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## **Modal Characteristics and Damping in a Vibrating Tower Framework Structure**

*Caractéristiques modales et amortissement dans des tours vibrantes en treillis*

*Modale Charakteristiken und Dämpfung in vibrierenden Turm-Fachwerken*

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Tower shaped structures, such as television towers, tall chimneys, water towers etc. are a very special form of civil engineering structure. The structure is susceptible to abnormal loadings, and, in particular, it is its response to dynamic loads in the form of wind gusts and seismic forces which are of serious concern. The role of structural and associated damping is becoming increasingly important in this respect. The inherent structural damping in a structure not equipped with additional dynamic absorbers is the controlling parameter on the maximum stresses and deformations experienced by that structure when subjected to dynamic loads. A knowledge of the characteristics and effects of structural damping is therefore highly desirable when assessing the probable behaviour of such tower shaped structures under normal environmental conditions. Although considerable knowledge exists on the damping characteristics of typical structures encountered in the aircraft and aerospace industries, there exists a need for a special study of vibration damping and modal characteristics in very tall and slender tower shaped civil engineering structures. Such towers are often of bolted joint triangulated framework basic construction.

This paper reports an experimental investigation of the dynamic behaviour of a small scale bolted joint triangulated framework tower structure. The response to steady-state and transient excitation was studied together with the effects of additional concentrated masses. The variation of overall structural damping with frequency and magnitude of excitation was examined using two methods of damping factor measurement. The nature and effects

of damping on natural mode shapes and frequencies is discussed, since it is well known that the presence of non-linear damping means that the use of natural modes as generalised coordinates in a theoretical dynamic analysis is not strictly valid.

## 2. Experimental Rig and Measuring Systems

The test model was constructed from angle section brass members, bolted together using gusset plates and ensuring a concurrent line of centres at the joints. The bolts were tightened to a constant torque over the whole of the structure. The overall height of the tower was 1.83 metres and the base was 20.3 cms square. The base was firmly bolted to a concrete floor using steel angle section to constrain the tower in a fixed-free condition. The model is shown in its external framework in Fig. 1.

Static bending and torsion can be applied through a loading disc fitted at the top of the tower and locating on the four main vertical members. Similarly static axial load is applied by weights positioned on a vertical shaft mounted centrally on the loading disc.

Dynamic loading was applied by an electro magnetic vibrator driven by a signal generator through an amplifier. A sinusoidal signal produced a harmonic

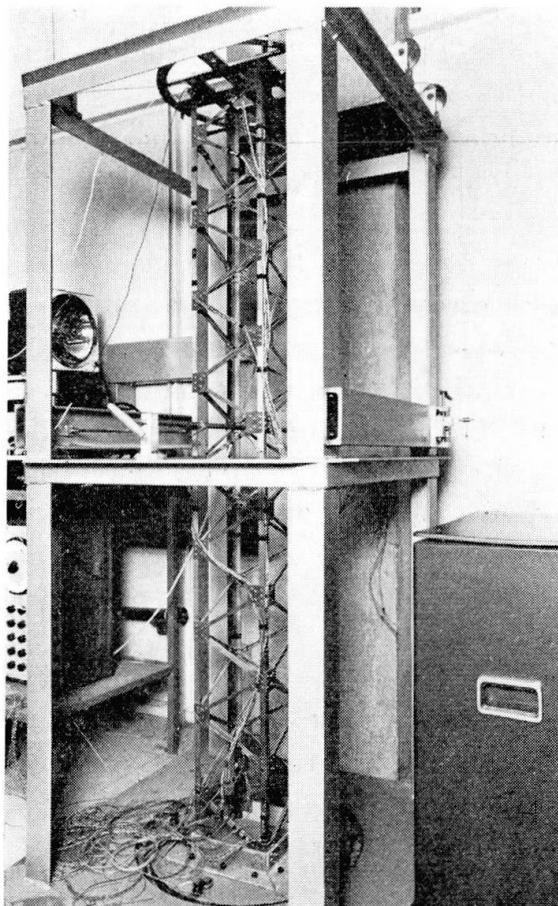


Fig. 1. Tower Model Shown within the External Framework.

load on the top of tower along a horizontal axis perpendicular to the vertical tower axis and parallel to two sides of the tower through a link incorporating a quartz crystal force transducer. The output from this transducer was amplified, and represented directly the input force transmitted to the tower by the vibrator. The vibrator itself was mounted on a concrete pillar to minimise any vibration being transmitted through the tower mountings. Displacements of the structure during vibration were determined by mounting an accelerometer on the tower. This piezo-electric transducer gives an output signal proportional to acceleration. The signal may be electronically integrated twice to give an output signal proportional to the local displacement. Individual structural member distortions were monitored by thirty-six foil resistance strain gauges positioned on the tower. Strains were read directly by a multi-channel dynamic strain gauge bridge. Direct visual observation of dynamic behaviour under harmonic loading was achieved by stroboscopic illumination of the tower.

Transient behaviour of the tower was examined by applying known static bending and torsional loads and then suddenly releasing these loads. The character of the subsequent decaying oscillations in the structure can be examined using accelerometers and strain gauges as previously indicated. Recordings of these transient phenomena were made using a storage oscilloscope and polaroid camera facilities.

A diagram of the experimental rig and measuring systems is given in Fig. 2.

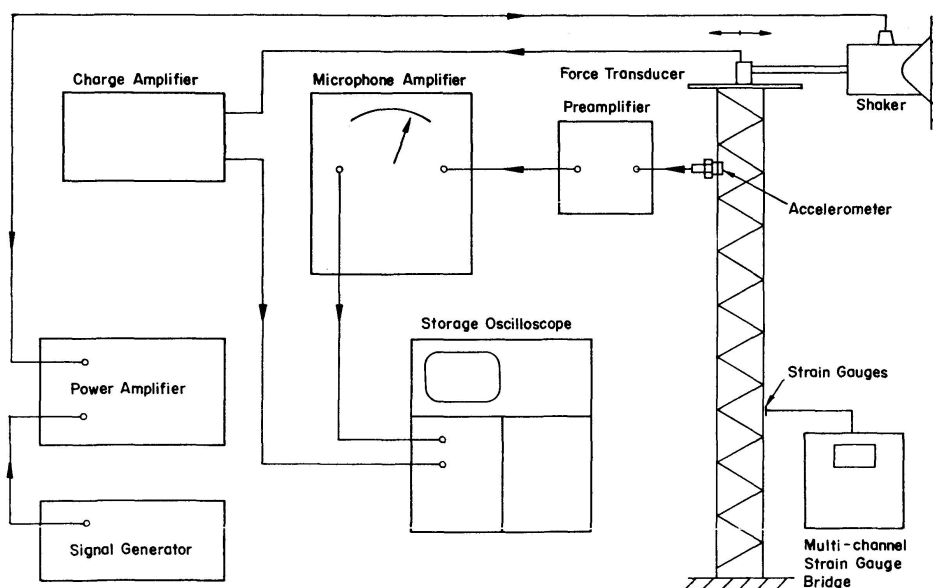


Fig. 2. Diagram of Tower Arrangement and Instrumentation Systems.

### 3. Modal Characteristics of the Tower under Steady State Harmonic Loading

In order to examine the basic dynamic characteristics of the structure, a series of tests were carried out under steady state harmonic loads. Firstly



a frequency sweep with constant input load amplitude identified the principal resonant frequencies of the structure. The mode shapes of the first three principal resonances corresponded well with the first three normal modes in transverse vibrations of a cantilever beam with a uniform distribution of mass and inertia. Incorporating the additional concentrated mass of the loading ring at the top of the tower gave acceptable correlation between measured resonant frequencies and those calculated from elementary beam theory. Increasing the frequency beyond the first principal resonance in the tower, a number of subdominant modes were apparent and these subdominant modes became increasingly prominent and complex as the loading frequency increased. Indeed, it was quite difficult to obtain a pure second mode as additional bending and torsional oscillations of small magnitude were always present, albeit in relatively small amplitudes. An indication of the complexity of the motion in the structure is given by the graph of amplitude of oscillation against input frequency given as Fig. 3.

This curve illustrates the growth of modal density and severity as excitation frequency increases. Stroboscopic studies and strain gauge results show that the response at higher frequencies is so complex in character that a full matrix structural analysis is necessary for theoretical analysis. Considering the tower as a three dimensional structure with fixed joints would result in approximately 250 unknown parameters, which is obviously far in excess of the number of unknowns usually considered necessary for the theoretical study of such a conventional tower structure. In the case of tower-shaped framed structures, the theoretical model is a single vertical assembly of uniform structural beam elements connected at discrete joints. Each beam is assumed to have a uniform flexural stiffness over its length. This is a conventional matrix flexibility

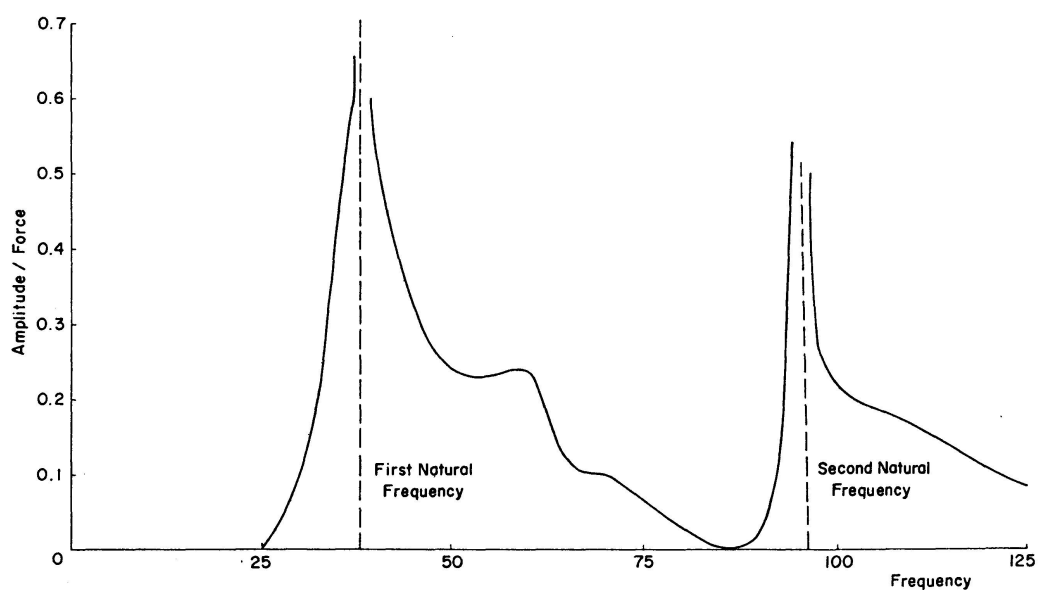


Fig. 3. Structure Frequency Response Curve.

analysis [1] and leads to a set of algebraic equations having as unknowns the transverse displacements of the continuous beam assemblage. Including inertia terms, this set of equations in the case of free oscillations may be written:

$$[M][\ddot{Y}] + [K][Y] = 0, \quad (1)$$

where  $[M]$  = Assembled mass matrix;  
 $[Y]$  = Vector of unknown transverse displacement;  
 $[K]$  = Assembled structure stiffness matrix.

Eq. (1) represents a standard eigenvalue problem and may be solved to give the natural frequencies of the structure, the eigenvalues, and the natural mode shapes, the normalised eigenvectors. Such a theoretical analysis will only predict the principal modes in transverse vibration of the tower, i.e. the analysis will identify as important only those frequencies marked as principal mode frequencies in Fig. 3. It is apparent from this graph that at the higher frequencies there are other important resonant modes which cannot be identified by continuous beam representations. This is particularly notable in the case of tall slender structures since it is not only the first mode of vibration which is examined during the design stage.

Further tests to evaluate the usefulness of simple beam theory were conducted by examining the effects of additional mass at the top of the tower on its fundamental frequency. The mass were located on the vertical column at the centre of the top loading ring and firmly bolted in position. The frequency of the fundamental mode was then determined for various additional masses. Applying Dunkerley's empirical formula permits a ready theoretical evaluation of the fundamental frequency of the combined system i.e.

$$\frac{1}{w^2} = \frac{1}{w_1^2} + \frac{1}{w_2^2}, \quad (3.2)$$

where  $w$  = fundamental frequency of combined system;  
 $w_1$  = fundamental frequency of uniform beam alone;  
 $w_2$  = fundamental frequency of massless beam with lumped mass at free end.

This empirical formula gave good correlation between measured and predicted frequencies as can be seen in the following table:

Additional Mass (Kg)	Predicted Fundamental Frequency (c.p.s.)	Measured Fundamental Frequency (c.p.s.)
1.75	21.2	19.8
3.34	18.9	18.0
4.25	16.9	16.9
5.60	15.2	15.0
6.50	14.6	14.0

This table again indicates the validity of elementary beam theory when examining the behaviour of the structure in the fundamental mode.

### *Damping*

The maximum amplitudes of oscillations in deflections and stresses occurring in a structure under dynamic loads are directly related to the damping forces experienced by that structure. Unlike mass and stiffness however, damping is not necessarily an inherent property of the structure. The overall damping mechanism in a tower structure will be a combination of material, air, and Coulomb friction damping. The material damping is the inherent phenomenon whereby energy is dissipated during the vibration of any system consisting only of a volume of solid material. This property is measurable as a material characteristic, but for structural materials it is often a function of the stress level [3], and is affected by the stress and temperature histories of the specimen. Material damping is determined from the area of the hysteresis loop generated during one stress cycle, and it is this hysteretic relationship between stress and strain which is a source of the system nonlinearity. The air damping is caused by the air pumping effects as the structure oscillates. This property is highly dependent on the surface area and vibration amplitudes of the structure. In the case of tower frameworks, this effect is usually too small to identify. Coulomb friction damping is due to the friction between components at the joints in the structure, e.g. between cross-member and gusset plate compressed together by the bolts. The magnitude of the damping force depends upon the normal pressure between the sliding surfaces and the coefficient of kinetic friction. The normal pressure is related to the tightening torque in the bolts. Dynamic similarity of the joints in the model structure was advised by adjusting all bolts to a uniform torque. The major contribution towards the overall structural damping factor is due to interfacial slip and friction in the structural joints. Although considerable information is available on the individual damping mechanisms, there is little existing fundamental data on the overall damping in built-up structures. This part of the study gives results on the variation in damping with force amplitude and frequency measured by both the oscillation decay curve method and the energy input method [2]. In addition the variation in damping in bending and torsion of the tower is discussed.

The damping factor was initially evaluated using the oscillation decay curve method. A known bending moment was applied to the top of the tower and then suddenly released. The resulting oscillations were in the first natural mode and examination of the decay curve permits the overall damping factor to be determined. For linear damping, the oscillation amplitudes should decrease exponentially. A typical decay curve for free bending oscillation is given as Fig. 4.

A study of a sequence of such decay curves shows that the damping factor is dependent on the amplitude of oscillation as determined from the applied initial bending moment. In addition, as the oscillations decay then the damping mechanism also alters in character. The primary large amplitude damping is

due to considerable interfacial movement at the joints, whereas for smaller amplitudes the damping is altered in character as the stick-slip motions become paramount. A distinct variation in the damping mechanisms is known by the plot of damping factor (decay oscillation number given as Fig. 5. The overall variation in damping factor (i. e. between  $\eta = 0.005$ ) will however result in fairly insignificant variations in the structural response calculated by theoretical methods.

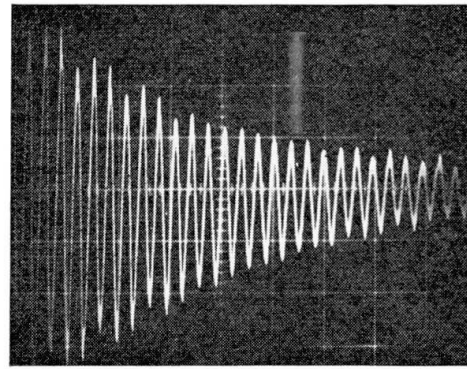
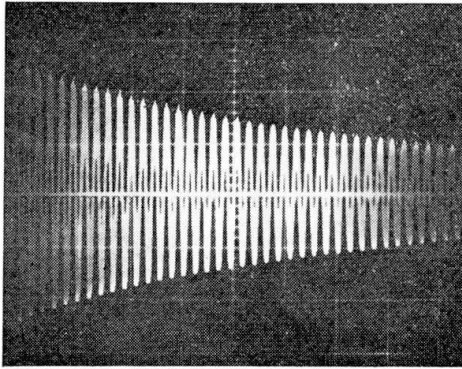


Fig. 4. Oscillation Decay Curve in Bending. Fig. 6. The Oscillation Decay Curve in Torsion.

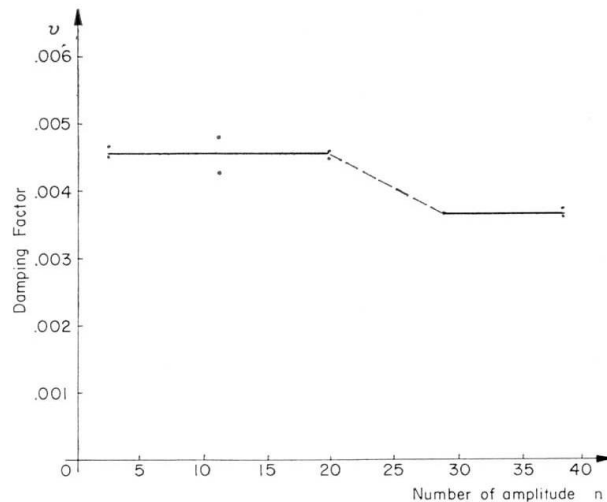


Fig. 5. Damping Factor/Decay Oscillation Number.

The decay curve method was also used to examine the damping characteristics of the tower in torsional oscillation. A typical decay curve is given as Fig. 6.

A comparison of the decay curves in free bending and in torsion shows a marked difference in the rates of energy absorption in the structure. The modal purity of the torque decay curve is not absolute, however the oscillation decay rate indicates a damping factor approximately three times higher in torsional oscillation than in the first free bending mode. This damping also

increases with amplitude, as can be seen from the plot of damping factor; applied initial torque given as Fig. 7.

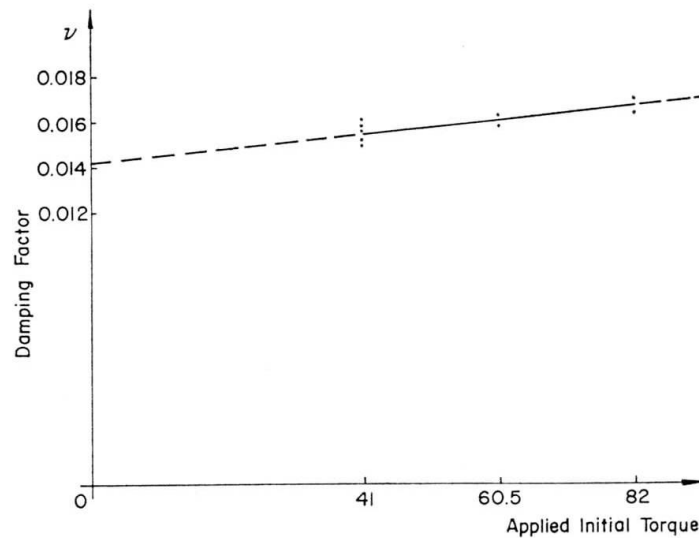


Fig. 7. Damping Factor/Applied Initial Torque.

The energy input method [2] was used to examine the characteristics of the damping mechanism and its variation with frequency. The method is based on the fact that the total energy input to maintain a system in a state of steady vibration is equal to the energy being dissipated within the system by the damping forces. For example, if the force exciting the system is  $P(t)$ , and the velocity at the point of excitation is  $\dot{y}(t)$ , then the work done in a time  $t_1$  is

$$\int_0^{t_1} P(t) \dot{y}(t) dt.$$

If both exciting force and system response are harmonic, i. e.  $P(t) = P_1 \cos \omega t$  and  $y = y_1 \cos(\omega t + \epsilon)$ , then the energy input per cycle is:

$$-\pi P_1 y_1 \sin \epsilon.$$

This quantity also represents the area of the closed hysteresis loop formed by plotting  $P(t)$  against  $y(t)$ . For harmonic excitation and response this plot is an ellipse as shown in Fig. 8.

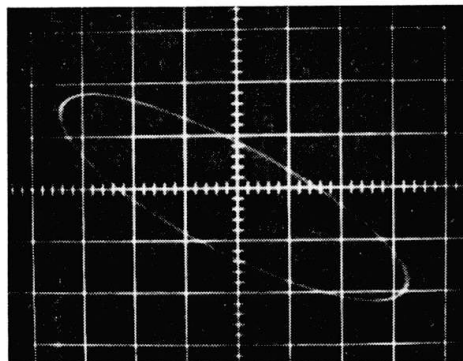


Fig. 8. Applied Force/Displacement  $P \omega t$  for Harmonic Force and Response.

An important characteristic of damping mechanisms is the variation of energy dissipation with input load amplitude. The sequence of  $P(t)/y(t)$  plots shown in Fig. 9 were obtained by gradually increasing the power input to the vibrating tower.

It has been shown [3] that certain damping/stress amplitude characteristics may be represented by the relation:

$$D = J \sigma^n,$$

where  $D$  = energy dissipated/cycle;  $J$  = damping constant,

as damping energy dissipated/cycle at unit stress amplitude;  $n$  = damping exponent. At low amplitudes of structural deformation and stress the overall mechanism is such that a damping exponent  $n=2$  is suitable. This is known as quadratic damping and leads to the elliptic hysteresis loop as shown in Fig. 8. This is also the response typical of a viscous damper in a linear system. As the input forcing amplitude is increased, the hysteresis loop ceases to be

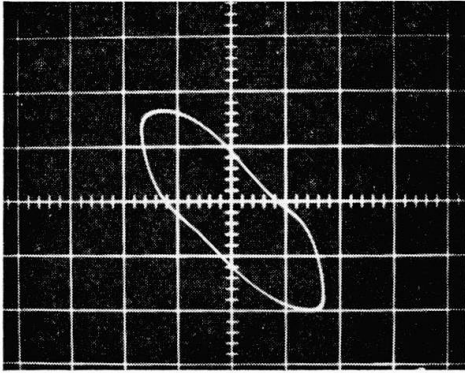


Fig. 9a.

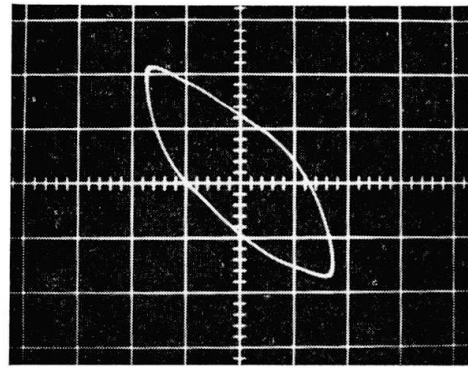


Fig. 9b.

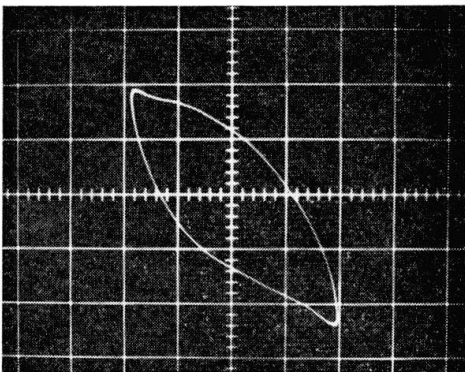


Fig. 9c.

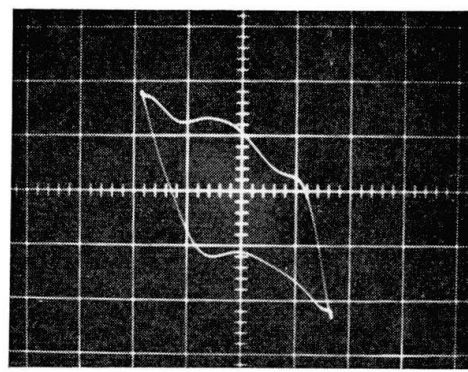


Fig. 9d.

Fig. 9. Applied Force/Displacement Plots for Increasing Force Inputs.

elliptical and develops into a loop typical of systems with a damping exponent  $2 < n < 3$  as shown in Fig. 9a and 9b. This is indicative of a force/displacement hysteresis loop in a system damped by Coulomb friction due to sliding at the

structural joint interfaces. The increasing non-linearity of the system with further increase in input power is reflected in the hysteresis loops Fig. 9c and 9d. These loops are typical of jointed structures under severe dynamic loads and corresponding to a damping index  $n = 3$ . It appears therefore that theoretical models for the damping mechanism typical of the tower framework should be of the form of equation but taking into account the fact that the damping index itself increases with increasing overall stress amplitude.

### Conclusion

The model tests described herein illustrate a number of important points in the dynamic behaviour of triangulated framework tower structures.

In the case of steady state vibrations, the use of elementary theory predicts the principal natural bending modes and frequencies quite closely but does not give any indication of the increasing complexity of the motion as the forcing frequencies increase. In the case of the triangulated framework, this increased complexity takes the form of coupled bending and torsional oscillations.

The vibration damping tests demonstrated an important difference in the damping factors associated with bending modes or torsional modes. In addition, the oscillation decay method indicated a variation of damping factor with oscillation amplitude. This variation was further investigated using the energy dissipation method and illustrations of the hysteresis characteristics at various energy levels are shown. Examination of the form of these hysteresis loops permits theoretical models of the damping mechanism to be postulated.

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### Summary

This paper describes experimental investigations into dynamic behaviour of bolted joint tower frameworks under impulse and steady state excitation. Comparisons are made with simple theory for natural frequencies and influence of additional static loads discussed. Damping characteristics are demonstrated and theoretical models for various input energy levels are discussed.



### **Résumé**

Ce travail décrit la recherche expérimentale du comportement dynamique des tours en treillis boulonnées, sous l'influence de secousses et excitations constantes. On compare les résultats de la théorie simplifiée pour les fréquences naturelles avec l'influence de charges statiques supplémentaires. En outre on présente des caractéristiques d'amortissement et on discute des modèles théoriques pour des entrées de niveau d'énergie différents.

### **Zusammenfassung**

Die Arbeit beschreibt die experimentelle Untersuchung des dynamischen Verhaltens von zusammengebolzten Turm-Fachwerken unter Stößen und konstanter Anregung. Es werden Vergleiche mit der einfachen Theorie für natürliche Frequenzen gezogen und der Einfluss zusätzlicher statischer Belastungen diskutiert. Ferner werden Dämpfungscharakteristiken dargelegt und theoretische Modelle für verschiedene Eingabe-Energieniveaux diskutiert.



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