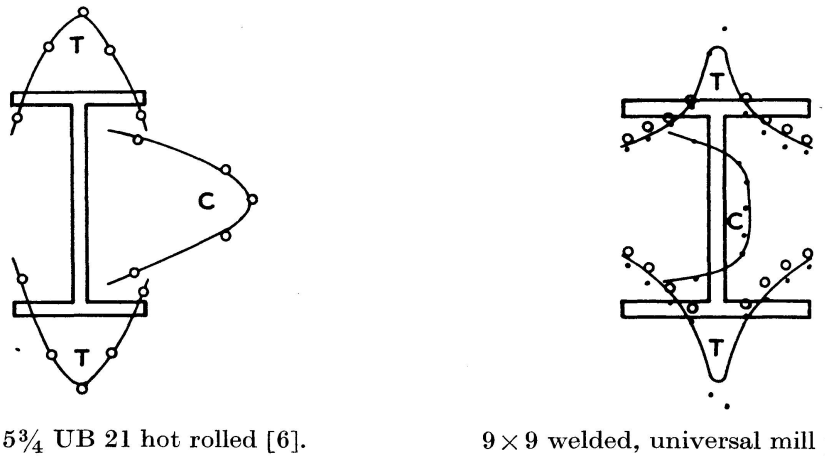
σ stress.
σ_{rc} compressive residual stress.
σ_y yield stress.

2. Introduction

Failure of laterally unsupported steel beams usually occurs by lateral-torsional instability, the fully plastic moment M_p being attainable only for very short spans. Over most of the range of practical slenderness ratios (span/minor radius of gyration) buckling will not occur until parts of the cross section have yielded under the action of the applied major axis loading system. When residual stresses are present, as they will be in most practical situations, the consequent reduction in the value of the applied load necessary to initiate yielding M_y, extends this inelastic range. Studies [1–5] of the buckling of beams containing patterns of residual stresses typical of those found in hot rolled sections have shown that, since yielding can occur at applied loads corresponding to less than one half of M_p, the range of spans for which buckling is inelastic may be at least twice that for a similar stress free section and that within this range the critical moment may be reduced by over forty per cent as a direct result of the presence of the residual stresses. Moreover, this range of spans comprises a major portion of the range of slenderness commonly used in practice.

Measurements of the residual stresses present in welded I and H shapes [6–9] have shown these to be significantly different to those found in hot rolled sections and an example for each type of section is given in Fig. 1. Because of the considerable influence of residual stresses on inelastic buckling strength it is to be expected that the behaviour of welded sections will be rather different to that of geometrically similar rolled sections.

In this paper theoretical studies of the inelastic buckling of bisymmetrical I section beams containing welding type residual stresses are reported. From



$10 \times 5\frac{3}{4}$ UB 21 hot rolled [6]. 9×9 welded, universal mill plates [8].

Fig. 1. Typical measured residual stress patterns (not to scale, refer to original refs. for details).

the results obtained a simple method of approximating the beam buckling curve is suggested and compared favourably with available experimental results. In order to be able to compare directly the behaviour of welded and rolled beams, standard cross-sectional shapes have been used in the numerical examples.

3. Method of Analysis

The beam is assumed to be of doubly symmetric I section and to be loaded in the plane of the web by equal end moments, the critical value of which is given by:

$$M_{cr} = \frac{\pi}{k\,L}\left[B_y\,C_T\left(1+\frac{\bar{K}}{C_T}\right)\right]^{1/2}\left[1+\frac{\pi^2\,C_w}{K^2\,L^2\,C_T\left(1+\frac{\bar{K}}{C_T}\right)}\right]^{1/2}, \qquad (1)$$

where B_y is the minor flexural rigidity,
C_T is the torsional rigidity,
C_w is the warping rigidity,

$$\bar{K} = \int_A \sigma\,a^2\,dA\,,$$

where a is the distance from the shear centre to the point where the total stress σ acts
and k is an effective length factor whose value depends upon the degree of restraint at either end.

In the numerical examples that follow the value of k has been taken as unity which corresponds to supports at both ends that prevent lateral deflexion and twist but offer no restraint to either lateral bending or warping. The treatment of other support conditions is discussed later in the paper.

For inelastic buckling the values of the stiffnesses B_y, C_T, C_w and $\bar{K}$ must be modified in accordance with the degree of plasticity present in the beam i.e. the value of $\frac{M_{cr}}{M_p}$. When residual stresses are present yielding occurs as a result of the combined effects of the residual stress system and the applied stress system and a convenient technique for tracing the spread of yield through the section is to consider it as being composed of a large number of small regions each of which, depending upon the value of the total i.e. residual plus applied, strain at its centroid, may be either elastic or plastic. The stiffness terms B_y etc. are then obtained by summing the contributions of all the regions and, since a different value of residual strain may be specified in each region, almost any residual stress pattern may be accommodated. Inelastic material behaviour was modelled using the suggestions of TRAHAIR and KITIPORNCHAI [10]. Briefly, these consist of replacing E by its strain hardening value E_{st} when calculating B_y and C_w, using the concept of a two phase material to determine the effective value of C_T and tracing the movement of the shear centre of the effective section when calculating $\bar{K}$ and C_w. Once relationships between the effective values of each of the stiffnesses have been established, Eq. (1) may be solved for the span L corresponding to a compatible set of values of M_{cr}, B_y, C_T, C_w and $\bar{K}$. Fuller details of the complete approach, including assessments of its numerical accuracy, have been reported elsewhere [5]. Convergence tests for the present examples have indicated the use of 200 elements for one half of the section to be adequate and this mesh has been used in all the numerical examples.

4. Results

a) Rectangular Stress Block

Measurements of residual stresses in welded sections (6–9) have consistently shown the areas adjacent to the welds to be stressed up to yield in tension, the remainder of the section containing smaller, balancing compressive stresses. Theoretical investigations at Cambridge [11] supported by tests [6] have produced a simple method of relating the width of the (assumed) rectangular tension block c, see Fig. 2, to either the heat input or the size of the weld(s). Using this process residual stress distributions for a series of geometrically

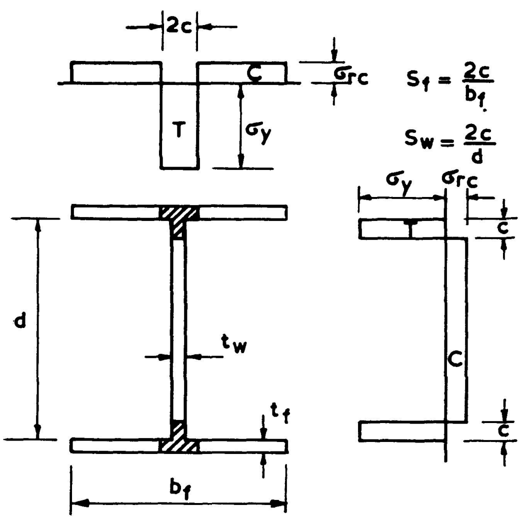


Fig. 2. Rectangular residual stress block (6).

different sections have been determined for use in the beam buckling analysis described above. Variations in the severity of welding have been included by using a number of values of s_w, a measure of the width of the tension block. Although axial equilibrium of the patterns has been checked, no subsequent adjustments have been made to try and ensure torsional equilibrium [5].

Two complete beam buckling curves for a $10 \times 5^3/_4$ UB section are illustrated in Fig. 3, values of s_w of 0.2 and 0.05 corresponding approximately to "moderate" and "light" welding respectively having been used in the calculations. Inclusion of the effects of residual stresses causes the critical moment to fall

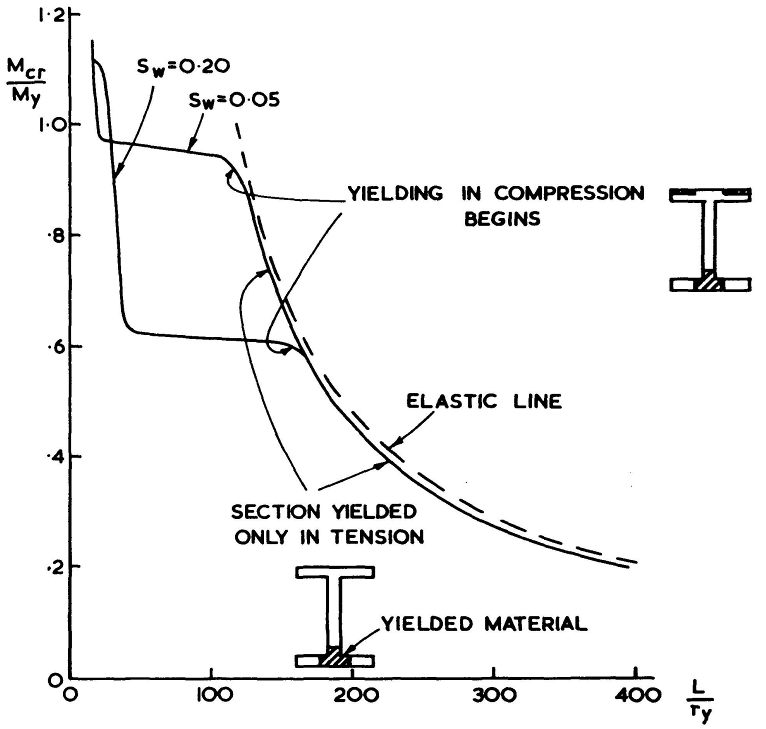


Fig. 3. Buckling curves for welded $10'' \times 5\frac{3}{4}''$ UB 21 sections.

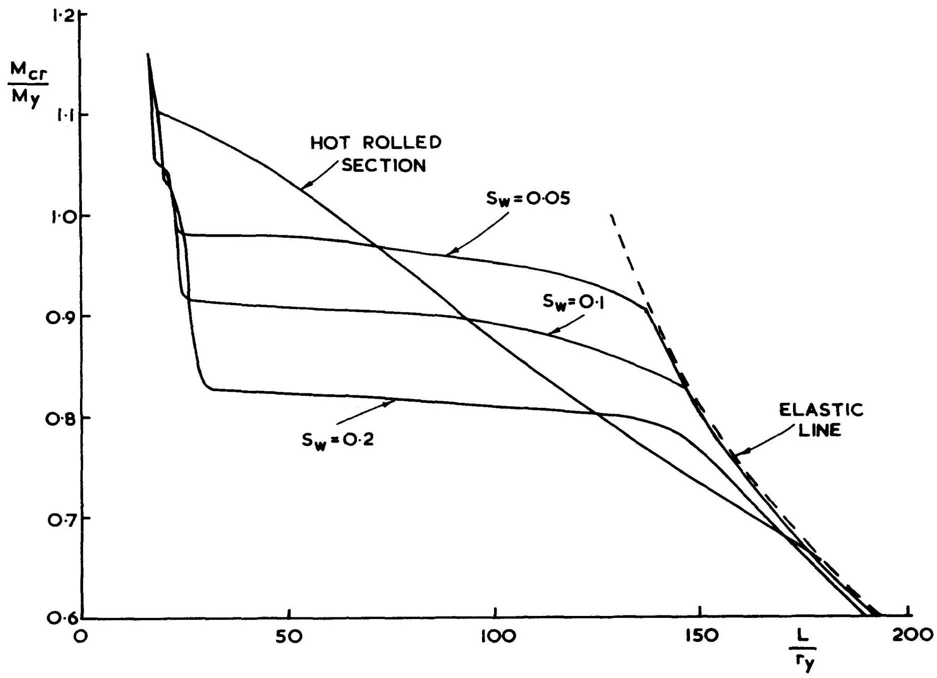


Fig. 4. Buckling curves for $8'' \times 8''$ UC 31 section.

below that based upon wholly elastic behaviour even for very long beams. This is a result of tensile yielding in the region of the web to tension flange junction which occurs immediately any load is applied. However, its effect upon the section's stiffness is small and the corresponding decrease in M_{cr} amounts to only a few per cent. On the other hand, when yield of the outer parts of the compression flange occurs, it is accompanied by a major reduction in stiffness with the result that the buckling curve dips sharply below the elastic line. Eventually progressive yielding of the web causes the section to become almost completely strain hardened and the fully plastic moment M_p is attainable for sufficiently short spans.

Figs. 4 and 5 show portions of the buckling curves for two further shapes, a compact "column type" shape and a slender "beam type" shape. Taking British UB and UC sections as a representative set, for a given value of s_w, the balancing compressive residual stresses in all UC sections are very similar, whilst the 12×4 UB represents a beam section in which they are particularly high and the $10 \times 5\frac{3}{4}$ UB a beam section in which they are particularly low. The three sections considered in Figs. 3–5 therefore cover a wide range. For the 12×4 section slightly different elastic curves are obtained for different values of s_w due to differences in the value of the $\bar{K}$ term for the residual stresses acting alone. This is a consequence of their not indentically satisfying the condition $\bar{K} = 0$ and is discussed more fully in ref. [5]. For the two other

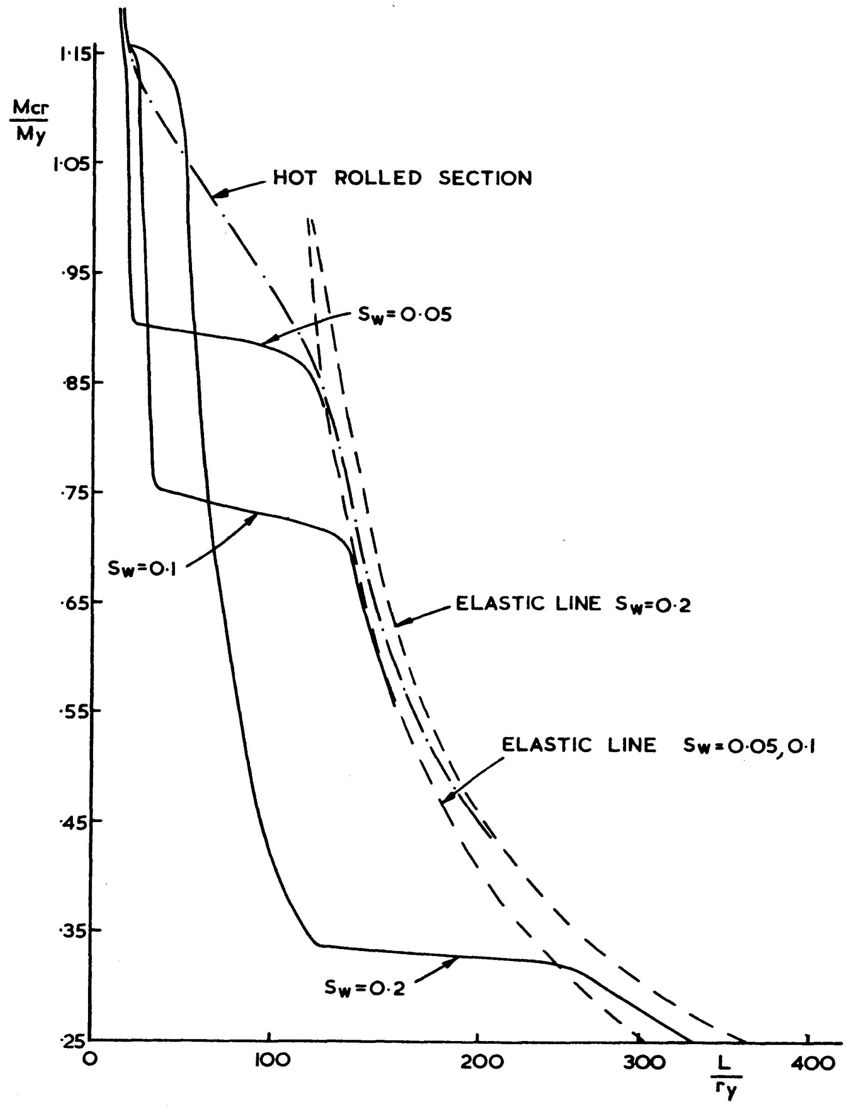


Fig. 5. Buckling curves for 12″ × 4″ UB 19 section.

sections this effect is sufficiently small that only one elastic curve need be plotted.

Figs. 4 and 5 also contain inelastic buckling curves for the equivalent hot rolled sections, YOUNG's [6] parabolic residual stress patterns having been employed in the generation of these curves. In both cases the differences between this curve and the set for the geometrically identical welded section clearly demonstrate the important role played by residual stresses in beam buckling. Whereas sections containing cooling type patterns lose stiffness gradually with increasing load, welded beams suffer a much more rapid decrease in stiffness due to the sudden loss of a considerable portion of the compression flange.

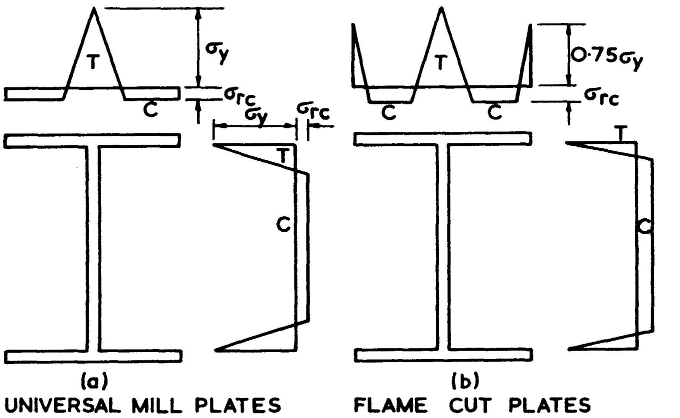


Fig. 6. Triangular residual stress block [12].

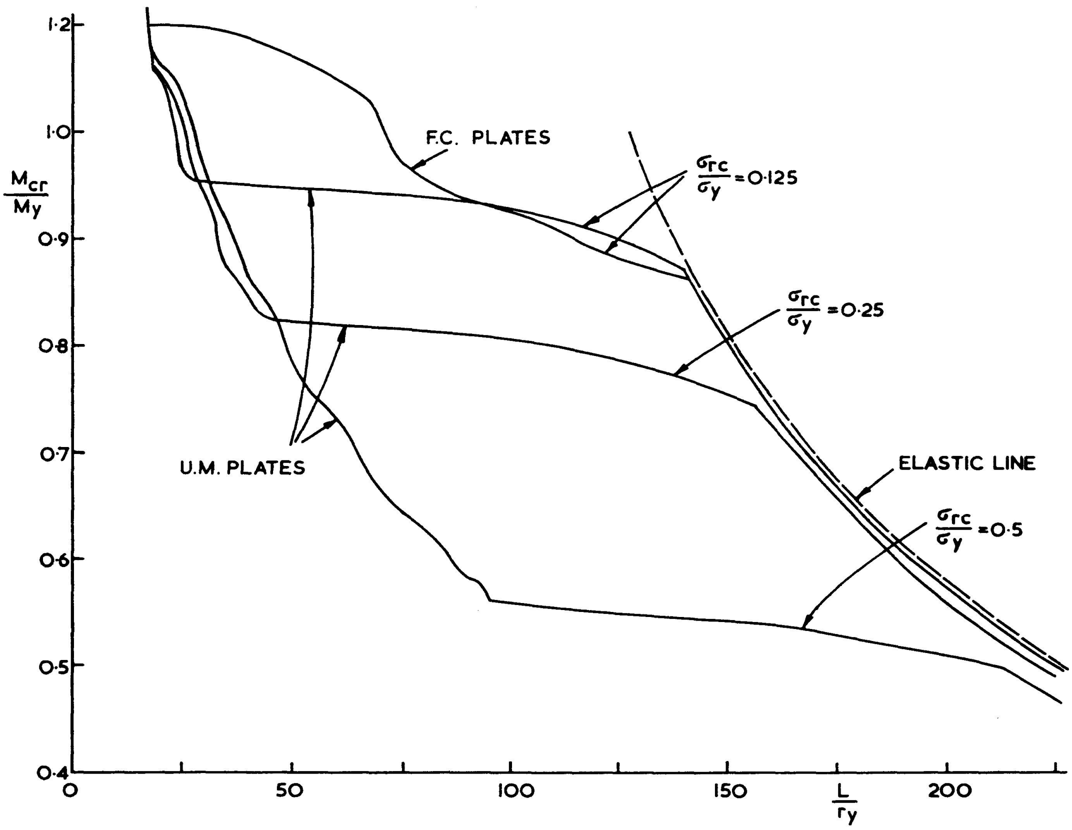


Fig. 7. Buckling curves for 8″ × 8″ UC 31, triangular stress block, UM or FC plates.

b) Triangular Stress Block and Flame Cut Sections

Opinions differ as to the actual shape of the tensile residual stress block and TALL and his co-workers at Lehigh prefer a triangular shape [12], see Fig. 6. Beam buckling curves calculated on the basis of this alternative residual stress pattern are given in Figs. 7 and 8. Over most of their range the shape of these curves is very similar to that corresponding to the use of the rectangular stress block for the same value of compressive residual stress in the flanges. For very short spans, however, since the proportion of the compression flange containing residual compression is smaller, the curves approach the fully strain hardened state more gradually, although the span at which M_p is reached is about the same. Therefore, for a given value of compressive residual stress, the assumption of a triangular stress block rather than a rectangular one leads to a slightly higher beam buckling curve in the region of very short spans.

Until now it has been assumed that the plates from which the section is fabricated are cut to size by mechanical means. Unless these plates are very

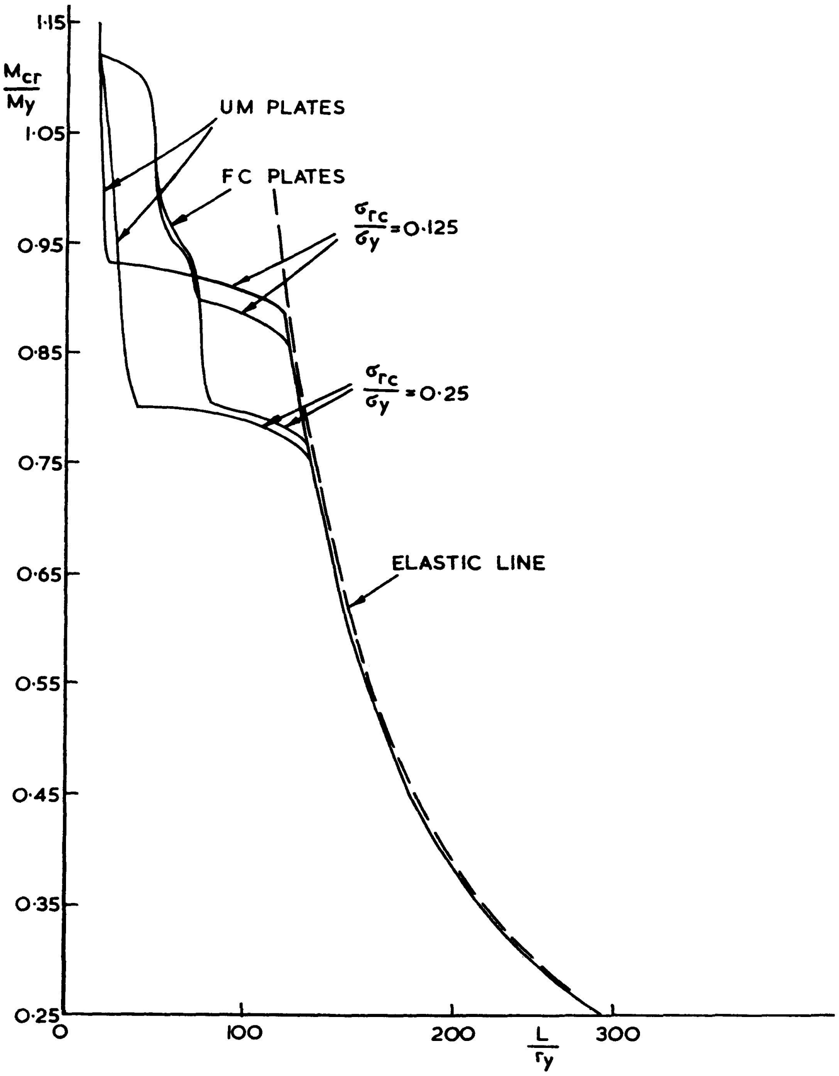


Fig. 8. Buckling curves for 12″ × 4″ UB 19 section triangular stress block and UM or FC plates.

thick (greater than about 25 mm) any residual stresses contained in them prior to welding will be comparitively small [13], and will be absorbed into the final residual stress pattern. However, plates with flame cut edges contain considerable tensile residual stresses in the vicinity of the cut [7]. For the web plate of a *I* section these will be absorbed into the stress block associated with the welding but at the flange tips a tensile stress block will remain. Some actual measurements are reported in ref. [7] and the type of pattern that has previously been used in column calculations [12] is shown in Fig. 6.

Using this pattern results have been obtained for the lateral buckling of the 8×8 and 12×4 sections and these are shown in Figs. 7 and 8. Since a balance of axial forces is required this type of pattern can only exist with fairly low values of compressive flange residual stress. For a similar value of σ_{rc} the section built up from flame cut plates possess a higher beam buckling curve than a similar section fabricated from universal mill plates. This is hardly surprising since the tensile stresses at the edges of the flanges produced by flame cutting postpone the yielding of that region which contributes most to the section's lateral stability; the compression flange tips.

c) Plate Girders

Few measurements of the residual stresses present in plate girders have been reported. OWEN [14], in Australia, has measured the stresses in two fairly small girders with unstiffened webs and has found that these conform to the expected pattern of stresses in welded *I*-sections. Using this data, beam buckling curves have been obtained for both of Owen's sections and these are shown in Figs. 9 and 10. These curves are similar to those derived previously for the more compact sections, although for the 24×6 girder in parti-

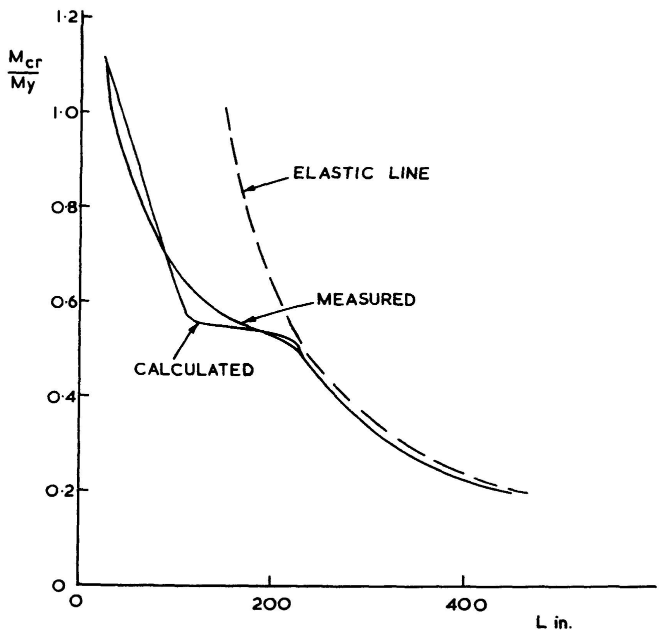


Fig. 9. Buckling curves for a $24'' \times 6''$ plate girder [14].

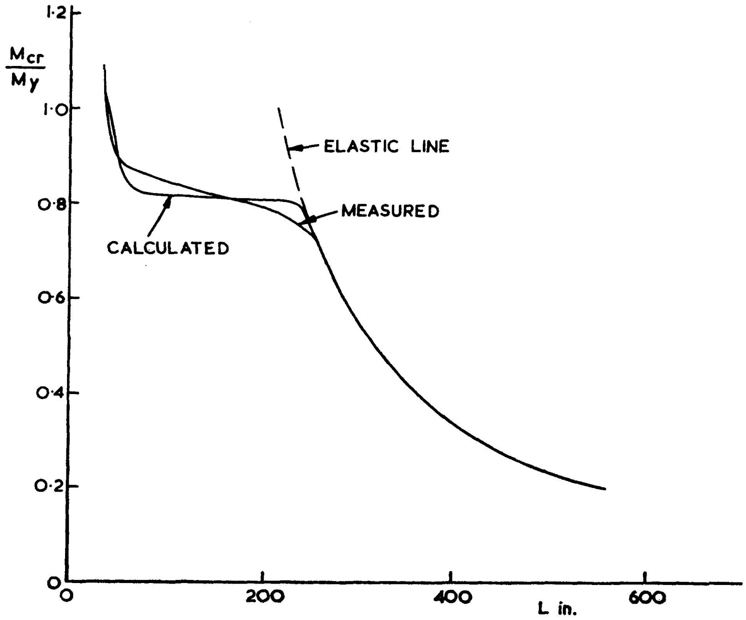


Fig. 10. Buckling curves for a $36'' \times 9''$ plate girder [14].

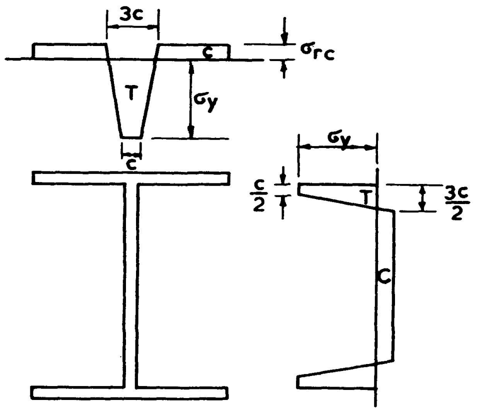


Fig. 11. Modified trapezoidal stress block [15].

cular, a slightly less rapid drop in stiffness at the point corresponding to the initiation of compressive yielding is indicated. This is a result of the actual variations in residual stresses over the cross section being somewhat smoother than the abrupt variations assumed in the patterns of Figs. 2 and 6.

Recently Dwight [15] has suggested that for column work it is preferable to modify the rectangular stress block of Fig. 2 into the trapezoidal shape of Fig. 11. Using the welding data given by Owen [14], together with this variant of the Cambridge approach, theoretical residual stress distributions have been obtained for both of Owen's sections and the resulting beam buckling curves are shown on the appropriate figures. Agreement between the two curves is, in both cases, excellent.

5. Simplified Approach and Comparison with Experimental Results

In ref. [5] it was shown that, for hot rolled sections, a close approximation to the actual beam buckling curve could be obtained from a knowledge of the moment required to initiate yielding at the tips of the compression flange

M_{yc} together with the elastic buckling curve. Moreover, values of M_{yc} calculated from simple bending theory neglecting the effects of any tensile yielding elsewhere in the section were shown to be sufficiently accurate for use with this process.

The shape of the buckling curve for welded beams also suggests an approximation of this type. In this case, however, it is not appropriate to join the point on the elastic curve corresponding to M_{yc} to the point $(M_p, 0.15\,L_y)$ i.e. the limiting span for which M_p may be attained, but rather to use the construction shown in Fig. 12.

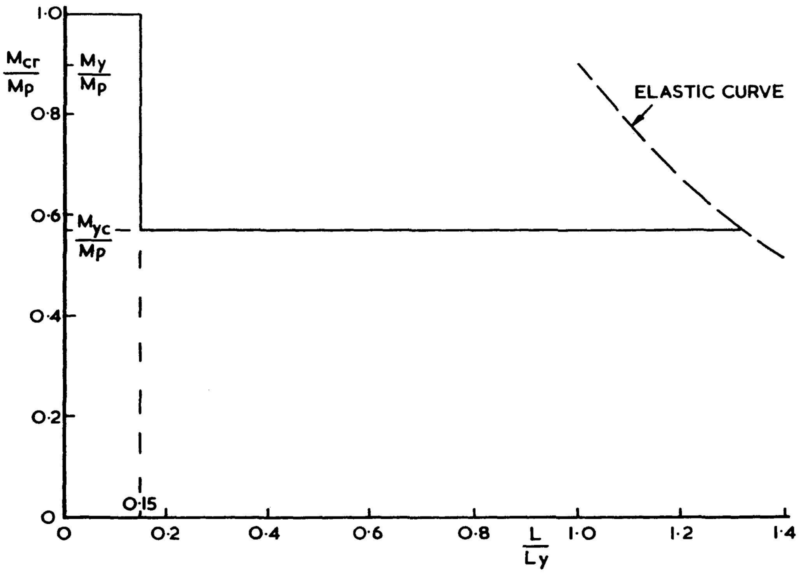


Fig. 12. Construction of approximate buckling curve for a welded beam.

Although the numerical examples considered previously were all for simply supported ends, ref. [4] showed that, providing L_y was calculated on the basis of the actual support conditions provided, this type of approach could be applied to beams with other forms of end support. The approximate curve of Fig. 12 is therefore applicable to welded beams with any degree of end fixity.

Inspection of the spans at which M_p is attained for each of the examples considered earlier suggests an average value of $0.15\,L_y$, where L_y is the span at which $M_{cr} = M_y$, the effects of residual stresses being neglected. The approximate curve is therefore obtained by combining a vertical line passing through this point $(M_p, 0.15\,L_y)$ with a horizontal line through the point on the elastic buckling curve at which compressive yielding is initiated. Table 1, which compares actual values of M_{yc} taken from the examples in the paper with the approximate values shows that good approximations to this point may readily be calculated. Since the value of residual flange compressive stress in the section may be calculated from a knowledge of the welding parameters using the Cambridge approach [6, 15], the approximate curve may be directly related to the severity of welding.

Table 1. Comparison of approximate and exact Values of $\frac{M_{yc}}{M_y}$

Section	Calculated Value of $\frac{M_{yc}}{M_y}$	"Exact" Value of $\frac{M_{yc}}{M_y}$	Calculated Value of $\frac{M_{yc}}{M_y}$	"Exact" Value of $\frac{M_{yc}}{M_y}$	Calculated Value of $\frac{M_{yc}}{M_y}$	"Exact" Value of $\frac{M_{yc}}{M_y}$
Value of s_w	0.2		0.1		0.05	
8×8 UC 31	0.78	0.77	0.83	0.83	0.95	0.94
12×4 UB 19	0.37	0.33	0.76	0.72	0.89	0.88
10×5¾ UB 21	0.61	0.60	0.84	0.84	0.93	0.91
Value of $\frac{\sigma_{rc}}{\sigma_y}$	0.125		0.25		0.50	
8×8 UC 31 } UM plates	0.87	0.87	0.75	0.74	0.50	0.50
12×4 UB 19 } UM plates	0.87	0.89	0.75	0.76	—	—
8×8 UC 31 } FC plates	0.87	0.87	—	—	—	—
12×4 UB 19 } FC plates	0.87	0.86	0.75	0.76	—	—
Plate girder	Measured residual stresses		Calculated residual stresses			
36×9	0.76	0.74	0.76	0.79		
24×6	0.53	0.50	0.53	0.51		

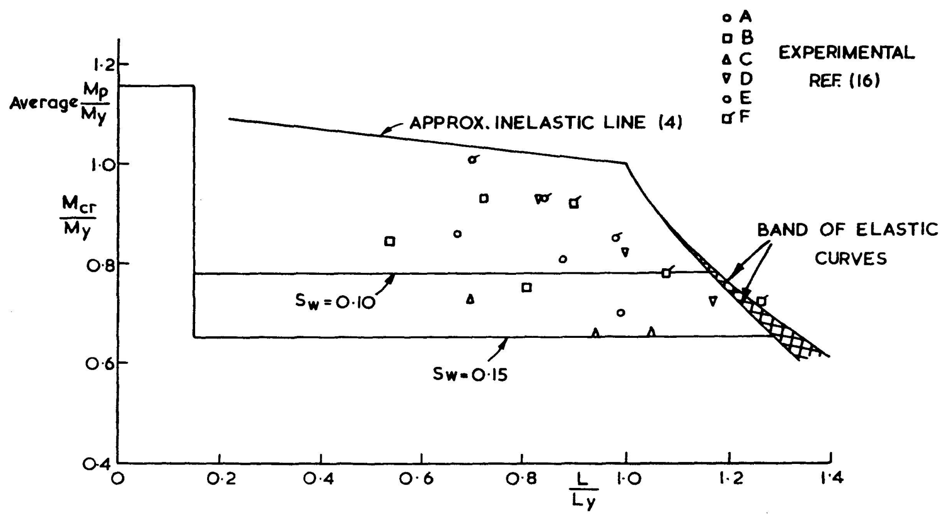


Fig. 13. Comparison of approximate buckling curves with test results [16].

Fig. 13 compares this approach with the results of a series of Japanese tests [16]. Complete fixity was provided at both ends of the test beams and the values of L_y used in plotting the results have therefore been modified accordingly. Since only limited welding data was available it has not been possible to use the above process to calculate values of M_{yc} for each section. Instead, average lines corresponding to values of s_w of 0.10 and 0.15 have been used. These may be considered appropriate for the "moderate" welding conditions suggested by the data given in ref. [16]. The scatter of the test results reflects the variability of the residual stresses developed in geometrically similar sections even for apparently similar welding conditions. With this in mind, agreement between the test points and the approximate theoretical lines $s_w = 0.1$, representing an average curve, and $s_w = 0.15$, representing an approximate lower bound, may be regarded as satisfactory. Unfortunately, no results for tests on short welded beams could be found, although experiments on rolled sections [17] tend to support the view that $0.15\,L_y$ is a reasonable limiting span for the development of M_p.

6. Conclusions

The influence of the residual stresses produced during the fabrication of welded *I* sections on the lateral buckling strength of such beams has been investigated.

Even for long spans the yielding which occurs as a result of the presence of tensile residual stresses approaching the material yield stress in magnitude causes a reduction in buckling strength but, since this yielding is confined to a portion of the section which contributes little to its lateral stiffness, such reductions are small. Compressive yielding, however, as in the case of rolled beams, causes a more rapid decrease in stiffness. Indeed, for welded sections the sudden loss of a large part of the compression flange means that the inelastic curve diverges very rapidly from the elastic curve, decreases in stability of the order of fifty per cent being possible in heavily welded sections.

Studies of the effect on the buckling curve of the exact shape of the assumed residual stress pattern have shown this to be small except in the range of very short spans. What is of importance, however, is the magnitude of the balancing compressive stress in the flanges, which is itself dependent upon the size of the tension block. Use of the Cambridge method of relating the size of this tension block to the welding data has been shown to yield buckling curves in close agreement with those determined from measured residual stress patterns.

Sections fabricated from flame cut plates have been shown to possess higher buckling curves than geometrically similar sections fabricated from mechanically prepared plates due to the beneficial effect of the additional tension blocks at the flange tips.

From the numerical results a simple method of constructing beam buckling curves for welded sections has been proposed. These curves, which make due allowance for the severity of welding and the end fixity of the member, have been checked against available test results.

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Summary

The inelastic lateral buckling of I beams loaded in uniform bending and containing patterns of residual stresses typical of those found in welded sections is studied. Variations in the assumed shape of the residual tension block are shown to be less important than the magnitude of the balancing compressive stresses and the beneficial effects of using flame cut plates are examined. The rapid construction of approximate beam buckling curves for welded sections, including the use of the Cambridge relationships between welding parameters and residual stresses, is discussed and compared against available rest results. Such curves are therefore directly related to weld severity.

Résumé

On étudie le flambement latéral inélastique de poutres en double Té soumises à une flexion uniforme et contenant des tensions résiduelles telles que l'on en trouve dans les sections soudées. On montre que les variations dans la forme admise pour la répartition des tensions résiduelles sont moins importantes que l'amplitude des tensions de compression équilibrantes et on examine les effets bénéfiques de plaques découpées au chalumeau. On discute la construction de courbes de flambage pour des sections soudées, en tenant compte des relations de Cambridge entre les paramètres de soudure et les tensions résiduelles et on compare les résultats obtenus par essais. De telles courbes sont ainsi directement en relation avec la rigueur de la soudure.

Zusammenfassung

Die Arbeit untersucht das unelastische Kippen von I-Balken, die durch konstantes Biegemoment belastet sind und typische Eigenspannungen wie die in geschweissten Querschnitten aufweisen. Es wird gezeigt, dass Variationen der angenommenen Form der Eigenspannungszugzone weniger wichtig sind als die Grösse der ausgewogenen Druckspannung; die vorteilhaften Effekte bei Anwendung heiss zugeschnittener Platten werden untersucht. Das schnelle Konstruieren angenäherter Balken-Kippkurven für geschweisste Querschnitte, einschliesslich des Gebrauchs der Cambridge-Beziehung zwischen Schweiss-Parametern und der bleibenden Eigenspannungen, wird diskutiert und mit vorhandenen Test-Resultaten verglichen. Solche Kurven stehen somit in direkter Beziehung zur Schweisshärte.

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