Automatic Optimum Plastic Frame Design

Automatisches optimales plastisches Dimensionieren von Rahmen

Dimensionnement optimal automatique de cadres par moments plastiques

B. O. KUZMANOVIĆ
Professor of Civil Engineering, University
of Kansas, Lawrence, Kansas, U.S.A.

N. WILLEMS
Professor of Civil Engineering, University
of Kansas, Lawrence, Kansas, U.S.A.

1. Introduction

The proposed method is based on the plastic moment distribution technique and the subdivision of multistory frames *).

1.1. Basic Assumptions

The following assumptions are made:

1. Frames are rectangular and have rigid joints.
2. Members are prismatic.
3. Loads are static.
4. Vertical loads are represented by single concentrated loads acting at beam midspans.
5. Horizontal loads are represented by concentrated loads acting at floor levels.
6. A continuous spectrum range of member sizes is available.
7. Frame stability does not control.

In addition it is assumed that the simple plastic theory for rigid frames applies and that effects such as caused by axial loading, unequal column shortening, the spreading of the yield zone and the influence of shear stresses and strain hardening [1] can be considered as secondary for an initial design.

---

*) References are given at the end of this paper.

1.2. Objective Function

To solve a deterministic structural optimum design problem involves finding a position vector $\vec{D}$, which locates one point in an n-dimensional design space such that all equality or inequality constraints $f_k(\vec{D}) \gtreqless 0$ (where $k = 1, 2, 3, \ldots K$) are satisfied while an objective function $W(\vec{D})$ at the same time is a minimum. The objective function is a linear expression approximating the frame weight in terms of the full plastic moment capacity of each member $(M_p)_i$ and its length l_i. Thus

$$W(\vec{D}) = \sum_{i=1}^{i=n} l_i (M_p)_i , \tag{1}$$

where n is the number of members. The plastic moments represent the design variables although some of them may be given preassigned values such as would be the case in an "equal" or "minimum stanchions" design.

When using a linear weight function there may be a range of designs which all have the same minimum weight. In most structural optimization problems the contours of the objective function prove to be very flat. This implies that there is a broad range of designs which differ by only a small percentage from the optimal weight [2]. As the final member size selection is affected by availability and uniformity of sizes and several additional constraints, the simplification due to using a linear weight function is justified.

1.3. Proposed Method

The first two stages of the plastic moment distribution method [5] consist of constructing an admissible moment distribution and as such do not form part of the optimization procedure. The third stage however, which consists of reducing and equalizing moments, represents the actual optimization procedure. When two moments acting at two different sections in a frame member are equalized by reducing the larger and increasing the smaller moment, two goals are achieved at the same time – one additional hinge is formed while also the frame weight is reduced.

If the virtual work equations (Fig. 1a) for beam, sway and joint collapse mechanisms are first written and then differentiated, the following four equations governing the adjustments dM_i result:

$$\begin{aligned}
-dM_3 + 2\,dM_4 + dM_5 &= 0, \\
dM_1 + dM_2 + dM_6 + dM_7 &= 0, \\
dM_2 + dM_3 &= 0, \\
dM_5 + dM_6 &= 0.
\end{aligned} \tag{2}$$

The coefficients of the dM's represent the carry-over factors and will be either 0, ± 1 or $+2$. The moment adjustment is carried out progressively at
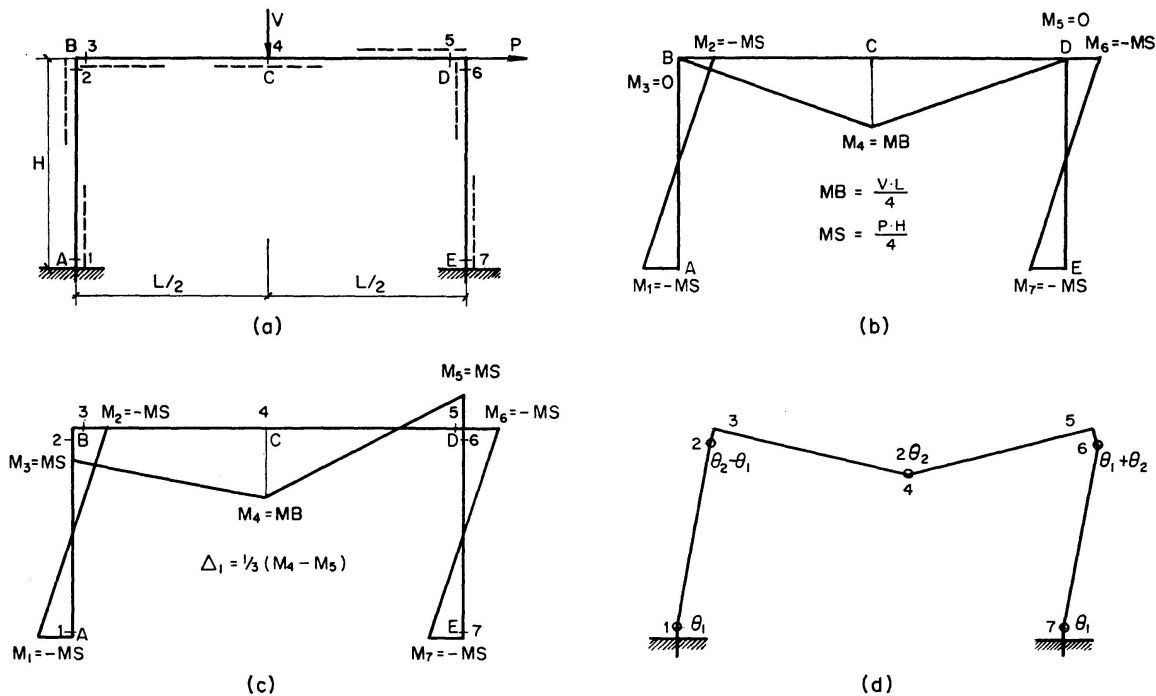
Fig. 1. Simple portal frame.

(a) Loads. Intermittent lines denote tensile side of a member under the action of a positive moments. (b) Moments from independent failure mechanisms. (c) Statically admissible moments. (d) Collapse mechanism.

a number of "potentially critical sections" [6] so as to satisfy all the equilibrium conditions (Eqs. 2) simultaneously until the structure or part thereof is transformed into a kinematical mechanism with one degree of freedom. By equalizing moments, plastic hinges are formed and this progressive cycling process is therefore dynamic in character. The proper sequence of creating hinges automatically until a minimum weight design is achieved with a minimal number of steps, forms the most important part of the proposed method. By partitioning a frame into basic units the designer is freed from trying to assume and try several collapse mechanisms. The successive creation of plastic hinges can be obtained by using correction moments which represent terms of a decreasing geometric series. The sum of such a series can easily be determined thus yielding the final moment values without carrying out corrections by successive cycling.

Partitioning of multistory frames was first suggested by N. S. BOULTON [7]. He had to assume a collapse mechanism and use a modification to the DINES [8] method to obtain a solution. The partitioning is possible because even in elastic analysis of such frames it is known that the member end moments in a story have a successively diminishing effect on stories above or below.

The proposed method can easily be extended to include distributed loads and several concentrated loads and considerations of instability and drift as well as the effect of axial and shear forces.

Following the first programs [3,4] linear programming for plastic design optimization is now widely used. Most of the optimization search methods are of the feasible direction type using either the kinematical or the statical approach. The plastic moment distribution method although known, has not been used for developing linear optimization programs [5].

2. Optimal Design of a Single Frame

Consider the frame shown in Fig. 1a under the action of loads V and P. Stage one consists of constructing moment diagrams from independent simple beam (BCD) and sway failure mechanisms $(ABDE)$ (Fig. 1b).

Stage two consists of establishing rotational balance at joints B and D so as to obtain an admissible bending moment distribution as shown in Fig. 1c. Setting $M_4 = MB$ and $M_1 = M_2 = M_6 = M_7 = -MS$ as shown in Fig. 1c, the "weight" at the start of the optimization process, depending on the relative values of MB and MS, is thus

$$MB > MS: \quad W_0 = 2H(MS) + L(MB), \tag{3a}$$

$$MB < MS: \quad W_0 = (2H + L)MS, \tag{3b}$$

$$MB = MS: \quad W_0 = (2H + L)(MS) \text{ or } (MB). \tag{3c}$$

2.1. $MB > MS$

Hinges will form in the frame at the end of stage two as shown in Fig. 1d, representing a two-degree of freedom mechanism. To equalize M_4 and M_5 while holding M_2 and M_3 constant, a correction Δ_1 is subtracted from M_4 as shown in Fig. 2a. To maintain equilibrium $2\Delta_1$ is then added to M_5 and subtracted from M_6 while Δ_1 is added to M_1 and M_7. From the equation that $M_4 - \Delta_1 = M_5 + 2\Delta_1$, Δ_1 is obtained as:
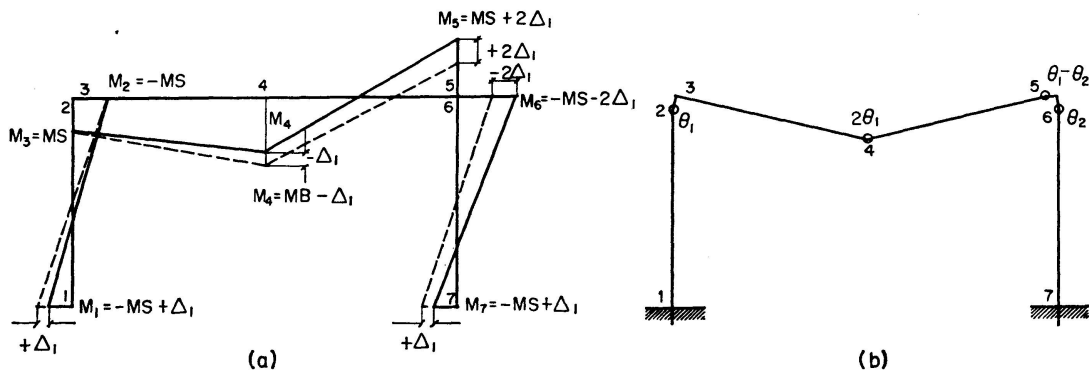

Fig. 2. First half cycle, Case $MB > MS$.
(a) Equalization of beam moments M_4 and M_5. Intermittent lines = admissible moments.
(b) Collapse mechanism after the first half cycle.

$$\Delta_1 = \frac{M_4 - M_5}{3} = \frac{MB - MS}{3}. \tag{4}$$

The corresponding collapse mechanism is shown in Fig. 2b. The moments can now be further modified by equating the moments M_1, M_6 and M_7 while holding M_4 constant. By adding $1.5\Delta_1$ to M_2 and M_6 and substracting it from M_3 and M_5 to balance joints B and D and subtracting $1.5\Delta_1$ also from M_1 and M_7 to satisfy the sway condition, the moments are

$$\begin{aligned}
M_1 &= M_6 = M_7 = -MS - 0.5\Delta_1, \\
M_2 &= -M_3 = -MS + 1.5\Delta_1, \\
M_4 &= MB - \Delta_1 = \overline{MB}, \\
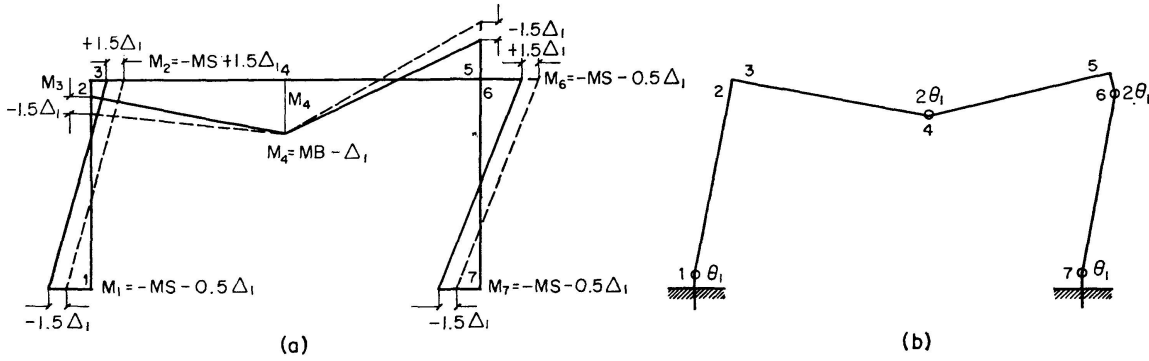M_5 &= -M_6 = MS + 0.5\Delta_1 = \overline{MS}.
\end{aligned} \tag{5}$$

Fig. 3. Second half cycle. Case $MB > MS$.

(a) Equalization of stanchion moments. Intermittent lines = moments after first half cycle. (b) Final collapse mechanism.

This results in the collapse mechanism shown in Fig. 3b and the frame weight is $W_1 = 2H(MS + 0.5\Delta_1) + L(MB - \Delta_1)$

or $$W_1 = W_0 - \Delta_1(L - H). \tag{6}$$

This completes the first cycle.

The same procedure is now repeated again and a new Δ_2 is used in terms of the new values of M_4 and M_5 which yields

$$\Delta_2 = \frac{\overline{MB} - \overline{MS}}{3} = \frac{\Delta_1}{2}. \tag{7}$$

Repeating the two previous steps of the first cycle yields a new distribution at the end of the second cycle. The new moment values are (analogous to the expressions given in Eqs. (5)):

$$\begin{aligned}
M_1 &= M_6 = M_7 = -MS - 0.5\Delta_1(1 + \tfrac{1}{2}), \\
M_2 &= -M_3 = -MS + 1.5\Delta_1(1 + \tfrac{1}{2}),
\end{aligned} \tag{8}$$

$$M_4 = MB - \Delta_1(1 + \tfrac{1}{2}) = \overline{MB} - \frac{\Delta_1}{2},$$

$$M_5 = -M_6 = MS + 0.5\,\Delta_1(1 + \tfrac{1}{2}) = \overline{MS} + \frac{\Delta_1}{4}. \tag{8}$$

The correction moment for the third cycle is therefore

$$\Delta_3 = \frac{\overline{MB} - \frac{\Delta_1}{2} - \overline{MS} - \frac{\Delta_1}{4}}{3} = \Delta_1\left(\frac{1}{2} - \frac{1}{6} - \frac{1}{12}\right) = \frac{\Delta_1}{2^2}. \tag{9}$$

In general the correction for the n-th cycle is

$$\Delta_n = \frac{\Delta_1}{2^{n-1}}. \tag{10}$$

Thus, the total sum of corrections for an infinite number of cycles is

$$S_n\Delta_1 = \lim_{n\to\infty}\left(1 + \frac{1}{2} + \cdots + \frac{1}{2^{n-1}}\right)\Delta_1 = 2\Delta_1. \tag{11}$$

The final value of the moments after an infinite number of cycles thus is (Fig. 4)

$$M_1 = -M_5 = M_6 = M_7 = -MS - 0.5\,\Delta_1 S_n = -MS - \Delta_1,$$
$$M_2 = -M_3 = -MS + 1.5\,\Delta_1 S_n = -MS + 3\Delta_1, \tag{12}$$
$$M_4 = MB - \Delta_1 S_n = MB - 2\Delta_1.$$
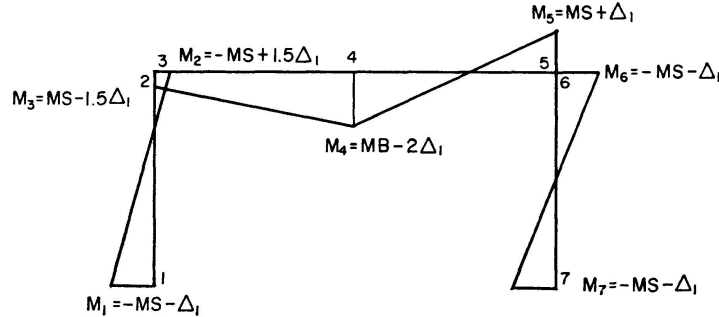
Fig. 4. Final moments after infinite cycling. Case $MB > MS$.

The above procedure can be generalized by assuming a correction Δ_1 necessary to equalize M_4 and M_5 as done before and given by Eq. (4). Subsequent corrections to equalize M_1, M_6 and M_7 are assumed to be $y\Delta_1$. Again calling the sum of all corrections $S_n\Delta_1$ the final moments are

$$M_1 = M_7 = -MS + (1-y)\Delta_1 S_n,$$
$$M_2 = -M_3 = -MS + \Delta_1 y S_n,$$
$$M_4 = MB - \Delta_1 S_n, \tag{13}$$
$$M_5 = -M_6 = MS + (2-y)\Delta_1 S_n.$$

Setting $M_4 = M_5 = -M_6$ and $M_1 = M_6 = M_7$ yields

$$MB - \Delta_1 S_n = MS + (2-y)\Delta_1 S_n, \tag{14a}$$

$$-MS + (1-y)\Delta_1 S_n = -MS - (2-y)\Delta_1 S_n. \tag{14b}$$

Using the expression given for Δ_1 in Eq. (4) yields $y = 1.5$ and $S_n = 2$ which is the same result as given before.

Making use of the geometric progression it is thus possible to find the final moment diagram without carrying out any cycling and the final weight is given as

$$W_n = 2H(MS + \Delta_1) + L(MB - 2\Delta_1) = W_0 - 2\Delta_1(L - H). \tag{15}$$

The corresponding final moment diagram is shown in Fig. 4. From Eq. (15) it is obvious that the weight decreases only if

$$L > H. \tag{16}$$

In most practical cases this is true. If however $L < H$ a frame can always be modified so as to satisfy Eq. (16) by increasing L and decreasing the vertical load [10]. As will be shown later, if such a modification is made the best value for L is twice the height H.

The above procedure in its final result as expressed in Eqs. (12) and (15) is only correct if at no time during the cycling process $|M_3| > M_4$. In such a case the collapse mechanism becomes a typical three hinged beam mechanism. This would not occur, according to Eqs. (12), when

$$-MS + 3\Delta_1 < MB - 2\Delta_1 \tag{17}$$

or

$$\frac{MB}{MS} < 4. \tag{18}$$

If this inequality is satisfied the true mechanism is the one shown in Fig. 3b. It can be easily shown that Foulkes theorem in this case is satisfied and therefore the distribution and corresponding mechanism yields a minimum value for the linearized weight function [5].

In case $MB/MS > 4$ it will occur that after say k cycles (where $0 < k < n$), $|M_3| = M_4$. The cycling should be stopped at this point. Using Eqs. (12) and replacing S_n by S_k and setting $|M_3| = M_4$ yields

$$S_k = \frac{6(MB + MS)}{5(MB - MS)}. \tag{19}$$

If there is only vertical loading, $MS = 0$ and therefore S_k equals $6/5$.

The final values of the moments and the corresponding partial collapse mechanism are shown in Fig. 5. In this case the total weight is

$$W_k = (H + L)(MB - \Delta_1 S_k) + (MS + 0.5\Delta_1 S_k)H =$$
$$W_0 - 2MB\frac{L - 2H}{5} - 2MS\frac{L + 3H}{5}. \tag{20}$$
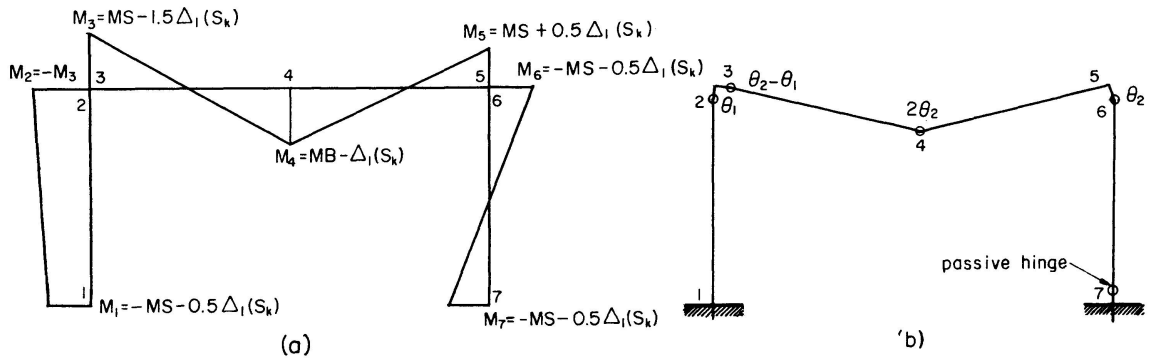
Fig. 5. Finite cycling, Case $MB > MS$.

(a) Moments after k-th cycle. (b) Collapse mechanism after k-th cycle.

2.2. $MB < MS$

Although this case is not likely to occur very frequently it will be briefly discussed here. The admissible moment diagram and collapse mechanism are shown in Fig. 6 while W_0 is given by Eq. (3b). As $M_4 < M_5, \Delta_1$ equals zero and geometric progression does not apply. No cycling is necessary and only one correction needs to be made by setting M_3 and M_5 equal to M_4 by reducing M_3 and M_5 by

$$\Delta = MS - MB. \tag{21}$$
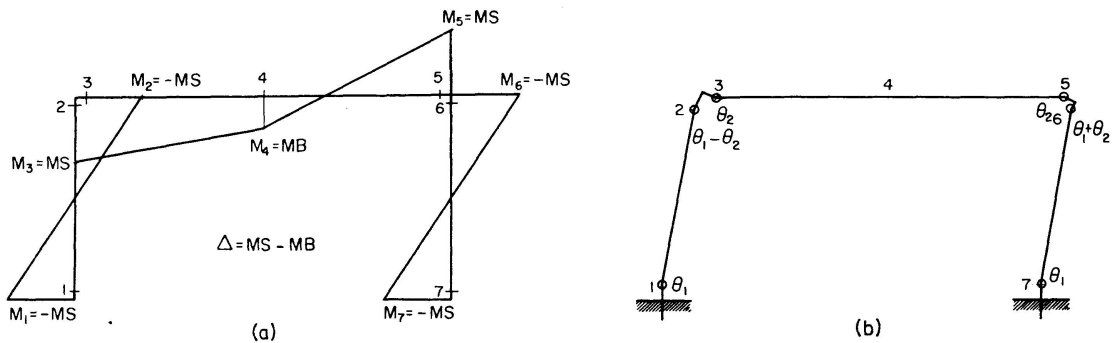
Fig. 6. Case $MB < MS$.

(a) Statically admissible moments. (b) Collapse mechanism for admissible moments.
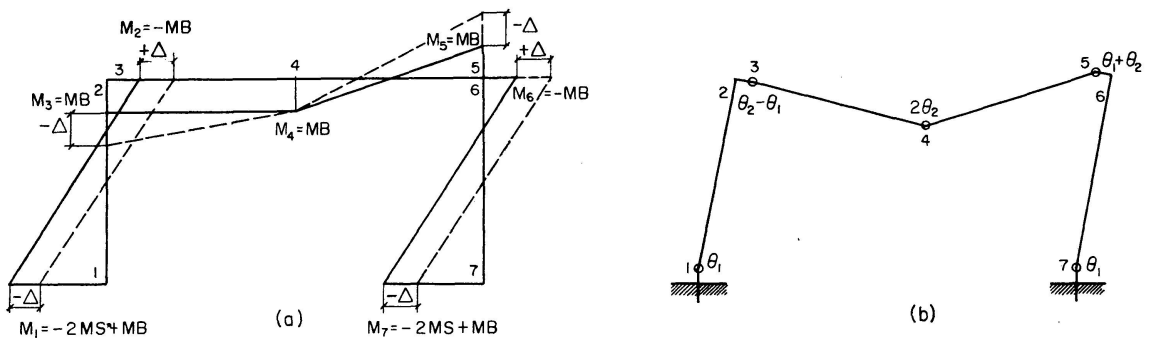
Fig. 7. Moment distribution. Case $MB < MS$.

(a) Equalization of beam moments. Intermittent lines = admissible moments. (b) Collapse mechanism after equalization.

The moments are given by

$$M_1 = M_7 = -MS - \Delta = -2MS + MB,$$
$$-M_2 = M_3 = M_4 = M_5 = -M_6 = MB. \tag{22}$$

The final weight is

$$W = L(MB) + 2H(2MS - MB) = W_0 - \Delta(L - 2H). \tag{23}$$

This value will only decrease if $L > 2H$. For the collapse mechanism shown in Fig. 7b, when it is tested to see if it satisfies Foulkes theorem, it is found that it only does so when $L > 4H$ because θ_2 has to be larger than θ_1. So only if $L > 4H$ is there a real minimum. If L is larger than $2H$ and smaller than $4H$ the design will be only at the vertex point of a permissible region.

If MB equals zero the frame collapse is a sway mechanism.

3.2. $MB = MS$

In this case a plastic hinge will develop at every joint and an overcomplete failure will occur. Obviously no optimization is needed.

2.4. Examples of Simple Portal Frame

As an illustration of case 1 ($M_4 > M_5$) two problems are solved.

a) Combined loading. This problem was originally solved by J. HEYMAN and W. PRAGER [11] using their method. The moment at midspan of simple beam BCD equals $M_4 = MB = VL/4 = 168(4)/4 = 168$. The sway moment is $MS = PH/4 = 84(3)/4 = 63$.

Because $MB > MS$ and $MB/MS = 168/63 = 2.67 < 4$ $\therefore n \to \infty$.

The correction moment now equals (Eq. (4)) $\Delta_1 = (168 - 63)/3 = 35$. Using Eq. (12), the final moments are

$$M_1 = -M_5 = M_6 = M_7 = -63 - 35 = -98,$$
$$M_2 = -M_3 = -63 + 3(35) = +42,$$
$$M_4 = 168 - 2(35) = +98.$$

In Figs. 8b and 8c the moment field and the collapse mechanism are shown. The weight is $W_n = 98(2 \times 3 + 4) = 980$ which is identical to previous results (11).

b) Vertical Load Only. This problem illustrates the case where $n \to k$, i.e., the cycling has to be terminated at the moment when $|M_3| = M_4$. This is the case of local beam collapse. The problem was solved by a different method by MASSONNET and SAVE [6]. Their optimum weight $W = 1008$ is exactly the same as obtained by the proposed method. The frame and its loading are shown in

$$M_4 = MB = \frac{168(4)}{4} = 168$$

$$M_5 = MS = \frac{84(3)}{4} = 63$$

(a)

(b)

(c)
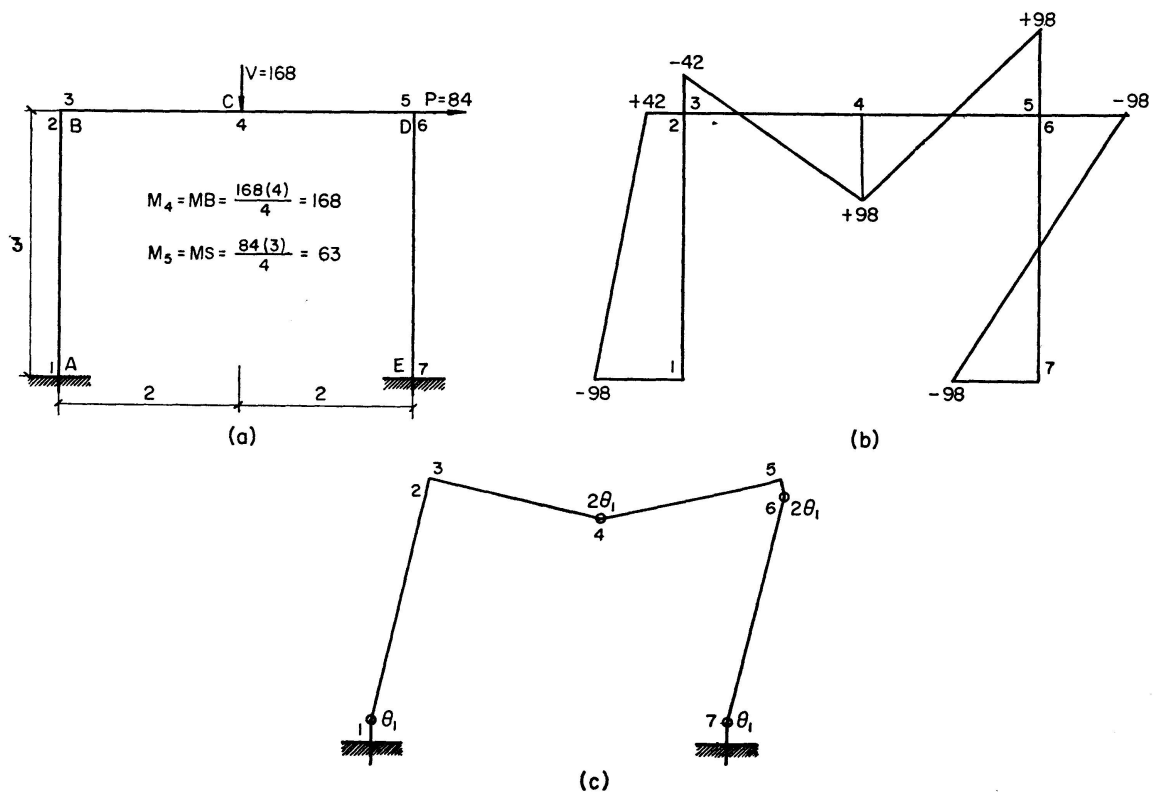
Fig. 8. Example (A).
(a) Combined loading. (b) Final moments. (c) Final collapse mechanism.

$$M_4 = MB = \frac{210(4)}{4} = 210$$

$$M_I = -MS = 0$$

(a)

(b)

passive hinge

(c)
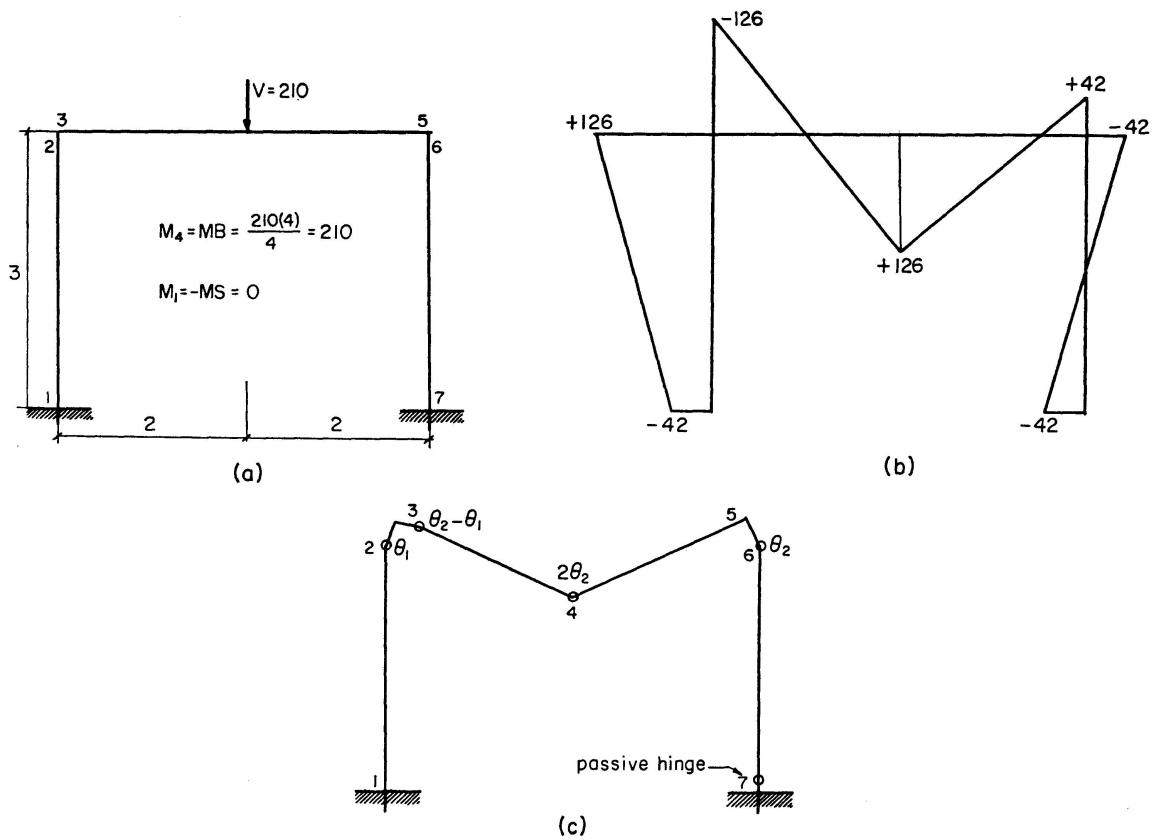
Fig. 9. Example (B).
(a) Vertical loading. (b) Final moments. (c) Final collapse mechanism.

Fig. 9 a. The simple beam failure moment is $M_4 = MB = 210 \times 4/4 = 210$. The sway moment is $MS = 0$ and the correction moment $\Delta_1 = (210)/3 = 70$; because $MB/MS > 4$ using Eq. (19) yields $S_k = 6/5 = 1.20$. From Eq. (12) the final moments are found to be

$$M_1 = -M_5 = M_6 = M_7 = -0.5\,(70)\,(1.2) = -42,$$

$$M_2 = -M_3 = 1.5\,(70)\,(1.2) = 126,$$

$$M_4 = 210 - 70\,(1.2) = 126.$$

In Fig. 9 b and 9 c the moment field and the collapse mechanism are shown.

3. Single-Bay Multi-Story Frames

The proposed method can be extended to apply to single-bay multi-story frames. Starting from the top story, the optimization is performed. The only difference is that any adjusting moments distributed to the bottom of stanchions of a story must be added with changed signs to the beam-end joints of the story immediately below it so as to preserve joint equilibrium.
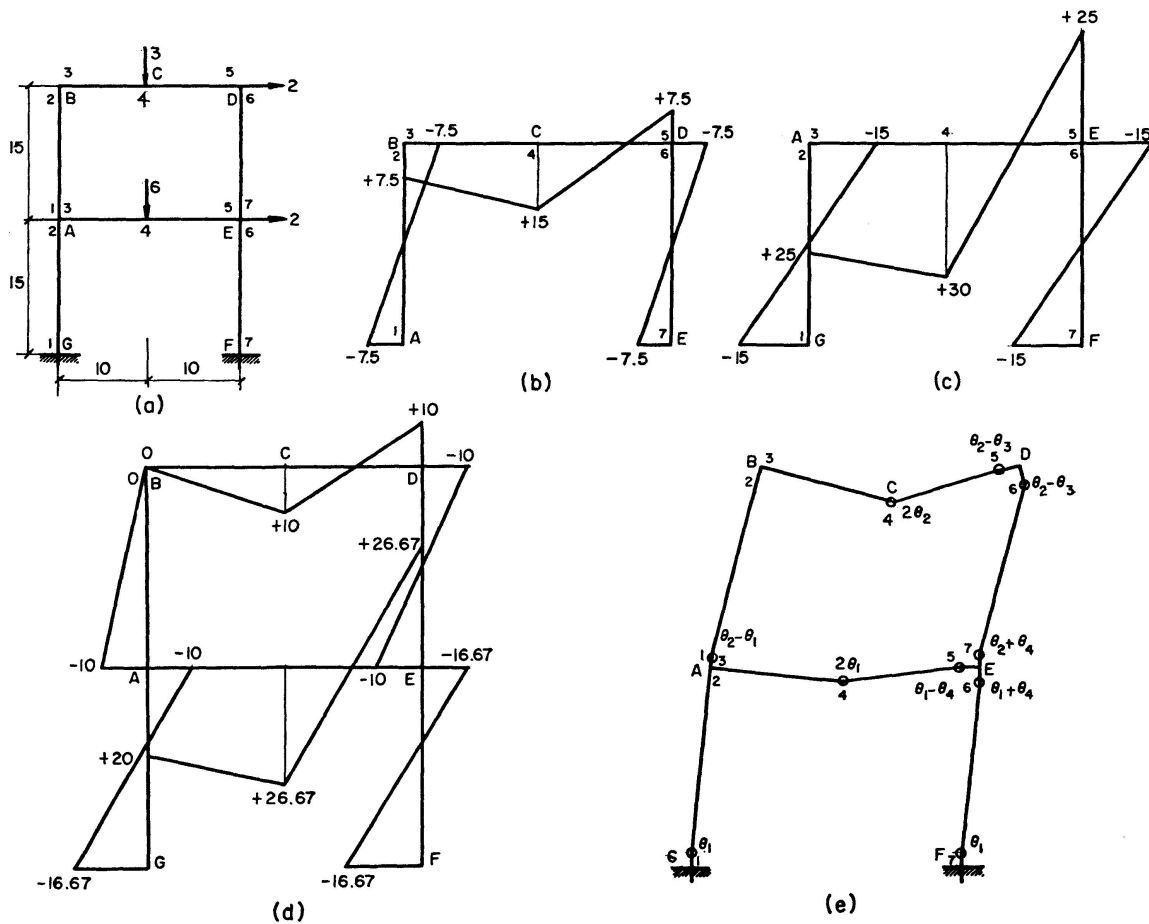

Fig. 10. Two-story, single-bay frame.

(a) Loading. (b) Top story admissible moments. (c) Bottom story admissible moments. Beam end moments increased by $\Delta_1 = 2.5$. (d) Final moments. (e) Final collapse mechanism.

As an example, the problem of a two-story single-bay rectangular frame (Fig. 10) is taken which is the same as given by B. G. NEAL [5]. His minimum value of the weight is 1533.3, exactly the same as obtained here.

a) *Top Story.* The admissible bending moments for the top story are reproduced in Fig. 10b where $M B = 3 \times 20/4 = 14$ and $M S = 2 \times 15/4 = 7.5$; $M B / M S = 2.0 < 4.0$ ∴ $S_n = 2.0$, and $\Delta_1 = (15 - 7.5)/3 = 2.5$.
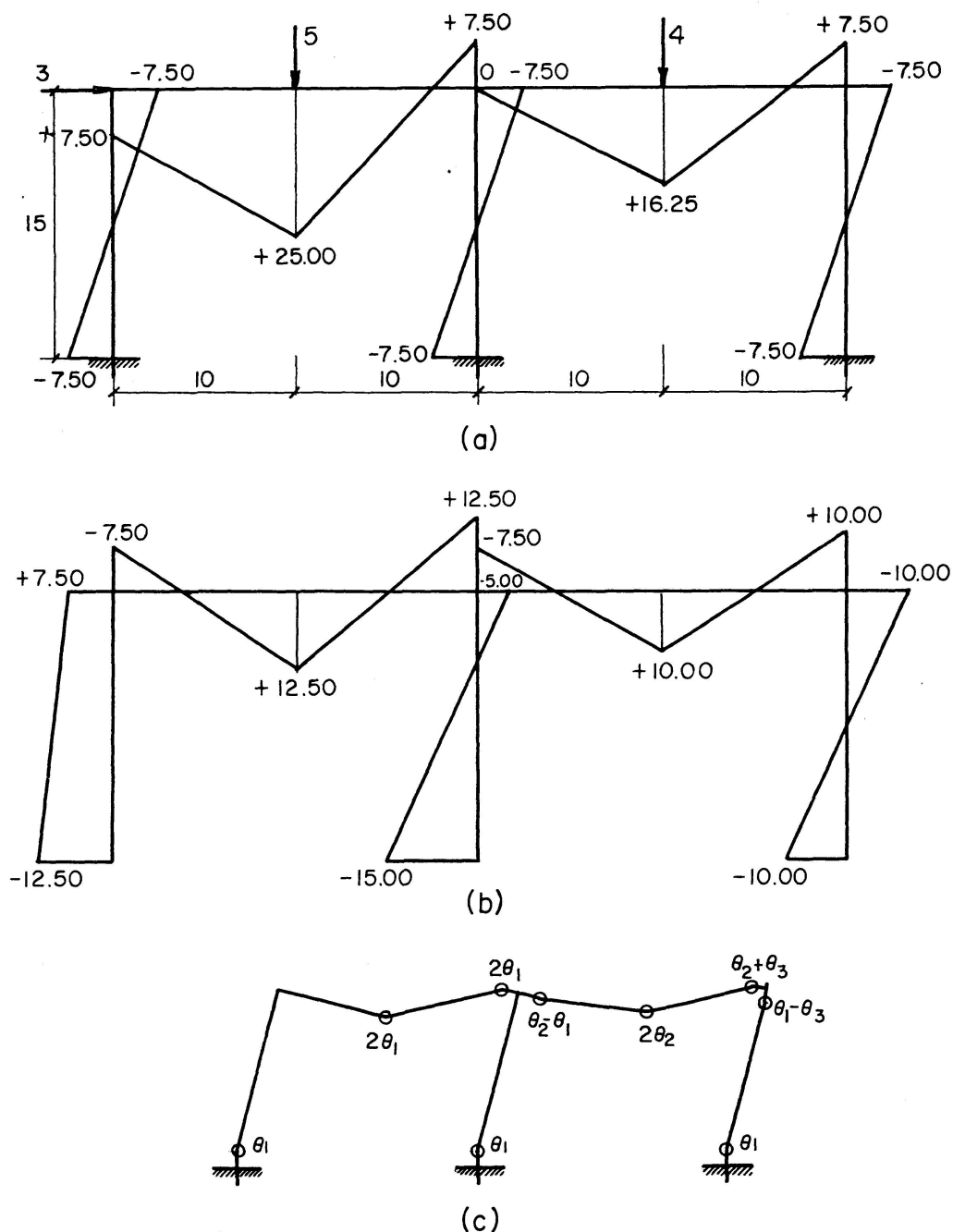

(a)

(b)

(c)

Fig. 11. Two-bay, single story frame.

(a) Loading and statically admissible moments. (b) Final moments, minimum beam design. (c) Final collapse mechanism, minimum beam design.

Using Eqs. (12) the final moments in the top story are obtained and are shown in Fig. 10d.

b) Bottom Story. The admissible bending moments are: $MB = 6 \times 20/4 = 30$; $MS = (2+2) \, 15/4 = 15$; $MB/MS = 2.0 < 4.0$ ∴ $S_n = 2.0$; $\Delta_1 = (M_4 - M_5)/3 = (30 - 25)/3 = 1.67$. The beam moments $M_3 = M_5 = +22.5$ had to be changed by $+2.5 = -\Delta_1$ of the top story, i.e., the negative of the correction moment -2.5 is added to the end-beam moments to re-establish rotational balance of joints A and E. Using the same Eqs. (12) the final moments are as shown in Fig. 10d. The collapse mechanism is shown in Fig. 10e. The final weight is: $W = 10 (20 + 2 \times 15) + 26.67 (20) + 2 (15) (16.67) = 1533.3$ which is same as obtained by NEAL [5].

4. Single Story Multibay Frames

The optimization of these frames depends to a large extent on the type of design that is selected. Two of the most common types are the so called "minimum beam" design and the "equal stanchion" design. The second type represents a more practical and realistic design. It is impossible to discuss in detail both approaches except to mention that several examples have been worked using the same principles as discussed above. An example of minimum beam design is shown in Fig. 11. The exact solution by NEAL [5] yields an
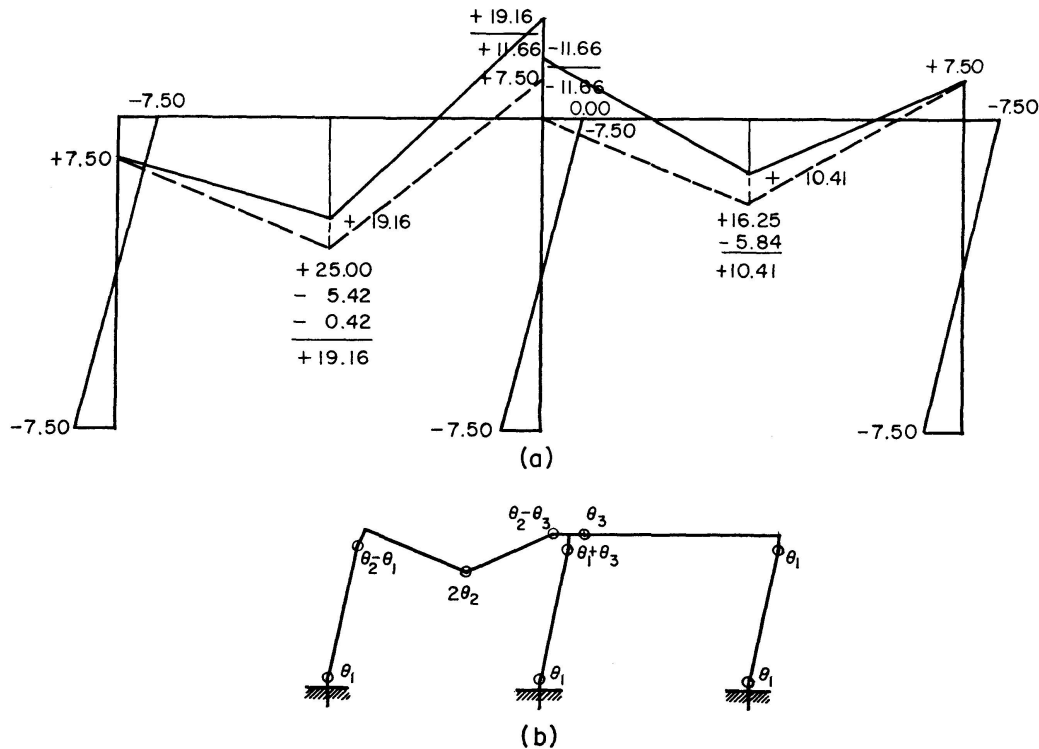

Fig. 12. Minimum stanchion design.

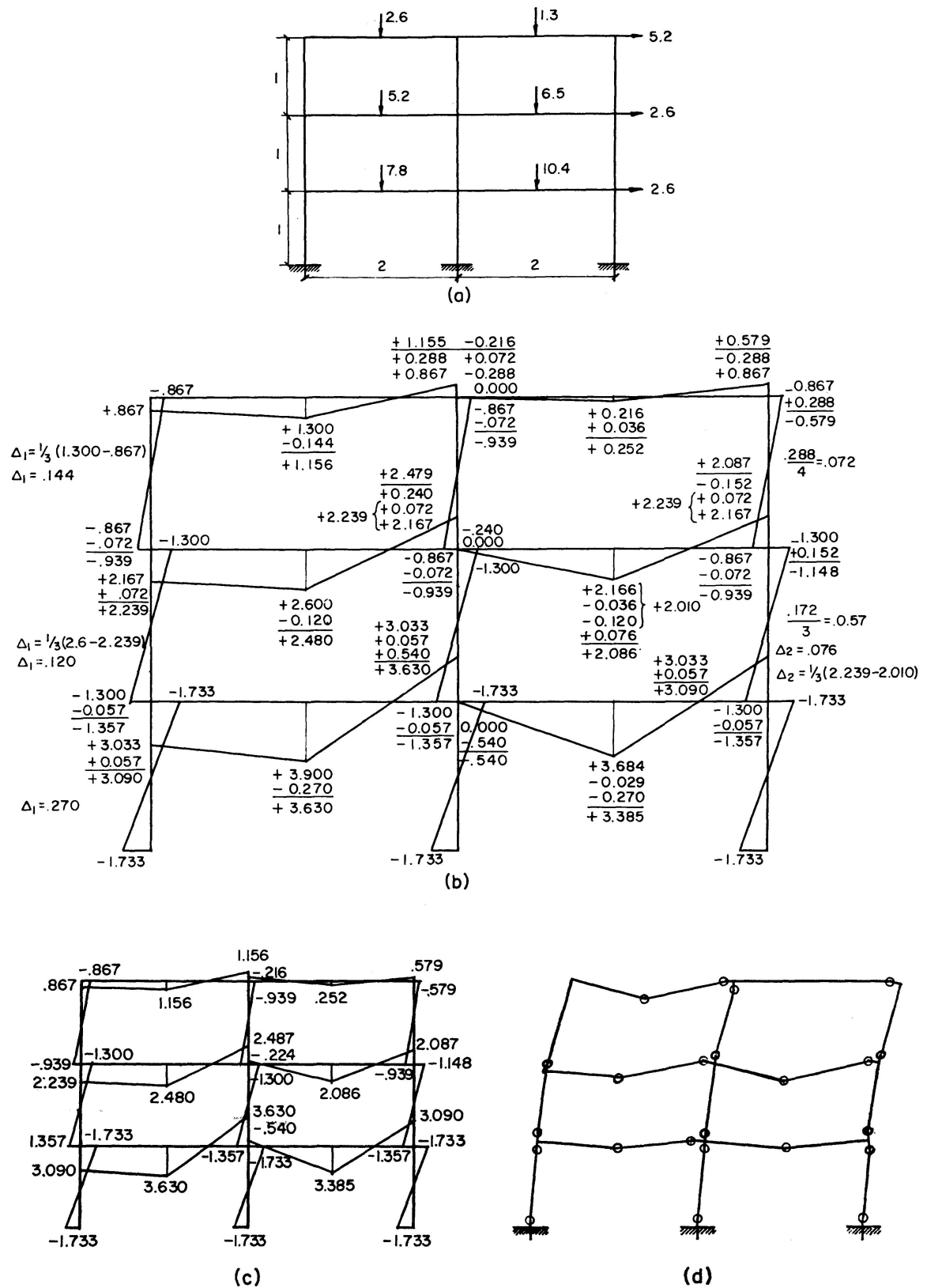(a) Moment distribution and final moments. (b) Collapse mechanism.

Fig. 13. Multistory frame, minimum stanchion design.

(a) Loading. (b) Admissible moments and moment distribution. (c) Final moments. (d) Final collapse mechanism.

overall weight of 987.5. The proposed method gives 1012.5 (+2.54%). The same problem solved as an equal stanchion design is shown in Fig. 12. The weight thus obtained is 953.9 while Neal obtained 954.1.

5. Multistory, Multibay Frames

The procedure discussed in sections 3 and 4 above can be combined to solve multistory-multibay frames. In Fig. 13 an example is shown using the equal stanchion design approach. To demonstrate how short and effective the proposed method is, all the necessary calculations are shown in Fig. 13b. This problem was originally discussed by NEAL and SYMONDS [12] and later by BOULTON [7]. The weight obtained by the proposed method is 38.72 as compared with 38.22 found by Boulton.

6. Conclusions

The proposed method makes use of the well known plastic moment distribution method. Cycling is eliminated by recognizing the existence of geometric progressive correction moments. Although the method is well suited for linear programming, the main purpose of this paper is to show that even fairly complicated frames can easily be designed by hand when making use of geometric progression. The method is not always exact but even when approximate is very close to other exact methods which are much more complicated and require large computer storage for relatively simple frames. The method, like most available minimum weight designs is only developed for preliminary design purposes. By considering additional structural constraints the design can be checked and corrected as for any iterative design procedure.

List of Notations

i	Subscript designating i-th member or section
k	Subscript designating k-th cycle
l_i	Length of i-th member
n	Number of members or cycles
$y\Delta_1$	Correction moment
$\vec{D}$	Position vector
H	Story height
L	Span
M_i	Bending moment at i-th section
dM_i	Correction moment

$(M_p)_i$ Full plastic moment of i-th member

MB Midspan bending moment of a simply supported beam

MS Sway moment

P Horizontal concentrated force

$S_k \varDelta_1$ Sum of correction moments after k-th cycle

$S_n \varDelta_1$ Sum of correction moments after n-th cycle

V Vertical concentrated force

W_0 Frame weight before optimization

$W(\vec{D})$ Objective function

θ Angle of rotation at plastic hinges

$\varDelta$ Correction moment in case $MS > MB$

$\varDelta_1$ Correction moment in case $MS < MB$

$|a|$ Numerical value of a

$a > b$ a greater than b

$a < b$ a smaller than b

References

1. BEEDLE, L. S., et al.: Recent Developments in Plastic Design Practice. Journal of the Structural Division, Proc. ASCE, Vol. 95, No. ST 9, Sept. 1969, p. 1915.
2. CORNELL, C. ALLEN: Examples of Optimization in Structural Design, Study No. 1. Solid Mechanics Division, Univ. of Waterloo, Canada, 2nd printing 1970, p. 149.
3. CHARNES, A. and GREENBERG, H. J.: Plastic Collapse and Linear Programming. American Mathematical Society, Abstracts No. 506, 1951.
4. LIVESLY, R. K.: The Automatic Design of Structural Frames. Quarterly Journal of Mech. and Appl. Math., Vol. 9, Pt. 3, Sept. 1956, p. 257–278.
5. NEAL, B. G.: The Plastic Methods of Structural Analysis. 2nd ed., New York, John Wiley & Sons, 1963, p. 146, 153, 247.
6. MASSONNET, C. E. and SAVE, M. A.: Plastic Analysis and Design. Vol. 1, 1st English ed., Blaisdell Publishing Company, New York, 1965, p. 272.
7. BOULTON, N. S., written discussion of the paper "Plastic Analysis and Design of Steel-Framed Structures", by J. Heyman, Final Rep. 4th Congr. Intern. Assn. Bridge and Struct. Engng., Cambridge, p. 113–118, 1953.
8. DINES, L. L.: Systems of Linear Inequalities. Annals of Mathematics, Series 2, Vol. 20, p. 191, 1918–1919.
9. TOAKLEY, A. R.: Optimum Design Using Available Sections. Journal of the Structural Division, Proc. ASCE, Vol. 94, No. ST 5, May 1968, p. 1219–1241.
10. FOULKES, J. D., written discussion of the paper, "The Rapid Calculation of the Plastic Collapse Load for a Framed Structure", by Neal, B. G. and Symonds, P. S., Proc. Instr. Civ. Engrs., Part III, April 1952, p. 88.
11. HEYMAN, J. and PRAGER, W.: Automatic Minimum Weight Design of Steel Frames. J. Franklin Inst., Vol. 266, p. 339–364, 1958.
12. NEAL, B. G. and SYMONDS, P. S.: The Rapid Calculation of the Plastic Collapse Load for a Framed Structure. Proc. Instn. Civil Engrs., Vol. 1, Part III, p. 60.

Summary

An approximate method based on plastic moment distribution and geometric progressions is developed for the optimal design of rectangular frames. The optimization consists of reducing the required full plastic moment capacities of frame members by the successive creation of plastic hinges. For a single frame "unit" the distribution of correction moments is developed in such a way that they represent terms of a decreasing geometric series. Expressions for the final moments can thus be found in closed form without cycling. Using subdivision of frames, the method is adapted to the design of multi-story-frames.

Résumé

Une méthode approchée est développée pour le calcul optimal des cadres rectangulaires. Cette méthode est fondée sur la distribution des moments plastiques et les progressions géométriques. Le procédé d'optimisation consiste dans la réduction des capacités nécessaires des moments entièrement plastiques des membres des cadres par la création successive de rotules plastiques. Pour un cadre "unité" la distribution des moments correctifs est développée de telle façon que ces moments représentent les termes d'une série géométrique descendante. On peut écrire les expressions pour les moments définitifs sans aucune répétition cyclique. En utilisant la subdivision des cadres, la méthode est adaptée au calcul des cadres à plusieurs étages.

Zusammenfassung

Es wird eine Näherungsmethode für das optimale Dimensionieren rechteckiger Rahmen abgeleitet. Diese Methode stützt sich auf die plastische Momentverteilung. Die Optimierung besteht aus der Reduktion der notwendigen vollplastischen Momentenkapazitäten der Rahmenglieder durch fortlaufende Schaffung plastischer Gelenke. Für eine einzelne Rahmeneinheit wird die Verteilung der Korrekturmomente so entwickelt, dass diese die Terme einer abnehmenden geometrischen Reihe darstellen. Die Ausdrücke für die endgültigen Momente lassen sich alsdann in geschlossener Form ohne irgendeine zyklische Wiederholung anschreiben. Unter Benutzung der Unterteilung der Rahmen wird die Methode der Berechnung mehrstöckiger Rahmen angepasst.

Leere Seite

Blank page

Page vide