Inelastic analysis of reinforced concrete panels: theory

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Inelastic Analysis of Reinforced Concrete Panels: Theory[1])

Analyse non-élastique de plaques acier/béton: Théorie[1])

Unelastische Analyse von Stahlbetonscheiben: Theorie[1])

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Introduction

This study is concerned with the inelastic analysis of reinforced concrete planar elements which are subjected to in-plane forces. Planar elements, such as walls or deep beams, often form parts of complex structural systems, for example, shear walls in multistory buildings, shells, folded plate roofs, or box girders.

A large number of special analytical methods developed for this type of structure can be divided into two groups. In the first group are the methods in which the walls or deep beams are treated as ordinary beams. This greatly simplifies analysis and allows the use of various well-developed beam-type analyses. Many methods for shear wall-frame systems are using this approach. Most of these analyses are elastic solutions, assuming homogeneous isotropic material [1], [2], [3]. Recently, limit analysis methods based on plastic hinge theory were applied to shear wall-frame structures in order to investigate the ultimate behavior and collapse patterns [4], [5].

The required reinforcement is usually designed for the internal forces resulting from the elastic analysis. In the case of shear walls either working stress or ultimate strength methods are used for reinforcement design. However, these design methods were developed for ordinary beams and there is neither much experimental nor theoretical evidence to support its extension to thin

[1]) This material is presented in two parts. This is the first paper which describes the theoretical aspects. A second paper will present experimental verification and applications.

deep walls. Since shear walls are responsible for lateral stability of multistory buildings, it seems useful to study their behavior in the inelastic stages, and to investigate the effect of cracks and the nature of the failure mechanisms.

The second analytical approach considers the planar elements in general plane stress state. This analysis involves a solution of a two-dimensional continuum problem (shells, folded-plate roofs, walls). The usual assumption in these analyses is that of an elastic, homogeneous, and isotropic material. The elastic solution can be obtained either in closed form, or by means of numerical computer methods. However, the cracking of the concrete at design loads causes stress redistribution, so that reinforcement design for elastic stresses may be questioned. Furthermore, yielding of the steel and cracking or crushing of the concrete should be considered for a realistic assessment of the ultimate strength of such structures. Thus, a two-dimensional theory capable of including these inelastic effects appears useful.

Modern analytical methods and available data about the behavior of basic materials make such a task possible. In particular, the recent development of the finite element method permits solutions to be obtained for any rationally conceived constitutive laws of the materials. An insight into the inelastic behavior of planar elements would give confidence to structural engineers in their use of various approximate methods.

The aim of this work is to help clarify the above mentioned questions by providing a tool for more detailed analysis. The purpose of the proposed analysis is to predict the response of reinforced concrete panels at all load stages throughout their load history including failure for both monotonic and cyclic loadings.

The considered reinforced concrete panel is a planar element with variable thickness, reinforced in its midplane. The panel is in plane stress state; only in-plane external forces are considered. The effect of bending moments is not included in this work.

The Characteristics of Reinforced Concrete Behavior

The characteristic stages of reinforced concrete behavior can be illustrated by means of a typical load-displacement relationship as shown in Fig. 1. This relationship can be, for example, result of a beam test. Similar diagrams can be obtained for the load-deformation relations of any other reinforced concrete structure. This highly nonlinear relationship may be roughly divided into three intervals; the uncracked elastic stage, crack propagation, and the plastic stage. The non-linear response is caused by two major material effects, cracking of the concrete, and plasticity of the reinforcement and of the compression concrete. Only these two material non-linearities will be considered in the proposed analysis. Obviously, cracking and plasticity can occur simultaneously which must be accommodated in the proposed theoretical model.

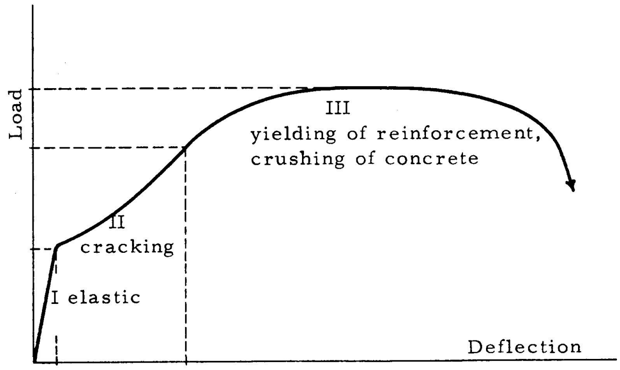

Fig. 1. Typical Load-Displacement Diagram for a Reinforced Concrete Element.

There are other nonlinear effects that are not included in this model. Among these, the bond slip between reinforcement and concrete, the effects of dowel action, aggregate interlocking of crack faces, and the deterioration of crack surfaces which prevent complete closing of cracks under load reversals may be mentioned.

Analytical Model of the Reinforced Concrete

The analytical model of the structure is capable of describing the interaction of uncracked or cracked concrete, and reinforcing in any number of arbitrary directions. The assumed action is as follows:

The uncracked concrete itself is assumed as isotropic homogeneous material in the plane stress state. The cracked concrete is considered anisotropic and capable of resisting only normal stresses parallel to the crack direction. This implies that the crack spacing is infinitesimal. The crack direction is perpendicular to the principal tension in the concrete just prior to crack formation.

Similarly, the reinforcement is considered as continuous medium; no individual bars are considered; instead, continuous distribution of reinforcement area within any one concrete element is assumed. Both cracked concrete and reinforcement are in uniaxial stress state.

It follows from the above assumptions that the reinforced concrete in the cracked state can be visualized as a planar lattice structure with infinitesimal mesh size. One set of lattice links is formed by the concrete columns and one is formed for each reinforcement direction. These links form a truss structure through which all internal normal and shear forces are transmitted. Perfect compatibility of deformation between concrete and reinforcement is assumed.

Any rational constitutive relation can be applied to the components of the above defined material. In this study the simplest bi-linear laws were adopted.

The uniaxial stress-strain relationship for concrete is assumed elastic-perfectly plastic in compression, and elastic and brittle in tension as shown in Fig. 2. The concrete plasticity is limited by maximum concrete strain ϵ_u. The yield criterion for two-dimensional plasticity of concrete is assumed of the Von Mises form. The maximum normal stress theory is applied to tensile failure. The complete assumed strength criterion is shown in Fig. 3. The dashed line indicates the experimental criterion obtained in Ref. [6].

The stress-strain relationship for reinforcement is assumed elastic-perfectly plastic, as shown in Fig. 4.

The validity of these assumptions must be determined by tests such as will be described later. In many cases they lead to acceptable results.

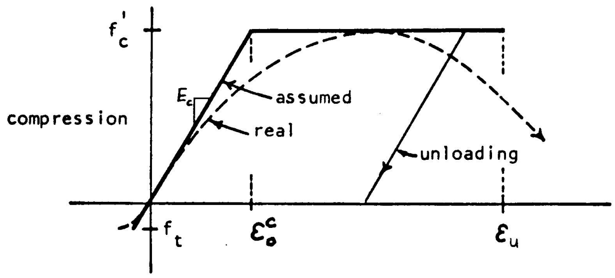

Fig. 2. Uniaxial Stress-Strain Relation for Concrete.

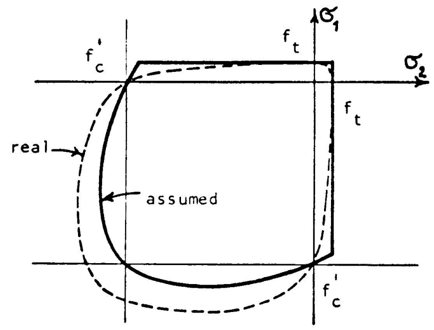

Fig. 3. Biaxial Strength of Concrete.

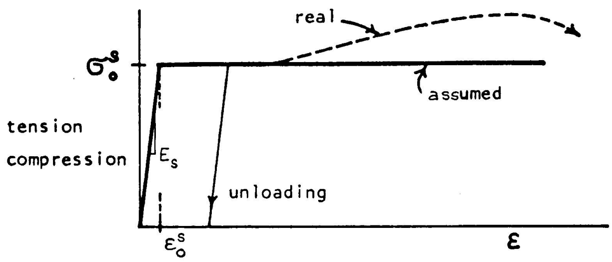

Fig. 4. Uniaxial Stress-Strain Relation for Steel Reinforcement.

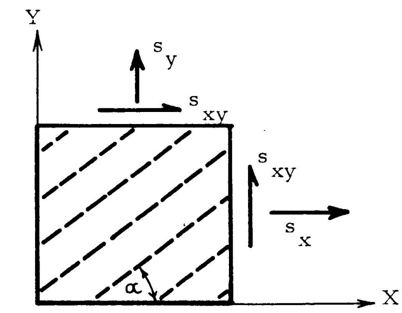

Fig. 5. Total Stresses $\{s\}$ in the element of Reinforced Concrete Panel.

Material Stiffness Formulation

The term "material stiffness" is used for stress-strain relations of an infinitesimal element in analogy with the designations "element stiffness" and "structure stiffness" which determine force-displacements relation of elements and structures. The material stiffness is derived on the basis of the above defined theoretical model of the reinforced concrete.

The material stiffness of the composite element is obtained by superposition of material stiffnesses of the individual material components, concrete and reinforcement. For the purpose of this derivation, let us consider an element of a reinforced concrete panel subjected to plane stress, as shown in Fig. 5. The side dimensions and the thickness of the element are equal to unity. A stress-strain relation for the element can be written in the form

$$\{s\} = [D]\{\epsilon\}, \tag{1}$$

where $\{s\} = [s_x\, s_y\, s_{xy}]^T$ is the total stress vector, $[D]$ is the composite material stiffness matrix, and $\{\epsilon\} = [\epsilon_x\, \epsilon_y\, \epsilon_{xy}]^T$ is the strain vector. The strains are common for all component materials, while the total stress vector is the sum of the component stress vectors

$$\{s\} = \{s^c\} + \sum_1^n \{s^i\}. \tag{2}$$

$\{s^c\}$ is the concrete stress vector and $\{s^i\}$ is the reinforcement stress vector for the reinforcement in the i-th direction. Stresses $\{s\}$, $\{s^c\}$, $\{s^i\}$, act on a unit area of the composite cross section of the panel; it should be noted that the total stresses $\{s\}$ do not represent real stresses, but internal forces acting on a composite element.

These stresses can be found from the strains by

$$\{s^c\} = [D^c]\{\epsilon\}, \tag{3}$$

$$\{s^i\} = [D^i]\{\epsilon\}. \tag{4}$$

in which $[D^c]$ and $[D^i]$ are the concrete and reinforcement material stiffness matrices respectively. Substituting Eq. (3) and (4) in Eq. (2) and comparing Eq. (1) and (2), the composite material stiffness matrix can be formed by superposition of component material stiffness matrices as follows:

$$[D] = [D^c] + \sum_1^n [D^i], \tag{5}$$

where n is the number of reinforcing directions.

The component material stiffness matrices must reflect all stages of assumed material behavior, namely, elastic and plastic stages of the concrete and of the reinforcement and the uncracked and cracked stages of concrete. Matrices for all these cases are listed in the next section. A detailed derivation of component stiffness matrices is given in Ref. [7].

Component Material Stiffness Matrices

Elastic Uncracked Concrete

This matrix is immediately available because it represents Hooke's Law for plane stress in an isotropic material

$$[D^c] = \frac{E_c}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}. \tag{6}$$

Plastic Uncracked Concrete

In this case the approach of the Theory of Plasticity is adopted which takes into account the Von Mises yield condition and associated flow rule. The material stiffness matrix including these plasticity conditions was derived in detail in Ref. [8] and [9] for incremental stresses $\delta\{s^c\}$ and incremental strains $\delta\{\epsilon\}$

$$\delta\{s^c\} = [D]_{ep}\,\delta\{\epsilon\}, \tag{7}$$

where the elasto-plastic matrix $[D]_{ep}$ is

$$[D]_{ep} = [D]([I]-\{g\}\{g\}^T[D](\{g\}^T[D]\{g\})^{-1}). \tag{8}$$

In this, $[D]$ is the elastic matrix given in Eq. (6), and

$$\{g\}^T = \frac{1}{\sigma_0^c}[(s_x^c - \tfrac{1}{2}s_y^c)\,(s_y^c - \tfrac{1}{2}s_x^c)\,(3\,s_{xy}^c)]. \tag{9}$$

The concrete material stiffness matrix for the plastic stage of the uncracked concrete, $[D^c]=[D]_{ep}$. It depends on the current concrete stresses, $\{s^c\}$ but is constant for a sufficiently small load increment.

Elastic Cracked Concrete

The cracked concrete is subjected to normal stress $\{s_u{}^c\}$ parallel to the cracks. The uniaxial stress-strain relation in the U-direction, as indicated in Fig. 6, is

$$s_u^c = E_c\,\epsilon_u. \tag{10}$$

Transformation of Eq. (10) from U, V- to the X, Y-coordinates using appropriate stress and strain transformation gives

$$\{s^c\}_x = [D^c]\{\epsilon\}_x, \tag{11}$$

where the material stiffness matrix of the cracked concrete is

$$[D^c] = E_c\begin{bmatrix} \cos^4\beta & \cos^2\beta\sin^2\beta & \cos^3\beta\sin\beta \\ \cos^2\beta\sin^2\beta & \sin^4\beta & \cos\beta\sin^3\beta \\ \cos^3\beta\sin\beta & \cos\beta\sin^3\beta & \cos^2\beta\sin^2\beta \end{bmatrix}. \tag{12}$$

The angle β indicates the direction of the cracks as shown in Fig. 6.

Plastic Cracked Concrete

In the plastic range the increment of strain does not produce any additional stresses. The instantaneous modulus E_c can be considered zero. Consequently, it follows from Eq. (12) that the component material stiffness matrix is a zero matrix

$$[D^c] = [0]. \tag{13}$$

Elastic Reinforcement

The reinforcement is subjected to uniaxial stress. The material stiffness derivation for the i-th reinforcement inclined by an angle α with the X-axis is very similar to the case of cracked concrete. The uniaxial stress-strain relation in the U-direction (Fig. 7) is

$$s_u^i = p^i E_s \epsilon_u, \tag{14}$$

where p^i is the steel ratio.

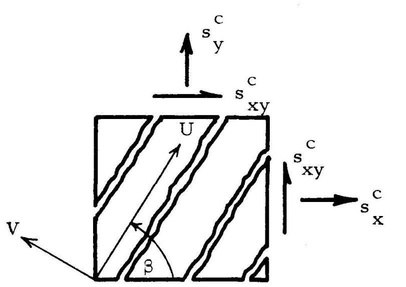

Fig. 6. Concrete Stresses $\{s^c\}$ in the Cracked Concrete.

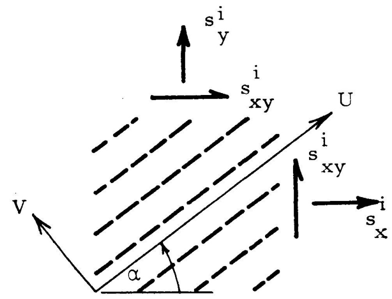

Fig. 7. Reinforcement Stresses $\{s^i\}$.

The transformation of Eq. (14) to the X, Y-coordinate system gives

$$\{s^i\}_x = [D^i]\{\epsilon\}_x, \tag{15}$$

where the material stiffness matrix of i-th reinforcement is

$$[D^i] = p^i E_s \begin{bmatrix} \cos^4\alpha & \cos^2\alpha\sin^2\alpha & \cos^3\alpha\sin\alpha \\ \cos^2\alpha\sin^2\alpha & \sin^4\alpha & \cos\alpha\sin^3\alpha \\ \cos^3\alpha\sin\alpha & \cos\alpha\sin^3\alpha & \cos^2\alpha\sin^2\alpha \end{bmatrix}. \tag{16}$$

Plastic Reinforcement

Similar to the case of cracked concrete, the instantaneous modulus E_s is zero and the component material stiffness matrix is a zero matrix

$$[D^i] = [0]. \tag{17}$$

All listed component material stiffness matrices, except the one for two-dimensional plasticity of concrete, are valid over finite load intervals. For example, Eq. (6) holds throughout the elastic uncracked stage, and Eq. (17) is valid for the entire period in which the yield stress of the reinforcement is reached. Only the elasto-plastic matrix given by Eq. (8) is derived for small load increments. However, it was shown in Ref. [9] that relatively large load increments can be applied in this case as well.

Crack Direction

It is assumed that the concrete cracks when the concrete principal tension reaches the tensile strength. The direction of the concrete cracks is perpendicular to the principal tension in the uncracked concrete just prior to crack formation. It is, therefore, necessary to determine this direction.

The composite material stiffness given by Eq. (5) may be regarded as Hooke's Law generalized for anisotropic materials. The degree of anisotropy depends on the amount and the mechanical properties of the reinforcement. In general, anisotropy causes deviation between principal stress and principal strain direction. This implies that the principal directions of concrete stresses, which are identical with the principal strain directions, deviate from the principal directions of the total stresses.

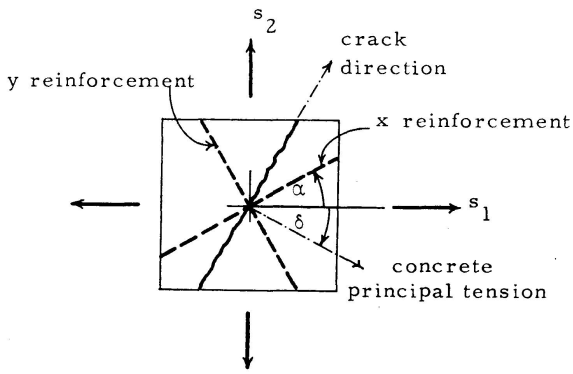

Fig. 8. Deviation of Crack Direction from Total Principal Stress Direction Caused by Reinforcement.

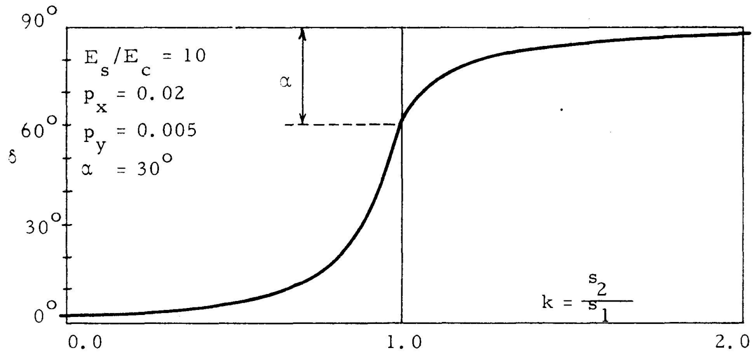

Fig. 9. Effect of Stress State on the Deviation of Concrete Principal Directions.

The magnitude of this deviation depends on the reinforcing ratios, and also on applied total stresses. An example for an orthotropically reinforced element is used to examine the deviation quantitatively. The notation of the angles is indicated in Fig. 8; the reinforcement parallel to the X and Y axes is inclined by the angle α with the direction of the total principal stress s_1; the angle δ denotes the deviation of the concrete principal stress σ_1^c from the total principal stresses s_1.

The effect of the stress state on this deviation is shown in Fig. 9 as a function of the stress ratio $k = s_2/s_1$ for the case of an element with $p_x = 0.02$, $p_y = 0.005$, $\alpha = 30°$, $E_s/E_c = 10$. Fig. 9 shows that the deviation is strongly affected by the ratio of principal stresses k. In the case of equal stresses, $k = 1.0$, the concrete principal directions coincide with the axes of orthotropy and therefore, $\delta = 90° - \alpha$. However, the deviation is relatively small for a wide range of stresses which are likely to occur in structures.

In some other approaches [10], [11] no distinction is made between the concrete and the total principal stresses, leading to crack directions different from those in this work in which the concrete stresses are considered separately. The magnitude of this difference is the same as the deviation of the concrete principal stress direction from that of the total principal stresses.

Comparison of the Theoretical Reinforced Concrete Model with Tests

The theoretical model of reinforced concrete which was derived deviates from real behavior in many respects, in order to simplify the mathematical formulation. Therefore, it is appropriate to compare theoretical predictions with experimental results. The data of PETER [10) are used for this purpose.

The test specimens in Ref. [10] contained uniformly distributed orthogonal reinforcement inclined with respect to the principal tension axis as shown in Fig. 10. A total of 9 panels was tested in uniaxial tension; 7 panels had equal

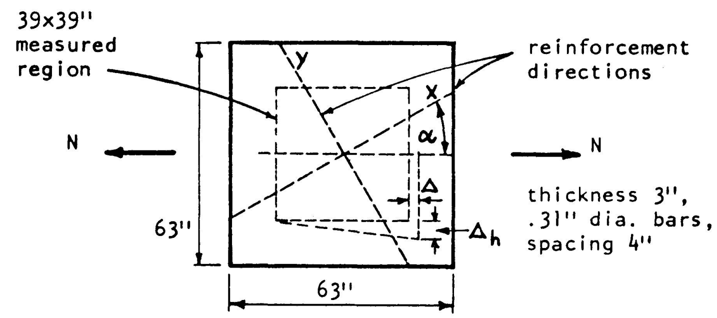

Fig. 10. Peter's Test Panel Subjected to Uniaxial Tension.

reinforcement in both directions and variable angle α from 0° to 40°. Two panels were orthotropically reinforced. The tensile force N was transferred to a panel edge by a mechanism ensuring uniform stress distribution.

The load-extension relationships of tested panels compared with theoretical results are shown in Fig. 11. The theoretical lines are obtained from Eq. (5) and (1).

The comparison of the theoretical and experimental load-extension diagram in Fig. 11 shows that the theoretical stiffness in the cracked stage underestimates the real stiffness. This could be expected, because in the test speci-

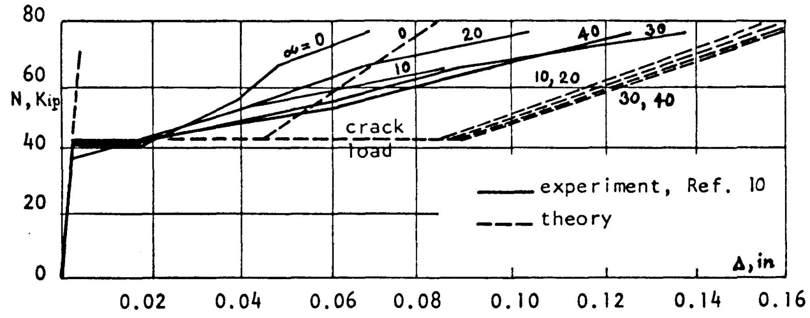

Fig. 11. Load-Extension Relationships.

mens the reinforcing bars between adjacent cracks are embedded in the concrete which contributes to the stiffness. This effect diminishes with bond slip progression at higher load and this leads to the better agreement between the theoretical and experimental stiffness at higher load.

Unfortunately, the arrangement of the test is an exceptional case in which the theoretical stiffness varies discontinuously with the angle α. Such a case arises only when the cracks are perpendicular to the applied principal tension; then the stiffness for $\alpha = 0°$ is about twice of that for $0° < \alpha < 45°$.

The transverse displacement Δ_h, defined in Fig. 10, essentially represents the shear deformation of the panel caused by uniaxial tension. The relationship between the transverse displacement and the reinforcement direction α is shown in Fig. 12 for both theoretical and experimental results. Good agreement between the analytical and experimental transverse displacements is observed for $30° < \alpha < 45°$. However, when the angle α approches $0°$ the theoretical model basically differs from the real behavior. This discrepancy is mainly caused by the lattice model of the cracked element.

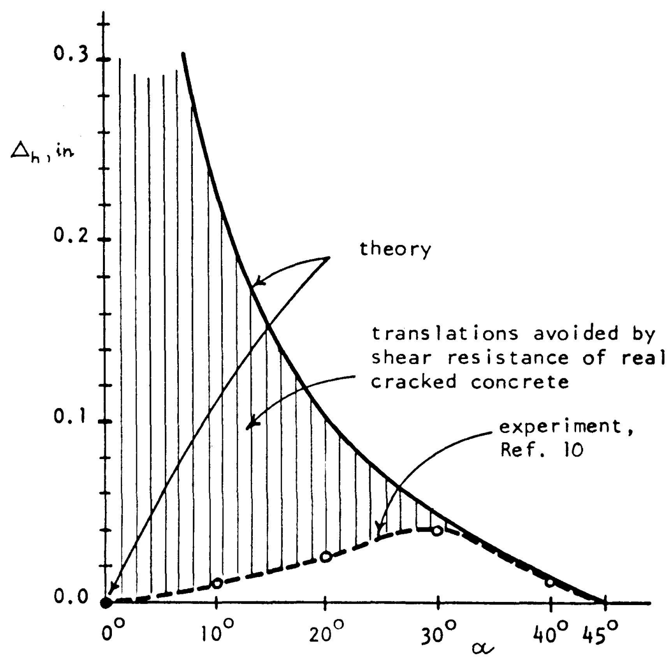

Fig. 12. Effect of Reinforcement Direction on the Transverse Displacement at Load $N = 77$ Kips.

The shear stiffness of the theoretical lattice model for the shear parallel to the cracks is very small as $\alpha \to 0°$. Therefore, the big shear displacements of the theoretical model can be eventually prevented by very small forces, so that from the viewpoint of stress analysis the discrepancy indicated by Fig. 12 is not so serious.

In the real element the shear deformations are prevented by the shear resistance of the cracked concrete owing to the irregular cracks (aggregate interlocking) and the dowel action of reinforcement [12], [13]. The shaded area in Fig. 12 therefore indicates the discrepancy caused by neglecting the aggregate interlocking and dowel action effects. Similar behavior was observed from comparison of reinforcement stresses.

On the basis of this comparison, some general conclusions can be drawn. The theoretical model is capable of qualitative simulation of the real element. The stiffness of the model is generally smaller than that of the real element. The magnitude of this deviation depends on the applied stress state. The underestimation of the theoretical stiffness in the cracked stage is caused by neglect of concrete interaction between adjacent cracks (or infinitesimal spacing of cracks) in the model, and by neglect of the shear resistance of the cracked concrete owing to the aggregate interlocking and to the reinforcement dowel effect.

Finite Element Analysis

The derived stress-strain relations, represented by the material stiffness matrices, can now be incorporated in the analysis of reinforced concrete panels. The analysis based on these material stiffnesses includes the nonlinear effects of cracking and plasticity. The problem can be classified as an inelastic nonlinear analysis of a nonhomogeneous anisotropic body. A direct solution for such problems is in general impossible to obtain. Hence, for this type of problem a numerical analysis must be employed.

The numerical analysis is performed by load increments as shown in Fig. 13.

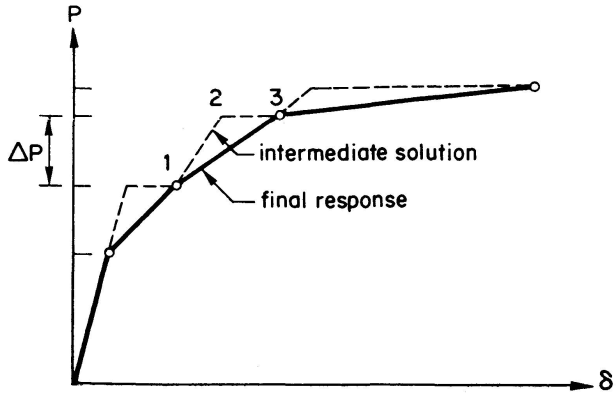

Fig. 13. Load-Displacement Diagram of an Incremental Analysis with Force Increments ΔP.

The boundary value problem within one load increment is solved by the finite element analysis based on the stiffness method. A panel is subdivided into a number of finite elements which are connected in the nodal points. A triangular constant strain element is used in this work. The derivation of this method is presented elsewhere [14], [15] and will not be repeated here. The stiffness method requires a solution of the matrix equation

$$\{\Delta X\} = [K]\{\Delta U\}, \tag{18}$$

where $\{\Delta X\}$ is the vector of load increments, and $\{\Delta U\}$ is the vector of unknown displacement increments. The size of these vectors is equal to the number of discrete displacements considered in the analysis. The stiffness matrix $[K]$ is assembled from the element stiffness $[k]$ matrices by the direct stiffness procedure. The element stiffness matrix depends on the material properties of the element, and is given by the matrix triple product [7], [14]

$$[k] = \int_v [B]^T [D] [B] \, dv$$

Matrix $[B]$ specifies the strain-displacement relations. The material properties are introduced by means of the material stiffness matrix $[D]$. The element stiffness matrix changes with every change of the material stiffness, such as, for example, during the transition from the uncracked to the cracked state.

The procedure involving one load increment can be illustrated by means of the load-displacement diagram shown in Fig. 13. The solution is first performed assuming an initial stiffness (Point 2 in Fig. 13). Then all material criteria are checked in all elements. In those elements in which the tensile strength or yield stress are exceeded, the material stiffness and also the corresponding element stiffness matrices are changed. The forces acting on these elements must be redistributed accordingly to maintain equilibrium. It may take several displacement solutions (between Points 2 and 3), before all material criteria are satisfied. Any number of elements can be treated simultaneously for cracking or plasticity during one load increment.

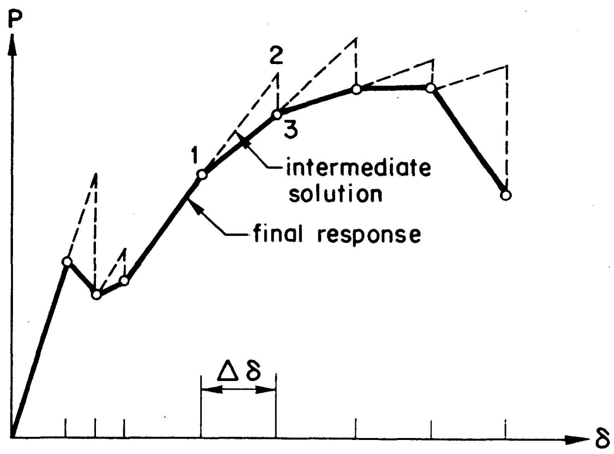

Fig. 14. Load-Displacement Diagram of an Incremental Analysis with Displacement Increments $\Delta\delta$.

If the displacement (instead of force) increment is specified the load-displacement diagram may appear as shown in Fig. 14. Thus, the response in the load intervals in which the structure is unstable with respect to specified forces can be obtained. Such regions can occur either just after crack formation, or at the limit load.

The outlined analysis was programmed for computer [7]. In order to be able to analyze both monotonic and cyclic load histories, a variety of different cracking modes representing the opening and closing of cracks have to be considered. Those used in this program are shown in Fig. 15.

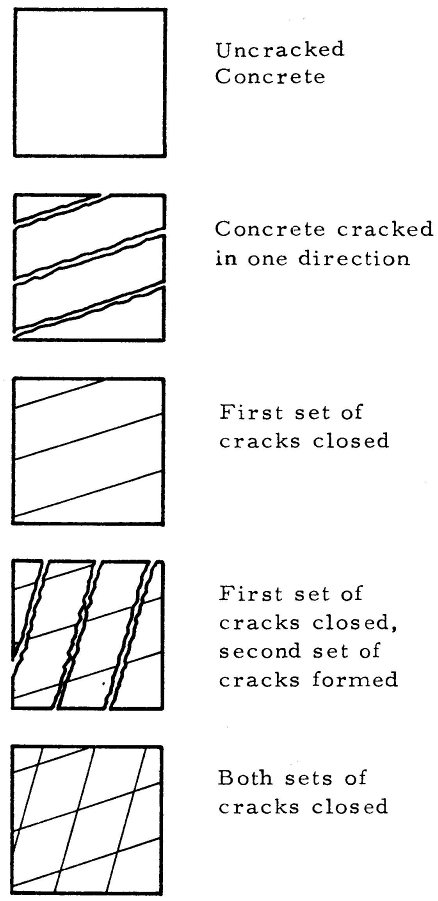

Fig. 15. Crack Modes.

The output of programs of this type can be very voluminous, so that it becomes necessary to obtain the output data in manageable form. In this case, plotting equipment was used to map crack patterns and directions as well as to mark plastified elements for every load step. Thus, a complete record of the crack propagation was obtained.

Examples of analytical solutions along with experimental verification will be presented in a companion paper.

Acknowledgement

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Keywords

Cracking; cyclic load; finite element method; lattice model; limit analysis; reinforced concrete; shear wall; stress analysis; stress-strain relations; structural analysis; panels; plasticity.

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Summary

Two nonlinear effects, cracking and plasticity, are considered in the analysis of reinforced concrete panels under in-plane forces. The stress-strain relations for an infinitesimal element of a panel are formed for an uncracked and cracked element in elastic and plastic stages. An incremental finite element method is used for the nonlinear analysis of the panels.

Résumé

On considère deux effets non linéaires, la fissuration et le comportement plastique, dans l'analyse de dalles (plaques) en béton armé sous l'effet de forces dans leur plan. Les relations entre la tension et la sollicitation pour un élément infinitésimal de la dalle (plaque) sont données pour un élément fissuré et un non fissuré dans les stades élastique et plastique. Pour l'analyse non-linéaire des plaques on utilise, en plus, une méthode d'éléments finis.

Zusammenfassung

Man untersucht zwei nichtlineare Einflüsse, Rissbildung und plastisches Verhalten bei der Analyse von Stahlbetonscheiben unter Einwirkung der in ihrer Ebene wirkenden Kräfte. Die Spannungs-Dehnungs-Beziehungen für ein infinitesimales Element einer Platte werden für ein gerissenes und ungerissenes Element im elastischen und plastischen Zustand untersucht. Für die nichtlineare Analyse der Platten wird eine Methode der inkrementalen endlichen Elemente benützt.

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