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The Flexural Ductility of Reinforced Concrete Sections

La capacité de déformation de sections en béton armé due à l'effort de flexion

Die Verformungsfähigkeit armierter Betonquerschnitte infolge Biegung

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Introduction

While the inelastic flexural behaviour of reinforced concrete members and structures has been recognized for a long time [1], its adoption in design practice is still a controversial matter: some codes allow for plastic redistribution of up to 30% of the elastic stress distribution [2], some codes do not recognize plastic action at all [3]. Between these two extremes, some other codes allow an arbitrary degree of redistribution varying from 0 to 30% [4], [5].

In a debate initiated by the Joint ACI-ASCE Committee 428, Limit Design, a few years ago, some basic questions on the admissibility and features of non-linear analysis and design were investigated [6], [7]. From that debate and other similar discussions in the literature, it became obvious that the role, nature and extent of ductility in flexural concrete structures is not fully understood and that some tentative conclusions on the subject were based on insufficient factual data.

It was suggested that high grade steels were not suitable for inelastic design, that the use of compression reinforcement (to increase ductility) eliminates the economic advantages of inelastic design, and that elastic action should not be allowed in reinforced concrete members subject to combined bending and axial loads.

Some years ago one of the authors remarked [7]:

... "It would be ignoring reality to neglect the existence of strength reserve due to the inelasticity of reinforced concrete and not to take advantage of it only because imperfect rather than ideal plasticity is proper to this material. The problem raised by the particular type of concrete ductility is not *whether* but *how* it should be considered. That concrete, unlike steel, displays only a limited adaptability does not preclude its exploitation, but rather requires a deeper study of its physical significance and limitations." ...

This paper is an attempt in this direction, and has the following objectives:

1. A proper definition of flexural ductility for reinforced concrete sections.
2. An exhaustive study of the main variables affecting ductility.
3. Some conclusions on conditions and limitations of plastic adaptability in structural concrete.
4. Possible practical guidelines on the applicability of limit design to reinforced concrete structures.

The approach used in this analytical investigation is a computer simulation of the behaviour of over 1700 reinforced concrete section specimens under pure and combined bending. Starting from reliable stress-strain characteristics for steel and concrete [8], moment-curvature relationships and ductility factors are derived for an extensive range of variable combinations. Results obtained justify a more positive view of the potential use of inelastic design methods, when their limitations and the effects of major variables are well understood.

Definition of Ductility

Ductility is recognized as a factor governing the rotation capacity of hinging zones and the redistribution of moments in a structure [9]; the adaptability of structures to foundation settlements and volume changes [6]; and the energy absorption capacity of structures subject to dynamic (wind, earthquake, blast) loads [10], [11], [12]. Ductility safeguards a structure against sudden overloads, impact and load reversals. For this reason it is desirable that structures be capable of mobilizing a reasonable amount of ductility whenever actions such as those mentioned above are foreseen. Experience shows that the members of a structure are sufficiently ductile, for all practical purposes, when they resist only transverse loads, are moderately reinforced in tension, moderately to heavily reinforced in compression and shear, use mild or intermediate grade steels, and high grade concretes. It is also an established fact that careful joint detailing and a high standard of execution in the field contribute to the achievement of high degrees of ductility [13].

Beyond these general qualitative facts and except for some attempts to study the ductility of reinforced concrete sections [10], [11], [14], [15], [16], [17], there is only a limited knowledge of the problem.

In a broad sense, ductility is taken to be the ability to sustain deformations beyond the elastic range without a significant variation of the resistance capacity. Such a qualitative description of ductility is broad enough to accommodate the response of plastic, strain-hardening or strain-softening materials, as long as precise limits of deformation and strength variations are not specified.

More precise definitions of ductility have to be dependent on at least the following: a) level of study – material, sectional and structural ductility should be defined in terms of strains, curvatures, and rotations or deflections, respectively; b) type of stress – ductility under axial loading, flexure, shear and torsion should be defined in terms of longitudinal strains, curvatures, shearing strains, and angles of twist, respectively; c) nature of study – depending upon which, it may be more or less suitable to define ductility in terms of limiting deformations (e.g. ϵ_u, ϕ_u), differences or ratios between limiting and idealized elastic limit deformations (e.g. $\epsilon_u - \epsilon_y$, $\phi_u - \phi_y$, ϵ_u/ϵ_y, ϕ_u/ϕ_y), or areas under load-deformation diagrams up to limiting deformations or between limiting and idealized elastic limit deformations. While the second alternative may be satisfactory in the limit analysis and design of concrete structures, the last may be more meaningful in earthquake engineering; d) nature of loading (static, dynamic).

Some possible and serious confusions may arise from an interchange of ductility definitions. Here are two typical examples:

a) The effect of high grade concrete is favorable on *sectional* ductility, [14], [15] but is unfavorable on *material* ductility [8].

b) Lateral reinforcement is more efficient than compression reinforcement in increasing the *material* ductility (of concrete), [18]; compression reinforcement is more efficient than lateral reinforcement in increasing *sectional* ductility [19].

This study is concerned with the ductility of reinforced concrete sections, on the assumption that the properties of steel and concrete are known. The investigation is limited to pure and combined bending, because these are the most common cases when a designer faces inelastic action in structural concrete. Further studies should provide similar data on r.c. ductility in shear and torsion. As an index of sectional ductility, the *ductility factor* is defined as the ratio of ultimate to yield curvatures, ϕ_u/ϕ_y. It is found that this definition is the most widely used for evaluating ductility under static loads and is equally significant for both steel [20] and reinforced concrete [21], [22]. In brief, this investigation is concerned with the ductility (a) of reinforced concrete sections, (b) under flexural action, (c) defined as a ratio of curvatures, (d) for static loading only.

Having defined the meaning of the ductility factor in the context of this paper it is necessary to further define the curvatures, ϕ_u and ϕ_y. The current practice is to assume that the ultimate curvature is associated with a conventional limiting value of the concrete strain at the extreme fibre i.e. $\epsilon_u = 0.3\%$, $\epsilon_u = 0.35\%$ and $\epsilon_u = 0.38\%$ according to the ACI Code [23], CEB Recommendations [4] and some earlier investigations at the University of Illinois [14], [24], respectively. These ϵ_u values are considered to be independent of such factors as the longitudinal and lateral reinforcement, strain-hardening, strain gradient, etc. A more satisfactory definition of the ultimate strain, proposed by RÜSCH [25] is adopted in this study: since the primary function of a structure is to carry loads, ϵ_u is defined as the strain corresponding to the ultimate stage, i. e. at which the section reaches its maximum load or moment carrying capacity. Similarly, the ultimate curvature, ϕ_u, is the one associated with the strain, (ϵ_u), load, (P_u), or moment, (M_u), at the ultimate stage.

The yield curvature, ϕ_y, is defined as the curvature at which the tension steel reaches its yield point stress. The stress-strain relationships used for steel in the present investigation are characterized by well defined yield points. Thus, when the tension steel in a section does not yield before the section reaches its ultimate stage, it is either because the section is highly over-reinforced or because it carries a heavy axial load. Instead of attempting to arbitrarily define an idealized yield stage, the ductility factors of such sections, possessing very little ductility, are assumed to be equal to unity in the present study.

Factors Affecting Ductility

The major factors affecting the ductility of a reinforced concrete section can be classified as follows:

1. Material Variables:
   - a) Concrete quality.
   - b) Grades of tension and compression reinforcement.
   - c) Grade of lateral reinforcement.
   - d) Strain-hardening of steel.
   - e) Bond.
   - f) Tensile strength of concrete.
2. Geometric Variables:
   - a) Shape and size of sections.
   - b) Amount of tension reinforcement.
   - c) Amount of compression reinforcement.
   - d) Amount and spacing of lateral reinforcement.
   - e) Cover thickness.
3. Loading Variables:
   - a) Duration of loading.
   - b) Axial loading.
   - c) Prestressing.
   - d) Repetition of loading.
   - e) Loading reversal.

The effects of the above factors on sectional ductility were investigated by using a nonlinear sectional theory, realistic stress-strain relationships for concrete and steel and a numerical method of computation developed in [8]. These have been described in a recent paper by the authors [26] and are briefly reviewed in the next section.

Sectional Analysis

a) Stress-Strain Relationship for Concrete in Compression

The main factors affecting concrete behaviour are: concrete strength, lateral reinforcement, creep, strain gradient, size of specimen and type of loading. A stress-strain relationship for concrete in compression, proposed by Sargin [8], takes all these factors into account by a proper choice of five governing parameters: the concrete cylinder strength, f'_c; the initial Young modulus, E_c; the ratio of maximum stress to cylinder strength, k_3; the strain corresponding to maximum stress, ϵ_0; and a parameter, D, which mainly affects the descending branch of the stress-strain curve. By denoting $A = E_c \epsilon_0 / k_3 f'_c$ and $x = \epsilon / \epsilon_0$, Sargin's relationship can be expressed as:

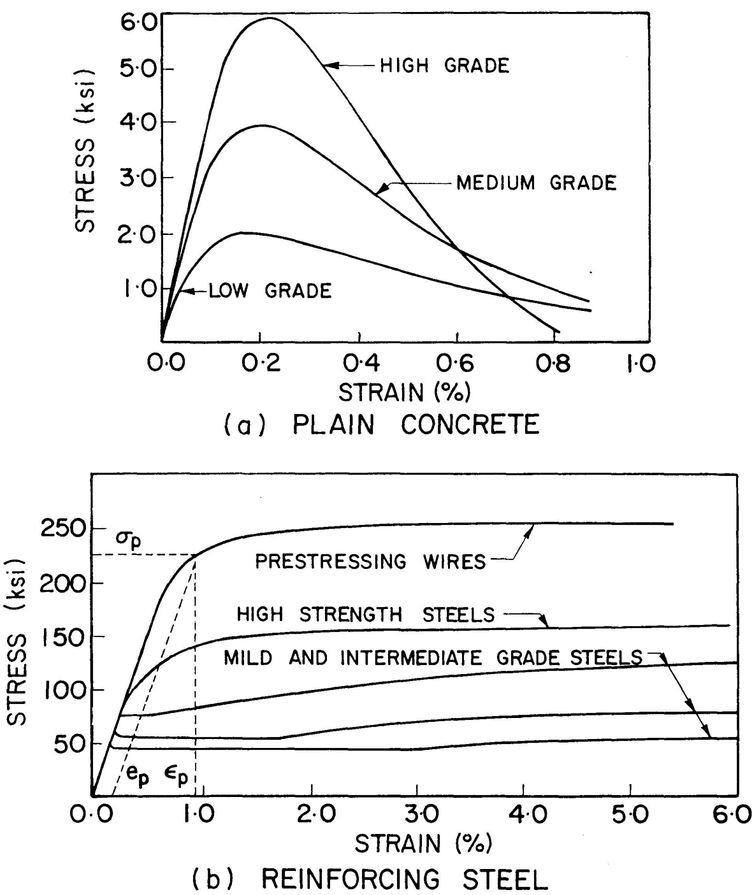

Fig. 1. Typical stress-strain curves for (a) concrete in compression and (b) reinforcing steel.

$$\sigma = k_3 f_c' \frac{A\,x + (D-1)\,x^2}{1 + (A-2)\,x + D\,x^2}. \tag{1}$$

Equations expressing E_c, k_3, ϵ_0 and D in terms of the factors affecting them are given in [8] and are used in numerical calculations. For the sake of brevity, these are not reproduced here. Typical stress-strain curves for concrete in compression, Eq. (1), are illustrated in Fig. 1(a).

b) Stress-Strain Relationship for Concrete in Tension

The behaviour of concrete in tension is assumed to be elastic-brittle and can be expressed by the following equations:

$$\begin{aligned} \sigma_t &= E_c\,\epsilon_t \quad (\text{for } \epsilon_t \leqq \epsilon_{tr}), \\ \sigma_t &= 0 \qquad\;\; (\text{for } \epsilon_t > \epsilon_{tr}), \end{aligned} \tag{2}$$

where $\epsilon_{tr} = \sigma_{tr}/E_c$ is the cracking strain and σ_{tr} is the modulus of rupture of concrete. An equation expressing σ_{tr} in terms of the factors governing it is also given in [8].

c) Stress-Strain Relationships for Reinforcing Steels

The following idealized relationships, consisting of three parts corresponding to the elastic, yield and strain-hardening ranges, and considered applicable to most American steel grades with yield limits not in excess of 75 ksi, are adopted in this study:

$$\begin{aligned} \sigma_s &= E_s\,\epsilon_s \quad (\text{for } 0 \leqq \epsilon_s \leqq \epsilon_y), \\ \sigma_s &= f_y \qquad (\text{for } \epsilon_y < \epsilon_s \leqq \epsilon_{sh}), \\ \sigma_s &= f_y + E_{sh}\,(\epsilon_s - \epsilon_{sh})\,1 - \frac{E_{sh}\,(\epsilon_s - \epsilon_{sh})}{4\,(\sigma_{su} - f_y)} \quad (\text{for } \epsilon > \epsilon_{sh}), \end{aligned} \tag{3}$$

where E_s is the Young modulus for steel, f_y is the yield limit, ϵ_{sh} is the strain at the onset of hardening, E_{sh} is the strain-hardening modulus and σ_{su} is the ultimate stress.

Typical stress-strain curves for steel, Eq. (3), are illustrated in Fig. 1(b), along with stress-strain curves for high strength steels (proof stress > 75 ksi) and prestressing wires, which are not used in the present investigation.

d) Nonlinear Sectional Theory

With the notations and assumptions of Fig. 2, the force and moment equilibrium equations for a reinforced concrete section, symmetrical about

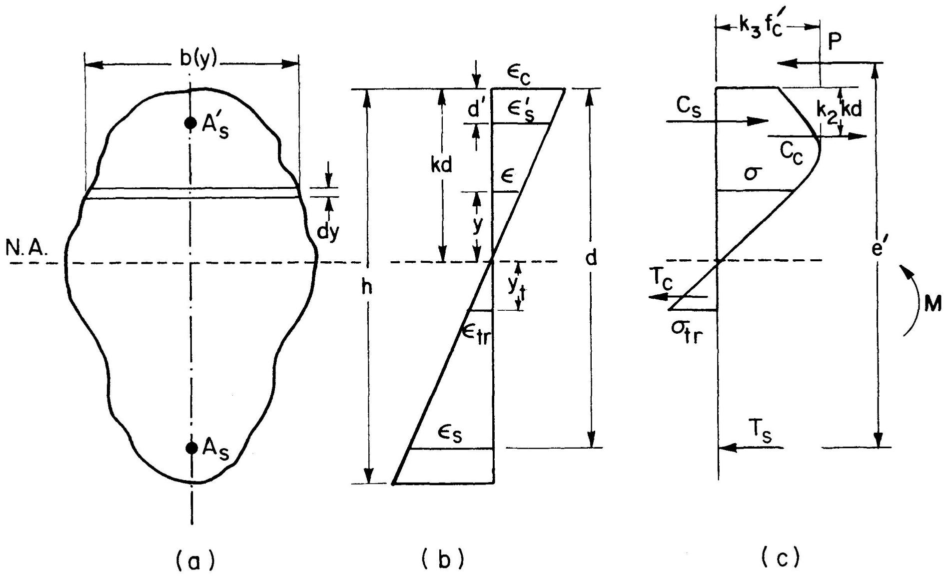

Fig. 2. Basic notations in the flexural analysis of reinforced concrete sections.

one axis and loaded in the plane of symmetry, can be expressed as follows:

$$\int_0^{kd} \sigma(\epsilon)\, b(y)\, dy + A'_s \sigma'_s - \int_0^{y_t} \sigma_t(\epsilon_t)\, b(y)\, dy - A_s \sigma_s = P, \tag{4}$$

$$\int_0^{kd} \sigma(\epsilon)\, b(y)\,(d - kd + y)\, dy + A'_s \sigma'_s (d - d') - \int_0^{y_t} \sigma_t(\epsilon_t)\, b(y)\,(d - kd - y)\, dy = P e' + M. \tag{5}$$

The assumption of linear strain distribution implies:

$$\frac{\epsilon_c}{kd} = \frac{\epsilon'_s}{kd - d'} = \frac{\epsilon_s}{d - kd} = \frac{\epsilon}{y} = \frac{\epsilon_{tr}}{y_t}. \tag{6}$$

Eqs. (1), (2) and (3) are used to eliminate σ, σ_t and σ_s, σ'_s, respectively, and Eq. (6) to eliminate y and y_t from Eqs. (4) and (5).

e) Numerical Method of Solution

A numerical method is developed to solve Eqs. (4) and (5) simultaneously in the following steps (Fig. 3):

a) Starting from zero, increase ϵ_c at some chosen interval.
b) For any given value of ϵ_c, find a value of k by successive approximation such that Eq. (4) is satisfied with a specified tolerance.
c) Solve Eq. (5) for M with the known values of ϵ_c, k and the given P.
d) Calculate all other behaviour parameters: ϕ, EI (flexural rigidity), etc.
e) Continue to increase ϵ_c up to and beyond the value ϵ_u at which the moment reaches a maximum. ϵ_u corresponds to the ultimate state of the section.

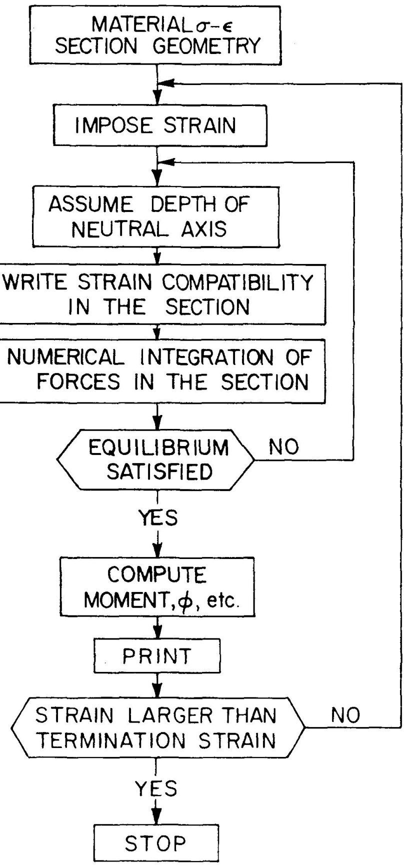

Fig. 3. Flow diagram for numerical analysis.

Using the above method of sectional analysis, 1734 sections were analyzed under various combinations of the factors enumerated in the preceding section (Tables 1 and 2). Some of the results of this investigation are presented herein. The effects of the various factors on the ductility of the sections analyzed are discussed in the next three sections.

Material Variables

The effects of *concrete and tension reinforcement qualities* on the $M-\phi$ relationships of reinforced concrete sections are shown in Fig. 4. This figure shows that, irrespective of the reinforcement percentage p, sectional ductility increases with increasing concrete and decreasing tension reinforcement strengths. This can be seen more clearly in Fig. 5, in which the ductility ratio ϕ_u/ϕ_y is plotted against the reinforcement percentage p for various grades of concrete and tension reinforcement. Each curve has a little arrowhead attached to it, which corresponds to the maximum percentage of tension reinforcement, p_{max}, that can be used in sections designed according to the ACI ultimate strength theory [23]. Fig. 5 shows that although for low reinforcement percentages fairly high ductility ratios are available for most grades of concrete and steel, this ratio may be as low as 2.5 for some steel and concrete grades,

Table 1. Variable combinations for various sections investigated

	SECTION # P/P_u =0.0 T	P/P_u =0.1	P/P_u =0.2	P/P_u =0.3	P/P_u =0.4	P/P_u =0.45	P/P_u =0.5	P/P_u =0.6	P/P_u =0.8	f'_c ksi	f_y ksi	p %	p'/p	E_{sh} $x10^3$ksi	d'' in.	s in.	t min.	c in.	F	σ_t
1	57											0.5								
2	58																30			
3																	10			
4																	10^2			
5												1.0					10^3			
6													0				10^4			
7																	10^5			
8																	10^6			
9	59											1.5								
10	60											2.0					30			
11												2.5								
12	61																			
13																	10			
14																	10^2			
15												3.0					10^3			
16																	10^4			
17																	10^5			
18																	10^6			
19												3.5								
20	62											4.0								
21												0.5								
22										3	45	1.0		1.25	1/4	9		2	1.0	CONSIDERED
23												1.5								
24	63											2.0	0.25							
25												2.5								
26												3.0								
27												3.5								
28	64											4.0								
29	65											1.5	0.33							
30	66											3.0								
31												0.5								
32	67	73	83	93	103	113	123	133	143			1.0								
33												1.5								
34	68	74	84	94	104	114	124	134	144			2.0	0.5				30			
35												2.5								
36		75	85	95	105	115	125	135	145			3.0								
37												3.5								
38	69											4.0								
39	70	76	86	96	106	116	126	136	146			1.5	0.67							
40	71	77	87	97	107	117	127	137	147			3.0								
41												0.5								
42												1.0								
43												1.5								
44												2.0	0.75							
45												2.5								
46												3.0								
47												3.5								
48												4.0								
49		78	88	98	108	118	128	138	148			0.5								
50	72	79	89	99	109	119	129	139	149			1.0								
51		80	90	100	110	120	130	140	150			1.5								
52		81	91	101	111	121	131	141	151			2.0	1.00							
53												2.5								
54		82	92	102	112	122	132	142	152			3.0								
55												3.5								
56												4.0								

Sections 153-204, 205-456, 457-608, 1127-1278, 1279-1430, 1431-1582, and 1583-1734 correspond to sections 1-152 for f'_c, f_y = 3,60; 3,75; 4,45; 4,75; 5,45; 5,60; and 5,75 ksi, respectively.

as p approaches p_{max}. It must be remembered, however, that Fig. 5 is for singly reinforced sections containing nominal amounts of lateral reinforcement (No. 2 ties at 9″ spacing). Ductility can be increased somewhat by reducing the spacing and increasing the diameter of the ties. It an be improved considerably by the addition of suitable amounts of compression reinforcement.

Sectional behaviour is affected much more by the spacing and cross-sectional area of lateral reinforcement than by its grade. The latter was, therefore, not studied in the present investigation.

Table 2.: Variable combinations for various sections investigated

SECTION #	P/P_u =0.0 T	P/P_u =0.1	P/P_u =0.2	P/P_u =0.3	P/P_u =0.4	P/P_u =0.45	P/P_u =0.5	P/P_u =0.6	P/P_u =0.8	f'_c ksi	f_y ksi	p %	p'/p	E_{sh} x10^3ksi	d'' in.	s in.	t min.	c in.	F	σ_t
609	831									4	60	0.5	0	0	1/4	9	30	2	1.00	CONSIDERED
610														1.25						
611														2.5						
612														1.25		2				
613																4				
614																6				
615																9	10			
616																	10^2			
617																	10^3			
618																	10^4			
619																	10^5			
620																	10^6			
621																	30	1		
622																		3		
623																		2	0.25	
624																			0.50	
625																			0.75	
626																			1.00	N.C.
627	832											1.0		0						CONSIDERED
628														1.25						
629														2.5						
630														1.25		2				
631																3				
632																4				
633																5				
634																6				
635																7				
636															3/8	2				
637																3				
638																4				
639																5				
640																6				
641																7				
642																9				
643															1/4	9	10			
644																	10^2			
645																	10^3			
646																	10^4			
647																	10^5			
648																	10^6			
649																	30	1		
650																		3		
651																		2	0.25	
652																			0.50	
653																			0.75	
654																			1.00	N.C.
655	833													0						CONSIDERED
656														1.25						
657														2.5						
658														1.25		2				
659																4				
660																6				
661																9	10			
662																	10^2			
663																	10^3			
664																	10^4			
665																	10^5			

SECTION #	P/P_u =0.0 T	P/P_u =0.1	P/P_u =0.2	P/P_u =0.3	P/P_u =0.4	P/P_u =0.45	P/P_u =0.5	P/P_u =0.6	P/P_u =0.8	f'_c ksi	f_y ksi	p %	p'/p	E_{sh} x10^3ksi	d'' in.	s in.	t min.	c in.	F	σ_t
666										4	60	1.5	0	1.25	1/4	9	10^6	2	1.00	C
667																	30	1		
668																		3		
669																		2	0.25	
670																			0.50	
671																			0.75	
672																			1.00	N.C.
673	834											2.0		0						CONSIDERED
674														1.25						
675														2.5						
676														1.25		2				
677																4				
678																6				
679																9	10			
680																	10^2			
681																	10^3			
682																	10^4			
683																	10^5			
684																	10^6			
685																	30	1		
686																		3		
687																		2	0.25	
688																			0.50	
689																			0.75	
690																			1.00	N.C.
691												2.5								CONSIDERED
692	835											3.0		0						
693														1.25						
694														2.5						
695														1.25		2				
696																3				
697																4				
698																5				
699																6				
700																7				
701															3/8	2				
702																3				
703																4				
704																5				
705																6				
706																7				
707																9				
708															1/4	9	10			
709																	10^2			
710																	10^3			
711																	10^4			
712																	10^5			
713																	10^6			
714																	30	1		
715																		3		
716																		2	0.25	
717																			0.50	
718																			0.75	
719																			1.00	N.C.
720												3.5								C
721	836											4.0		0						
722														1.25						

SECTION #	P/P$_u$ =0.0	P/P$_u$ =0.1	P/P$_u$ =0.2	P/P$_u$ =0.3	P/P$_u$ =0.4	P/P$_u$ =0.45	P/P$_u$ =0.5	P/P$_u$ =0.6	P/P$_u$ =0.8	f'_c ksi	f_y ksi	p %	p'/p	E_{sh} x10^{3}ksi	d'' in.	s in.	t min.	c in.	F	σ_t
	T																			
723										4	60	4.0	0	2.5	1/4	9	30	2	1.00	CONSIDERED
724														1.25		2				
725																4				
726																6				
727																9	10			
728																	10^2			
729																	10^3			
730																	10^4			
731																	10^5			
732																	10^6			
733																	30	1		
734																		3		
735																		2	0.25	
736																			0.50	
737																			0.75	
738																			1.00	N.C
739												0.5	0.25							CONSIDERED
740												1.0								
741												1.5								
742	837											2.0								
743																		1		
744																		3		
745												2.5						2		
746												3.0								
747												3.5								
748	838											4.0								
749																		1		
750																		3		
751	839											1.5	0.33					2		
752																		1		
753																		3		
754	840											3.0						2		
755																		1		
756																		3		
757												0.5	0.5					2		
758	841	847	882	917	952	987	1022	1057	1092			1.0								
759																		1		
760																		3		
761												1.5						2		
762	842	848	883	918	953	988	1023	1058	1093			2.0								
763																		1		
764																		3		
765												2.5						2		
766		849	884	919	954	989	1024	1059	1094			3.0								
767												3.5								
768	843											4.0								
769																		1		
770																		3		
771	844	850	885	920	955	990	1025	1060	1095			1.5	0.67					2		
772																		1		
773																		3		
774	845	851	886	921	956	991	1026	1061	1096			3.0						2		
775																		1		
776																		3		
777												0.5	0.75					2		
778												1.0								
779												1.5								

SECTION #	P/P$_u$ =0.0	P/P$_u$ =0.1	P/P$_u$ =0.2	P/P$_u$ =0.3	P/P$_u$ =0.4	P/P$_u$ =0.5	P/P$_u$ =0.6	P/P$_u$ =0.7	P/P$_u$ =0.8	f'_c ksi	f_y ksi	p %	p'/p	E_{sh} x10^{3}ksi	d'' in.	s in.	t min.	c in.	F	σ_t
	T																			
780										4	60	2.0	0.75	1.25	1/4	9	30	2	1.00	CONSIDERED
781												2.5								
782												3.0								
783												3.5								
784												4.0								
785		852	887	922	957	992	1027	1062	1097			0.5	1.0							
		853	888	923	958	993	1028	1063	1098					0						
786														1.25			10			
787																	10^2			
788																	10^3			
789																	10^4			
790																	10^5			
791																	10^6			
792	846	854	889	924	959	994	1029	1064	1099			1.0					30			
		855	890	925	960	995	1030	1065	1100					0						
793														1.25			10			
794																	10^2			
795																	10^3			
796																	10^4			
797																	10^5			
798																	10^6			
799		856	891	926	961	996	1031	1066	1101								30	1		
800		857	892	927	962	997	1032	1067	1102							2				
801		858	893	928	963	998	1033	1068	1103							9		3		
802		859	894	929	964	999	1034	1069	1104							2				
803		860	895	930	965	1000	1035	1070	1105			1.5				9		2		
		861	896	931	966	1001	1036	1071	1106					0						
804		862	897	932	967	1002	1037	1072	1107					1.25		2				
805		863	898	933	968	1003	1038	1073	1108							3				
806		864	899	934	969	1004	1039	1074	1109							4				
807		865	900	935	970	1005	1040	1075	1110							5				
808		866	901	936	971	1006	1041	1076	1111							6				
809		867	902	937	972	1007	1042	1077	1112							7				
810		868	903	938	973	1008	1043	1078	1113							9	10			
811		869	904	939	974	1009	1044	1079	1114								10^2			
812		870	905	940	975	1010	1045	1080	1115								10^3			
813		871	906	941	976	1011	1046	1081	1116								10^4			
814		872	907	942	977	1012	1047	1082	1117								10^5			
815		873	908	943	978	1013	1048	1083	1118								10^6			
816		874	909	944	979	1014	1049	1084	1119								30			
		875	910	945	980	1015	1050	1085	1120					0						
817												2.0		1.25			10			
818																	10^2			
819																	10^3			
820																	10^4			
821																	10^5			
822																	10^6			
823												2.5					30			
824		876	911	946	981	1016	1051	1086	1121			3.0								
		877	912	947	982	1017	1052	1087	1122					0						
825		878	913	948	983	1018	1053	1088	1123					1.25				1		
826		879	914	949	984	1019	1054	1089	1124							2				
827		880	915	950	985	1020	1055	1090	1125							9		3		
828		881	916	951	986	1021	1056	1091	1126							2				
829												3.5				9		2		
330												4.0								

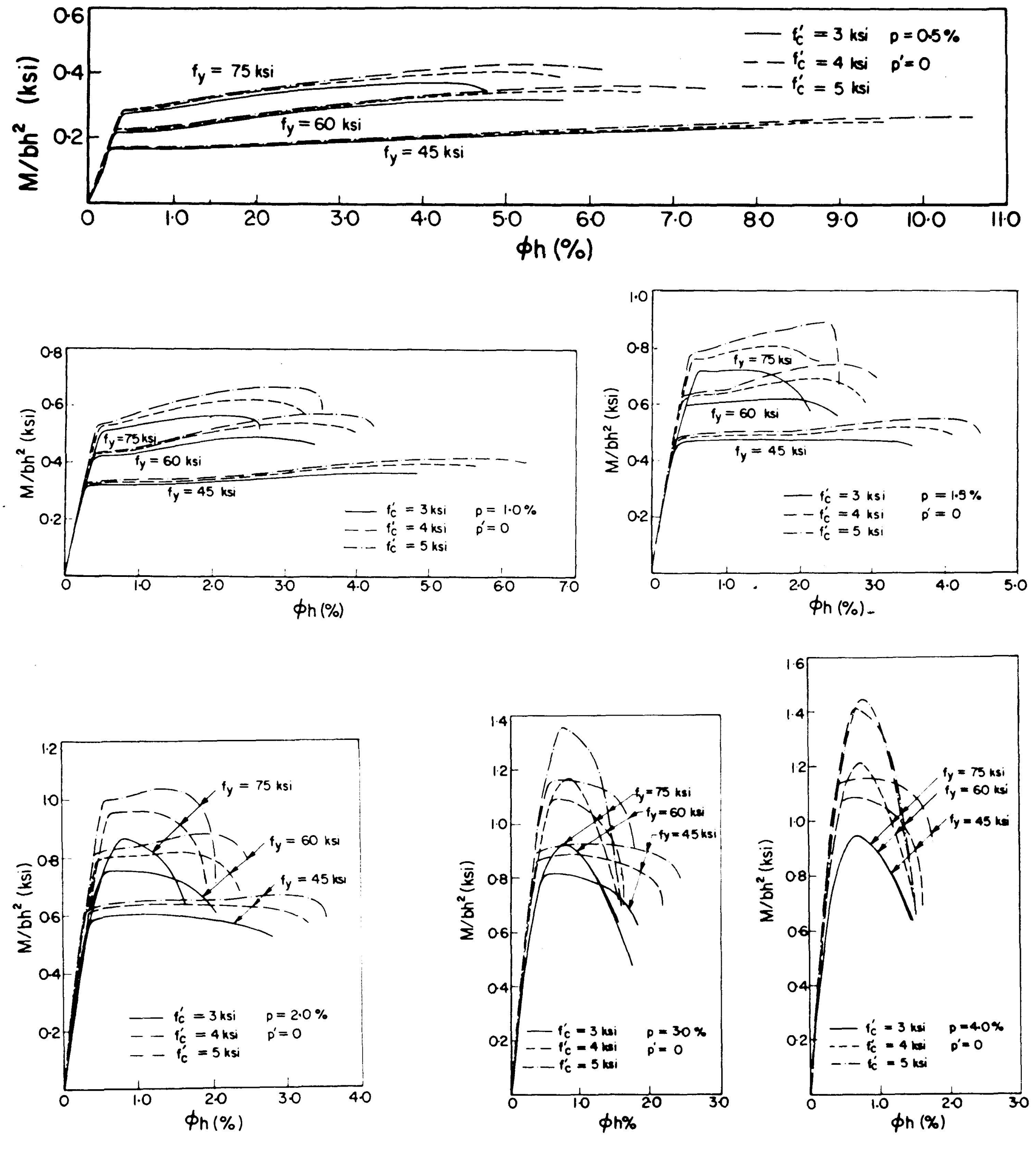

Fig. 4. Effect of concrete and steel grades and of steel percentages on ductility: M-ϕ diagrams.

Fig. 6 shows the effects of *strain-hardening* of steel on the $M-\phi$ relationships of singly reinforced sections. All reinforcing steels had the same modulus of elasticity, yield strength and strain at the onset of hardening; only the strain-hardening modulus E_{sh} was varied. The ductility ratio ϕ_u/ϕ_y is plotted against p for three different E_{sh} in Fig. 7. It can be seen that strain-hardening of steel improves the ductility of lightly reinforced sections, but has a negligible effect on heavily reinforced sections. Fig. 7 also shows that ductility increases as E_{sh} is increased from 0 to 1.25×10^3 ksi, but then it decreases as E_{sh} is further increased to 2.5×10^3 ksi. This would suggest that there is an optimal values of E_{sh} that maximizes the sectional ductility.

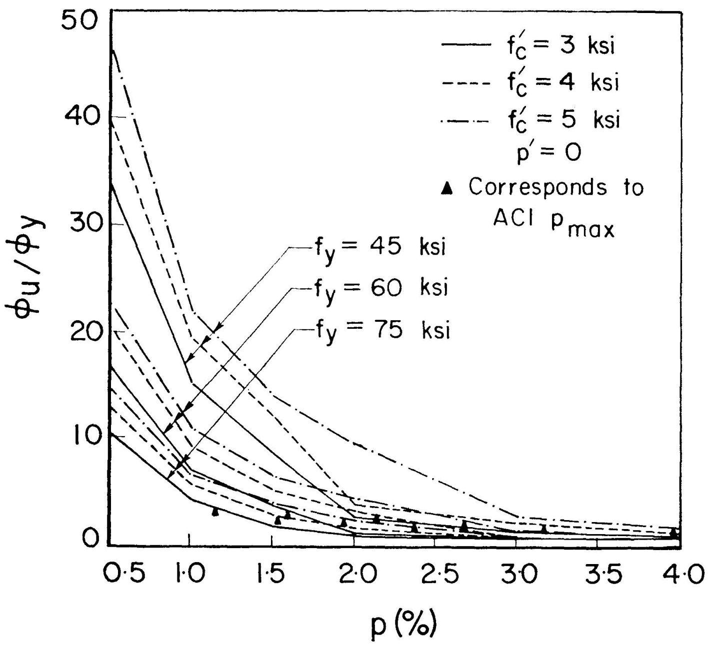

Fig. 5. Effect of concrete and steel grades and of steel percentages on ductility: ϕ_u/ϕ_y-p diagrams.

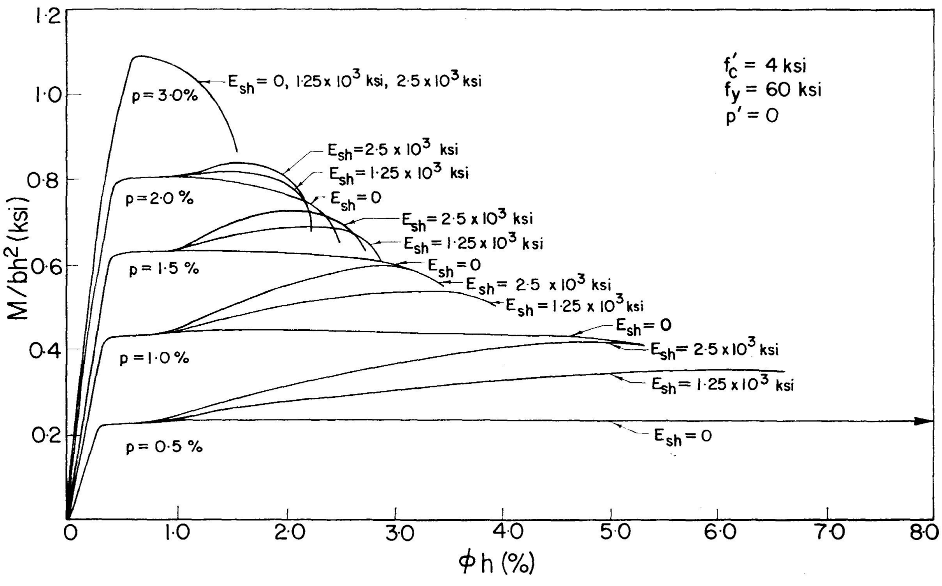

Fig. 6. Effect of strain-hardening of steel on ductility: M-ϕ diagrams.

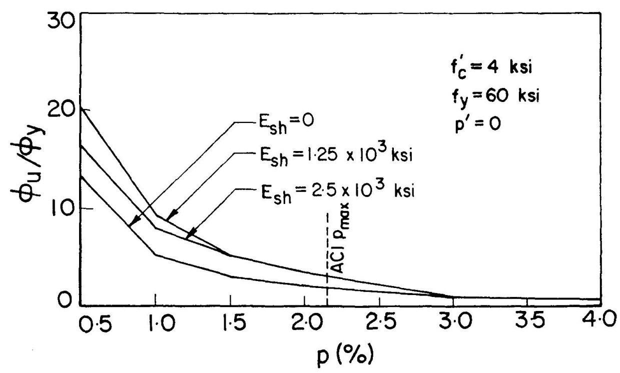

Fig. 7. Effect of strain-hardening of steel on ductility: ϕ_u/ϕ_y-p diagrams.

The effect of *bond* was investigated through BAKER's [26] bond factor F, which is defined as the ratio of steel and virtual concrete strains at the same level, i.e.

$$F = \frac{\epsilon_s k}{\epsilon_c (1-k)}. \tag{7}$$

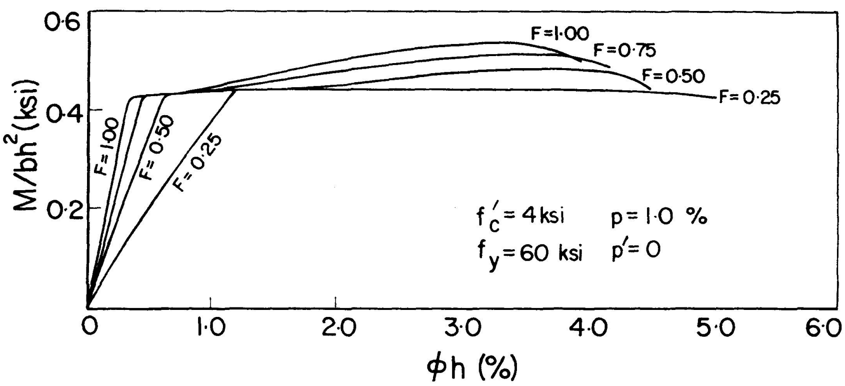

Fig. 8. Effect of bond on ductility: M-ϕ diagrams.

Fig. 8 shows the $M-\phi$ relationships of four singly reinforced sections with $F=0.25$, 0.50, 0.75 and 1.00, respectively, and identical in all other respects. The ductility ratios of these sections are plotted in Fig. 9, which shows that ductility is the highest for the section with full bond ($F=1.00$) and that it decreases steadily as F is reduced from 1.00 to 0.25.

$M-\phi$ relationships for sections in which the *tensile strength of concrete* was totally neglected and in which it was accounted for were found to be nearly identical. It was, therefore, concluded that the tensile strength of concrete has no significant effect on the sectional behaviour.

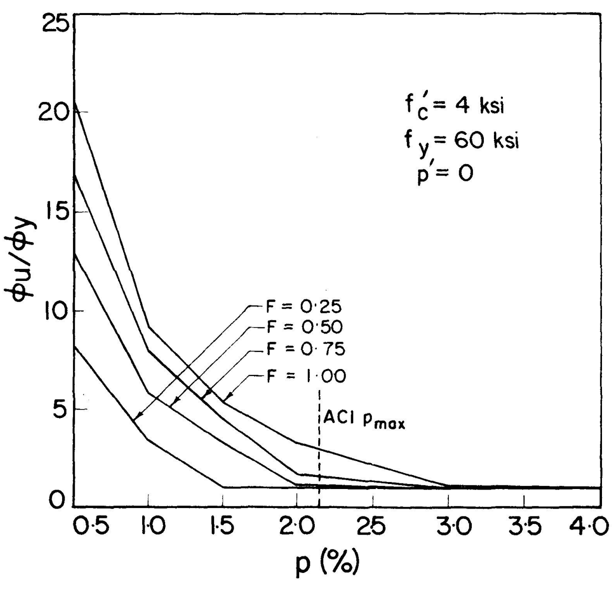

Fig. 9. Effect of bond on ductility: ϕ_u/ϕ_y-p diagrams.

Geometric Variables

NEWMARK and HALL [10] studied the effect of variations in depth on the ductility of rectangular sections and concluded that ductility was unaffected by such variations. The effects of sectional width and effective depth variations on the rotation capacity (hence ductility) of rectangular sections were investigated by CORLEY [28]. He concluded that ductility was not significantly affected either by depth or by width. These conclusions, based on reliable experimental evidence, are accepted in this study; the effects of *sectional size* on ductility are not investigated.

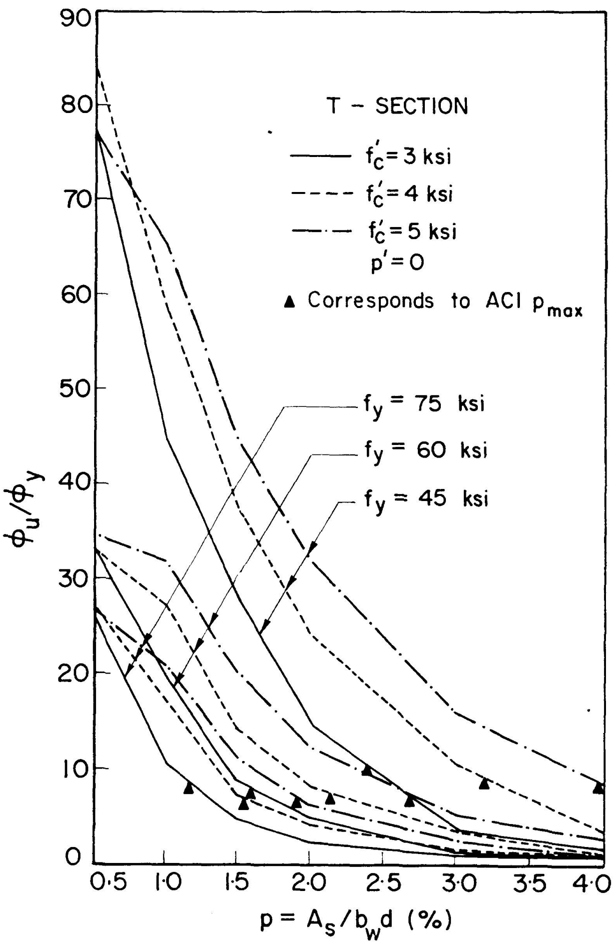

Fig. 10. Effect of sectional shape on ductility: ϕ_u/ϕ_y-p diagrams for T-sections.

Fig. 10 shows $\phi_u/\phi_y - p$ (with $p = A_s/b_w d$, and b_w = web width) diagrams for T-sections, the overall depth, effective depth and web width of which were equal to the corresponding depths and width (20″, 18″ and 10″, respectively) of the rectangular sections studied so far. The flange width and thickness were 30″ and 2.5″, respectively. It can be seen by comparison with Fig. 5 that the overhanging flanges provide a substantial improvement in ductility. This is not surprising in view of the fact that the overhanging flange area can be considered as an equivalent compression steel area and compression reinforcement is known to have a favourable effect on ductility. *Sectional shapes* other than rectangle and *T* were not investigated.

The effect of the *amount of tension reinforcement* on ductility can be observed in Figs. 4 and 5. Fig. 11 illustrates sectional $M-\phi$ relationships for various amounts of tension steel, corresponding to a particular quality of concrete

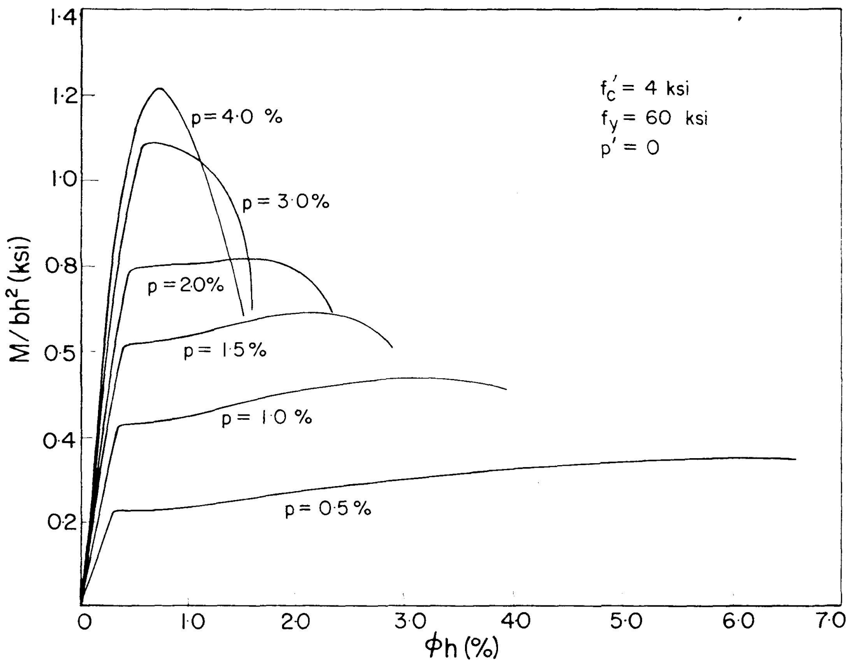

Fig. 11. Effect of tension steel percentage on ductility: M-ϕ diagrams.

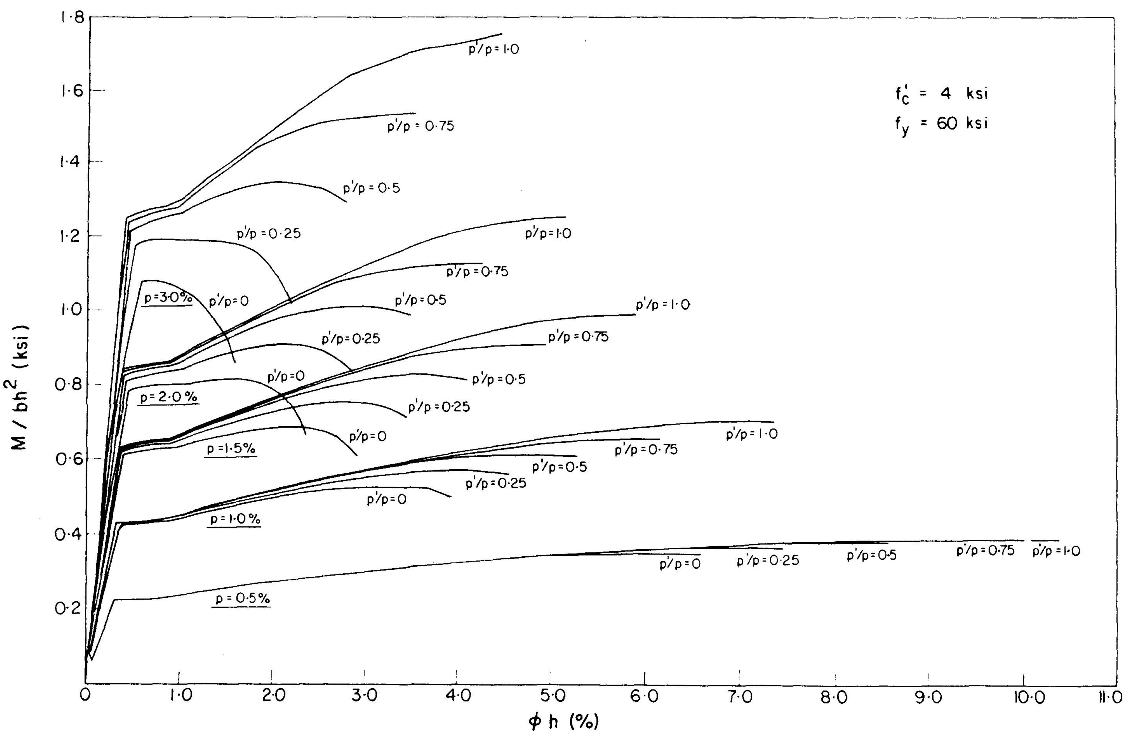

Fig. 12. Effect of compression reinforcement on ductility: M-ϕ diagrams.

and of tension reinforcement. Figs. 4, 5 and 11 confirm that ductility decreases with increasing amounts of tension reinforcement, and that very little or no ductility is available for sections with very high steel percentages (e.g. 4%). This is why most codes of practice [23] impose an upper limit on the amount of tension reinforcement that should be used in design.

Fig. 12 shows the effect of *compression reinforcement* on sectional $M-\phi$ relationships. For a particular quality of concrete and of reinforcing steel, $M-\phi$ diagrams are drawn for various percentages of tension reinforcement and for various ratios of compression and tension steel areas. ϕ_u/ϕ_y-p diagrams for various p'/p ratios and for various qualities of concrete and steel are plotted in Fig. 13. These figures show clearly that sectional ductility can be improved considerably by the addition of suitable amounts of compression reinforcement. This is also evident from Table 3, in which ϕ_u/ϕ_y values are tabulated for different concrete and steel qualities, various amounts of tension

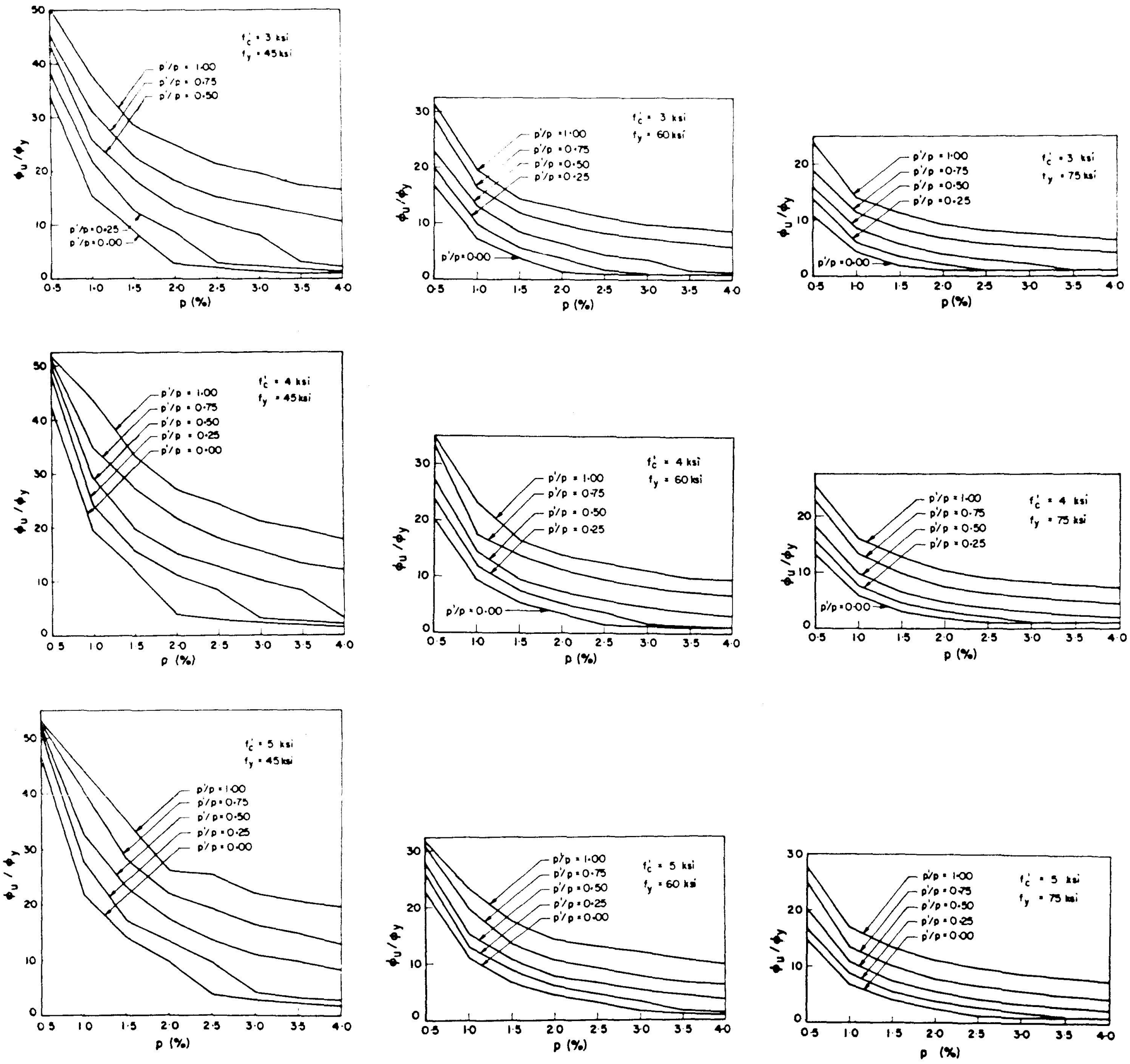

Fig. 13. Effect of compression reinforcement on ductility: ϕ_u/ϕ_y-p diagrams.

reinforcement and various p'/p ratios. For flexural members, the ductility of support sections is usually more critical than that of span sections [22], because span sections very often act as T-sections, so that their ductility is improved considerably by the overhanging flange areas (Fig. 10). However, support sections usually contain an amount of compression reinforcement. In flexural members, the amount of positive moment steel is usually about 75 to 80% of the tension steel provided for negative moments. If half of the steel for positive moments is bent up, the other half automatically provides compression steel in the amount $A_s' = 0.5 \times 0.8\, A_s = 0.4\, A_s$, where A_s' is the tension steel for negative moments. This amount of compression steel can considerably improve the ductility of a section. Table 3 shows that the ductility ratio of a section with $p = 1.5\%$, $f_c' = 4$ ksi and $f_y = 60$ ksi nearly doubles from 5.35 to 9.48 when compression steel in the amount $A_s' = 0.5\, A_s$ is added. Incidentally, Table 3 also indicates that sections reinforced with usual, economic percentages of high grade steel, when made of commensurately high grades of concrete, are capable of mobilizing reasonable levels of ductility. For instance, a section with $f_c' = 5$ ksi, $f_y = 75$ ksi, $p = 1.5\%$, and $p' = 0.25\, p$, has a ductility factor of 5.48. This would appear to suggest that inelastic design is not necessarily unsuitable for structures made of high grade steels, as has sometimes been claimed in the past. Table 3 also indicates that the unfavourable effect of an increase in steel strength on sectional ductility is usually more pronounced than the favourable effect of an increase in concrete strength.

The effect of *tie spacing* on sectional $M-\phi$ relationships is illustrated in Fig. 14. Fig. 15 shows ϕ_u/ϕ_y variation with p for different tie spacings. It can be seen that ductility increases somewhat as tie spacing is reduced, but that

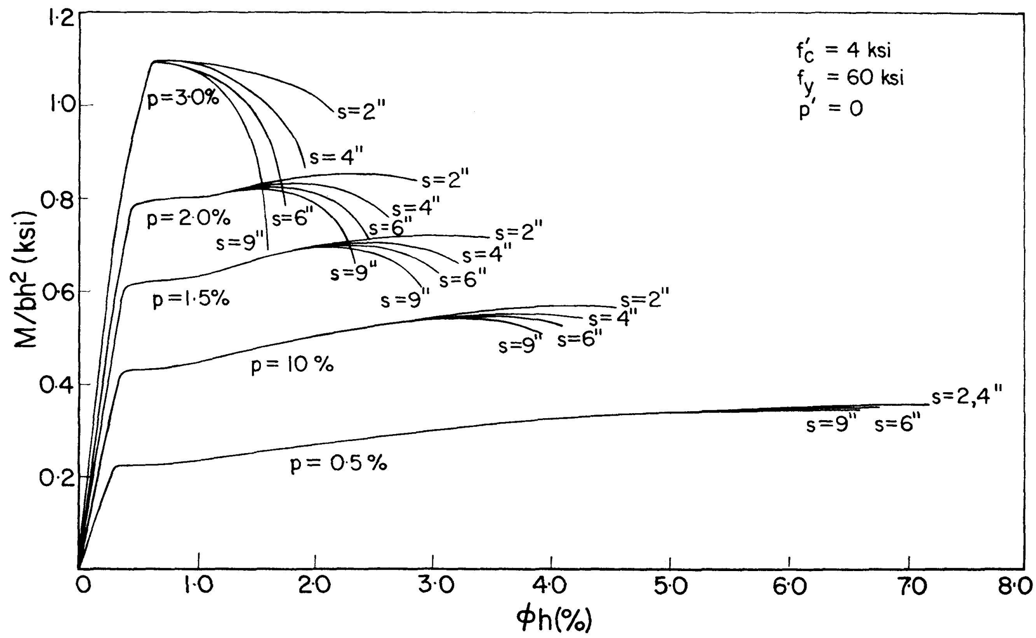

Fig. 14. Effect of tie spacing on ductility: M-ϕ diagrams.

Table 3. Ductility factor, ϕ_u/ϕ_y, for sections with various steel and concrete strengths and tension and compression reinforcement areas

f'_c ksi	f_y ksi	p %	$p'/p=0$	$p'/p=0.25$	$p'/p=0.5$	$p'/p=0.75$	$p'/p=1.0$
			ϕ_u/ϕ_y				
3	45	0.5	33.80	38.20	43.70	45.20	50.30
		1.0	15.20	21.80	25.80	31.00	37.60
		1.5	8.75	12.60	18.60	22.80	28.50
		2.0	2.78	8.63	13.20	18.25	25.00
		2.5	2.00	2.89	10.20	15.25	21.00
		3.0	1.39	2.32	8.18	13.90	19.70
		3.5	1.00	1.76	3.00	12.10	17.45
		4.0	1.00	1.33	2.47	10.65	16.60
	60	0.5	16.75	19.95	22.90	28.60	31.40
		1.0	7.10	9.61	12.95	16.20	19.40
		1.5	3.77	5.52	8.30	11.70	14.30
		2.0	1.42	3.74	6.43	9.65	12.70
		2.5	1.00	1.10	4.40	8.30	11.05
		3.0	1.00	1.00	3.54	7.32	9.85
		3.5	1.00	1.00	1.64	6.52	9.38
		4.0	1.00	1.00	1.33	5.92	8.59
	75	0.5	10.60	13.70	15.85	18.60	23.90
		1.0	4.25	5.83	8.39	11.30	13.88
		1.5	1.90	3.41	5.60	8.41	11.10
		2.0	1.00	2.00	3.89	6.77	9.12
		2.5	1.00	1.00	2.90	5.85	8.12
		3.0	1.00	1.00	2.28	5.21	7.61
		3.5	1.00	1.00	1.00	4.72	7.00
		4.0	1.00	1.00	1.00	4.30	6.47
4	45	0.5	42.40	48.10	50.10	51.20	51.80
		1.0	19.40	24.10	29.30	34.60	43.80
		1.5	12.13	15.65	19.80	27.30	33.40
		2.0	3.94	11.12	15.05	21.60	27.00
		2.5	2.89	8.37	12.60	17.95	24.40
		3.0	2.21	3.06	10.10	15.55	21.05
		3.5	1.72	2.61	8.35	13.20	19.80
		4.0	1.5	2.10	3.17	12.10	17.64
	60	0.5	20.30	23.70	27.20	33.40	34.70
		1.0	9.22	11.80	14.35	17.35	23.00
		1.5	5.35	7.42	9.48	13.70	16.40
		2.0	3.26	5.05	7.15	11.16	13.65
		2.5	1.50	3.56	5.67	9.50	12.48
		3.0	1.10	1.64	4.59	8.34	11.05
		3.5	1.00	1.33	3.66	7.41	9.95
		4.0	1.00	1.03	3.08	6.65	9.53
	75	0.5	13.05	15.50	18.30	22.60	25.45
		1.0	5.82	7.50	9.68	13.20	15.95
		1.5	2.96	4.48	6.46	9.70	12.60
		2.0	1.77	3.00	4.65	7.44	10.20
		2.5	1.00	2.03	3.61	6.27	9.00
		3.0	1.00	1.00	2.97	5.52	8.44
		3.5	1.00	1.00	2.39	4.97	7.69
		4.0	1.00	1.00	2.00	4.48	7.07
5	45	0.5	46.40	51.00	52.00	52.50	53.00
		1.0	21.80	27.70	32.60	40.40	44.30
		1.5	14.00	17.10	23.00	28.20	35.20
		2.0	9.60	13.30	17.20	21.90	26.10
		2.5	3.82	9.56	13.50	19.20	25.40
		3.0	2.84	4.06	10.90	16.20	21.90
		3.5	2.32	3.30	9.70	14.80	20.50
		4.0	1.86	2.72	8.18	12.70	19.50
	60	0.5	22.60	25.90	27.90	30.90	31.80
		1.0	10.95	12.90	15.25	19.60	23.40
		1.5	6.53	8.73	10.80	13.33	17.50
		2.0	4.43	6.05	8.21	10.50	14.30
		2.5	3.13	4.57	6.60	9.15	13.00
		3.0	1.60	3.48	5.38	7.80	12.10
		3.5	1.14	1.83	4.50	6.71	10.85
		4.0	1.00	1.46	3.74	6.24	9.85
	75	0.5	14.73	16.70	20.20	25.00	27.60
		1.0	6.64	8.56	10.70	13.30	16.95
		1.5	3.96	5.48	7.30	10.00	13.20
		2.0	2.48	3.76	5.50	7.67	11.10
		2.5	1.19	2.61	4.23	6.52	9.75
		3.0	1.00	1.87	3.39	5.73	8.72
		3.5	1.00	1.09	2.79	4.96	8.28
		4.0	1.00	1.00	2.37	4.54	7.57

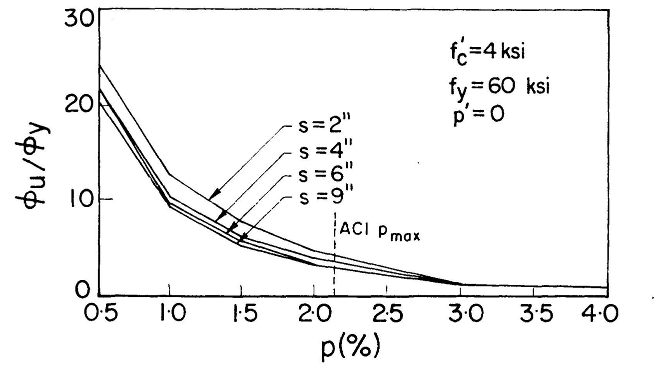

Fig. 15. Effect of tie spacing on ductility: ϕ_u/ϕ_y-p diagrams.

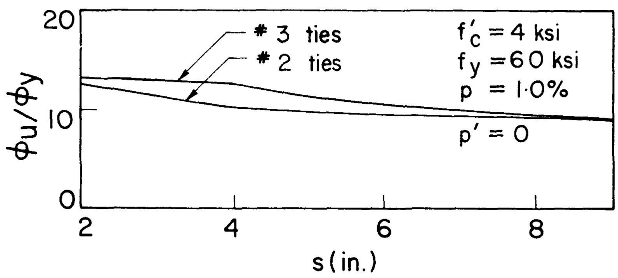

Fig. 16. Effect of tie size on ductility: ϕ_u/ϕ_y-p diagrams.

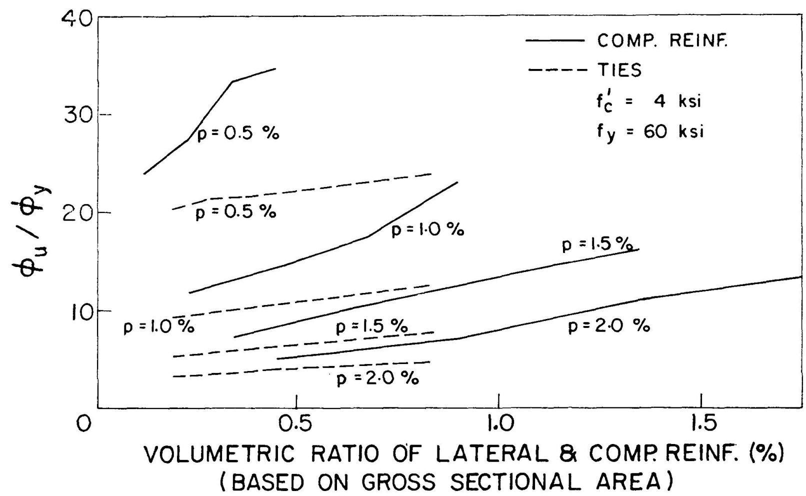

Fig. 17. Relative effect of compression and lateral reinforcements on ductility.

this is not a very effective way to improve ductility. Fig. 16 shows that ductility increases slightly with an increase in the cross-sectional area of lateral reinforcement, at all tie spacings.

Fig. 17 illustrates the relative efficiency of compression and lateral reinforcements in improving the ductility of reinforced concrete sections. It can be seen clearly that compression reinforcement is considerably more efficient than ties at all percentages of tension reinforcement. In a recent investigation, [18] Shah and Vijay Rangan studied the relative efficiency of compression reinforcement, rectangular ties and randomly oriented short steel fibres in improving the ductility of compression concrete in flexural members. Rectangular ties were found to be by far the most efficient among the three. This conclusion may seem to be contradictory to that suggested by Fig. 17. It has been pointed out, however, that this is not the case [19]. As has been mentioned while discussing various ductility definitions, the ductility of steel and concrete as materials is very different from the ductility of a reinforced concrete section.

Fig. 18 shows the effect of cover thickness on sectional $M-\phi$ relationships. ϕ_u/ϕ_y-p diagrams for different thicknesses of cover are plotted in Fig. 19. It can be seen that the influence of cover thickness on sectional ductility is negligible.

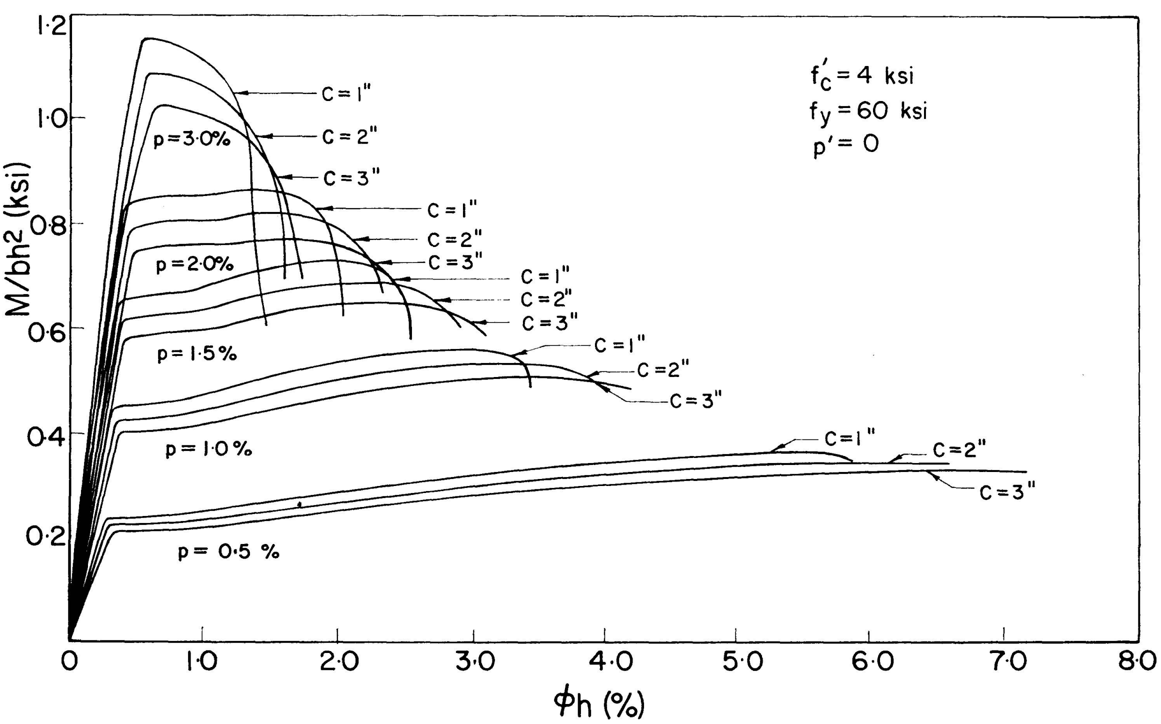

Fig. 18. Effect of concrete cover on ductility: M-ϕ diagrams.

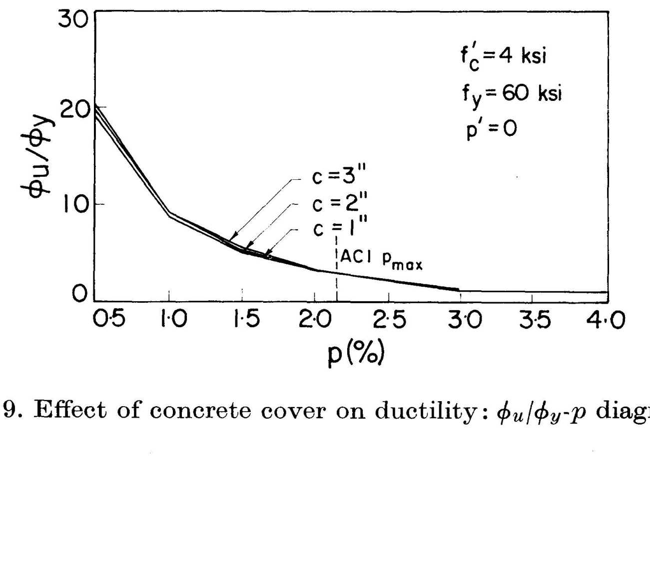

Fig. 19. Effect of concrete cover on ductility: ϕ_u/ϕ_y-p diagrams.

Loading Variables

In creep analysis, the major factors are the rate of loading, loading duration and the age of concrete at the time of loading. Since control of the rate of loading is difficult in actual structures, a conventional loading rate should be adopted. A practical proposal, due to RÜSCH [25] and adopted in this study, is to assume that the load is applied in about 20 minutes at constant rates and sustained subsequently up to failure. The age of concrete at the time of loading was assumed to be 28 days for all the sections studied. Fig. 20 illustrates $M-\phi$ relationships for different *durations of loading*, corresponding to various percentages of tension reinforcement. $\phi_u/\phi_y - p$ diagrams are plotted for different loading durations in Fig. 21. It can be seen that the effect of creep

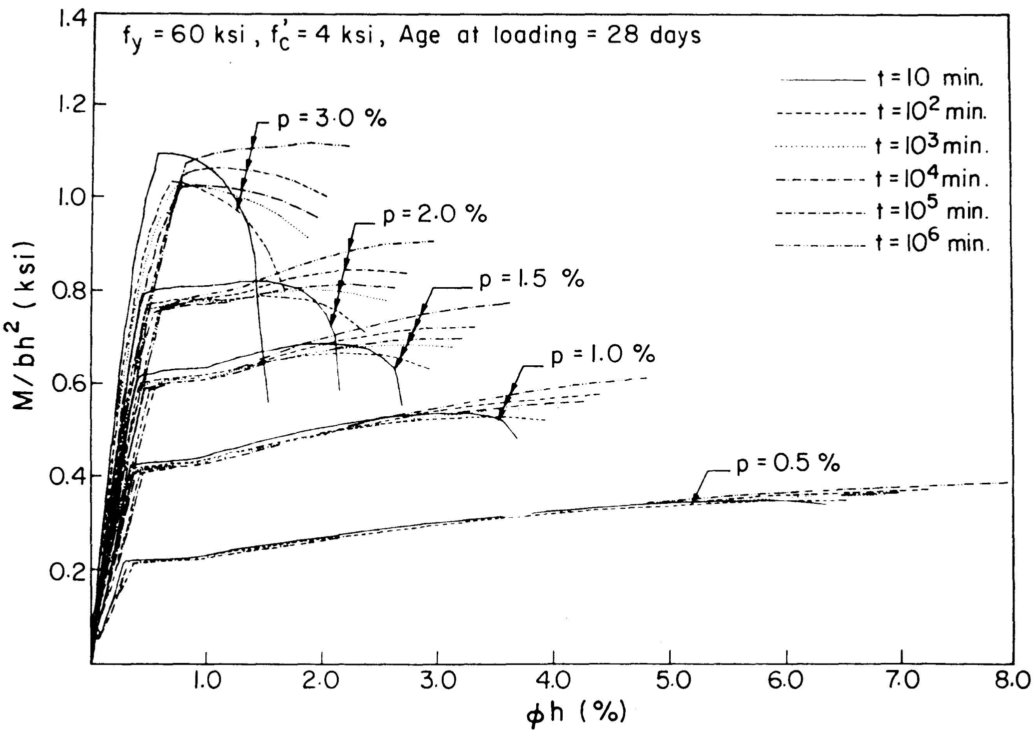

Fig. 20. Effect of loading duration on ductility: M-ϕ diagrams.

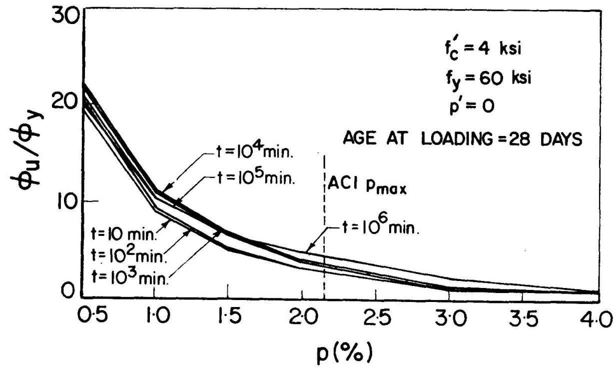

Fig. 21. Effect of loading duration on ductility: ϕ_u/ϕ_y-p diagrams.

on ductility is, in general, favourable, although for low reinforcement percentages, ductility drops a little when loading duration exceeds 10^4 min. ($\doteq$ 1 week). On the whole, the effect of loading duration on ductility is found not to be very significant.

Fig. 22 shows the effect of *axial loading* on sectional $M-\phi$ relationships for a particular quality of concrete and of reinforcing steel and for various amounts of reinforcement. $\phi_u/\phi_y - p\,(=p')$ diagrams for various levels of axial load and various qualities of steel and concrete are plotted in Fig. 23 for symmetrically reinforced rectangular sections. Load-moment interaction diagrams for sections with $f_c' = 4$ ksi, $f_y = 60$ ksi and different $p = p'$ are illustrated

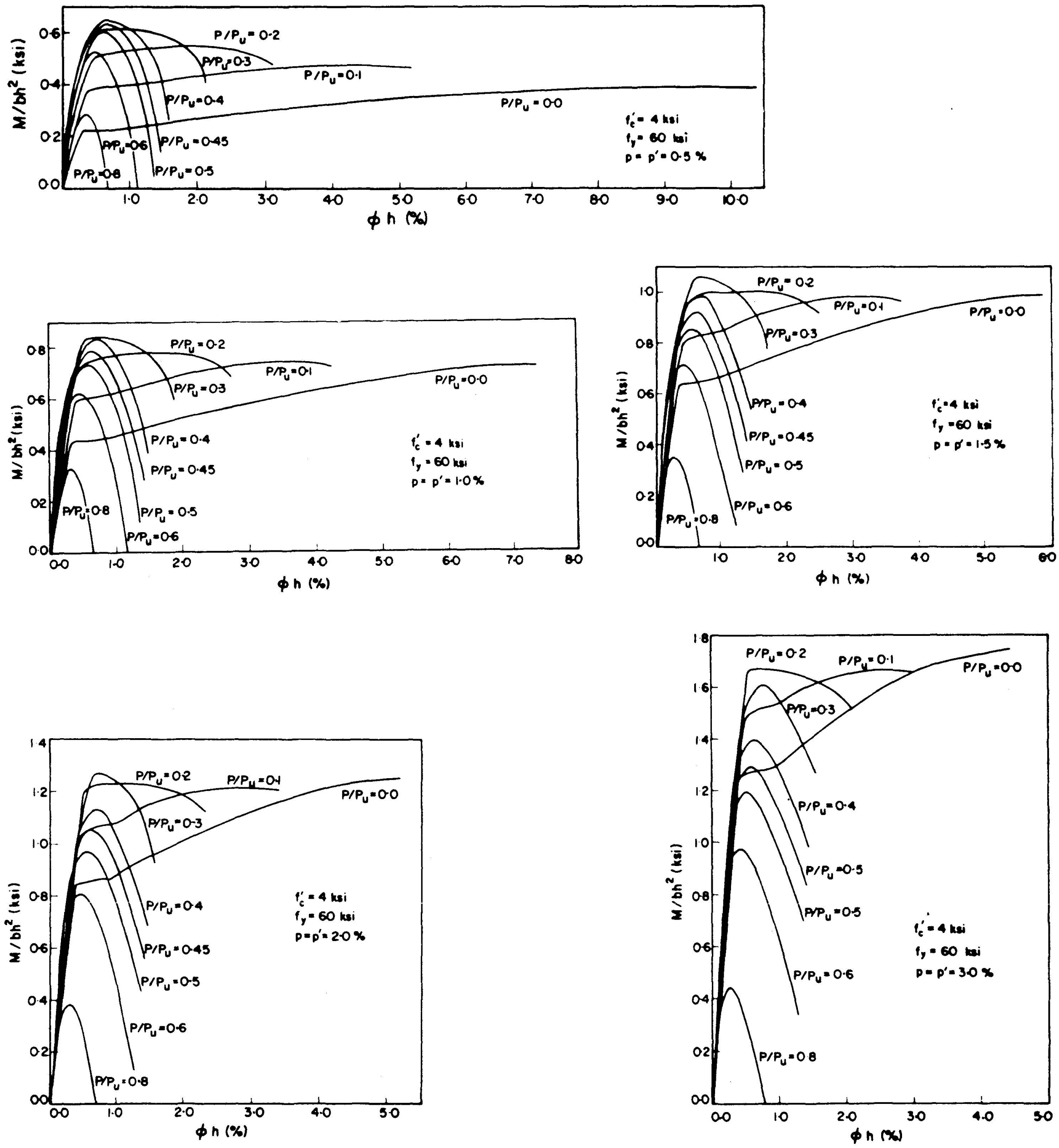

Fig. 22. Effect of axial loads on ductility: M-ϕ diagrams.

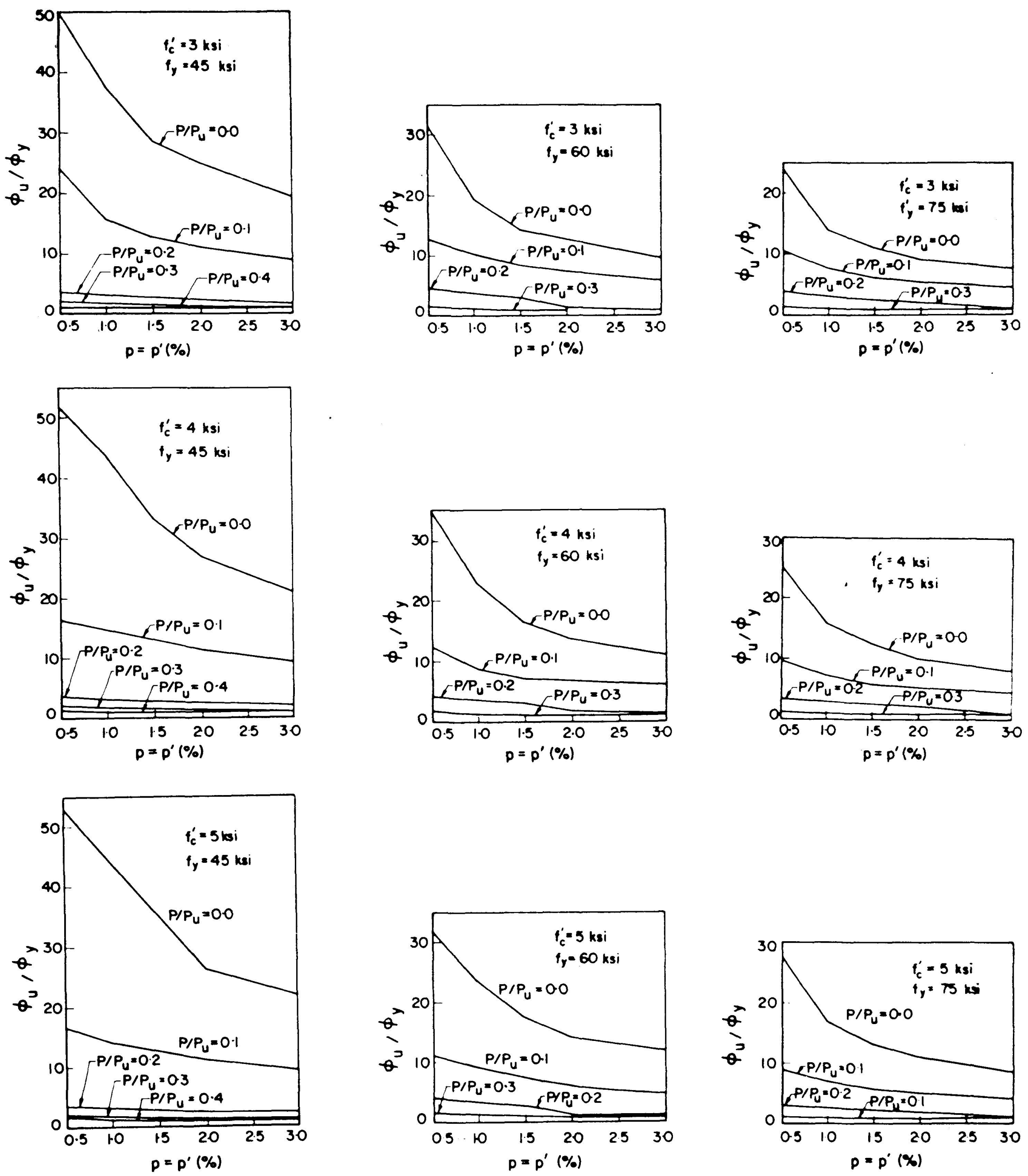

Fig. 23. Effect of axial loads on ductility: ϕ_u/ϕ_y-p diagrams.

(in full lines) in Fig. 24. The shape of these diagrams in the region which corresponds to tension failure is to be noted. This shape is a consequence of the ultimate stage definition adopted in this study. The interaction diagrams drawn in full lines change to those indicated by dotted lines when the ultimate stage is defined by the extreme compression fibre strain reaching a value of 0.3%. Fig. 24 indicates that sectional behaviour is governed by tension only as long as axial load levels do not exceed 20 to 35% (depending upon the amount of reinforcement) of the axial load carrying capacity of the section. Fig. 23 indicates that as long as failure is governed by tension, a section is

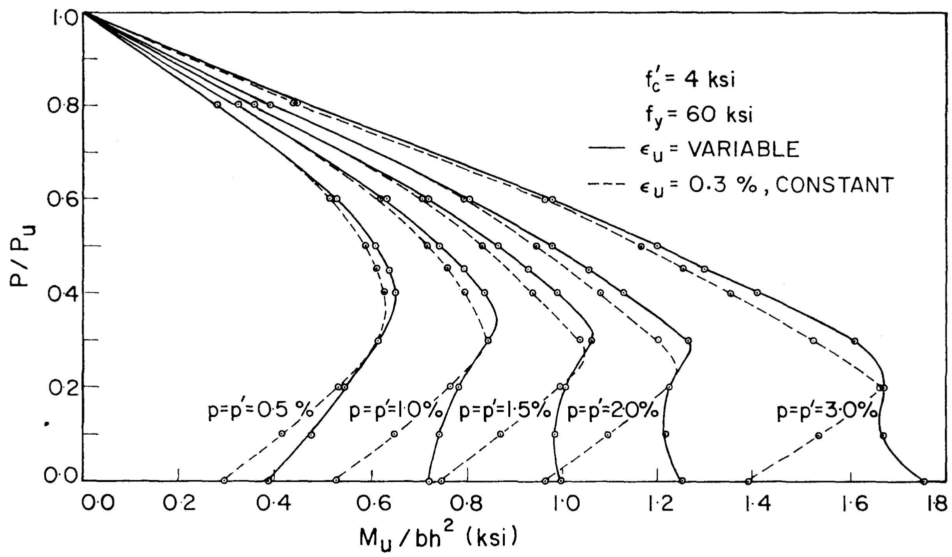

Fig. 24. Interaction diagrams for symmetrically reinforced sections.

capable of mobilizing a certain amount of ductility, although this decreases drastically as the axial load on the section approaches the level corresponding to balanced failure. Fig. 23 also indicates that for the low levels of axial load usually carried by flexural members in reinforced concrete frames (not exceeding 10–15% of the axial load carrying capacity), fairly high amounts of sectional ductility are always available.

The effects of *repeated loading and load reversals* were not investigated by the authors. They would like, however, to refer here to an experimental investigation into these effects carried out by NEWMARK and HALL [10]. Several singly and doubly reinforced beams were tested such that the load on each beam was removed completely and then reapplied at several stages during a test. The results indicated that the removal and reapplication of load had little or no effect on either the load carrying capacity or the ductility. A few symmetrically reinforced beams were tested under repeated reversals of load. There were some indications that while load carrying capacity remained unchanged, ductility was impaired, albeit only slightly, by such repeated load reversals.

Summary and Conclusions

The findings of the present investigation can be summarized as follows:

1. The various factors affecting the ductility of a reinforced concrete section can be divided into three groups: material, geometric and loading variables.

2. a) Among the material factors, the quality of concrete and the grade of reinforcement appear to be the most important. Sectional ductility increases with higher concrete grades and lower tension reinforcement strengths.

b) Sectional ductility increases as the strain-hardening modulus of tension reinforcement, E_{sh}, is increased from zero to a certain value, but then decreases as E_{sh} is further increased.

c) Ductility is the highest for sections with full bond.

d) The tensile strenght of concrete does not appreciably affect sectional ductility.

3. a) Among the geometric variables, sectional width and depth do not appear to affect sectional ductility.

b) The overhanging flange areas of T-sections improve sectional ductility considerably.

c) The ductility of a section decreases as the amount of tension reinforcement increases, very little ductility being available as p approaches the balanced steel percentage.

d) Sectional ductility can be improved by decreasing the spacing and increasing the amount of lateral reinforcement, and also by the addition of suitable amounts of compression reinforcement. The latter is found to be much more efficient than the former in providing sectional ductility. When an improved ductility is desired, the addition of suitable amounts of compression reinforcement seems to be the best way to provide it.

e) The influence of cover thickness on sectional ductility appears to be negligible.

4. Among the loading factors, only the effects of duration of loading and of axial loading are studied in detail.

a) The effect of loading duration on sectional ductility is found not to be very significant.

b) A section is found capable of mobilizing some ductility as long as failure is governed by tension, although the amount of this available ductility decreases drastically as the level of axial load approaches that corresponding to balanced failure. However, fairly high amounts of sectional ductility are found to be available for sections subject to axial loads in the order of 10–15% of their carrying capacities.

The following conclusions can be drawn from this study:

1. The ductility of reinforced concrete sections in bending is primarily influenced by p, p', f'_c and f_y. Other variables have a secondary effect. (However, the amount and distribution of lateral reinforcement have an essential role in improving shear behaviour and preventing premature shear failure.)

2. The ductility factor is significantly reduced for high grade steels. However, when such steels are combined with relatively high grade concretes, the ductility factor is sufficiently large to accommodate plastic redistribution for usual, economical steel percentages. (For $\phi_u/\phi_y \geqq 5$ with $f_y = 75\,\text{ksi}$, $f'_c = 5\,\text{ksi}$ and $p' = 0$, $p \doteq 1.4\%$ follows.)

3. Since most reinforced concrete sections contain some compression rein-

forcement (because of construction or code requirements), the available ductility is usually adequate and larger than it would appear.

4. If a minimum value of $\phi_u/\phi_y = 5$ is accepted as a prerequisite for practical inelastic design, plastic action can be permitted in reinforced concrete columns, provided that the sections fail in tension under combined bending and compression.

Notations

A — $E_c \epsilon_0 / k_3 f'_c$.
A_s — area of tension steel.
A'_s — area of compression steel.
b — width of section.
c — thickness of cover.
C_c — resultant force in compression concrete.
C_s — resultant force in compression steel.
d — effective depth or depth of tension steel from extreme compression fibre.
d' — depth of compression steel from extreme compression fibre.
d'' — tie diameter.
D — parameter of compression concrete stress-strain relationship.
e' — eccentricity of external load from centroid of tension steel.
e_p — proof strain of high strength steel.
E_c — initial tangent modulus of elasticity of concrete.
E_s — modulus of elasticity of steel.
E_{sh} — strain-hardening modulus of steel.
EI — $dM/d\phi = \Delta M / \Delta \phi$, flexural rigidity of section.
f'_c — standard cylinder strength of concrete.
f_y — yield strength of steel.
F — $\epsilon_s k / \epsilon_c (1-k)$, bond factor.
h — total depth of section.
k — relative depth of neutral axis.
k_2 — parameter indicating position of resultant compression concrete force.
k_3 — ratio of maximum stress to cylinder strength.
M — sectional moment.
M_u — ultimate moment.
p — A_s/bd, tension reinforcement ratio.
p' — A'_s/bd, compression reinforcement ratio.
p_{max} — maximum permissible tension reinforcement ratio.
P — axial load on section.
P_u — ultimate load.
s — spacing of lateral reinforcement.
t — loading duration in minutes.
T_c — resultant force in tension concrete.
T_s — resultant force in tension steel.

x ϵ/ϵ_0.
y distance of a sectional fibre away from neutral axis.
y_t depth of uncracked area in tension from neutral axis.
ϵ longitudinal strain, specifically compressive strain in concrete.
ϵ_0 compression concrete strain corresponding to maximum stress.
ϵ_p $e_p + \sigma_p/E_s$.
ϵ_s strain in tension steel.
ϵ_s' strain in compression steel.
ϵ_{sh} steel strain at the onset of strain-hardening.
ϵ_t tensile strain in concrete.
ϵ_{tr} cracking strain of concrete.
ϵ_u ultimate strain, specifically in compression concrete.
ϵ_y yield strain, specifically in compression concrete.
σ compressive stress in concrete.
σ_p proof stress of high strength steel.
σ_s stress in tension steel.
σ_s' stress in compression steel.
σ_t tensile stress in concrete.
σ_{tr} modulus of rupture of concrete.
ϕ sectional curvature.
ϕ_u curvature corresponding to ultimate moment.
ϕ_y curvature corresponding to yield moment.

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Summary

An analytical investigation simulating tests on some 1700 specimens is performed, in order to evaluate the effect of a large number of factors on the ductility of reinforced concrete sections in pure and combined bending.

The analysis is based on reliable stress-strain characteristics for steel and concrete and enables moment-curvature and ductility factor-steel percentage diagrams to be prepared for a broad range of combinations of the variables.

The study demonstrates that reinforced concrete sections possess a significant amount of ductility which can be used advantageously within a wide spectrum of design variables.

Résumé

Une étude analytique simulant environ 1700 essais est effectuée afin de trouver l'influence d'un grand nombre de facteurs sur la capacité de déformation des sections en béton armé sous flexion pure et composée.

L'analyse est basée sur des diagrammes contraintes-déformations éprouvés pour l'acier et le béton. Elle offre la possibilité de trouver pour beaucoup de combinaisons de variables les diagrammes moments-courbures et les diagrammes capacité de déformation-pourcentage d'acier.

L'étude montre que les sections en béton armé possèdent une capacité de déformation accentuée dont on peut profiter dans un large spectre des variables.

Zusammenfassung

Es wird eine analytische Untersuchung, die Versuche an ungefähr 1700 Proben simuliert, durchgeführt, um den Einfluss einer Grosszahl von Faktoren auf die Verformungsfähigkeit armierter Betonquerschnitte unter reiner sowie zusammengesetzter Biegung zu ermitteln.

Die Berechnung beruht auf zuverlässigen Spannungs-Dehnungs-Diagrammen für Stahl und Beton und ermöglicht das Erstellen der Momenten-Krümmungs- sowie der Verformungsfaktor-Stahlgehalt-Diagramme für viele Kombinationsmöglichkeiten der Variablen.

Die Studie zeigt, dass armierte Betonquerschnitte eine ausgeprägte Verformungsfähigkeit besitzen, welche innerhalb eines breiten Spektrums von konstruktiven Annahmen vorteilhaft ausgenützt werden kann.

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