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Incremental Collapse under Conditions of Partial Unloading

Défaillance incrémentale sous des conditions de décharges partielles

Zunehmendes Versagen unter teilweisen Entlastungsbedingungen

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Introduction

It has been shown by GRUNING [1] and KAZINCZY [2] that a hyperstatic structure may collapse when subjected to a relatively small number of cycles of repeated loads even though none of the loadings applied is sufficiently severe to cause failure by plastic collapse in the first cycle. This type of failure is known as "incremental collapse" and it has been studied extensively, both theoretically and experimentally, during the past thirty years. In particular, the work of BLEICH [3], NEAL and SYMONDS [4], HODGE [5], HORNE [6], MASSONNET [7], POPOV and McCARTHY [8], and KOITER [9] is particularly note-worthy in this regard.

The phenomena associated with this particular form of structural collapse may be briefly summarized as follows. If the maximum intensities of the loads applied to a particular structure are each expressed in terms of a single magnitude W, then it can be shown that if W exceeds a certain intensity W_H, "residual moments" due to rotation at "plastic hinge" locations are induced in the structure after each cycle of loading. If the magnitude W exceeds W_H but remains smaller than a certain critical value W_s, then the increases in the residual moments which remain in the structure after each cycle of loading become progressively smaller as the number of cycles of loading increases. Eventually, a condition is reached where no further changes in the pattern of "residual moments" takes place. That is, no further changes in plastic hinge rotations occur and subsequent applications of these loads causes only completely reversible elastic changes of bending moment in the structure. When this happens, the structure is said to have *shaken down*. If the maximum intensity of the loads exceeds the critical value W_s, then the structure never shakes down, and definite irrecoverable rotations take place at the plastic hinges during each cycle of loading. If the maximum load intensity is unchanged from cycle to cycle, an unvarying regime emerges in which the change in the rotation at any given hinge is a constant from cycle to cycle, so that in each cycle the irrecoverable or residual deflections of the structure increase by a definite amount. After a certain number of cycles of load application has taken place, the residual deflections will have risen to such high values that the structure is rendered useless. For this reason, the structure is then said to have failed by *incremental collapse*. The particular load intensity above which incremental collapse can occur and below which incremental collapse cannot occur is called the *incremental collapse load*, W_s, or, sometimes, the shakedown load. This shakedown load, of course, is influenced by the geometry of the structure and its support conditions, the mechanical properties of its material, and the manner and intensity of load applications.

If a structure shakes down, then after many cycles of load have been applied to it there remain a set of residual bending moments m_i locked into the structure which must satisfy, according to MASSONNET and SAVE [10], the following conditions:

1. The bending moments m_i represent a state of self-stress of the structure; that is, these bending moments are in equilibrium with each other in the absence of external loads acting on the structure.
2. When the elastic bending moments M_e^i corresponding to any particular state of loading are superimposed on the residual bending moments m_i, then the resulting bending moments nowhere exceed the plastic moment M_p in absolute value.
3. As the external loads are permitted to vary within their prescribed ranges, the magnitude of the change in the elastic bending moment ΔM_e^i nowhere exceeds the value $2 M_y^i$.

Residual bending moments m_i that satisfy these three conditions may be said to represent a *virtual shakedown state of the structure for the given ranges of the load*. These three conditions may be expressed quantitatively as,

$$\begin{aligned} m_i + (M_e^i)_{max} &\leqq M_p^i, \\ m_i + (M_e^i)_{min} &\geqq -M_p^i, \\ (M_e^i)_{max} - (M_e^i)_{min} &\leqq 2 M_y^i, \end{aligned} \tag{1}$$

in which $(M_e^i)_{max}$ and $(M_e^i)_{min}$ are the extreme values of the *elastic* bending moment at the corresponding (or, i-th) cross-section of the structure for all states of loading under consideration. The third of these three conditions, of course, restricts the external loads to a range which avoids the onset of alternating plasticity.

The fundamental theorem of shakedown analysis is an assertion that the three conditions (1) which are, of course, *necessary* for shakedown to occur, are also *sufficient*. This may be stated as a theorem.

Theorem 1. If there exists a distribution of virtual, that is, statically admissible, bending moments m_i that represents a virtual shakedown state for the given ranges of the loads, then the structure will shake down.

The first version of the shakedown theorem was given by BLEICH [3]; modern proofs may be found in the books of HODGE [11], NEAL [12] and HEYMAN [13], among others. It must be remarked that the distribution of residual moments m_i after the structure has shaken down need not coincide with the original virtual shakedown distribution from which it was deduced that the structure will indeed shake down. This point is discussed in the proof of the shakedown theorem given by HEYMAN [13] among others.

Partial Unloading

Equations 1 together with the application of the appropriate static equilibrium conditions to the residual moments m_i are sufficient to determine the shakedown load W_s for an uncomplicated structure subjected to a relatively simple pattern of applied loads. If the structure is complicated (e.g. a multibay rigid framework) and/or the pattern of applied loads is complex (e.g. if partial unloading and/or complete load reversals can occur) then computations of the traditional type needed to obtain the shakedown load or loads may become exceedingly complex even if aided by the digital computer. For this reason, it is worthwhile to explore alternate formulations of the conditions defining shakedown in an attempt to render the computations more tractable, if for no other reason.

ERBER, GURALNICK and LATAL [14] have demonstrated that an alternate way of characterizing shakedown and of defining the incremental collapse load arises from a consideration of the energy imparted to and recovered from a structure during each cycle of a long series of varying-intensity load applications. This alternate approach may be summarized in the form of a general shakedown principle.

Principle 1. Suppose that an elasto-plastic, rigidly-jointed framework structure is subjected to a cyclically-recurring, varying-intensity set of loads in which the maximum and the minimum intensity of each of the loads can be expressed in terms of a mean load intensity $\overline{W}$. Furthermore, suppose that the maximum intensity of each load may be expressed as a definite multiple of the quantity $W_{max} = \overline{W} + R/2$ and the respective minimum intensity of each load may be written as the same multiple of the quantity $W_{min} = \overline{W} - R/2$. Then there exists a pair of limiting values W^*_{max} and W^*_{min} ($W^*_{max} > W^*_{min}$), for each set of load multiples, for a given structure, which are uniquely determined by the value of $\overline{W}$, such that if $W_{max} < W^*_{max}$ and $W_{min} > W^*_{min}$, then the series composed of all terms representing the energy losses encountered at each cycle is convergent. Furthermore, if $W_{max} > W^*_{max}$ and $W_{min} < W^*_{min}$, then the energy loss (or "hysteresis") for each cycle of loads becomes a constant after a few cycles of loading has been completed and, hence, the series composed of all terms representing the energy losses encountered at each cycle is divergent and the structure fails.

The converse statement that W^*_{max} and W^*_{min} do not exist or that if they do exist they are not uniquely-determined by $\overline{W}$ and the given set of load multiples is contrary to all evidence regarding the behavior of real structures, therefore we may regard the principle as being established by contradiction.

Without explicit recognition of its existence, of course, the use of this principle to assess the resistance of structures to earthquake loads is implicit in the work of HOUSNER [15], BLUME [16] and MEDEARIS [17], among others. It is the purpose of this paper to show that this principle may also be used to determine the alternating plasticity load W_a, the shake-down load W_s and the incremental collapse load for conditions in which the live loads are only partially removed during each cycle.

If $W_{min} = 0$ (i.e. complete load removal in each cycle), then this principle is merely a statement of the shakedown theorem in a new guise and $W^*_{max} = W_s$, the *shakedown load*, as has been shown by ERBER, GURALNICK and LATAL [14]. Similarly, it may be shown that if $W_{max} = -W_{min}$ (i.e. complete load reversal in each cycle), then carrying out a set of computations of energy losses cycle by cycle for a particular structure will simply lead to the conclusion that, $W^*_{max} = -W^*_{min} = W_a$, the *alternating plasticity load*. Furthermore, if the cyclic load degenerates to a single cycle of load application to collapse then $W^*_{max} = W^*_{min} = W_c$, the *plastic collapse load*.

Let us now define a "range parameter" R such that,

$$R = W_{max} - W_{min} \tag{2}$$

and a "limiting range parameter" R^* such that

$$R^* = W^*_{max} - W^*_{min}. \tag{3}$$

If we define the corresponding mean load $\overline{W}$ to be

$$\overline{W} = \frac{W_{max} + W_{min}}{2} \tag{4}$$

then we may write,

$$W_{max} = \overline{W} + \frac{R}{2}$$

and

$$W_{min} = \overline{W} - \frac{R}{2} \tag{5}$$

Furthermore, given the definitions of W^*_{max}, W^*_{min} and R^*, we may write,

$$
\begin{aligned}
W^*_{max} &= \overline{W} + \frac{R^*}{2},\\
W^*_{min} &= \overline{W} - \frac{R^*}{2}.
\end{aligned}
\tag{6}
$$

Principle 1 guarantees that there exists an R^*, which is a continuous function of the mean load $\overline{W}$, for any particular structure subjected to any given load pattern (or set of load multiples). Hence, it should be possible to construct a graph, or a family of graphs, for any particular structure of the form shown in Fig. 1. In this graph, R^* is shown as a continuous monotone

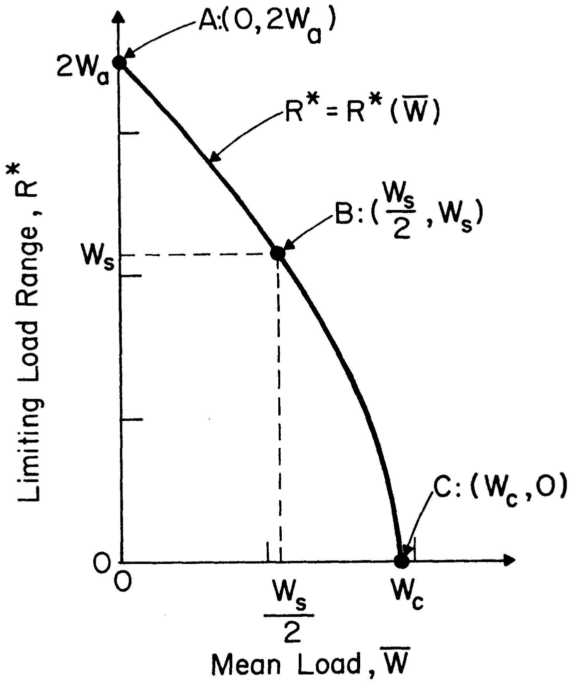

Fig. 1. Load Range Versus Mean Load.

decreasing function of $\overline{W}$. From the previous discussion of the alternating plasticity load W_a, the shakedown load W_s and the plastic collapse load W_c, it is clear the points labelled A, B and C lying on the graph $R^* = R^*(\overline{W})$ in Fig. 1 must have, respectively, the coordinates $(0, 2\,W_a)$, $(W_s/2, W_s)$ and $(W_c, 0)$. It is certainly true for all structures of the type discussed herein, that

$$W_c > W_s > W_a \tag{7}$$

and that,

$$2\,W_a > W_s > 0 \tag{8}$$

and, furthermore, the points $(0, 2\,W_a)$, $(W_s/2, W_s)$ and $(W_c, 0)$ must lie on the graph $R^* = f(\overline{W})$. Hence, if $\overline{W}$ is in the range, $0 \leqq \overline{W} \leqq W_c$, then R^* must be a continuous, monotone decreasing function of $\overline{W}$. This result is, of course, obvious on purely intuitive grounds as well. This is so because as $\overline{W}$ increases, the ability of the structure to tolerate excursions of cyclically-varying load above and below $\overline{W}$ must, of necessity, eventually decrease to zero; otherwise, plastic collapse in a single cycle to the maximum load W_c could not take place.

The existence of the function or family of functions $R^* = R^*(\overline{W})$ for a given structure together with Eqs. (6), suggests that a graph or family of graphs of the kind shown on Fig. 2 may be constructed. The upper branch of the graph shown in Fig. 2 is, from Eq. (6), given by

$$W^*_{max} = \overline{W} + \frac{R^*}{2}. \tag{9}$$

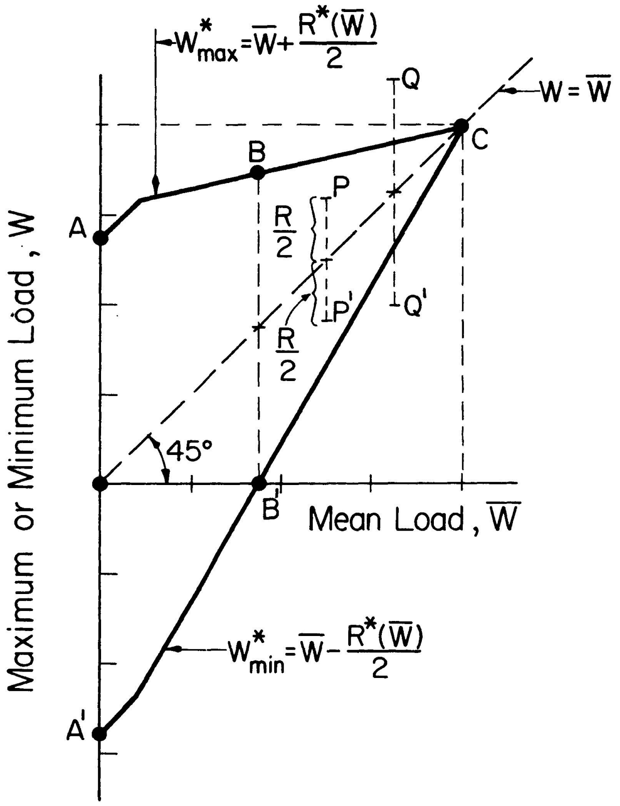

Fig. 2. Maximum or Minimum Load Versus Mean Load.

But it has been shown that R^* is a function of $\overline{W}$. Hence, Eq. (9) may be written as

$$\overline{W}_{max} = \overline{W} + \tfrac{1}{2} R^*(\overline{W}). \tag{10}$$

Similarly, the lower branch of the graph shown in Fig. 2 is given by

$$W^*_{min} = \overline{W} - \tfrac{1}{2} R^*(\overline{W}). \tag{11}$$

Thus W^*_{max} and W^*_{min} are likewise functions of the mean load $\overline{W}$.

From the previous discussion regarding the alternating plasticity load W_a, the shakedown load W_s and the plastic collapse load W_c, it is clear that the points A, A', B, B' and C of Fig. 2 have, respectively, the coordinates $(0, W_a)$, $(0, -W_a)$, $(W_s/2, W_s)$, $(W_s/2, 0)$, and (W_c, W_c). Furthermore, in view of Principle 1 and the definition of $R^*(\overline{W})$, any pattern of loads such as that characterized by points P and P' lying wholly within the W^*_{max} and W^*_{min} "envelope" as shown in Fig. 2 represents a "safe" (i. e. non-collapse) loading pattern. On the other hand, any pattern of loads such as that characterized by points Q and Q' lying outside the W^*_{max} and W^*_{min} "envelope" as shown in Fig. 2 represents a state of loading that will, after a finite number of cycles has elapsed, result in failure by incremental collapse. This latter condition, therefore, represents an extended definition of the concept of incremental collapse embracing as it does all loading conditions which result in excursions beyond the $W^*_{max} - W^*_{min}$ "envelope" (as defined herein). This new treatment of incremental collapse obviates the necessity for drawing any real (or intrinsic) distinctions among the phenomena of "alternating plasticity", "shakedown" and "plastic collapse"; because in view of the foregoing, all of these phenomena are merely noteworthy points lying on a continuum represented by the $W^*_{max} - W^*_{min}$ envelope. The lack of intrinsic distinction between the alternating plasticity load and the shakedown load has also been remarked by FRANCIOSI, AUGUSTI and SPARACIO [18]. An explicit treatment of the entire $W^*_{max} - W^*_{min}$ envelope, however, is not, to the writer's knowledge, to be found apart from the treatment herein.

Principle 1, because of its basic simplicity and applicability to a wide class of structures, may also be generalized to include structures made of materials more complex than the simple elasto-plastic material. An explicit inclusion in the Principle of structures composed of more complex materials (e. g. the elasto-plastic-strain-hardening material) must await the results of further research. Despite this restriction, Principle 1 may be used as the basis for practical design computations of extended incremental collapse loads for the usual type of steel structures. Furthermore, it is implicit in this principal that a particularly simple and useful set of computations may be made (particularly, if a digital computer is available) by a consideration of load versus deflection response. For this reason, no matter how complex the cyclically varying load pattern may be, it is an inherently simple matter to predict whether or not extended incremental collapse can occur. All that is necessary, is to construct a computer program that will generate as output the "hysteresis" or irrecoverable energy ΔU imparted to the structure during each cycle of load. When such an output of a sequence of ΔU's is produced, it may be examined in the following manner.

If ΔU_n is the irrecoverable energy or hysteresis imparted to the structure during the nth cycle, then the series,

$$U = \Delta U_1 + \Delta U_2 + \cdots + \Delta U_n + \cdots, \qquad (12)$$

may be formed out of the sequence of ΔU's. If this series converges in the practical sense that

$$\lim_{n \to \infty} \Delta U_n = 0, \qquad (13)$$

then failure by extended incremental collapse will not occur. On the other hand, if this series diverges, or, in other words, if

$$\lim_{n\to\infty} \frac{\Delta U_n}{\Delta U_{n-1}} = 1, \tag{14}$$

then the structure will fail by extended incremental collapse. In practice, of course, only a relatively small number of cycles of loading ($n \leqq 40$) is needed to establish the validity of statements (13) or (14) in any particular instance as will be demonstrated in the example given in the next section.

It is interesting to note that the graph of R^* vs $\overline{W}$ given in Fig. 1 is analogous to the well-known observation concerning fatigue fracture that the range of stress σ_R necessary to produce fracture decreases as the mean stress σ_m increases. This observation was first made by WOHLER [19] on the basis of comprehensive and systematic fatigue experiments. Furthermore, it is noteworthy that the diagram of Fig. 2 in also analogous to the well-known GOODMAN or GOODMAN-GERBER [20] construction used in predicting fatigue strength of metals based on empirical observations relating fatigue strength, stress range (or amplitude) and mean stress. A fuller exploration of this remarkable analogue will be given in a subsequent paper.

Example

The simple portal frame, shown in Fig. 3, which has been extensively treated analytically by NEAL [12] and experimentally by NEAL and SYMONDS [21] will be analyzed by an approach involving only load-deflection response and hysteresis in order to illustrate the application of Principal 1. If the flexural members of the structure of Fig. 3 are of the same cross-section and made of an elasto-plastic material whose stress-strain behaviour is similar to that diagrammed in Fig. 5a and they display the bending moment versus curvature response (linear-perfectly plastic) shown in Fig. 5b and if the structure is subjected to repeated cycles of loading, the first sequence of which is shown in Fig. 4 then for $\beta = 1$, NEAL [12] has found by conventional methods that the alternating plasticity load is,

$$W_a = 2.759 \frac{M_p}{L}, \tag{15}$$

the shakedown load is,

$$W_s = 2.857 \frac{M_p}{L}, \tag{16}$$

and, the plastic collapse load is,

$$W_c = 3 \frac{M_p}{L}. \tag{17}$$

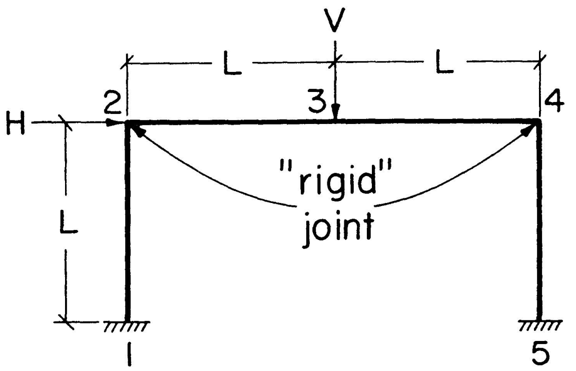

Fig. 3. Three-Bar Hyperstatic Portal Frame.

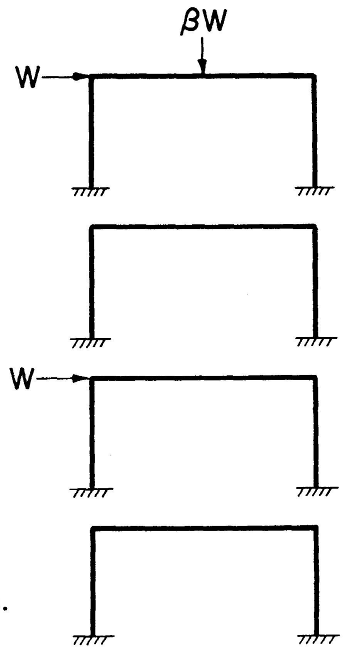

Fig. 4. One Cycle of Load Applications.

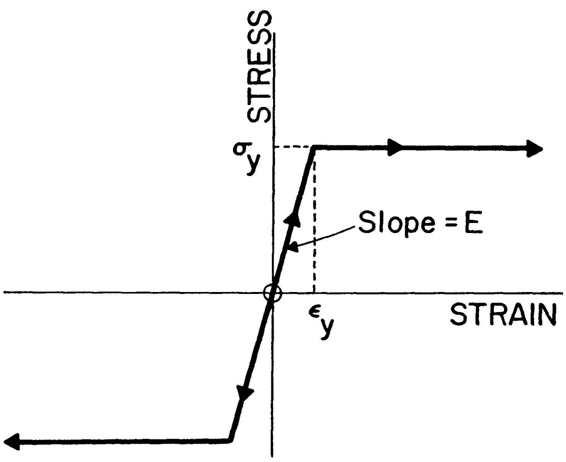

Fig. 5a. Stress Versus Strain.

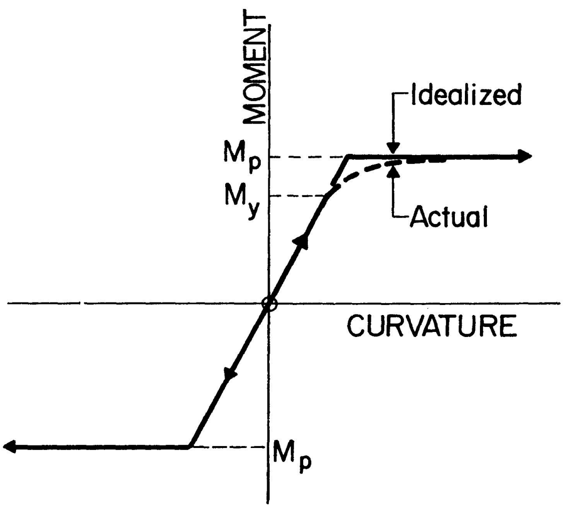

Fig. 5b. Bending Moment Versus Curvature.

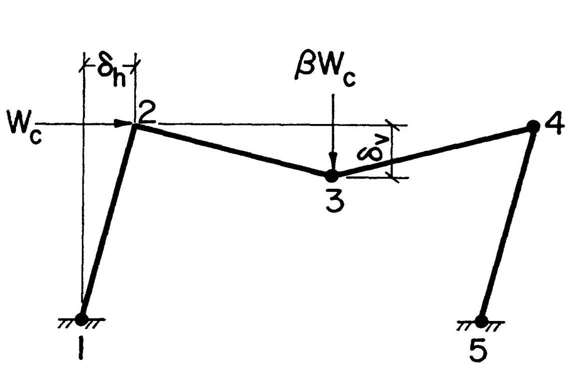

Fig. 6. Collapse Mechanism.

The mechanism of collapse corresponding to the load W_c is shown in Fig. 6. Additional values of W_a, W_s and W_c are given in Table 1 for $\beta = 0.5$, 1, 1.5 and 2.

Table 1

	Load	β			
		0.5	1.0	1.5	2.0
Alternating Plasticity	$\frac{W_a L}{M_p}$	2.750	2.425	2.049	1.665
Shakedown	$\frac{W_s L}{M_p}$	3.478	2.857	2.264	1.875
Plastic Collapse	$\frac{W_c L}{M_p}$	4.00	3.00	2.40	2.00

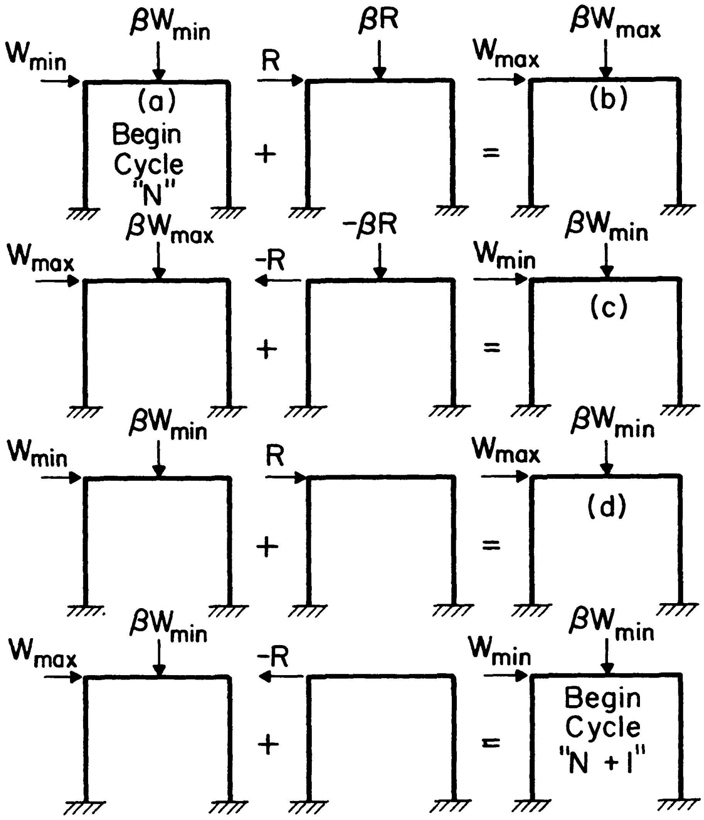

Fig. 7. The *N*th Cycle of Load Applications.

If β is allowed to take on values in the range $0.5 \leqq \beta \leqq 2.0$ and a more generalized cyclic load pattern, the first cycle of which is shown in Fig. 7, is applied to the structure of Fig. 3, then the results of a relatively simple digital computer program which determines energy loss at the end of each cycle of load for every predetermined load level may be plotted as a family of R^* vs $\overline{W}$ curves as in Fig. 8 or as a family of $W_{max}^* - W_{min}^*$ envelopes as shown in Fig. 9. In each of these two charts, the curves corresponding to $\beta = 1$ have been emphasized for clarity. From these $\beta = 1$ curves, it may be observed that

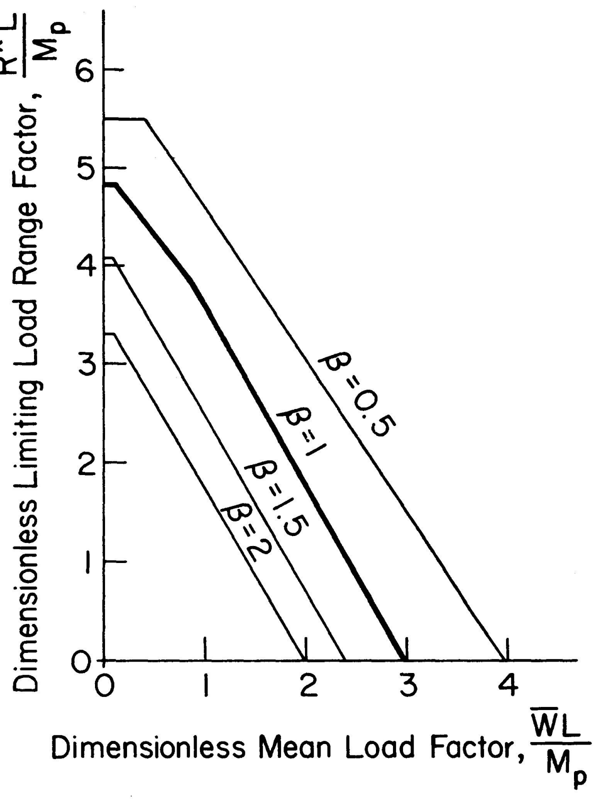

Fig. 8. Load Range Factor Versus Mean Load Factor Diagrams.

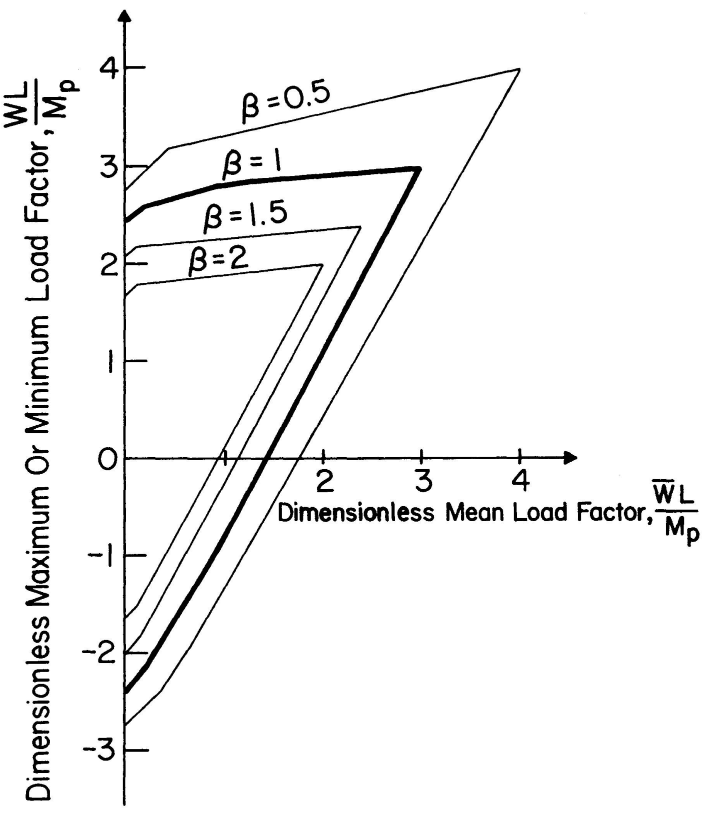

Fig. 9. Maximum or Minimum Load Factor Versus Mean Load Factor Diagrams.

values of W_a, W_s and W_c are in perfect agreement with the respective values given by Eqs. (15), (16) and (17) even though these latter three values were obtained by means of a completely different (i.e. by conventional methods based on theorem 1) type of computational process.

If the initial direction of loading shown in Fig. 7 is reversed and the pattern of loading becomes that shown in Fig. 10 and if this pattern of loading is applied to the portal frame of Fig. 3, then diagrams geometrically similar to those of Fig. 9 may be constructed (except that signs will be reversed). If such diagrams are combined with those of Fig. 9, then the "complete" load factor versus mean load factor diagrams of Fig. 11 will result. These diagrams, of course, are completely analogous to the conventional Goodman-Gerber diagrams mentioned previously.

If the minimum load shown in Fig. 7 is zero, then the load pattern degenerates to that shown in Fig. 4. If the maximum load is allowed to take on various values above and below the shakedown load W_s and if the energy loss per cycle is computed, then for the structure of Fig. 3 and $\beta = 1$ the results are those shown on Fig. 12. From the diagrams of energy loss versus number of cycles given in Fig. 12, it is clear that the shakedown load $W_s = 2.857\, M_p/L$ is the greatest load for which Eq. (13) still holds and that for all loads greater than W_s the condition stated by Eq. (14) pertains and failure by incremental collapse must eventually occur.

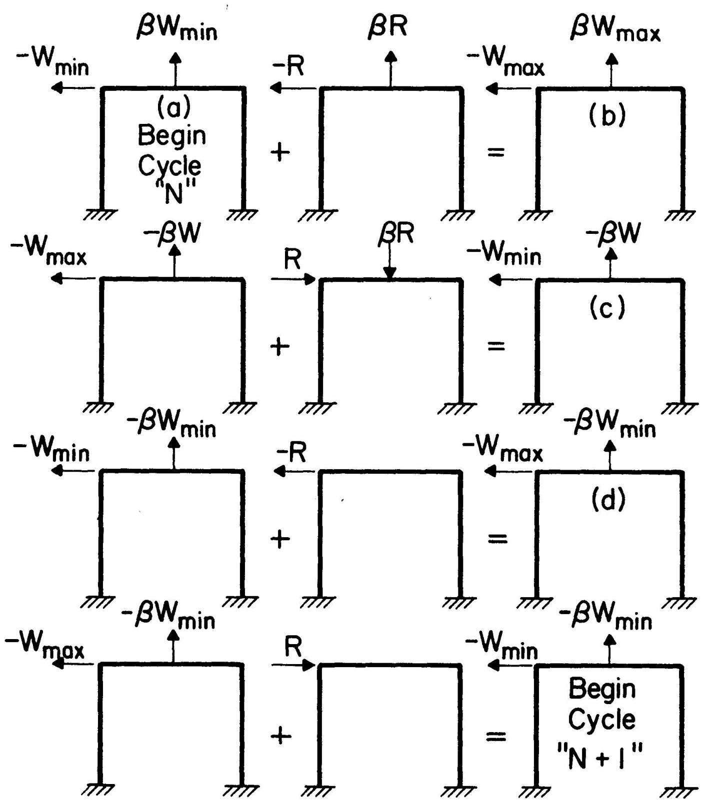

Fig. 10. The Nth Cycle of Reversed Load Applications.

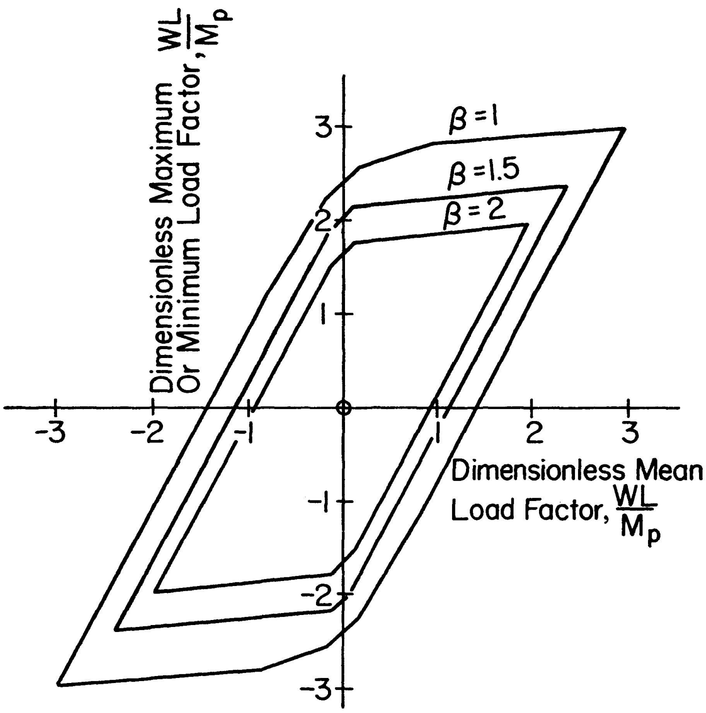

Fig. 11. Complete Load Factor Versus Mean Load Factor Diagrams.

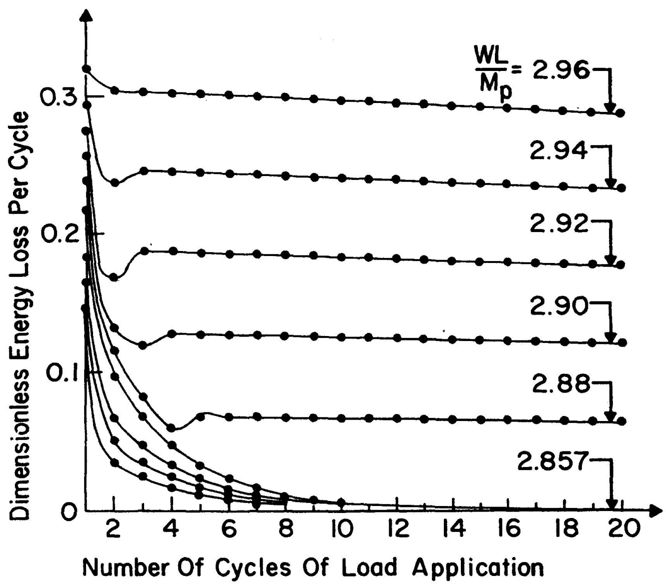

Fig. 12. Energy Loss per Cycle Versus Number of Cycles of Load Application.

Conclusions

It has been demonstrated that alternating plasticity, shakedown and collapse are interlinked phenomena which are part of a continuum herein defined as the $W^*_{max} - W^*_{min}$ "envelope" or "extended incremental collapse envelope". Furthermore, it has been shown that this envelope may be constructed by essentially elementary techniques in which the irrecoverable energy imparted (i.e. "hysteresis") to the structure is computed at the end of each cycle of load application. This technique is suggestive of the possibility of simplifying the conventional earthquake analysis of structures as well as conventional "plastic analysis" of structures subjected to complex load patterns.

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Notation

L	Length of member.
M_p	Bending moment at which a plastic hinge will form in member.
M_e^i	Elastic bending moment in a particular frame member at location i.
m_i	Residual bending moment in a particular frame member at location i after one or more cycles of load of intensity $W > 2.424\, M_p/L$ have been applied to the frame.
W_a	The alternating plasticity load.
W_c	The plastic collapse load.
W_s	The incremental collapse load or "shakedown" load.
W_{max}	Maximum load intensity applied to the structure.
W_{min}	Minimum load intensity applied to the structure.
$\overline{W}$	Mean load intensity applied to the structure.
R	Range of loads applied to the structure (equal to absolute value of difference between $\lvert W_{max}\rvert$ and $\lvert W_{min}\rvert$).
E	Modulus of Elasticity or Young's Modulus.
I	Second static moment of cross-sectional area of flexural member.
δ_h	Horizontal displacement of end 2 of member 1—2 relative to end 1 during the application of a horizontal load to the frame at location 2.
δ_v	Vertical displacement of member 2–4 at the point 3 relative to its ends during the application of a vertical load to frame at location 3.
ΔU_i	Irrecoverable energy imparted to the structure during the ith cycle of load application and removal.
β	Ratio of vertical load to horizontal load applied to the structure.
σ	Stress.
ϵ	Strain.

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Summary

It is well-known that a hyperstatic structure may collapse when subjected to a relatively small number of cycles of repeated loads even though none of the applied loadings is sufficiently severe to cause failure by plastic collapse in the first cycle. This type of failure is known as "incremental collapse". It is shown that alternating plasticity, shakedown and plastic collapse are interlinked phenomena which may appropriately be included in an extended definition of incremental collapse. Furthermore that the "extended incremental collapse envelope" may be constructed by essentially elementary techniques in which the irrecoverable energy imparted to the structure (i. e. the "hysteresis") is computed at the end of each cycle of load application.

Résumé

Il est connu qu'une structure statiquement indéterminée peut s'écrouler lorsqu'elle est soumise à un nombre restreint de cycles de charges, même si aucune des charges appliquées ne soit assez grande à provoquer une panne plastique au premier cycle. Ce processus est défini comme «défaillance incrémentale». Dans le présent article on démontre que la plasticité alternante, le «shake down» et la panne plastique sont des phénomènes liés entre eux et compris dans une définition élargie de l'écroulement incrémental. En plus, la «ligne limite élargie de défaillance incrémentale» peut être construite moyennant un procédé relativement élémentaire où l'énergie non récupérable de la structure (c'est-à-dire la «hystérèse») peut être calculée à la fin de chaque cycle de la charge appliquée.

Zusammenfassung

Bekanntlich kann ein statisch unbestimmtes Bauwerk bei einer verhältnismässig kleinen Anzahl von Zyklen wiederholter Belastungen zusammenbrechen, selbst dann wenn keine der wirkenden Lasten genügend gross ist, um ein plastisches Versagen im ersten Zyklus herbeizuführen. Dieser Vorgang wird als «inkrementales Versagen» bezeichnet. Hier wird gezeigt, dass wechselnde Plastizität, «shake down» und plastisches Versagen miteinander verkettete Erscheinungen sind, die dementsprechend in einer erweiterten Definition des inkrementalen Versagens enthalten sind. Ferner wird gezeigt, dass sich die «erweiterte inkrementale Versagens-Grenzwertlinie» durch ziemlich elementare Verfahren konstruieren lässt, bei denen die nicht wiedergewinnbare Energie (d.h. die «Hysteresis») am Ende jedes Zyklus der Lasteinwirkung berechnet wird.