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Nonlinear Buckling of Lattice Domes

Stabilité non-linéaire de coupoles à treillis

Nichtlineare Stabilität von Gitterkuppeln

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1. Introduction

In the design of modern large span geodesic domes, the determination of the buckling load is a problem of primary importance, normally the decisive factor in the whole design. In a previous publication [2], the problem of edge disturbances in lattice domes with triangular meshes has been studied. It is the aim of the present paper, to suplement the previous stress problem by means of a buckling theory, simple enough to be used in effective design. For the theory to give realistic results, it must be based on a nonlinear post-buckling approach, in the spirit of KÁRMÁN and TSIEN's pioneering work [1].

The structural behaviour of the shell lattice will be dealt with by means of a continuous analogue model, which will conveniently replace the discrete lattice members.

Both simple and double-layer lattice domes can be analysed by means of the intended theory. It should be particularly emphasized that, as it was already remarked for the stress problem [2], it may be dangerous to use simple analogies, obtained from the theory of isotropic shells. The bending and the membrane stiffness may differ considerably in the lattice model, whereas they bear a definite relationship to each other in the case of uniform shells.

We shall first review briefly some basic results for the analogue model. Next, the relevant equilibrium and kinematical equations including nonlinear terms will be stated. Appropriate expressions for the compatibility condition and the potential energy will also be derived.

Finally, an approximate solution for the nonlinear buckling problem of lattice domes will be proposed. The method of solution is similar to the one used by WOLMIR [6], for the non linear buckling problem in uniform shells.

The theory will be applied to a concrete example and the results will be compared with other formulas in the literature.

2. Analogue Model

Let us imagine a sphere (Fig. 1) made of a triangular lattice of stiff members. The lattice voids are closed by a continuous skin, so that the sphere can

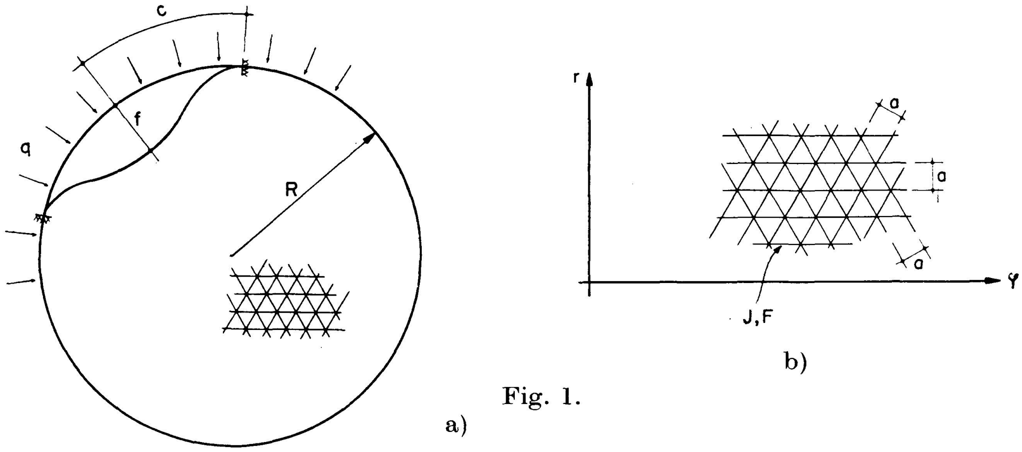

Fig. 1.

support an external pressure q. It is the aim of the present analysis, to determine the value of the external pressure, for which a portion of the sphere of radius c will "snap through" to a new buckled position, determined by the deflection f of its mid-point. We assume that the buckled zone behaves as a shallow shell. The lattice details are reproduced in Fig. 1b, the meshes being equilateral triangles of height a. The properties of lattice members are defined through the cross-sectional area F and the moment of inertia J. Members may be simple bars or trusses. The above lattice will be referred to a polar system of coordinates r, φ, which will be used in the subsequent analysis.

The continuum properties of the lattice model are obtained by subjecting the lattice to generalized unit deformations, as unit elongations and shears and unit changes of curvature and twist.

The contributions of different bars to the shell stress resultants and stiffness will be refered to the unit length of the shell middle surface.

We refer to [3] and [4] for detailed demonstrations. By neglecting the coupling between in-plane and bending contributions (see Fig. 2), the interesting constitutive equations can be written as

$$M_r = d_r^r k_r + d_r^\varphi k_\varphi, \qquad M_\varphi = d_r^\varphi k_r + d_\varphi^\varphi k_\varphi, \tag{1}$$

$$\epsilon_r = \Delta_r^r N_r + \Delta_r^\varphi N_\varphi, \qquad \epsilon_\varphi = \Delta_r^\varphi N_r + \Delta_\varphi^\varphi N_\varphi. \tag{2}$$

In the above relationships, k_r and k_φ are changes of curvatures and ϵ_r and ϵ_φ, membrane strains of the shell middle surface. It has been shown in [3] and [4] that the coefficients d_r^r, $d_r^\varphi \cdots \Delta_r^\varphi$, Δ_φ^φ in (1) and (2) are given by

$$\begin{aligned} d_r^r &= d_\varphi^\varphi = \frac{3\,E\,J}{8\,a}(3+\mu), & d_r^\varphi &= \frac{3\,E\,J}{8\,a}(1-\mu), \\ \Delta_r^r &= \Delta_\varphi^\varphi = \frac{a}{E\,F}, & \Delta_r^\varphi &= -\frac{a}{E\,F}, \end{aligned} \tag{3}$$

in which

$$\mu = \frac{G\,J_d}{E\,J} \tag{4}$$

and $G\,J_d$ and $E\,J$ are respectively Saint-Venant's torsional stiffness and the bending stiffness of a lattice member.

3. Equilibrium and Kinematical Relations for Rotationally Symmetric Bending of Shallow Spherical Shells with Large Deflections

With the notations of Fig. 2, the equilibrium equations of the symmetrically loaded spherical shell, by accounting for the influence of deflections on the

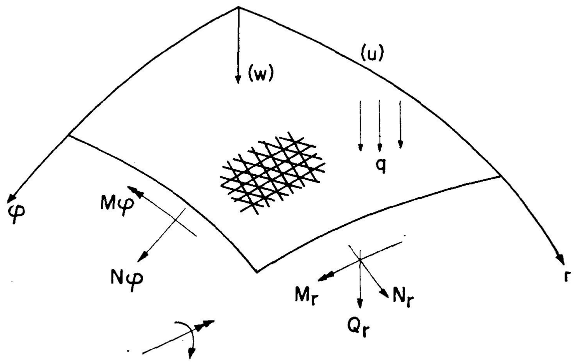

Fig. 2.

geometry are given by

$$\begin{aligned} &\frac{d}{dr}(r\,N_r) - N_\varphi = 0, \\ &\frac{d}{dr}(r\,Q_r) + r\left(k + \frac{d^2 w}{dr^2}\right)N_r + r\left(k + \frac{1}{r}\frac{dw}{dr}\right)N_\varphi + q\,r = 0, \\ &\frac{dM_r}{dr} + \frac{M_r - M_\varphi}{r} = Q_r, \end{aligned} \tag{5}$$

where

$$k = \frac{1}{R}. \tag{6}$$

By eliminating Q_r from the second of (5) by means of the third,

$$\frac{d^2 M_r}{dr^2} + \frac{2}{r}\frac{dM_r}{dr} - \frac{1}{r}\frac{dM_\varphi}{dr} + \left(k + \frac{d^2 w}{dr^2}\right)N_r + \left(k + \frac{1}{r}\frac{dw}{dr}\right)N_\varphi + q = 0. \tag{7}$$

The first equation of (5) will be satisfied identically by assuming

$$N_r = \frac{1}{r}\frac{d\Phi}{dr}, \qquad N_\varphi = \frac{d^2\Phi}{dr^2}, \tag{8}$$

in which Φ is a stress function. On the other hand, the well known formulas for rotationally symmetric in-plane strains are

$$\begin{aligned} \epsilon_r &= \frac{du}{dr} - k\,w + \frac{1}{2}\left(\frac{dw}{dr}\right)^2, \\ \epsilon_\varphi &= \frac{u}{r} - k\,w, \end{aligned} \tag{9}$$

from which we can eliminate the tangential displacement u to obtain

$$\frac{d\,(r\,\epsilon_\varphi)}{dr} - \epsilon_r + k\,r\frac{dw}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 = 0\,. \tag{10}$$

By accounting for (3) and (8), the constitutive Eqs. (2) for the extensional strains are rewritten as

$$\begin{aligned} \epsilon_r &= \frac{a}{E\,F}\left(\frac{1}{r}\frac{d\Phi}{dr} - \frac{1}{3}\frac{d^2\Phi}{dr^2}\right), \\ \epsilon_\varphi &= \frac{a}{E\,F}\left(\frac{d^2\Phi}{dr^2} - \frac{1}{3}\frac{d\Phi}{dr}\right). \end{aligned} \tag{11}$$

If we substitute (11) in (10), the new form of the compatibility relation will be

$$\frac{d}{dr}\left(r\frac{d^2\Phi}{dr^2}\right) - \frac{1}{r}\frac{d\Phi}{dr} + \frac{E\,F}{a}\left[k\,r\frac{dw}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2\right] = 0\,,$$

which can be rewritten as

$$\frac{d}{dr}(\nabla^2\Phi) = -\frac{E\,F}{a}\left[\frac{1}{2\,r}\left(\frac{dw}{dr}\right)^2 + k\frac{dw}{dr}\right] \tag{12}$$

by introducing the Laplacian operator

$$\nabla^2(\cdots) = \frac{d^2(\cdots)}{dr^2} + \frac{1}{r}\frac{d(\cdots)}{dr}. \tag{13}$$

By means of (1) and the well known formulas for the changes of curvature

$$k_r = -\frac{d^2 w}{dr^2}, \qquad k_\varphi = -\frac{1}{r}\frac{dw}{dr} \tag{14}$$

the equation of equilibrium (7) can be also rewritten as

$$d_r^r\,\nabla^2\nabla^2 w - \left(k + \frac{d^2 w}{dr^2}\right)\frac{d^2\Phi}{dr^2} - \left(k + \frac{1}{r}\frac{dw}{dr}\right)\frac{1}{r}\frac{d\Phi}{dr} = q\,. \tag{15}$$

Eqs. (12) and (15) are no-linear and direct methods of solution have slim chances of success. We supplement the above derivations by including an expression for the total potential energy of the buckled area in the shell, which will be helpful in obtaining approximate solutions.

4. Potential Energy

If the shell is deformed, it will store potential energy, which can be recovered upon unloading. The potential energy is made up partly of the strain energy and partly of the potential energy of the external loading.

The strain energy arises from two components, membrane effect and bending, which are given respectively by

$$U_m = \tfrac{1}{2}\iint (N_r \epsilon_r + N_\varphi \epsilon_\varphi)\, dS = \tfrac{1}{2}\iint (\Delta_r^r N_r^2 + 2\Delta_r^\varphi N_r N_\varphi + \Delta_\varphi^\varphi N_\varphi^2)\, dS \qquad (16)$$

and $$U_b = \tfrac{1}{2}\iint (M_r k_r + M_\varphi k_\varphi)\, dS. \qquad (17)$$

By substituting above (3), (8), (1) and (14) these formulas change into

$$\left.\begin{aligned} U_m &= \frac{a}{2EF}\iint \left[(\nabla^2\Phi)^2 - \frac{8}{3}\frac{1}{r}\frac{d\Phi}{dr}\frac{d^2\Phi}{dr^2}\right] dS, \\ U_b &= \frac{3EJ}{16a}(3+\mu)\iint \left[(\nabla^2 w)^2 - 4\frac{(1+\mu)}{(3+\mu)}\frac{1}{r}\frac{dw}{dr}\frac{d^2w}{dr^2}\right] dS. \end{aligned}\right\} \qquad (18)$$

We next evaluate the potential energy of the external loading. The displacement pattern of the shell is sketched in Fig. 3.

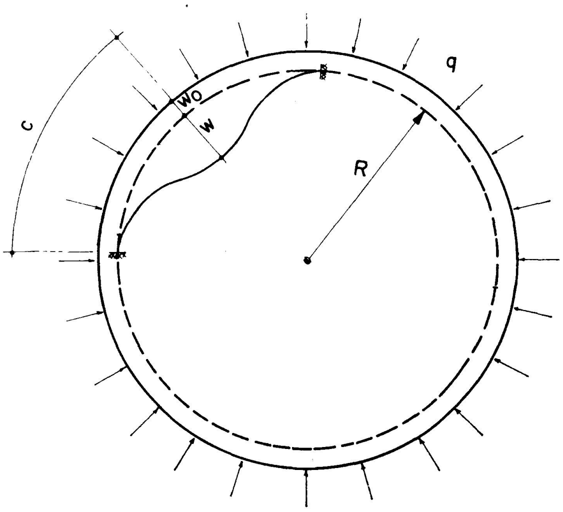

Fig. 3.

In the pre-buckling stage, the lattice sphere will be compressed by an amount w_0 and, in the buckling stage, an area of radius c will experience an additional deflection. Since, before buckling we have a uniform compression $N_r = N_\varphi = -\frac{qR}{2}$, the pre-buckling deflection is easily seen to be $w_0 = \epsilon_r R = \frac{1}{3}\frac{qa}{kEF}$, by using (2). We include the effect of this deflection in the potential energy W of the external loading. The strain energy in the pre-buckling stage will be accounted for later.

Then
$$W = -\iint q\,(w + w_0)\,dS \tag{19}$$

and the total potential energy of the buckled shallow shell will be
$$\Pi = U_m + U_b + W\,. \tag{20}$$

5. Boundary Conditions

We state the boundary conditions which are usually assumed for the buckled shallow shell (Fig. 4):

In $r = c$,
$$w = 0\,, \qquad \frac{dw}{dr}\,, \tag{21}$$

which is a perfect restraint. Two other boundary conditions must be added for Φ. By assuming that the tangential displacement u vanishes in $r=c$, we

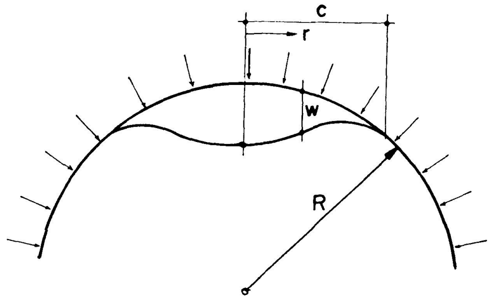

Fig. 4.

conclude from the second of (9) that $r\epsilon_\varphi + k\,r\,w = 0$, for $r=c$. On account of (21), this is the same as $\epsilon_\varphi = 0$ for $r=c$, so that
$$\frac{d^2\Phi}{dr^2} - \frac{1}{3}\,\frac{d\Phi}{dr} = 0 \tag{22}$$

by considering (11). The other condition on Φ is obtained by demanding that N_r be finite for $r=0$, i.e.,
$$\frac{d\Phi}{dr} = 0 \tag{23}$$

[see (8)].

In reality, we have an elastic restraint in $r=c$, which tends to decrease the buckling load based on the assumed conditions.

6. An Approximate Solution of the Buckling Problem

As the integration of the non-linear differential Eqs. (12) and (15) is difficult, we shall develop an approximate energy solution.

We assume for the normal deflection the expression
$$w = f\left(1 - \frac{r^2}{c^2}\right)^2, \tag{24}$$

in which f is the deflection at the mid-point and c the radius. The parameters f and c will be determined later, in such a way that the potential energy be a minimum.

We can make sure that (24) satisfies the boundary conditions (21) Next we insert (24) in the compatibility Eq. (12) and integrate for Φ and then substitute both w and Φ in the expression (20) for the potential energy. The constants c and f which define the shape and the size of the buckled region will be determined from the condition that the potential energy be a minimum.

In a second stage, we could improve the expression for w, by putting Φ in (15), and integrating for w, but the present approximation is considered satisfactory.

By putting now (24) in (12),

$$\frac{d}{dr}(\nabla^2\Phi) = -\frac{8\,E\,F}{a}\,\frac{f}{c^2}\left(\frac{r}{c^2}-2\frac{r^3}{c^4}+\frac{r^5}{c^6}\right)+\frac{E\,F}{a}\,\frac{4\,f\,k}{c}\left(\frac{r}{c}-\frac{r^3}{c^3}\right),$$

from which, after two integrations and by accounting for the identify

$$\nabla^2\Phi = \frac{1}{r}\,\frac{d}{dr}\left(r\,\frac{d\Phi}{dr}\right)$$

we find

$$\frac{d\Phi}{dr} = -\frac{8\,E\,F}{a}\,\frac{f^2}{c^2}\left(\frac{r^3}{8\,c^2}-\frac{r^5}{12\,c^4}+\frac{r^7}{48\,c^6}\right)+\frac{4\,E\,F}{a}\,\frac{f\,k}{c}\left(\frac{r^3}{8\,c}-\frac{r^5}{24\,c^3}\right)+C_1\,r+\frac{C_2}{r}. \quad (25)$$

The constants of integration C_1 and C_2 are determined from the boundary conditions (22) and (23):

$$C_1 = \frac{E\,F}{a}\,f\left(2\frac{f}{c^2}-\frac{5}{3}k\right), \qquad C_2 = 0. \quad (26)$$

The expression (25) must be completed, by adding the influence of the membrane stresses $N_r = N_\varphi = -\frac{q}{2\,k}$ in the pre-buckling stage. This will automatically account for the strain energy in the pre-buckling stage.

From the first of (8), we see that the above effect on $\frac{d\Phi}{dr}$ is given by $-\frac{q}{2\,k}\,r$.

Thus, the complete expression of (25) with (26) will be

$$\frac{d\Phi}{dr} = \frac{E\,F}{6\,a}\,\frac{f^2}{c}\left(6\frac{r}{c}-6\frac{r^3}{c^3}+4\frac{r^5}{c^5}-\frac{r^7}{c^7}\right)-\frac{E\,F}{6\,a}\,f\,c\,k\left(5\frac{r}{c}-3\frac{r^3}{c^3}+\frac{r^5}{c^5}\right)-\frac{q\,r}{2\,k}. \quad (27)$$

We also record the expressions for the membrane stresses:

$$\frac{1}{r}\,\frac{d\Phi}{dr} = N_r = \frac{E\,F}{6\,a}\,\frac{f^2}{c^2}\left(6-6\frac{r^2}{c^2}+4\frac{r^4}{c^4}-\frac{r^6}{c^6}\right)-\frac{E\,F}{6\,a}\,f\,k\left(5-3\frac{r^2}{c^2}+\frac{r^4}{c^4}\right)-\frac{q}{2\,k}, \quad (28)$$

$$\frac{d^2\Phi}{dr^2} = N_\varphi = \frac{E\,F}{6\,a}\,\frac{f^2}{c^2}\left(6-18\frac{r^2}{c^2}+20\frac{r^4}{c^4}-7\frac{r^6}{c^6}\right)-\frac{E\,F}{6\,a}\,f\,k\left(5-9\frac{r^2}{c^2}+5\frac{r^4}{c^4}\right)-\frac{q}{2\,k}.$$

We see that $\nabla^2\Phi = N_r + N_\varphi$ and now we are in a position to evaluate the potential energy.

We first introduce (27) and (28) in the first of (18) and, after some tedious but simple algebraic manipulations, we find

$$\begin{aligned} U_m = &\frac{5\pi}{21}\frac{EF}{a}\frac{f^4}{c^2} - \frac{4\pi}{9}\frac{EF}{a}kf^3 - \frac{\pi}{3}q\frac{f^2}{k} \\ &+ \frac{19\pi}{60}\frac{EF}{a}c^2k^2f^2 + \frac{\pi}{3}qc^2f + \frac{\pi}{6}\frac{a}{EF}\frac{q^2}{k^2}c^2. \end{aligned} \tag{29}$$

The strain energy of bending is found by inserting (24) in the second of (18):

$$U_b = \frac{4EJ}{a}(3+\mu)\frac{f^2}{c^2}. \tag{30}$$

The potential energy W of the external pressure is, with (19), (24) and $w_0 = \frac{1}{3}\frac{qa}{kEF}$.

$$W = -\frac{\pi}{3}qfc^2 - \frac{\pi}{3}\frac{q^2ac^2}{k^2EF}. \tag{31}$$

If, for greater facility in manipulations we introduce the dimensionless quantities

$$\chi = \frac{kc^2a}{F}, \qquad \sigma = \frac{qa^2}{2k^2EF^2}, \qquad \tau = \frac{af}{F}, \qquad \lambda = \frac{Ja^2}{F^3}. \tag{32}$$

The expression (20) for the total potential energy Π can be written as

$$\Pi = \frac{\pi}{3}\frac{kEF^4}{a^4}\left(\frac{5}{7}\frac{\tau^4}{\chi} - \frac{4}{3}\tau^3 - 2\tau^2\sigma + \frac{19}{30}\chi\tau^2 - 2\sigma^2\chi + 12(3+\mu)\frac{\lambda\tau^2}{\chi}\right). \tag{33}$$

The conditions for a minimum of Π are obviously

$$\frac{\partial\Pi}{\partial\tau} = 0, \qquad \frac{\partial\Pi}{\partial\chi} = 0$$

with a result that

$$\begin{aligned} \sigma &= \frac{5}{7}\frac{\tau^2}{\chi} - \tau + \frac{19}{60}\chi + 6(3+\mu)\frac{\lambda}{\chi}, \\ \sigma &= \tau\left[\frac{19}{60} - \frac{5}{14}\frac{\tau^2}{\chi^2} - 6(3+\mu)\frac{\lambda}{\chi^2}\right]^{1/2}. \end{aligned} \tag{34}$$

The above conditions must be satisfied simultaneously. The parameters τ, χ and λ depend on the geometry of the buckled region of the lattice. The parameter σ defines the buckling pressure through (32).

Eqs. (34) must be solved by trial and error. The parameter λ depends entirely on the mesh size a of the lattice and the cross sectional properties F and J of the lattice members. Hence it is given for a shell under consideration. We next choose values for τ and χ and obtain σ from both formulas (34). If σ

happens to have the same value from both formulas, τ and χ determine a possible buckled shape of the shell. The parameter σ will define the corresponding buckling pressure.

In reality, there are many pairs of values for τ and χ which will give the same values for σ, i.e., there are many possible buckling shapes corresponding to different buckling pressures.

The interesting solution will be the one which yields the smallest value for the buckling pressure.

Once the minimum value for σ is known, the buckling pressure q_{cr} and the associated shape of the buckled region are found from (32):

$$q_{cr} = \sigma_{min} \frac{2\,k^2\,E\,F^2}{a^2}, \qquad c = \sqrt{\frac{\chi\,F}{k\,a}}, \qquad f = \frac{\tau\,F}{a}. \tag{35}$$

The process of determining σ_{min} is well fitted for a computer program, involving the simultaneous Eqs. (34).

Graphs of solution of (34), from which we may obtain immediately the relavant buckling parameters are not feasible, since the solution depends heavily on the lattice properties, through λ and μ. These constants change in wide ranges for different lattices and a higher degree of precision is required in the calculations.

On these grounds, an elementary FORTRAN IV program was written for (34), in which associated values of σ were printed in table form. An inspection of the table would supply the wanted solution.

7. Stress Resultants in the Post-Buckling Stage

In order to assess the value of forces in the lattice members in the post-buckling stage, we shall derive formulas for the stress resultants for the buckled shallow shell of radius c.

Formulas for the shearing forces Q_r are particularly important for double-layer lattice domes, because they are determinant for estimating the cross-sections of the diagonal bars in the truss lattice members.

A formula for Q_r is found from the third equation of equilibrium (5) along with (1), (3) and (14). The result is

$$Q_r = -\frac{3\,E\,J\,(3+\mu)}{8\,a} \frac{d}{dr} (\nabla^2 w)$$

and, by substituting (24),

$$Q_r = -\frac{12\,(3+\mu)\,E\,J}{a} \frac{f\,r}{c^4} \tag{36}$$

the maximum of which $(r = c)$

$$Q_r = -12\,(3+\mu) \frac{E\,J\,f}{c^3}. \tag{37}$$

A similar calculation would give for the bending moments,

$$
\begin{aligned}
&M_r = M_\varphi = 6\frac{E\,J}{a}\frac{f}{c^2}, && (r=0),\\
&M_r = -9\frac{E\,J}{a}\frac{f}{c^2}, \qquad M_\varphi = -3\frac{E\,J}{a}\frac{f}{c^2} && (r=c).
\end{aligned}
\tag{38}
$$

8. A Numerical Application

We shall now apply the preceding theory to the double-layer lattice dome of Fig. 5.

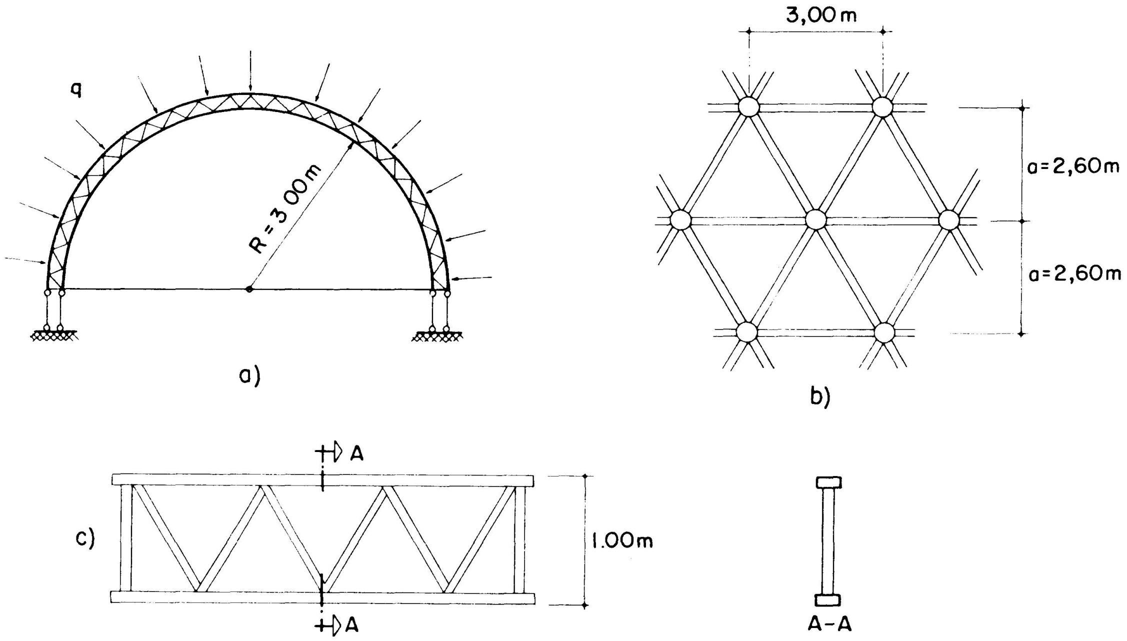

Fig. 5.

It is a dome of 300 m in diameter, whose members are steel trusses with 1 m depth.

The properties of the lattice in the upper region of the dome are found to be

$$F = 40\ \text{cm}^2, \qquad J = 81{,}000\ \text{cm}^4, \qquad a = 260\ \text{cm},$$

$$\lambda = \frac{J\,a^2}{F^3} = 85000.$$

If $E = 2.1 \times 10^6\ \text{kg/cm}^2$, the solution of (34), by means a FORTRAN IV program gives, for the minimum of σ,

$$\chi = 14400, \qquad \tau = 13400, \qquad \sigma_{min} = 170.$$

With the above numerical values in (35), we obtain

$$q_{cr} = \sigma_{min} \frac{2\,E\,F^2}{R^2\,a^2} = 170 \frac{2 \times 2.1 \times 10^6 \times 40^2}{15\,000^2 \times 260^2} = 7.6 \times 10^{-2}\ \mathrm{kg/cm^2} = 760\ \mathrm{kg/m^2},$$

$$f = \frac{\tau\,F}{a} = \frac{13\,400 \times 40}{260} = 2050\ \mathrm{cm} = 20.5\ \mathrm{m},$$

$$c = \sqrt{\frac{14\,400 \times 40 \times 15\,000}{260}} = 5800\ \mathrm{cm} = 58\ \mathrm{m}.$$

We next use these results in order to evaluate the stress resultants in the post-buckling stage, by means of (37) and (38) ($\mu \simeq 0$)

$$Q_{r(r=c)} = -36 \frac{2.1 \times 10^6 \times 8.1 \times 10^4 \times 2.05 \times 10^3}{5.8^3 \times 109 \times 2.6 \times 10^2} = -246\ \mathrm{kg/cm} = 24.6\ \mathrm{t/m},$$

$$M_{r(r=c)} = -9\,\frac{2.1 \times 10^6 \times 8.1 \times 10^4 \times 2.05 \times 10^3}{2.6 \times 10^2 \times 5.8^2 \times 10^6} = -1.74 \times 10^5\ \mathrm{kgcm/cm} = -174\ \mathrm{tm/m}.$$

The above value of Q_r would make an estimate of the diagonal bars in the trusses possible. We can see that the radial bending moment M_r would bring about plastic deformations.

9. Comparison with Other Theories

In order to check the results of the proceding theory, we compare it with approximate formulas proposed by other authors.

SCHÖNBACH [11] and WRIGHT [8] recommended the formula

$$q_{cr} = \frac{k\,E\,F\,J_x}{l\,r^2}, \tag{39}$$

in which k = 1,25 (SCHÖNBACH) or $k = 1{,}6$ (WRIGHT)
F cross-section of lattice members (40 cm²)
J_x = moment of inertia of lattice members
l = length of lattice members = 300 cm
r = radius of the dome = 15 000 cm

By inserting the appropriate numerical values in (39) we would obtain

$$q_{cr} = 700\ \mathrm{kg/m^2} \quad \text{for } k = 1.25 \quad \text{and} \quad q_{cr} = 890\ \mathrm{kg/m^2} \quad \text{for } k = 1.6.$$

BUCKERT [9], [10] in his buckling analysis of orthotropic shells proposed the formula

$$q_{cr} = 0.366\,E \left[\frac{t_m}{R}\right]^2 \left|\frac{t_B}{t_m}\right|^{3/2}, \tag{40}$$

in which

t_m = membrane thickness
t_B = bending thickness
R = radius of dome = 15000 cm

The membrane and the bending thickness will be obtained presently from the analogue model of the lattice shell, i. e., we assume

$$t_m = \frac{F}{a} \quad \text{and} \quad \frac{E\,t_B}{12} = \frac{3\,(3+\mu)\,E\,J}{8\,a}.$$

Then, $t_B = 2.38 \sqrt[3]{\frac{J}{a}}$ and, by inserting the numerical values,

$$t_m = \frac{40}{260} = 0.153 \text{ cm}, \qquad t_B = 2.38 \sqrt[3]{\frac{81{,}000}{260}} = 16.2 \text{ cm}. \tag{41}$$

By substituting all numerical values in (40), we would find

$$q_{cr} = 940 \text{ kg/m}^2.$$

We thus made sure that the present theory is in reasonable agreement with other theories and gives also the shape of the buckled zone.

10. Concluding Remarks

An important conclusion of the present study is that we should avoid applying buckling formulas derived from the theory of uniform shells to lattice domes.

For isotropic shells, membrane and bending thickness are identical. From (41), we can see that, for double-layer lattice domes, the difference between membrane and bending thickness may be considerable. The bending thickness for the above numerical example is larger than the membrane thickness by a factor of a hundred.

We should be also cautious in the choice of the safety factor for the determination of the permissible pressure.

The imperfections in geometry and boundary conditions, as well as the post-buckling deviations in the directions of the external pressure, tend to decrease the theoretical buckling load.

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Summary

The nonlinear buckling problem of lattice domes with triangular meshes is investigated. The buckled zone is treated as a shallow shell, by accounting for nonlinear terms in the kinematic relations.

The lattice properties are simulated by means of a continuous analogue model.

Results of the theory are compared with approximate formulas in the literature.

Résumé

Le problème du flambage non-linéaire des coupoles à treillis à subdivision triangulaire est étudié. La zone d'instabilité est traitée comme coque surbaissée, en considérant des termes non-linéaires dans les rélations cinématiques.

Les propriétés du treillis sont simulées par un modèle continu. Les résultats de la théorie sont comparés à d'autres formules existant dans la littérature.

Zusammenfassung

Das nichtlineare Stabilitätsproblem der Gitterkuppeln mit dreieckiger Ausfachung wird untersucht. Die Beulfläche wird als flache Schale behandelt, in dem nichtlineare Glieder in den kinematischen Beziehungen berücksichtigt werden.

Das Verhalten des Gitters wird mit einem kontinuierlichen Modell nachgebildet.

Die Ergebnisse der Theorie werden mit anderen Näherungsformeln in der Literatur verglichen.

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