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# **Differential Shrinkage Effects in Composite Structures**

*Influences différentielles du retrait aux structures composées*

*Differentielle Schwindeinflüsse bei Verbundbauten*

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## **Introduction**

Composite type of construction comprises use of in-situ concrete cast at a later date with precast or earlier-cast concrete member that may also be prestressed. The composite structure may sometimes be composed of in-situ concrete with steel members.

The shrinkage and creep characteristics of concrete are not unknown. The term "Differential shrinkage" represents the difference between the shrinkage characteristics of the in-situ concrete and the total strain characteristics of the precast concrete due to shrinkage and creep. The influence of the differential shrinkage between the two concretes in a composite member has been a subject of interest, and tests carried out in laboratories have revealed data for estimation of shrinkage and creep strains for concrete of various strengths and under various stresses. A thorough-going report on composite concrete beams with pertinent references has been presented in Ref. [1].

It would however appear that the approach to Analysis of Stresses in composite sections has not received the same kind of attention as that of the laboratory tests. As a result, the methods of analysis as currently adopted bear certain inconsistencies, that appear to require a review.

## **Behaviour of Concrete in Composite Unit**

The member in Fig. 1 is of composite design in which the in-situ concrete portion is cast later. As a result there would be a tendency for the upper

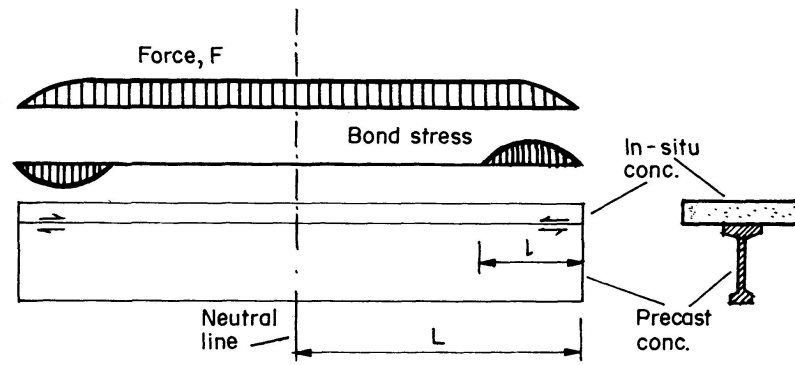


Fig. 1. Interface Force Between Two Concretes.

concrete to shrink differently from, say more than that in, the precast member resulting in inward movement of the concrete at both the ends. If sufficient bond is developed at the interface of the two concretes, the inward movement of the upper concrete would be restrained by a force developed near the ends. The magnitude of the force would be proportional to the developing shrinkage and would be maximum when full effect thereof is realized. The length,  $l$ , over which the bond will prevail will depend on the amount of force developed and the strength of bond at interface with or without use of shear connectors. The nature of distribution of bond stress at each end together with the resulting diagram of force,  $F$ , at each section along length at the interface are indicated. It would be understood that in the present case the bottom face of top concrete unit is in tension and the top face of bottom unit in compression due to the forces at interface, representing a "positive system". There would be resulting influence on the internal stresses in the two concrete sections.

If the bottom member is prestressed, it would occasionally happen that the overall strain in the bottom member due to creep effect is much more than the shrinkage strain in the upper in-situ concrete. The direction of interface forces in that event would become reversed to represent a "negative system" with consequent influence in internal stresses.

The relative movement or sliding between the two concretes at the ends of the member can be calculated when overall movements of the individual concretes are known. These would depend on the relative or differential strain,  $\delta$ , and the length of the member,  $L$ , upto the neutral point along length where the relative movement is Zero, such that the differential movement,

$$\Delta = \delta L. \quad (1)$$

The shrinkage and creep properties of a concrete are related to its strength characteristics, which in turn are related to the "water/cement" ratio used. The relationship under normal conditions at 70°F temperature is as indicated in Fig. 2 [2], that can be represented by

$$C_u = \frac{17400}{2.64^{2.5R}}, \quad (2)$$

where  $C_u$  = Cube strength of concrete at 28 days,  
 $R$  = Water/Cement ratio.

The desirable  $w/c$  ratio for a specified strength is obtainable from the graph.

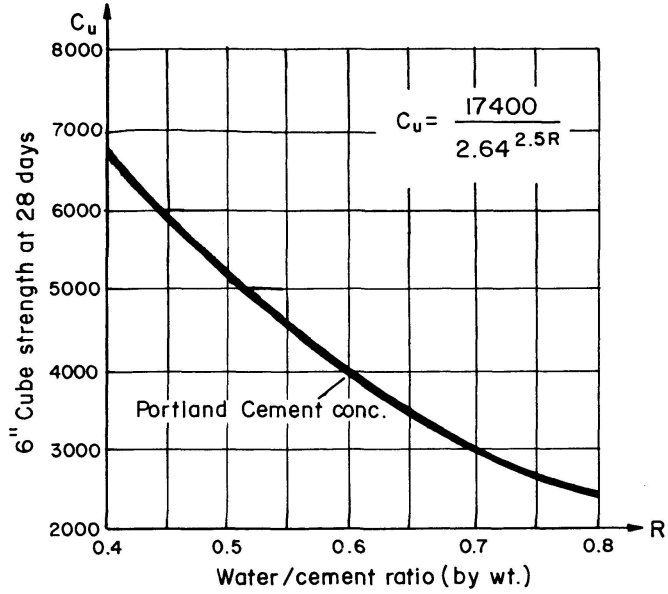


Fig. 2. Strength of Properly Compacted Concrete vs. Water/Cement Ratio.

### *Drying Shrinkage*

The ultimate free shrinkage strain of a plain concrete that takes place from the time of suspension of wet curing has been established from tests and is obtainable from the simple relation [1].

$$10^4 \epsilon_s^L = 5 + 11.7 R^4,$$

where  $\epsilon_s^L$  = Ultimate drying shrinkage strain.

If the concrete has some embedded steel reinforcement, the amount of its shrinkage would be restrained according to the degree of concentration of steel. The Reduction factor,  $C_r$ , for shrinkage value of concrete with steel is given in Fig. 3, based on research [3]. Thus,

$$\epsilon_s^L = (5 + 11.7 R^4) 10^{-4} C_r. \quad (3)$$

The variation of drying shrinkage in respect of age, from the time of suspension of wet curing, can be related by a Reduction factor to the ultimate  $\epsilon_s^L$  [6], such that at any time

$$C_{ts} = 0.225 \log_{10} 10 P + 0.55, \quad (4)$$

where  $P$  = Age of concrete in years from the time of suspension of wet curing ( $P > 10$ ).



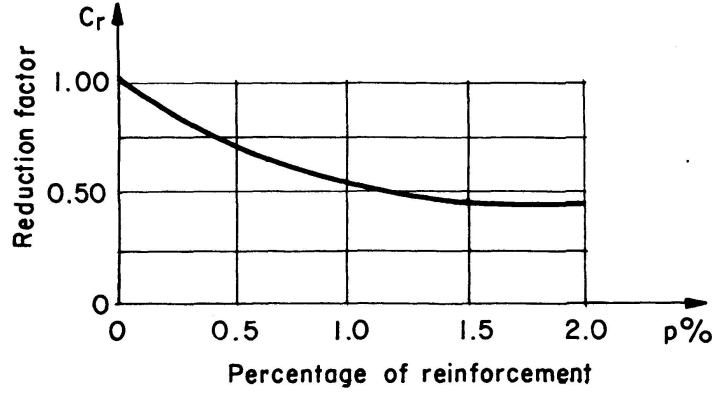


Fig. 3. Reduction Factor for Restraint in Strain vs. Percentage of Steel.

It is assumed that the ultimate shrinkage, as given by Eq. (3), takes place in a period of 10 years. Thus the shrinkage in respect of strength and age can be represented by

$$\begin{aligned}\epsilon_s &= (5 + 11.7 R^4) 10^{-4} C_r C_{ts} \\ &= \epsilon_s^L C_{ts},\end{aligned}\quad (5)$$

where  $\epsilon_s$  = Shrinkage strain at any given age.

#### Creep

The creep of concrete is the permanent strain induced by stress. Although it is known that this strain is not directly proportional to the induced stress, for practical purposes however this is assumed to be so.

The ultimate value of Creep strain of plain concrete can be represented, on the basis of data in [4], by

$$\begin{aligned}10^7 \epsilon_c^L &= 17.4 - 61 R + 77.5 R^2 \text{ per lb. per sq. in.} \\ &= 247 - 870 R + 1100 R^2 \text{ per Kg. per sq. cm.} \\ &= K_c,\end{aligned}$$

where  $\epsilon_c^L$  = Ultimate creep strain.

The restraint effect of steel reinforcement will be taken care of as before by the factor  $C_r$ , so that under such conditions

$$\epsilon_c^L = K_c 10^{-7} C_r \text{ (per unit stress).} \quad (6)$$

The limiting value of strain is assumed to reach in 10 years, and the value  $\epsilon_c$  at any intermediate period of time can be related by a Reduction factor to the limiting value, based on results in [4],

$$C_{tc} = \frac{Y^{0.07}}{1.175}, \quad (7)$$

where  $Y$  = Age of concrete in years ( $> 10$ ).

Thus, at any given age,

$$\begin{aligned}\epsilon_c &= K_c 10^{-7} C_r C_{tc} \\ &= \epsilon_c^L C_{tc} \text{ (per unit stress).}\end{aligned}\quad (8)$$

### Residual Strain

The "Residual strain",  $\epsilon^R$ , is designated to the difference between the ultimate strain and the instant strain at a particular stage or age, Fig. 4.

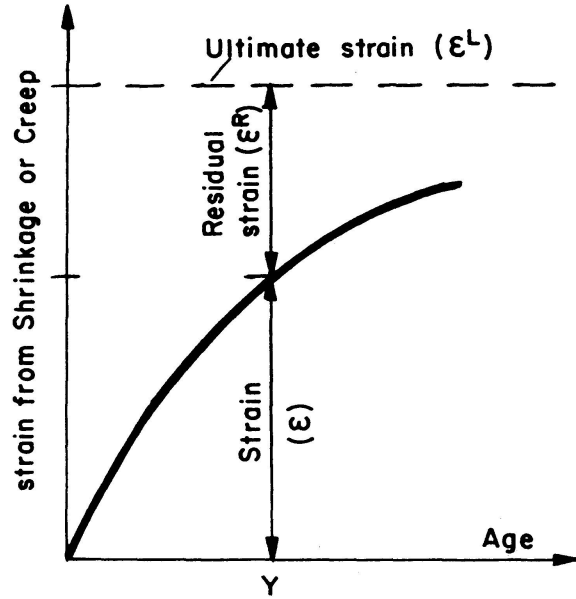


Fig. 4. Strain vs. Age of Concrete Due to Shrinkage or Creep Under Stress (Typical Diagram).

Thus, the Residual shrinkage strain,

$$\begin{aligned}\epsilon_s^R &= \epsilon_s^L - \epsilon_s \\ &= \epsilon_s^L (1 - C_{ts})\end{aligned}\quad (9)$$

and Residual creep strain per unit stress,

$$\begin{aligned}\epsilon_c^R &= \epsilon_c^L - \epsilon_c \\ &= \epsilon_c^L (1 - C_{tc}).\end{aligned}\quad (10)$$

### Mechanism of Strain Movement and the Interface Force

The process of analysis consists in the determination of the interface force,  $F$ , between the precast and the in-situ concretes on account of the differential strain. The strains from various causes in the two concretes would be evident from Fig. 5a.

The end of the precast unit is at vertical line 1 at time of casting. It is at line 2 at the time of prestressing, if adopted, due to shrinkage. The line changes to 3 due to bending effect of prestressing. The concrete continues to shorten

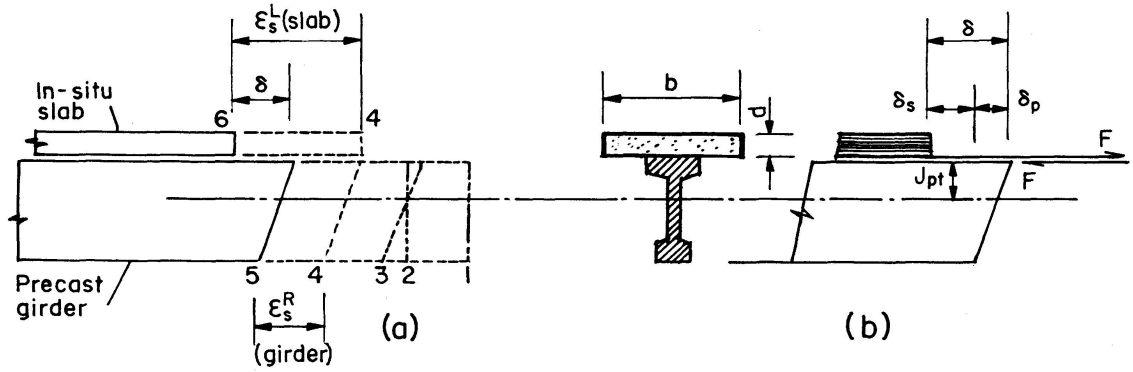


Fig. 5. Strain Movement of Concretes and Differential Shrinkage.

further due to shrinkage and effect of creep, and would have reached line 5 if left free. But at a stage while on line 4 the in-situ concrete is cast above, so that there still remains a residual strain to cover the distance 4-5.

The top in-situ concrete, if left free, would shrink upto line 6.

The force at interface operates to restrain the free movements of the two concretes due to the above strains such that the differential movement is prevented from realization, Fig. 5b. The corresponding differential strain,  $\delta$ , thus influences the value of interface force,  $F$ . The value of the differential strain is obtainable from consideration of the respective strains enumerated earlier, and represents the difference between the free shrinkage strain of in-situ concrete and the residual strain in precast concrete from line 4 due to shrinkage and creep.

The function of force  $F$  is to keep the in-situ concrete extended by an amount of strain  $\delta_s$ , as also the precast concrete contracted by an amount  $\delta_p$  in a compatible manner, Fig. 5b, such that the differential strain  $\delta$  is thereby made up. Thus,

$$\delta = \delta_s \sim \delta_p = |\delta_s + \delta_p|. \quad (11)$$

The value of differential strain  $\delta$  is initially known.

### Analysis of Stresses

#### Strain in In-situ Concrete

Total free shrinkage strain

$$\epsilon_s^L = (5 + 11.7 R^4) 10^{-4} C_r \quad (12)$$

and is as in Eq. (3).

Referring to Fig. 5b, the stress at bottom fibre of in-situ concrete (tensile) due to force  $F$ ,

$$f_{sb} = -\frac{F}{A_s} - F \frac{d}{2} \frac{1}{Z_s} = -F \left[ \frac{1}{b d} + \frac{d}{2} \frac{6}{b d^2} \right] = -\frac{4 F}{A_s}.$$

The resulting elastic strain in fibre

$$\epsilon = \frac{f_{sb}}{E_s} = -\frac{4F}{E_s A_s}. \quad (13)$$

The total Creep strain due to  $F$ , from Eq. (6),

$$\epsilon = -\epsilon_c^L \frac{4F}{A_s}. \quad (14)$$

If the tension is high and taken only by steel reinforcement, the creep strain may be non-existent.

Thus, total strain in bottom fibre of in-situ concrete due to  $F$ ,

$$\delta_s = -\frac{4F}{A_s} \left[ \frac{1}{E_s} + \epsilon_c^L \right]. \quad (15)$$

In the case of  $F$  being compressive, as in a negative system, the sign of the term would be positive.

#### *Strain in Precast Concrete*

Residual strain in top fibre from the time of casting of top concrete,

$$\begin{aligned} \epsilon^R &= \text{Residual shrinkage strain plus} \\ &\quad \text{Residual creep strain from prestress.} \\ &= \epsilon_s^R + \epsilon_c^R \sigma. \end{aligned} \quad (16)$$

The values are obtainable from Eqs. (9) and (10).

From Fig. 5b, the stress at top fibre of precast unit (elastic compressive) due to  $F$ ,

$$f_{pt} = \frac{F}{A_p} + \frac{F y_{pt}}{Z_{pt}}$$

and the corresponding elastic strain

$$\epsilon_{pt} = \frac{f_{pt}}{E_p} = \frac{F}{E_p} \left[ \frac{1}{A_p} + \frac{y_{pt}}{Z_{pt}} \right]. \quad (17)$$

The creep strain in the fibre due to  $F$ ,

$$\begin{aligned} \epsilon &= \epsilon_c^L f_{pt} \\ &= \epsilon_c^L F \left[ \frac{1}{A_p} + \frac{y_{pt}}{Z_{pt}} \right]. \end{aligned} \quad (18)$$

Thus, the total strain in top fibre of precast unit due to  $F$

$$\delta_p = F \left[ \frac{1}{E_p} + \epsilon_c^L \right] \left[ \frac{1}{A_p} + \frac{y_{pt}}{Z_{pt}} \right]. \quad (19)$$

In the event of  $F$  being tensile in a negative system, the sign would be negative.

### Stresses

Since the value of differential strain is a known quantity, as explained in previous section, the value of  $F$  can be determined through Eqs. (11), (15) and (19). Then the fibre stresses in the concrete sections due to  $F$ , when in positive system, would become

$$\begin{aligned}
 \text{In-situ section, top:} \quad f_{st} &= \frac{2 F}{A_s}, \\
 \text{In-situ, bottom:} \quad f_{sb} &= -\frac{4 F}{A_s}, \\
 \text{Precast section, top:} \quad f_{pt} &= F \left[ \frac{1}{A_p} - \frac{y_{pt}}{Z_{pt}} \right], \\
 \text{Precase section, bottom:} \quad f_{pb} &= F \left[ \frac{1}{A_p} - \frac{Z_{pt}}{Z_{pb}} \right].
 \end{aligned} \tag{20}$$

The stresses would have opposite signs in negative  $F$ -system. These represent the resulting stresses on account of differential shrinkage and creep.

### Resultant Behaviour of Structure

The foregoing fibre stresses in members would develop gradually, and would occur to represent the extreme values when the full effects of shrinkage and creep of both the concretes have taken place in about 10 year period. As long as the value of tensile stress  $f_{sb}$  in the positive  $F$ -system (or  $f_{pt}$  in the negative  $F$ -system) is within the tensile strength of corresponding concrete, the stress diagram due to differential shrinkage, as represented by the values in Eq. (20), would hold good.

If however the tensile strength of the corresponding concrete is exceeded, the concrete would develop tension cracks in the member along its length similar in nature to those developed on the underside of simply supported reinforced concrete beams. As soon as the concrete cracks on account of the shrinkage strain, the developing interface force  $F$  gets extinct together with the resulting stresses in the members.

The tensile stress in concrete as developed from  $F$  can be reduced by some extent by restricting the movement strain with reinforcement. It cannot, however, be completely eliminated and, as would be evident from Fig. 3, any reinforcing steel above 1.0% across section can hardly have any influence in this direction.

The reinforcing steel in the uncracked tensioned concrete under shrinkage becomes subjected to a secondary compressive stress,  $t_i$ . The actual stress would correspond to the resulting shrinkage strain (Fig. 5)

$$\epsilon_s^L \mp \delta_s$$

in the member. There would thus be secondary compressive stresses in steel at different layers, such that

$$t_i = E_s (\epsilon_s^L \mp \delta_s). \quad (21)$$

If the permissible tensile stress in steel is  $t$ , the available stress in such steel for basic loading designs would then be

$$t_d = t_i + t. \quad (22)$$

If however the concrete fails to take tension and develops cracks as afore-said, the strain in steel would correspond only to the differential strain,  $\delta$ , and the secondary stress to

$$t_i = E_s \delta \quad (21a)$$

for use in Eq. (22). Normally, the tensile strength of concrete [9]

$$f_t = \frac{C_u}{20} + 110 \text{ (psi)}. \quad (23)$$

For practical designs therefore, if the calculated value of tension in concrete is found to be in excess of the tensile strength, a stress diagram for the composite section corresponding to this limiting strength should only be considered, since nothing beyond this condition could occur during the progress of shrinkage. The secondary compressive value in steel reinforcement should be correspondingly considered.

### Example

In a bridge of 145' span of 3-girder system, the design data for a composite section are as follows:

#### *In-situ slab*

$$A_s = 1150 \text{ in}^2$$

$$C_u = 3500 \text{ psi}$$

$$E_s = 3 \times 10^6 \text{ psi}$$

Minimum reinforcement in  
slab and beam = 0.2%

#### *Precast girder*

$$A_p = 1504 \text{ in}^2$$

$$I_p = 3475 \times 10^3 \text{ in}^4$$

$$y_{pt} = 67.6 \text{ in}$$

$$y_{pb} = 70.4 \text{ in}$$

$$Z_{pt} = 51400 \text{ in}^3$$

$$Z_{pb} = 49400 \text{ in}^3$$

$$C_u = 5500 \text{ psi}$$

$$E_p = 5.5 \times 10^6 \text{ psi}$$

$$\sigma = 900 \text{ psi (after losses).}$$

The girder is to be cast in segmental units and prestressed in assembly after three weeks of casting. Slab concrete to be cast one week thereafter. It is required to find out the internal stresses due to differential shrinkage effects.

*Solution*

The concrete mix for precast concrete will be nominal 1:1½:3 with a  $w/c$  ratio of 0.37 (Fig. 2). The mix for in-situ concrete will be nominal 1:2:4 with the maximum  $w/c$  ratio for vibrated concrete of 0.55.

*Design Constants*

1. For slab section:

$$C_u = 3500 \text{ psi (min)}$$

$$\therefore C_u \text{ (average)} = 3500/0.75 = 4600 \text{ psi}$$

From Fig. 2,  $R = 0.55$ ; from Fig. 3,  $C_r = 0.90$

$$\text{From Eq. (12), } \epsilon_s^L = (5 + 11.7 \times 0.55^4) \times 10^{-4} \times 0.90 = 5.50 \times 10^{-4}$$

$$\text{From Eq. (6), } K_c = 17.4 - 61 \times 0.55 + 77.5 \times 0.55^2 = 7.27$$

$$\text{and } \epsilon_c^L = 7.27 \times 10^{-7} \times 0.90 = 6.60 \times 10^{-7}$$

2. For Beam section:

$$C_u = 5500 \text{ psi (min)}$$

$$\therefore C_u \text{ (average)} = 5500/0.75 = 7300 \text{ psi}$$

From Fig. 2,  $R = 0.37$ , and from Fig. 3,  $C_r = 0.90$

$$\text{From Eq. (3), } \epsilon_s^L = (5 + 11.7 \times 0.37^4) \times 10^{-4} \times 0.90 = 4.70 \times 10^{-4}$$

From Eq. (4), if  $P = \frac{1}{12}$  year, i.e., 1 month, from date of suspension of wet curing, then  $C_{ts} = 0.225 \times \log_{10} \frac{10}{12} + 0.55 = 0.532$

$$\text{From Eq. (9), } \epsilon_s^R = 4.70 \times 10^{-4} (1 - 0.532) = 2.20 \times 10^{-4}$$

$$\text{From Eq. (6), } K_c = 17.4 - 61 \times 0.37 + 77.5 \times 0.37^2 = 5.4$$

$$\text{and } \epsilon_c^L = 5.4 \times 10^{-7} \times 0.90 = 4.86 \times 10^{-7}$$

From Eq. (7), if  $Y = \frac{1}{52}$  year, i.e., 1 week after prestressing, then

$$Y^{0.07} = \left(\frac{1}{52}\right)^{0.07} = 0.76 \quad \text{and} \quad C_{tc} = \frac{0.76}{1.175} = 0.65$$

$$\text{From Eq. (10), } \epsilon_c^R = 4.86 \times 10^{-7} (1 - 0.65) = 1.70 \times 10^{-7}.$$

*Differential Strain*

Free shrinkage strain in bottom fibre of slab, Eq. (12),

$$\epsilon_s^L = 5.50 \times 10^{-4}.$$

Residual strain in top fibre of beam, Eq. (16),

$$\begin{aligned} \epsilon^R &= \epsilon_s^R + \epsilon_c^R \sigma \\ &= 2.20 \times 10^{-4} + 1.70 \times 10^{-7} \times 900 = 3.73 \times 10^{-4}. \end{aligned}$$

Thus, the differential strain from Fig. 6,

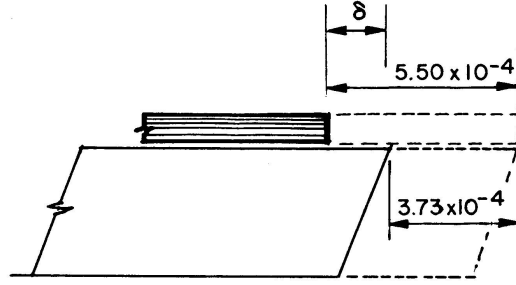
$$\delta = (5.50 - 3.73) \times 10^{-4} = 1.77 \times 10^{-4} \quad (\text{A})$$

and the  $F$ -system would be positive.

The anticipated differential movement would be from Eq. (1),

$$\Delta = \frac{1.77}{10^4} \times \frac{145}{2} \times 12 = 0.156''.$$

Fig. 6. Strain Movements at End of Member.



### Compatibility Strains

From Eq. (15), 
$$\delta_s = -\frac{4F}{1150} \left[ \frac{1}{3.00 \times 10^6} + \frac{6.60}{10^7} \right] = -\frac{F}{10^6} \times 0.00348.$$

From Eq. (19), 
$$\delta_p = -F \left[ \frac{1}{5.50 \times 10^6} + \frac{4.86}{10^7} \right] \left[ \frac{1}{1504} + \frac{67.6}{51400} \right]$$

$$= -\frac{F}{10^6} \times 0.00133.$$

Hence from Eq. (11), 
$$\delta = |\delta_s + \delta_p| = \frac{F}{10^6} \times 0.00481 = \frac{F}{207 \times 10^6}. \quad (\text{B})$$

Equating (A) and (B),

$$F = 1.77 \times 207 \times 10^2 = 3.66 \times 10^4 \text{ lb} \quad (\text{C})$$

Hence, Compatibility strain in slab from differential shrinkage,

$$\delta_s = 3.66 \times 10^4 \times \frac{0.00348}{10^6} = 1.28 \times 10^{-4}$$

and in beam, 
$$\delta_p = (1.77 - 1.28) 10^{-4} = 0.49 \times 10^{-4}.$$

### Fibre Stresses

From Eq. (20), 
$$f_{st} = \frac{2 \times 3.66 \times 10^4}{1150} = +63.5 \text{ psi},$$

$$f_{sb} = -\frac{4 \times 3.66 \times 10^4}{1150} = -127.0 \text{ psi},$$

$$f_{pt} = 3.66 \times 10^4 \left[ \frac{1}{1504} + \frac{67.6}{51400} \right] = 66.6 \text{ psi},$$

$$f_{pb} = -3.66 \times 10^4 \left[ \frac{1}{1504} - \frac{67.6}{49400} \right] = -26.1 \text{ psi}.$$



Fig. 7 gives the stress diagram, and the stresses occur at each section along the length of composite girder. The maximum tensile stress of 127 psi is within the strength limit of 285 psi for such concrete.

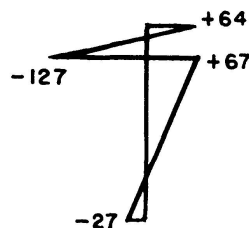


Fig. 7. Stress Diagram from Differential Shrinkage.

### Conclusion

The existing methods for determining the differential shrinkage effects on stresses in composite section contain propositions that are not consistent with the true mechanism of behaviour. According to these methods, the value of interface force is evaluated from the relation

$$F = E_s A_s \delta$$

with due consideration of creep effect. In the Example, wherein the creep effect is included in the differential shrinkage strain, this accordingly would lead to a value of

$$F = (3.00 \times 10^6) \times 1150 \times (1.77 \times 10^{-4}) = 61.0 \times 10^4 \text{ lb},$$

whereas the actual value is  $3.66 \times 10^4$  lb. according to the method herein presented Eq. (C); a considerably lower value. Besides, the assumed straight line variation of fibre strain from top to bottom of composite section analysis, that influences the fibre stresses, would also appear irrational. The two concretes in the section would actually be subjected to opposite kind of strains due to differential shrinkage.

The shrinkage and Creep properties of concrete of various strengths have been shown as related to the "water/cement" ratio used. If specific values of these properties are known for concretes in a project, these can be directly made use of for  $\epsilon_s$ ,  $\epsilon_c$  and  $E$ . The influence of Relative humidity of atmosphere in Shrinkage Coefficients has not been considered necessary having similar effects on both the concretes.

It is to be kept in mind that the interface force gains intensity with age as the shrinkage and creep effects continue, and ultimately achieves its maximum value, as considered herein, after a long period of time.

The Creep strain considered in Eq. (14) for slab may not be operative if the force is in tension and the concrete cracks due to high stress ( $> f_t$ ).

If the precast girder is of R.C. construction, the creep effect due to prestress considered in Eq. (16) will be non-existent.

The differential shrinkage coefficient in this instance has been found, Eq. (A), to be  $1.77 \times 10^{-4}$ , a reasonable value in the context of test results in Ref. [1].

The presented method of analysis forecasts the value of interface force and the resulting stresses in the composite section as considerably low.

### Notation

$A_s$	Area of in-situ concrete unit.
$A_p$	Area of precast concrete unit.
$C_u$	6" Cube strength of concrete at 28 days.
$C_r$	Reduction factor for restraint effect of reinforcement.
$C_{ts}$	Shrinkage factor due to age effect.
$C_{tc}$	Creep factor ditto.
$E_c, E_p$	Initial modulus of Concrete (psi), in-situ and precast.
$E_s$	Young's modulus of reinforcing steel.
$F$	Interface force between two concretes (lb).
$f$	Fibre stress in concrete section (psi) $f_{st}, f_{sb}, f_{pt}, f_{pb}$ .
$f_t$	Tensile strength of concrete (psi).
$K_c$	Term in Eq. (6).
$L$	Length of beam under influence of sliding at interface.
$l$	Length of bond in operation between two concretes at end of member.
$P$	Age of concrete from time of suspension of wet curing (year).
$R$	Water/Cement ratio.
$t$	Permissible tensile stress in steel.
$t_d$	Available design stress in steel.
$t_i$	Initial stress in steel reinforcement on account of shrinkage of surrounding concrete.
$Y$	Age of concrete from time of application of stress (year).
$y_{pb}$	Bottom fibre distance in precast concrete section from its N.A. (in).
$y_{pt}$	Top fibre ditto.
$Z_{pb}$	Section modulus of precast concrete in respect of bottom fibre (in <sup>3</sup> ).
$Z_{pt}$	Ditto top fibre.
$\delta$	Differential strain.
$\delta_p, \delta_s$	Compatibility strains in precast and in-situ concrete units.
$\Delta$	Movement of concrete member at ends due to $\delta$ (in.).
$\epsilon$	Strain.
$\epsilon_c$	Creep strain of concrete per unit stress.
$\epsilon_c^L$	Ultimate creep strain ditto, i. e., Creep coefficient per unit stress.
$\epsilon_c^R$	Residual creep strain.
$\epsilon_s$	Shrinkage strain of concrete.
$\epsilon_s^L$	Ultimate shrinkage strain, i. e., Shrinkage coefficient.

- $\epsilon_s^R$       Residual shrinkage strain.  
 $\sigma$       Average intensity of prestress as after losses (psi).

*Note:*      Cylinder strength of concrete,  $C_y \simeq 0.8 C_u$ .

### References

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### Summary

The existing methods for determining the differential shrinkage effects in composite section contain propositions that are not consistent with true mechanism of structural behaviour. These include:

1. Disregard of shrinkage and creep effects in precast member.
2. Arbitrary assumption for the lines of action of the interconnecting force in the two concrete sections.
3. Assumption of "plain section remaining plane" in composite section due to differential shrinkage effects – leading to arbitrary methods of analysis and resulting stresses.

While the presented method has a rational basis for analysis, its application brings about results that are not so significant as from current methods.

### Résumé

Les méthodes existantes de la détermination des influences différentielles dans les sections composées comprennent des propositions non compatibles avec le comportement réel. Elles se rapportent à :

1. La négligence des effets du retrait et du fluage dans les membres préfabriqués;
2. une supposition arbitraire des lignes d'action des forces d'interconnexion dans les deux sections du béton;
3. la supposition du maintien d'un état plan de la section composée, aussi pour des influences de fluage différentielles ce qui conduit à des méthodes arbitraires d'analyse et aux sollicitations en résultant.

La méthode présentée mène à une analyse plus exacte. Elle montre en même temps que les tensions résultantes dans les sections composées sont inférieures aux tensions calculées par les méthodes courantes.

### **Zusammenfassung**

Die bestehenden Methoden zur Bestimmung der differentiellen Schwind- einflüsse in Verbundquerschnitten enthalten Vorschläge, die mit dem wirklichen Verhalten nicht vereinbar sind. Diese beziehen sich auf:

1. Ausserachtlassen der Schwind- und Kriecheinflüsse in vorfabrizierten Gliedern;
2. eine willkürliche Annahme für die Wirkungslinien der Verbindungskräfte in den beiden Beton-Querschnitten;
3. Annahme eines Ebenbleibens des Verbundquerschnittes auch für differentielle Schwindeinflüsse, was zu willkürlichen Methoden der Analyse und der resultierenden Beanspruchungen führt.

Die vorliegende Methode führt zu einer genaueren Analyse. Sie zeigt zugleich, dass die resultierenden Spannungen in Verbundquerschnitten kleiner sind als die nach den gängigen Methoden errechneten.

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