Ultimate Load Behaviour of Longitudinally Reinforced Webplates Subjected to Pure Bending

Propriétés des charges de rupture de poutres âme pleine renforcées longitudinalement soumises à la flexion pure

Traglastverhalten längsversteifter Profilträger bei reiner Biegung

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List of Symbols

d	Clear depth of webplate between flanges
b	Width of webplate between centre line of adjacent transverse stiffeners
t	Thickness of webplate
δ	Lateral deflections of webplate, stiffeners or flanges
E	Young's modulus
μ	Poisson's Ratio
k	Massonnet efficiency factor
$D = \dfrac{E t^3}{12(1-\mu^2)}$	Flexural rigidity of plate
$\sigma_{cr} = K_b \dfrac{\pi^2 D}{d^2 t}$	Critical (edge) buckling stress for a webplate subjected to pure bending
σ_y	Yield stress of material
EI	Flexural rigidity of a single sided longitudinal stiffener about surface of webplate, and flexural rigidity about the mid surface of the webplate for a double sided longitudinal stiffener
$\gamma = \dfrac{EI}{Dd}$	Non dimensional stiffener rigidity parameter

γ^* Stiffener rigidity required according to the linear theory of web buckling if a symmetrical stiffener is to remain straight when the adjacent panels buckle

Subscripts

v Refers to vertical (transverse) stiffeners
l Refers to longitudinal stiffeners

I. Introduction

The earlier work of MASSONNET [1–4], COOPER [5, 6] and ROCKEY [7] has provided a great deal of information in respect of the behaviour of webplates loaded in pure bending and reinforced by double sided longitudinal stiffeners. In addition most of the above work was restricted to a study of webplates reinforced by a single longitudinal stiffener. Since all of this existing work has been well reported by MASSONNET in his recent paper [8], at the I.A.B.S.E. conference held in New York and also in the papers by COOPER [9] and others [10–12], no detailed survey of existing published work will be given in the present paper and references will be confined to those papers of direct association with the present study.

From his experimental work on all welded girders reinforced by double sided stiffeners, MASSONNET produced his well known efficiency factor concept. MASSONNET noted that for the longitudinal stiffeners on his all welded girders to remain effective up to the collapse of the girder it was necessary that they had a rigidity $\gamma = k\gamma^*$ where γ^* is the theoretical rigidity which, according to the linear theory of elastic web buckling, an ideal stiffener should possess if it is to remain straight when the adjacent webplate panels buckle.

What is also of importance is that MASSONNET showed that the value of his efficiency factor k varied with the position of the longitudinal stiffener; stiffeners close to the compression flange, requiring a higher k value than those closer to the neutral axis.

The k values recommended by MASSONNET are as follows:

Distance between horizontal stiffener and compression flange	k
$0.5\ d$	3
$0.33\ d$	4
$0.25\ d$	6
$0.20\ d$	7

Since the above data was obtained from tests on girders reinforced by double sided stiffeners – it has been considered necessary to carry out tests on webplates reinforced by single sided longitudinal stiffeners.

Tests on aluminium girders of bolted construction conducted by Corney and one of the present authors [13] has shown that when the webplate is initially plane the behaviour of webs reinforced by single sided stiffeners is significantly different from that of similar webs reinforced by double sided stiffeners. However, since a welded steel girder usually has large initial distortions as noted by GOODPASTURE and STALLMEYER [14], who give detailed values one would not necessarily expect similar behaviour from welded steel girders.

The present tests are of particular interest since COOPER and his colleagues [5, 6] in 1965 and 1966 reporting on tests carried out at Lehigh University on all welded steel girders reinforced by a single longitudinal stiffener commented that "the longitudinal stiffeners which were used in these tests had no significant effect upon the observed ultimate loads, except for girder LB 6 where a 11% increase in ultimate load was realised". Although this stagement would appear to conflict with the earlier findings of MASSONNET and others, examination of the test data presented by COOPER shows that failure of the girders occurred at less than twice the theoretical buckling load calculated on the assumption that the web was simply supported along its boundary. In this case, one would not except the influence of stiffener rigidity to have such a significant effect and furthermore it would be difficult to distinguish between the effects of stiffener rigidity and the many other factors such as residual stresses, initial deformations etc., which affect the ultimate load. In a more recent report by D'APICE, FULLDERY and COOPER [15] the authors have modified the earlier findings of COOPER and conclude that "if properly proportioned longitudinal stiffeners are provided, a significant increase in loading strength can result". The authors did not however present any data similar in form to that provided by MASSONNET.

In order to study the effects which the rigidity of single sided longitudinal stiffeners have upon the post buckled behaviour of webs loaded in bending the authors have tested two series of girders in which only the size of the longitudinal stiffener was varied, all other member sizes being kept constant.

The purpose of the present study was as follows:

1. To study the behaviour under pure bending of large all welded webplates reinforced by one line of single sided longitudinal stiffeners placed at the optimum position according to the linear theory of web buckling [20–23].

2. To study the behaviour of webplates reinforced by two lines of single sided longitudinal stiffeners placed at the optimum position according to the linear theory of buckling [16, 17].

3. To develop a simple collapse method of design for reinforced webs loaded in bending.

II. Design of Girders

Since the object of the investigation was to study the post buckled and collapse behaviour of longitudinally reinforced girders when subjected to pure bending, only that portion of the test girders so loaded was of interest. It was therefore decided to test girders having detachable end panels. These end panels were overdesigned to ensure that failure took place in one of the three central panels which were subjected to the uniformly applied bending moment. Two sets of end panels were manufactured and these were used throughout the complete investigation.

The general details of the girders tested are given in Fig. 1. It will be noted that three types of girders were tested. One girder TG 0 was fitted with only transverse stiffeners which had been designed to provide a value of γ_v $(\gamma = E I/Dd)$ equal to 3 times the γ_v^* [18, 19] value; the purpose of this test being two fold. First to provide a datum against which to gauge the performance of the longitudinally stiffened girders and secondly to check whether or not the value of 3 for the efficiency factor k for transverse stiffeners as proposed by MASSONNET is satisfactory. The second series comprised four girders having the central test section reinforced by one line of single sided longitudinal stiffeners together with two transverse stiffeners placed as shown in Fig. 1 b. The strength of the transverse stiffeners was kept constant and the girders only differed from each other in the strength of the longitudinal stiffeners. Three of the girders were fitted with single, one-sided, longitudinal stiffeners and the fourth with a single-double sided stiffener. The purpose of testing the latter girder was to provide a datum against which to judge the performance of the one-sided stiffeners. The details of the longitudinal stiffeners employed are given in Table 1.

Table 1

| Test girders	No. of longitudinal stiffeners	Position of stiffeners		Dimensions of stiffeners (in.)	
		First stiffener	Second stiffener	First stiffener	Second stiffener
TG 0	0	—	—	—	—
TG 1-1	1	0.2 d	—	0.713 × 0.187	—
TG 2-1	1	0.2 d	—	0.872 × 0.187	—
TG 4-1	1	0.2 d	—	1.323 × 0.187	—
TG 7-1*)	1	0.2 d	—	0.87 × 0.187	—
TG 1-2	2	0.123 d	0.40 d	0.765 × 0.187	0.758 × 0.187
TG 2-2	2	0.123 d	0.40 d	1.008 × 0.187	1.014 × 0.187
TG 3-2	2	0.123 d	0.40 d	1.262 × 0.187	1.261 × 0.187
TG 4-2	2	0.123 d	0.40 d	1.503 × 0.187	1.531 × 0.187
TG 5-2	2	0.123 d	0.40 d	1.741 × 0.187	1.746 × 0.187

*) Double sided stiffener
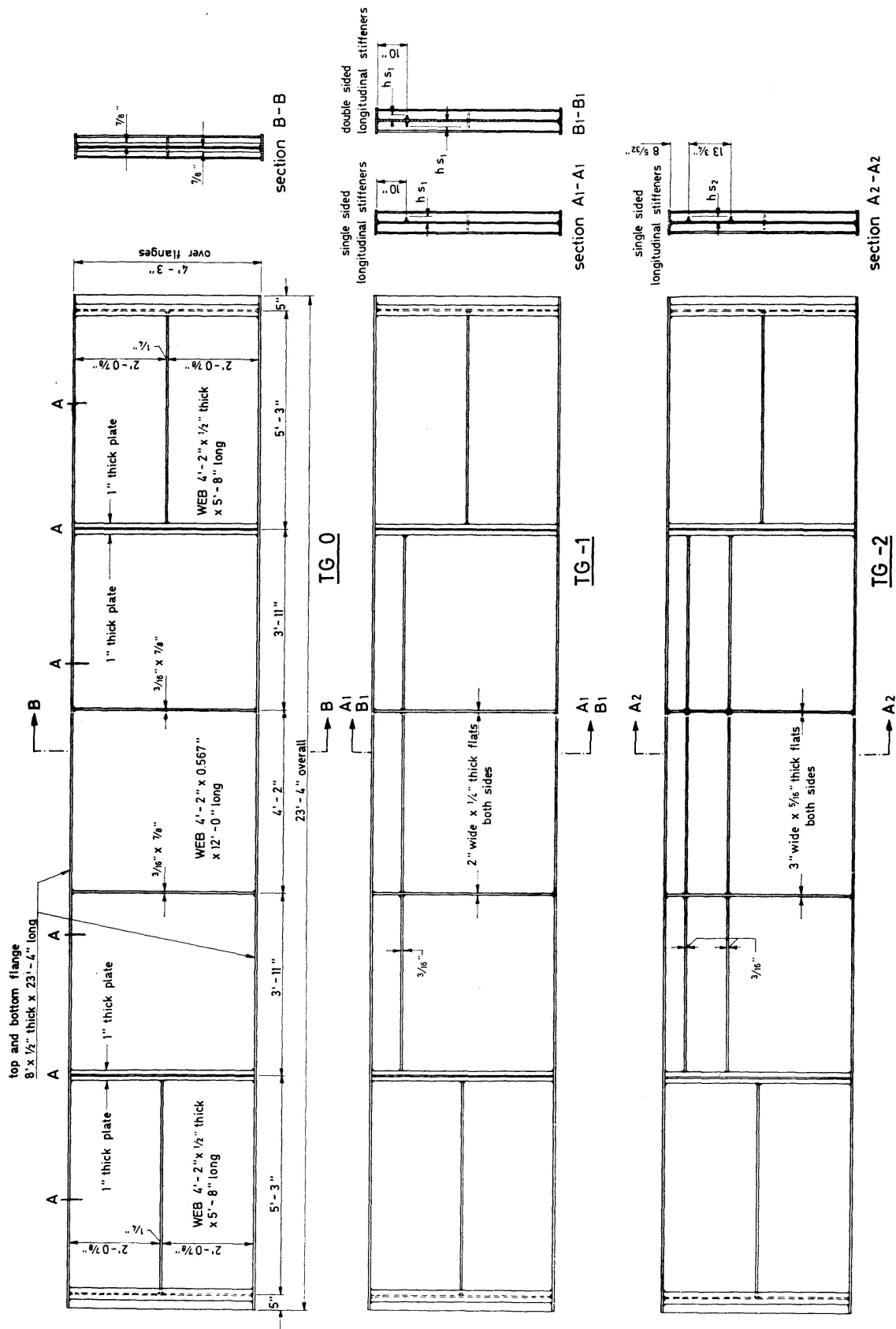
Fig. 1.

These longitudinal stiffeners were positioned at one fifth of the overall depth of the girder from the compression flange, which is the optimum position for a web which is simply supported on all 4 edges [20–23].

It has been established in references [20–23] that the value of γ^* varies the area parameter β, where β is the ratio of the cross sectional area of the stiffener to the area of the webplate (dt). It has been shown in reference [16] that the relationship between γ^* and the area parameter β is given by Eq. (1)

$$\gamma_L^* = \gamma_{L0}^* + k\,\alpha^2 \left(1 - \frac{2\,\eta}{d}\right)\beta_L,\qquad(1)$$

where γ_{L0}^* is the value of γ^* when $\beta_L = 0$.
η is the position of the stiffener from the compression flange.

In determining the value of γ^* given in Table 1, a practical longitudinal stiffener, of rectangular cross section, was designed subject to the requirement that its depth to thickness ratio should approach but not exceed 8.5 : 1.

The area of the actual stiffeners used in the tests varied and γ_{La}^* is the corresponding value of γ_L^* for the actual β value possessed by the stiffener.

The third test series consisted of five girders each having two lines of one-sided longitudinal stiffeners whose dimensions are also given in Table 1. It was again assumed that the flanges and transverse stiffeners provide a simple support to the web and the stiffeners positioned so as to give the maximum resistance against buckling. Thus the stiffeners were placed at $0.123\,d$ and $0.275\,d$ from the compression flange [16, 17].

Both the end and central panels were of welded construction. The flanges and the heavy transverse stiffeners at the end of the panels were continuously welded to the webplate but in an attempt to reduce the distortions of the web due to welding, the longitudinal and intermediate transverse stiffeners were welded to the web using a staggered welding procedure. This can be seen in a number of the photographs presented later in the report.

The end panels were attached to the central test section by means of 1″ diameter high tensile steel bolts distributed across the depth of the girder, these are clearly shown in Fig. 3.

Although a large number of bolts were used to connect the end panels to the central panel, it was found necessary to provide an additional connection between the tension flanges on either side of the junction between the end panel and the central test girder, since during the testing of girder TG 7-1 (the first test conducted) a weld fracture occurred at the junction of the tension flange with the heavy end stiffeners. A satisfactory solution was achieved by means of a short cover plate which was welded to the tension flange of the central section and bolted to the end panel.

The ratio of panel depth to web thickness was chosen to be 750 : 1, the web plate being 0.0666 in. thick and the clear web depth between flanges 50 in.

This ensured that the ultimate load, as calculated from simple plastic theory, was approximately three times the critical load for those girders with two lines of longitudinal stiffeners and six times the critical load for girders reinforced by a single line of stiffeners. By choosing such a high d/t ratio it was possible to make an adequate study of the post buckled behaviour of the web. The tension and compression flanges were of equal section each being $8''$ wide and $\frac{1}{2}''$ deep. The vertical or transverse stiffeners separating the three central panels were designed to provide an inertia 3 times the theoretical value required, according to the linear theory.

III. Experimental Apparatus

III.1

The girders were simply supported at their ends on case-hardened steel rollers and loaded vertically at the junction of the strong end panels and the central section by means of two 100 ton capacity hydraulically operated jacks, see Fig. 2 and 3, which show one of the test girders in position in the testing
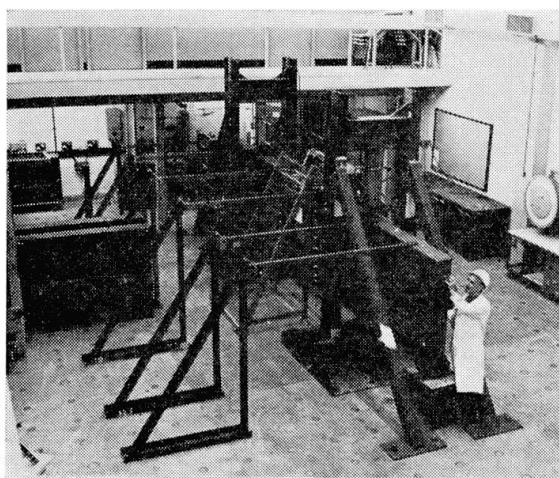

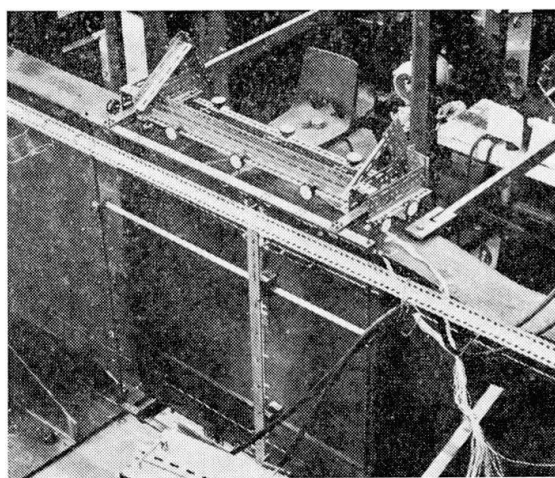Fig. 2. General view of testing arrangement.

Fig. 3. View of Central Bay of girder showing deflection measuring apparatus.

frame. Each jack reacted against a very rigid yoke which was bolted to the testing floor. By ensuring that the two jacks applied equal loads, the central test section was in a state of pure bending, the bending moment being constant along the length of the entire experimental section. The applied load was recorded by two load cells, connected to Elliott load indicators which can be seen in the right hand side of Fig. 2.

Since it was essential that no lateral buckling of the compression flange occurred, the compression flange was restrained against lateral deflection at six positions by the stabilising trusses which are also shown in Fig. 2; the

positions where the stabilising trusses were connected to the girders is given in Fig. 1. The stabilising arms were pin connected at one end to the compression flange of the girder and at the other end to the rigid trusses which are securely bolted to the floor at the other. Since these lever arms were 6 ft. long, they allowed vertical movement of the girder but prevented lateral movement of the compression flange. Reamered holes and machined pins were used to ensure good restriction in the lateral direction.

III.2. Deflection Measuring Apparatus

a) *Flanges*. The rotational, vertical and lateral movements of the compression flange over the central 13' 6" section were recorded by means of the dial gauge system shown in Figs. 2 and 3, each gauge being graduated in units of 0.0001". The measurements were made at five equally spaced sections as shown in Fig. 4. Since the frame supporting the dial gauges was firmly bolted
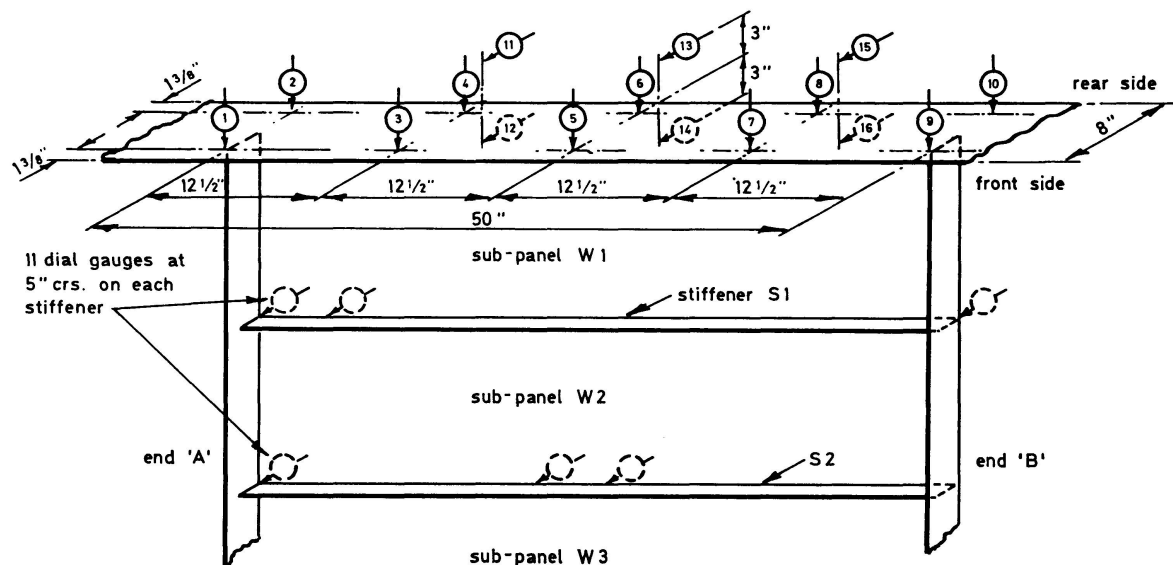

Fig. 4. Distribution of dial gauges.

to the floor, the over-all deflections of the girder were measured. The measurement of the lateral movement of the compression flange was complicated by the fact that the girders deflected vertically by an appreciable amount. To allow for this the dial gauges measuring this lateral movement recorded against vertical plates clamped to the edge of the compression flange as may be seen in Figs. 3, 4 and 5.

b) *Stiffeners*. Since one of the most important factors under investigation was the behaviour of the longitudinal stiffeners, it was essential that their deflection was measured with extreme accuracy. This was achieved by means of the dial gauge system shown in Fig. 5. This apparatus was clamped to the
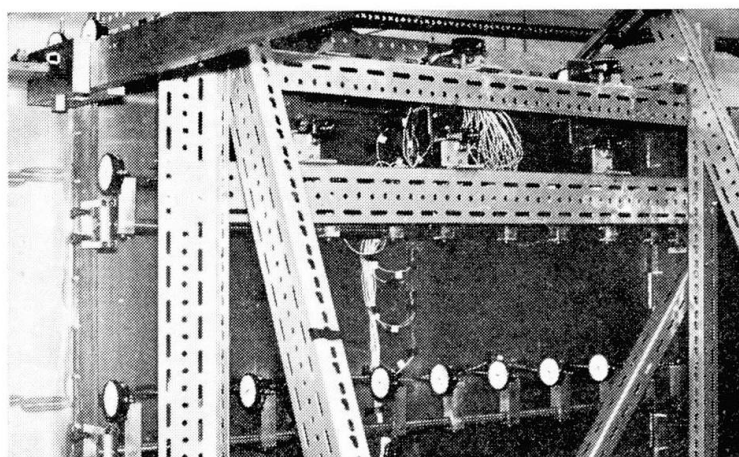
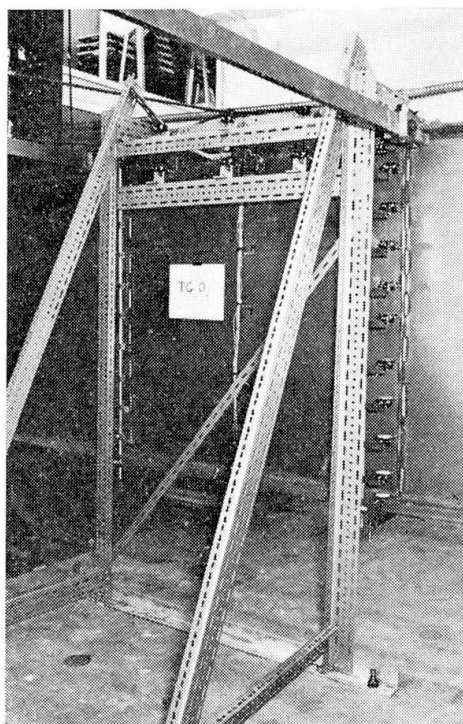Fig. 5. Dial gauges in position to record lateral deflection of longitudinal stiffeners.

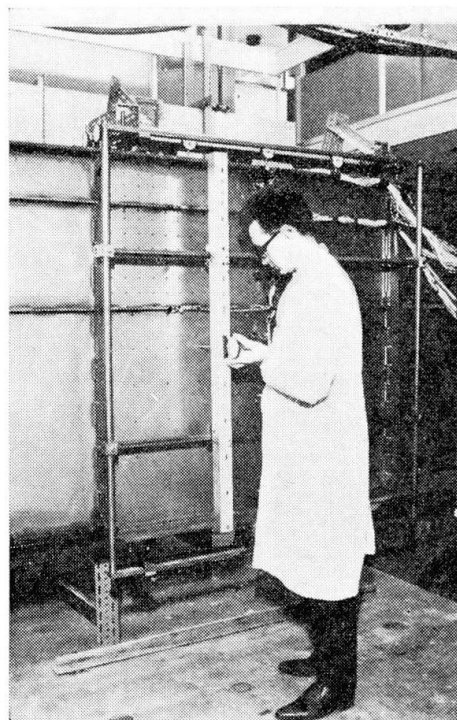Fig. 6. Central section of girder TG 0 under test.

Fig. 7.

transverse stiffeners, its construction being such that it remained unstressed by the deformation of the girder under load. Eleven dial gauges calibrated in intervals of 0.0001 in., were distributed at 5″ centres along each stiffener. For the testing of the girder TG 0, which had no longitudinal stiffeners, the same device was used to record the lateral displacement of the vertical stiffeners, see Fig. 6.

c) *Web Plate.* In order to be able to determine the effectiveness of the various stiffeners in preventing the lateral deformation of the web plate under loading, the device shown in Fig. 7, see also Fig. 3, was used to measure web

plate deflections over the whole of the centre panel. The frame was spring loaded on to the girder to avoid self stressing during the loading of the girder. The frame was attached to the tension, and compression flanges in the plane of the web plate, very small centering holes having been drilled in the flange to locate the framework. Thus the apparatus was completely unaffected by any rotation occurring in the flanges. The vertical bar, which was used as a datum for the depth gauge employed, was provided with slots to allow deflection readings to be made with a dial depth gauge at convenient vertical intervals. The depth gauge, which was calibrated in units of $0.001''$ had a $2''$ travel. This vertical reference bar, which was supported on two longitudinal guides, could be moved into any longitudinal position, thus allowing readings to be taken along any vertical section. The trial tests carried out with this apparatus indicated that good reproductibility of results was obtained.

The lateral deflection of the web plate in the central panel was measured at a sufficient number of points to enable a contour plot of the deformed shape to be drawn. For the large panel adjacent to the tension flange, readings were taken over a 5 in. square mesh. In the case of the panel adjacent to the compression flange for girders with one line of stiffeners and for the case of two stiffeners, the panel bounded by the stiffeners, readings were taken over a 2 inch square mesh. For those girders with two lines of longitudinal stiffeners, in the panel between the compression flange and the first stiffener, readings were taken at 2 in. intervals in the longitudinal direction and at $1\frac{1}{2}''$ intervals in the vertical direction.

IV. Strain Measurements

Each girder was instrumented with electrical resistance strain gauges. In each case, strain gauges were attached to the web, flanges and the longitudinal stiffeners. Wherever possible, orthogonal pairs of gauges were used to enable the evaluation of stresses from the strain readings. The gauges used were PL-10 polyester type gauges supplied by Electro Mechanisms Ltd., who claim an accuracy of $\pm 1.5\%$ for these gauges. The gauges were connected to a multipoint Bruel and Kjaer strain recording bridge which provided the strain readings direct in micro-strains.

V. Test Procedure

The girders were tested in the following order. The girders with one line of longitutinal stiffeners were tested first followed by those girders with two lines of longitudinal stiffeners. Finally, the unstiffened girder TG 0 was tested.

The strain gauge bridges were initially balanced to give zero readings and the initial readings of the various dial gauges were recorded.

The lateral deflection of the web and stiffeners was measured at zero load to enable a contour plot of the initial deformed slope of the web to be determined.

The girder was then loaded by the hydraulic jacks the load being applied in increments of $2\frac{1}{2}$ tons, all strain gauge readings being noted at these loads. At 10 ton intervals, the stiffener deflections were noted as well as the readings of the dial gauges used to measure the deformation of the compression flange. However, as soon as the strain gauge readings indicated that either the flanges or the web was yielding, these readings were recorded more frequently.

For the first girder tested, web deflection readings were taken at 10 ton intervals, but subsequent analysis of the results showed the lateral movement of the web to be small for values of the applied load below about 20 tons. Hence, for the remaining tests on girders with a single line of longitudinal stiffeners, lateral web deflection readings were taken at applied load values of 30 tons and 45 tons. Since the girders with two lines of longitudinal stiffeners were considered to be of prime importance in their case web deflection readings were taken at applied load values of 20, 30 and 40 tons.

As soon as the behaviour of the girder indicated that the ultimate load was being approached, all dial gauges were removed in order to avoid their being damaged. The girder was then further loaded slowly until collapse occurred, the ultimate load being noted.

An exception to the above procedure for the final stages of the test occurred during the first test conducted, where the girder failed prematurely, very suddenly, due to a weld failure; the tension flange breaking away from the vertical member. This weakness due to the attachment details was overcome in following tests by employing a flange strap as described in a previous section.

A large number of material tests were carried out on samples of flange and web plate materials for each girder. Tensile tests were performed on specimens cut from the sheet both along and at right angles to the direction of rolling. The principal results are listed in Table 2, average values being recorded.

Table 2

Material	Limit of proportionality tons/in²	Lower yield stress tons/in²	Upper yield stress tons/in²	Ultimate stress tons/in²
Web plate	8.68	13.66	13.98	21.31
Flange plate	12.36	15.66	15.98	27.37

VI. Discussion of Test Results

In the present paper full test details will not be given for all of the tests, comments will be restricted to presenting the overall behaviour of the girders together with a detailed discussion of the results which arise from the tests.

In order to provide a datum against which it would be possible to gauge the efficiency of longitudinal stiffeners, a girder TG 0, which did not have any longitudinal stiffeners, was tested. Except for the dimensions of the vertical stiffeners, which were designed to have a $\gamma_v = 3\gamma_v^*$, all other dimensions of this girder were equal to those of the other girders.

The measured relationships between flange strains and the applied load are given in Fig. 8. It will be noted that the compression load/strain plots depart from a linear relationship as early as 30 tons and that local yielding in the web occurs at 40 tons. In contrast, the tensile load/strain relationships remain reasonably linear and even at 45 tons the tensile strain has not reached the lower yield limit. Since the slopes of the load/strain plots for the gauges attached to the front and rear of the compression flanges are in close agreement this indicates that very little lateral bending of the compression flange occurred; likewise, the dial gauges also indicated that little lateral bowing occurred. The
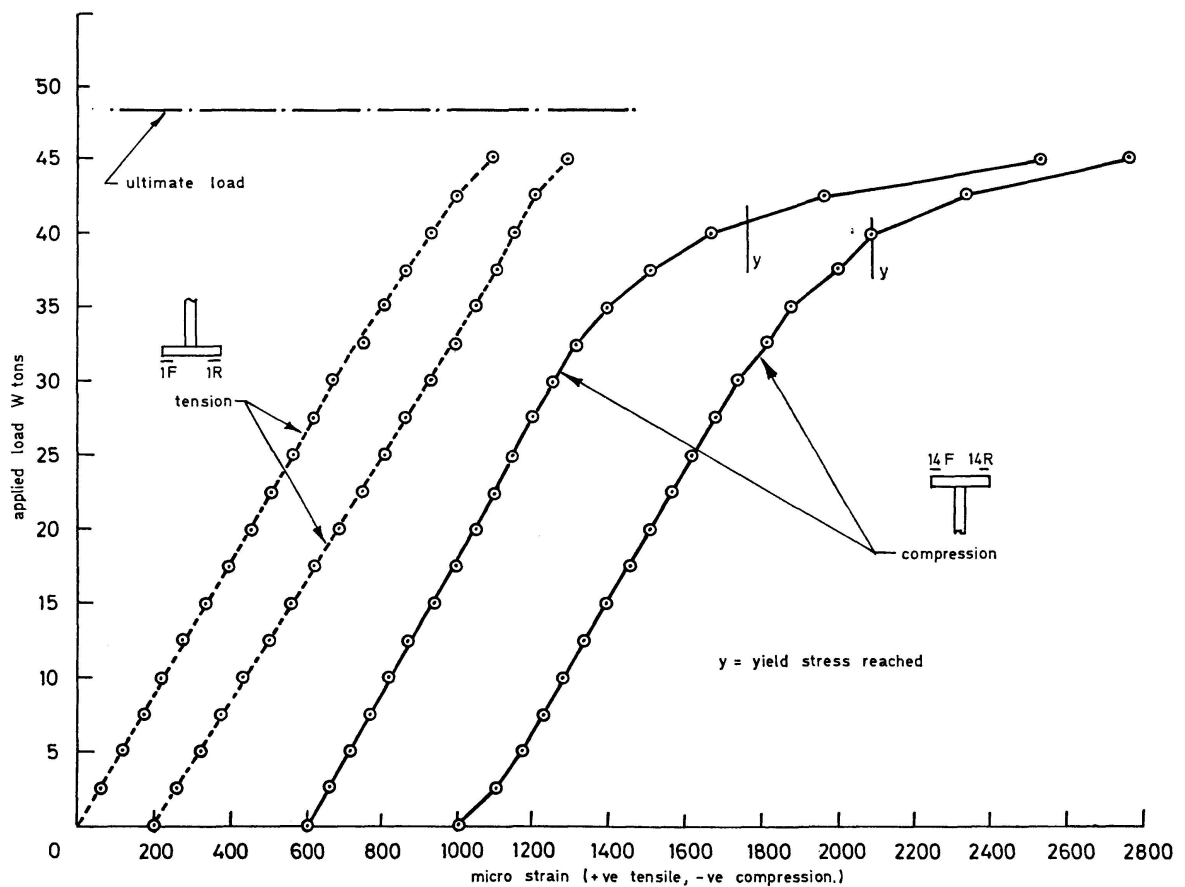
Fig. 8. Longitudinal strains in tension and compression flanges at mid section of girder TG 0.

early departure from linearity which occurred in the compression flange is due to load shedding causing a shift of the neutral axis with a resulting increase in the compressive strains.

The large initial lateral deflections caused by the welding process were found to dominate the post-buckled behaviour of the webplate, this being clearly shown in Fig. 9. It is of interest, however, to note that in the tension zone, the tensile forces have tended to reduce the initial deformations.
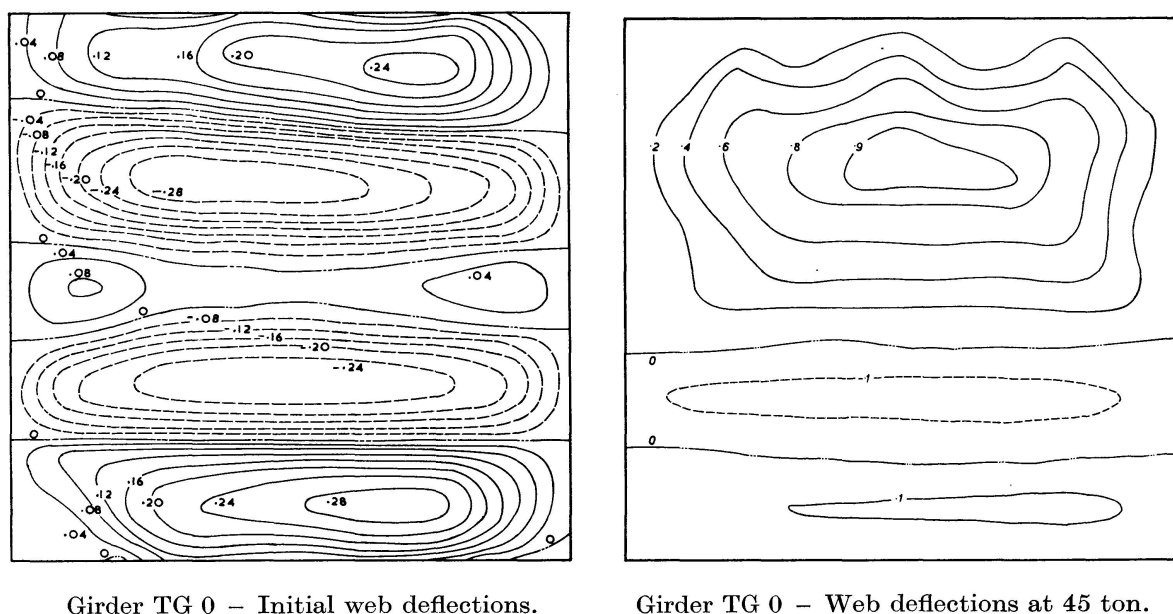

Girder TG 0 – Initial web deflections.   Girder TG 0 – Web deflections at 45 ton.

Fig. 9. Web deformation in web of girder TG 0 – all dimensions in inches.

The measurements of the lateral deflection of the two transverse stiffeners in the central section, indicated that slight bowing of them occurred during loading, see Fig. 10. However, this bowing was slight and even when the applied load had reached 37.5 tons, the maximum deflection was still less than the thickness of the webplate.

The girder finally failed at a load of 48.25 tons when the compression flange buckled vertically inwards, see Fig. 11. This load was 25.4 times the web buckling load calculated assuming simply supported edges. It was subsequently observed that the length of this flange buckle was somewhat greater than that which occurred in those girders having webs reinforced with longitudinal stiffeners.

The next test was conducted on girder TG 1-1 which was fitted with a longitudinal stiffener of size $0.713 \times 0.1875'$, see Table 1, which provided a value of γ_L equal to 0.89 times the value of γ_L^*, which according to the linear theory a stiffener should possess if it is to form a nodal line. Table 3 gives the maximum initial web deformations of this girder, from which it will be noted that these initial web deformations were very large, the maximum web plate
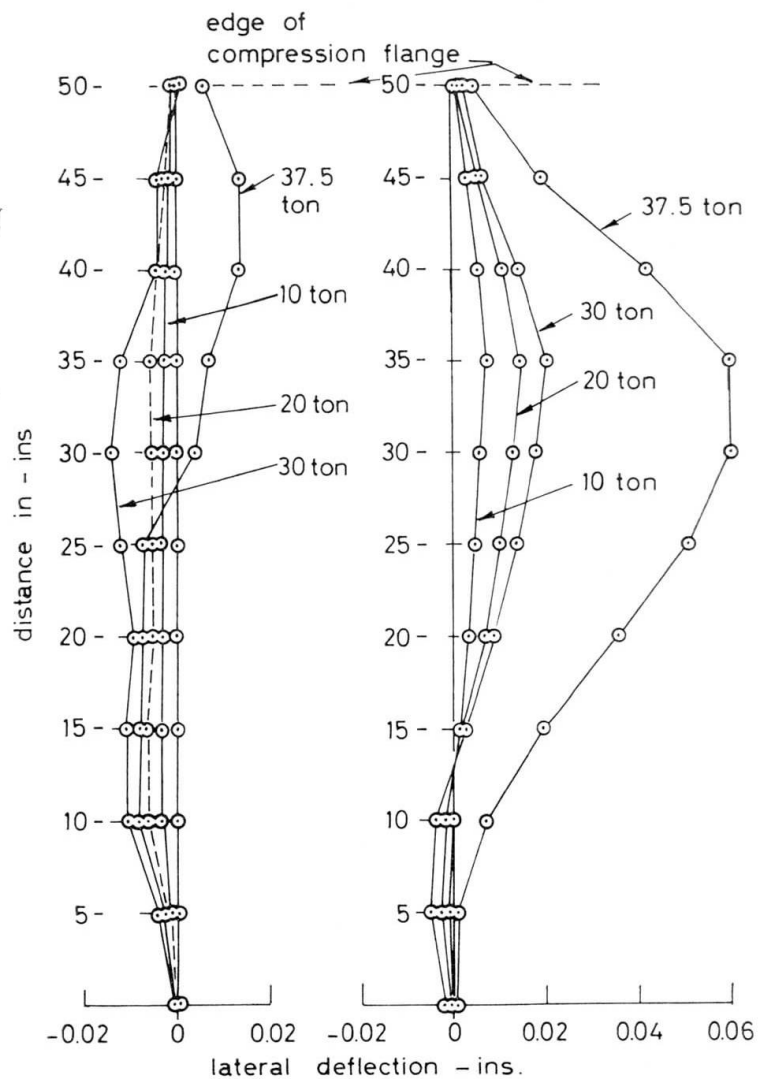
Fig. 10. Lateral deflection of vertical stiffeners in central section. Girder TG 0.

deflection being 4.25 times the web thickness. The stiffener itself had an initial deformation, single curvature in form, with a maximum deflection 2.89 times the web thickness.
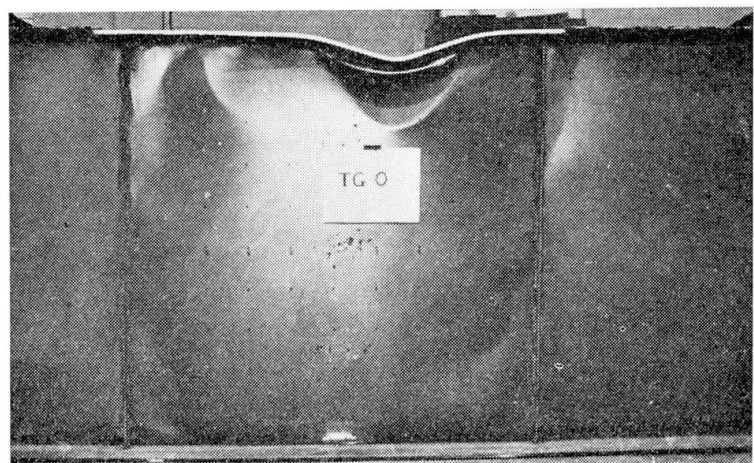

Fig. 11. Girder TG 0 after to failure.

Table 3. Maximum Initial Deformations (ins.)

	First stiffener	Second stiffener	Top panel W 1	Middle panel W 2	Lower panel W 3
TG 0	—	—	0.3	—	—
TG 1-1	0.192	—	-0.241	-0.281[1])	—
TG 2-1	0.152	—	0.158	-0.193[1])	—
TG 4-1	0.056	—	0.166	-0.338[1])	—
TG 1-2	-0.032	0.37	-0.078	-0.113	-0.092
TG 2-2	-0.110	0.138	-0.11	0.133	0.183
TG 3-2	0.092	-0.05	-0.091	0.138	-0.073
TG 4-2	0.020	0.06	-0.087	-0.155	-0.320
TG 5-2	0.040	0.117	-0.089	+0.204	0.268

[1]) Lower panel for girders fitted with a single longitudinal stiffener.

In view of the very heavy web deformations which the authors encountered in their test programme which are similar in magnitude to those experienced by other investigators [14] it is considered highly desirable that detailed studies should be made into the effect of welding processes upon the initial deformation of web plates. In view of these web plate deformations one would not expect to observe any buckling phenomenon and therefore the main interest in the test was concentrated upon the behaviour of the flange members and the longitudinal stiffener. The strains which occurred in the compression and tension flanges of this girder were similar in form to those given in Fig. 8, although the compression strains remained elastic up to a higher load.

A strain equal to the local yielding strain was noted in the flange at a load of 46 tons, whereas the tension flanges remained elastic until close to the ultimate load.

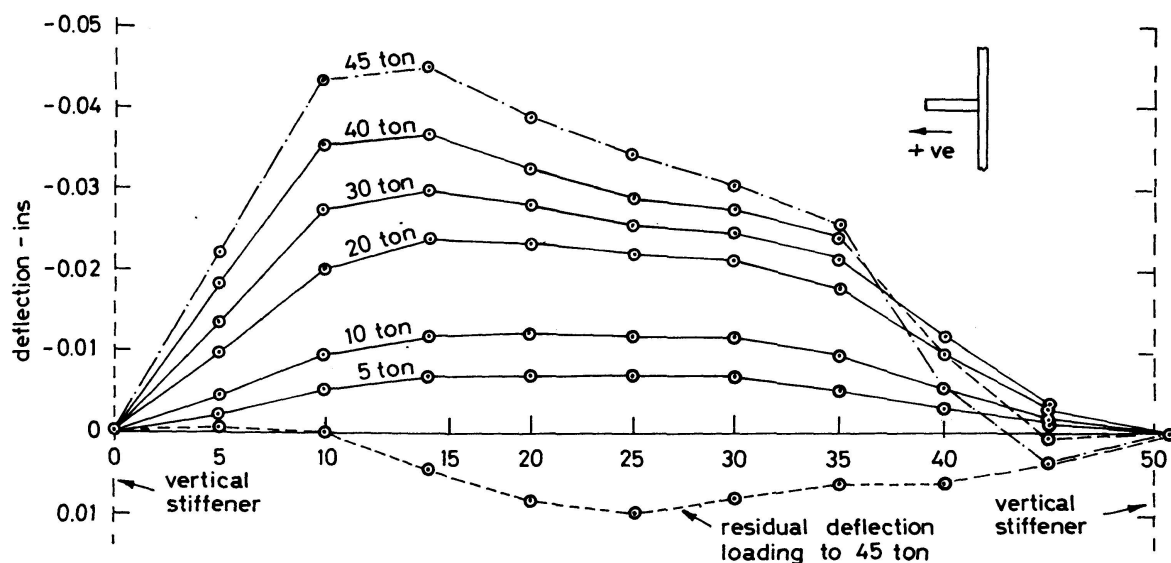Fig. 12 gives the additional lateral deflections of the stiffeners which

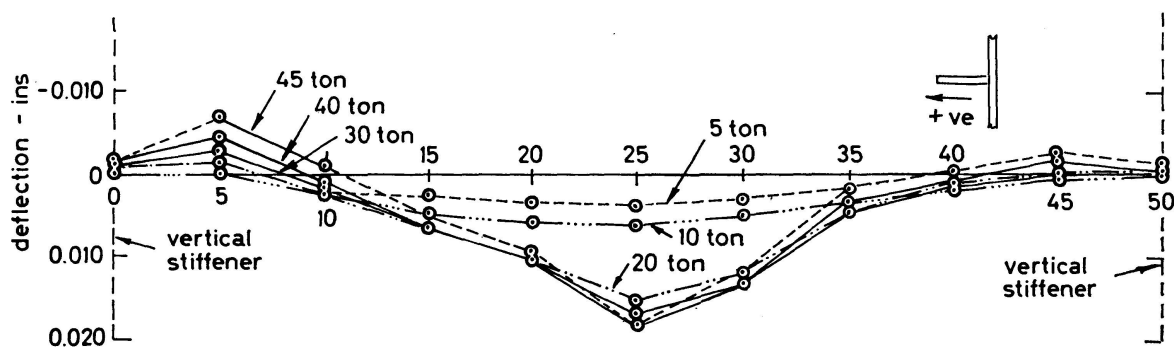Fig. 12 a. Lateral displacement of longitudinal stiffener in central panel. Girder TG 1-1.

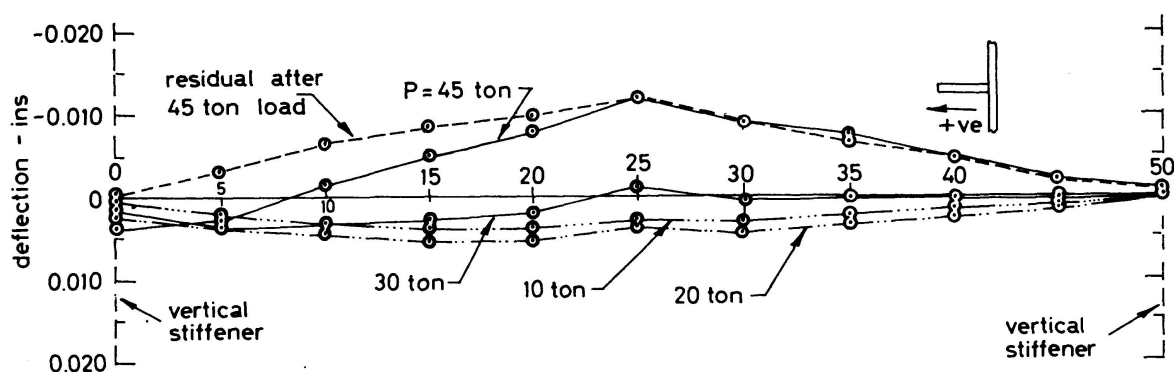Fig. 12 b. Lateral displacement of longitudinal stiffener in central panel. Girder TG 2-1.

Fig. 12 c. Lateral displacement of longitudinal stiffener in central panel. Girder TG 4-1.
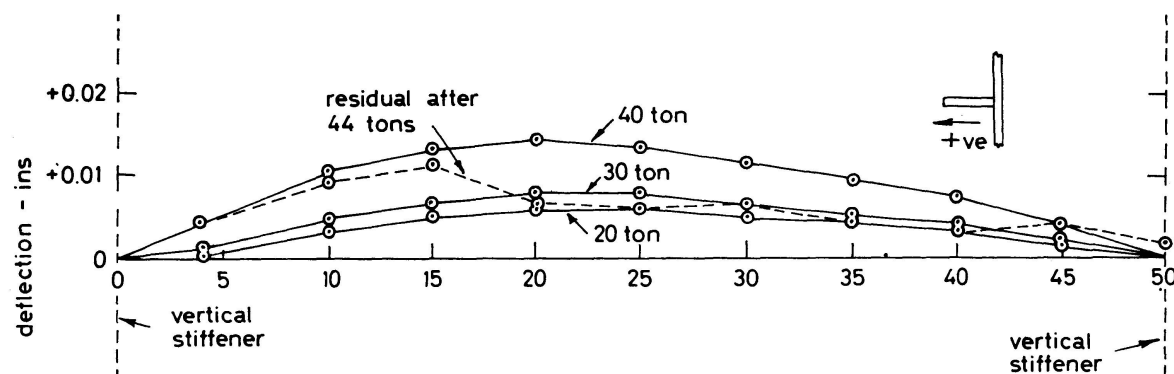

Fig. 12 d. Lateral displacement of longitudinal stiffener in central panel. Girder TG 7-1
(double sided stiffener).

occurred at different values of applied load from which it will be noted that the stiffener has tended to straighten, i.e. to reduce in the magnitude the initial deformation. Although at a load of 45 tons, these additional deflections are just in excess of 0.7 times the web plate thickness, they are still relatively small in comparison with the initial deformations of the stiffeners.

The dial gauges recording the vertical deflection of the compression flange remained linear up to loads in excess of 45 tons, it was also noted that no significant twisting of the compression flange occurred.

It was noted in all of the tests conducted that failure first occurred in one of the panels adjacent to end panels, due to the reduced buckling stress of this

panel due to the application of load at the junction of the test and end panels. The flange in this region was then reinforced by a structural member and the girder loaded to failure once more.

Failure finally occurred in the central panel when the applied load had reached 55.5 tons, the compression flange again buckling inwards in what is generally called "vertical buckling". At this load, the flange was yielding and the buckle in the flange had formed above the region where the web deflections were the greatest. At this section, there is lateral loading on the flange due to the tensile membrane stresses developed in the buckled web, this membrane action being in phase with the buckle pattern and greatest where the lateral deflections of the web are greatest.

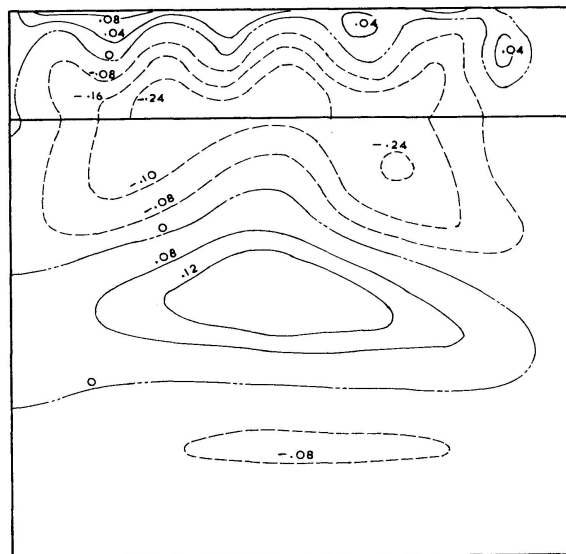Fig. 13 a shows the initial lateral deformations of the central web panel, it
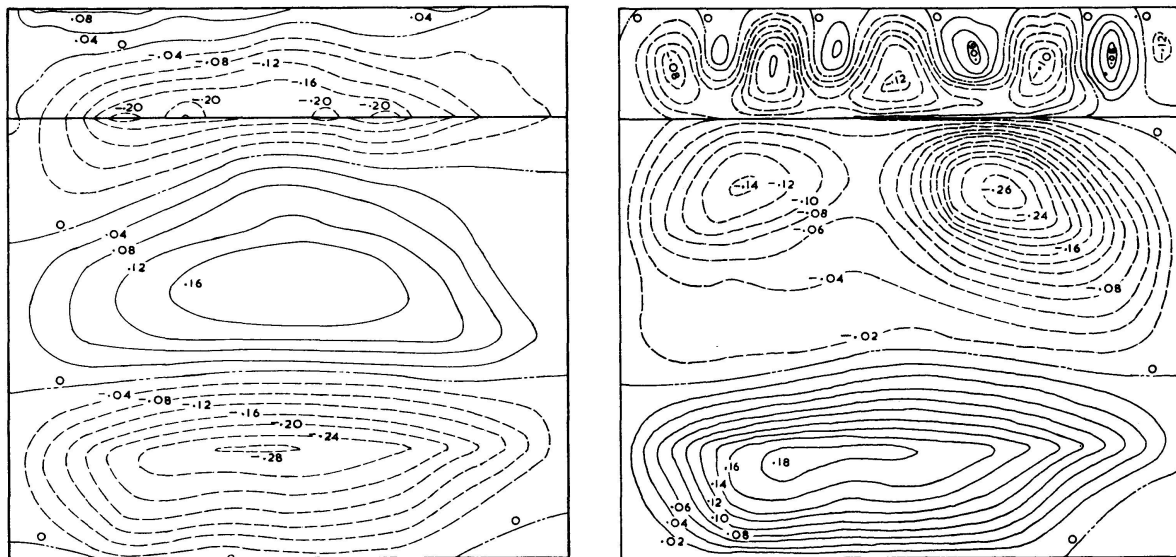
a) Initial deflections in web
of girder TG 1-1.

b) Additional deflections in web after loading
each jack to 45 ton.

Fig. 13.

c) Total web deflection after loading
each jack to 45 ton.

will again be noted that the initial web deflections, resulting from the welding process; are very large. Fig. 13 b gives the additional web deflections after the girder had been loaded to 45 tons. Fig. 13 b shows that the loading has tended to straighten the web in the tensile zone and to buckle the web in the compression zone into the characteristic sinusoidal wave pattern. However, as will be seen from Fig. 13 c the initial deformations still dominate the overall picture.

Fig. 14 shows girder TG 1-1 after it had failed, from which it will be noted that the web and the stiffener has failed in the direction of the initial deformations. It is also of interest to note that the inward collapse of the girder has occurred in the region where the highest lateral deflection of the web and stiffener occurred.

The second girder with a single longitudinal stiffener TG 2-1, had a longitudinal stiffener of size $0.872\,\text{in.} \times 0.1875\,\text{in.}$, this stiffener thus had a rigidity which was 1.84 times that of the stiffener on Girder TG 1-1 and equal to $1.63\gamma^*$ as defined and shown in Table 4. Due to the additional stiffness of this longitudinal stiffener, it will be noted from Table 3 that the initial deflections of the stiffener are reduced, as are also the initial web deflections, the latter being only 65% of these of TG 1-1. This girder behaved in a manner very similar to that of girder TG 1-1. The load/strain plots for the compression flange remained linear up to 40 tons and local yielding did not occur until a load of 50.5 tons was reached. As in the case of TG 1-1, the tensile flange load/strain plots remained linear until a much higher load was reached.

Fig. 12 gives the lateral deflection of the longitudinal stiffener for a number of different applied loads. It will be noted that in this case, the additional

Table 4

Test girder	No. of longitudinal stiffeners	γ/γ^* [1]	γ/γ_a^* [2]	W_{cr}	W_{exp}	W_{th}	$\dfrac{W_{exp}}{W_{th}}$	$\dfrac{W_{exp}}{W_{cr}}$
TG 0	0	—	—	1.89	48.25	56.9	0.848	25.52
TG 1-1	1	0.89	0.795	9.62	55.5	57.66	0.963	5.77
TG 2-1	1	1.63	1.412	9.66	57.15	57.79	0.989	5.92
TG 4-1	1	5.8	4.525	9.75	59.75	58.16	1.027	6.13
TG 7-1 [3]	1	3.28	2.645	—	—	—	—	—
TG 1-2	2	0.64	0.602	24.15	54.75	58.61	0.934	2.27
TG 2-2	2	1.51	1.323	24.25	52.50	58.92	0.891 [4]	2.17 [4]
TG 3-2	2	2.94	2.39	24.44	62.50	59.24	1.055	2.56
TG 4-2	2	5.11	3.89	24.60	64.0	59.55	1.075	2.60
TG 5-2	2	7.75	5.57	24.80	64.75	59.82	1.082	2.61

[1] γ^* is the flexural rigidity of an optimum longitudinal stiffness of rectangular cross-section that results from a design by the linear theory of web buckling.
[2] γ_a^* is the flexural rigidity resulting from the linear theory of web buckling and corresponding to the actual area factor β of the longitudinal stiffener.
[3] Double sided stiffener.
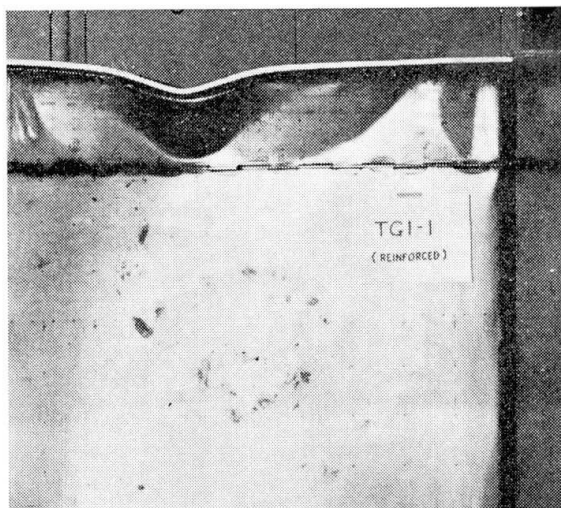[4] Lateral buckling influenced W_{exp}.

Fig. 14. View of front side of central panel
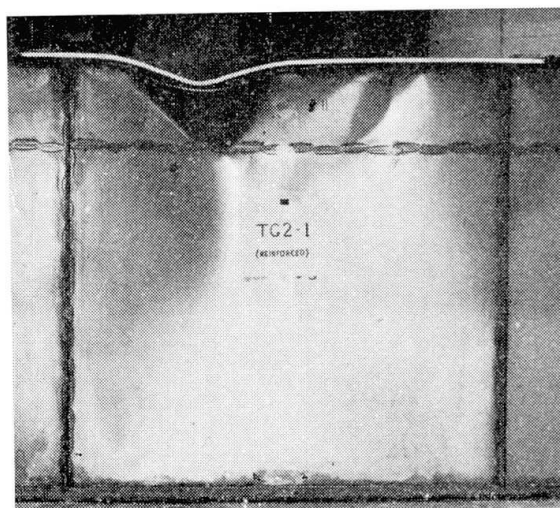of girder TG 1-1 after test to failure.

Fig. 15. View of girder TG 2-1
after test to failure.

deflections of the stiffeners are small, at a load of 45 tons the maximum
stiffener deflection being only some 30% of the web thickness.

Girder TG 2-1 finally failed in the central panel by inward buckling of the
flange at a load of 57.75 tons. Fig. 15 shows girder TG 2-1 after it was failed.
It was noted that the inward buckling of the flange again coincided with a
large web buckle, as in the case of girder TG 1-1. Furthermore, it is seen that
by increasing the size of the longitudinal stiffener the failure load of the girder
has been increased some 4%.

Girder TG 4-1 which was reinforced by an even heavier longitudinal stiffener,
see Table 1, behaved similarly to the other two girders having a single sided
stiffener, and finally failed in the central panel at an applied load of 59.75 tons,
this load being some 24% greater than the collapse load of girder TG 0.

Fig. 12 gives the additional lateral deflection of the longitudinal stiffener
on girder TG 4-1 at various applied loads. It will be noted that these additional
deflections are extremely small, indicating that the stiffener was remaining
quite straight, its maximum initial deflection being only 0.83 the web thickness.
The strain gauges readings also indicated that little bending of the stiffeners
occurred.

The behaviour of those girders fitted with two lines of single-sided stiffeners
was similar in character to that of the girders with a single longitudinal
stiffener. All girders failed by inward collapse of the flanges. It was also noted
that with an increase in the size of the longitudinal stiffeners there was an
increase in the ultimate load.

The load/strain plots for the compression and tension flanges followed the
same pattern as for girder TG 0 and of that for those girders with a single
longitudinal stiffener.

The relative behaviour of the two lines of longitudinal stiffener merits
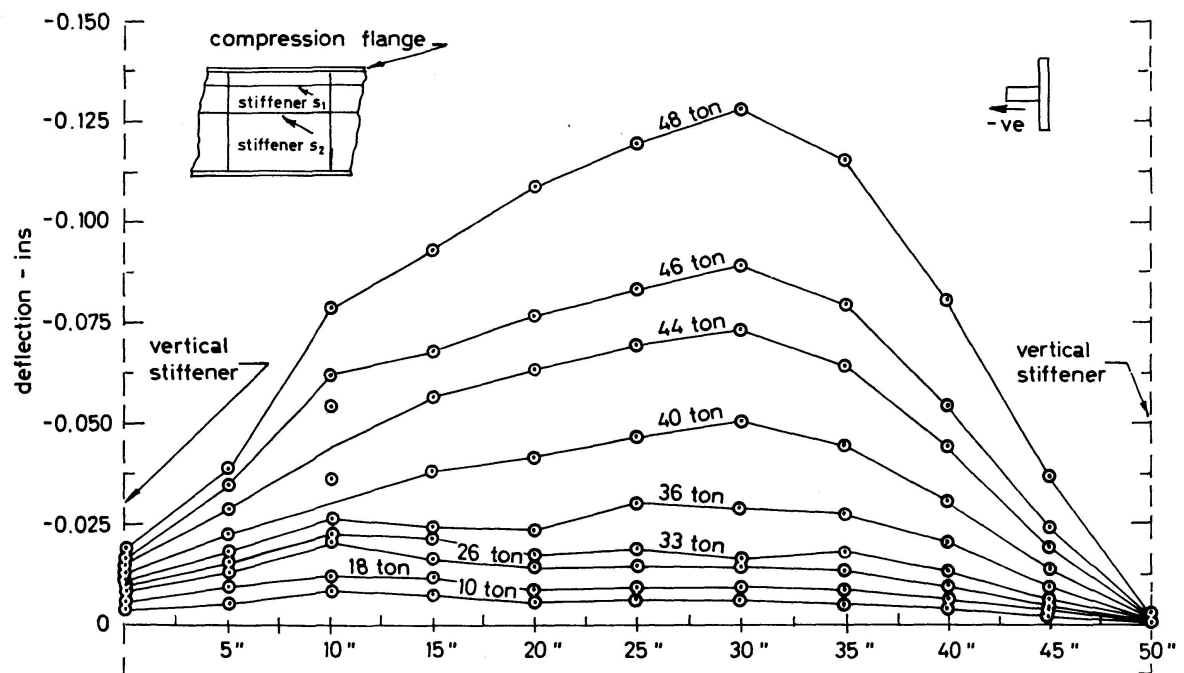
Fig. 16a. Lateral displacement of longitudinal stiffener S 1, in central panel of girder TG 1-2.
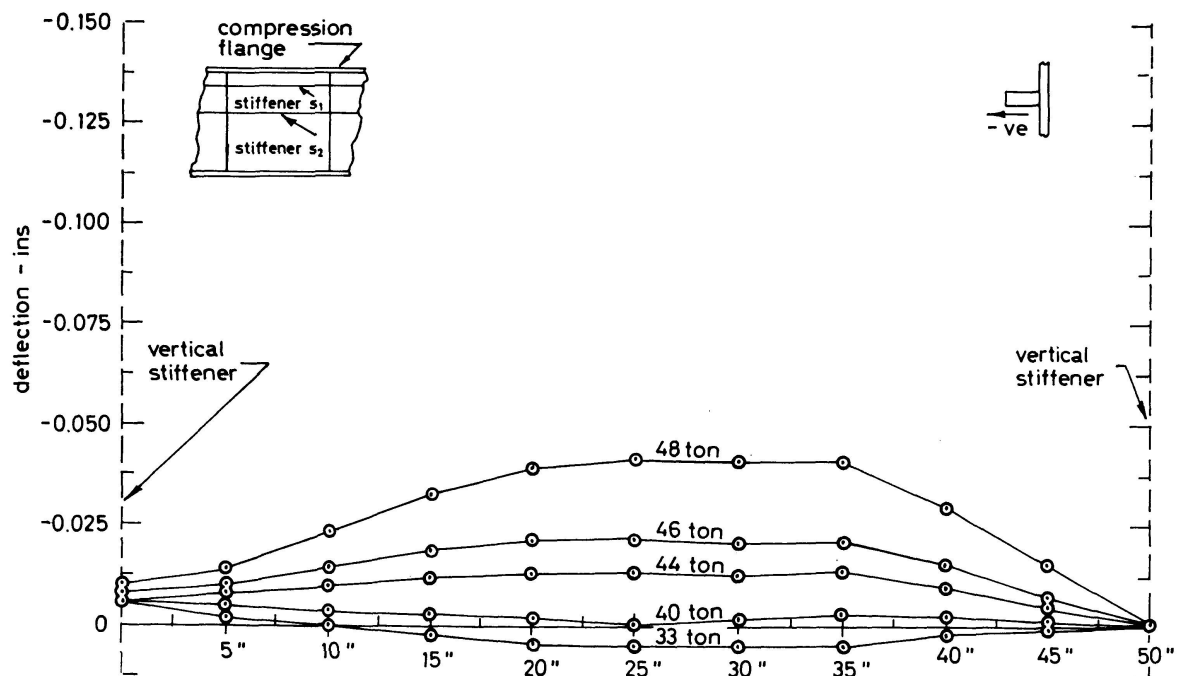
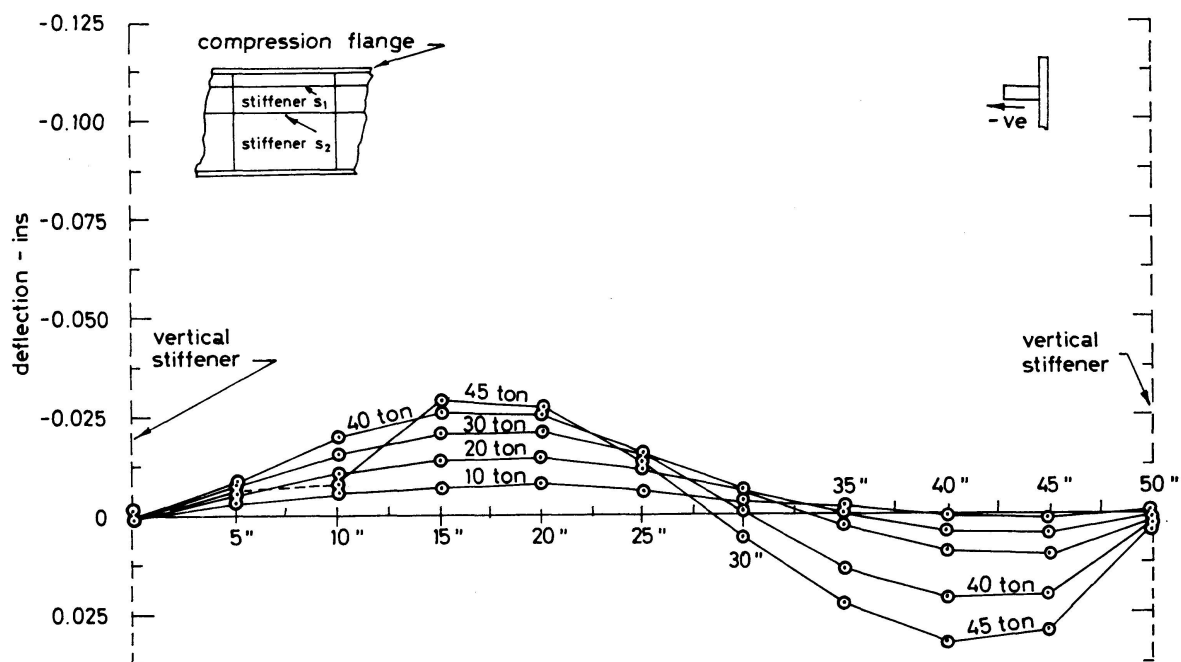Fig. 16b. Lateral displacement of longitudinal stiffener S 2, in central panel of girder TG 1-2.

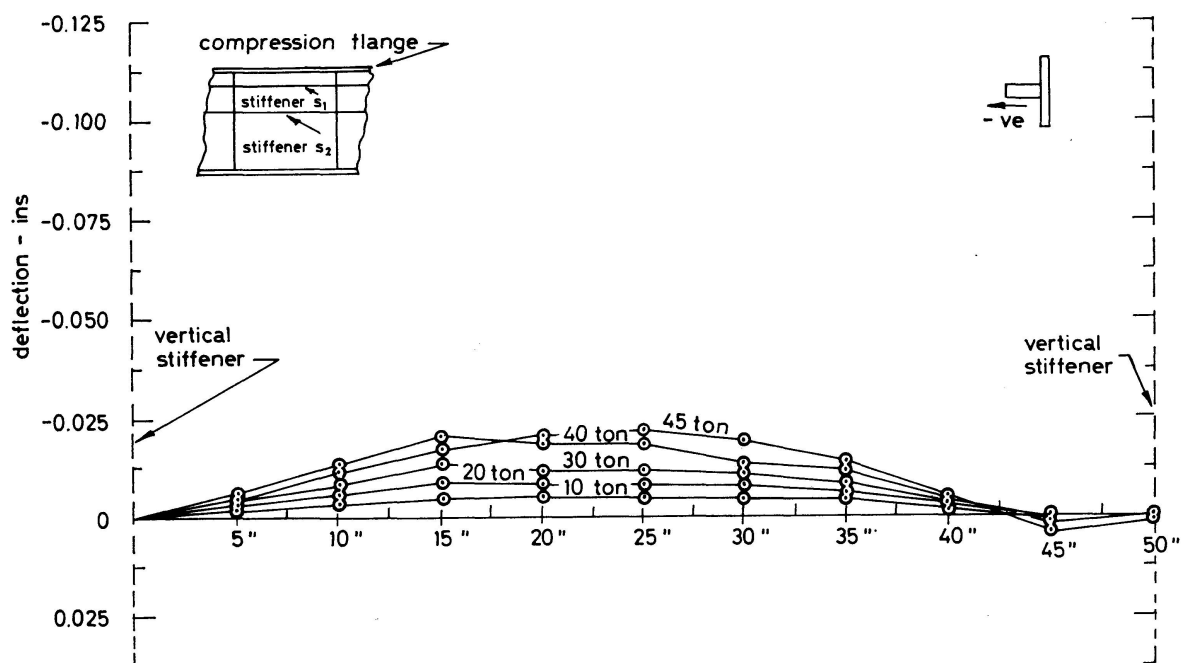Fig. 17a. Lateral displacement of longitudinal stiffener S 1, in central panel of girder TG 2-2.

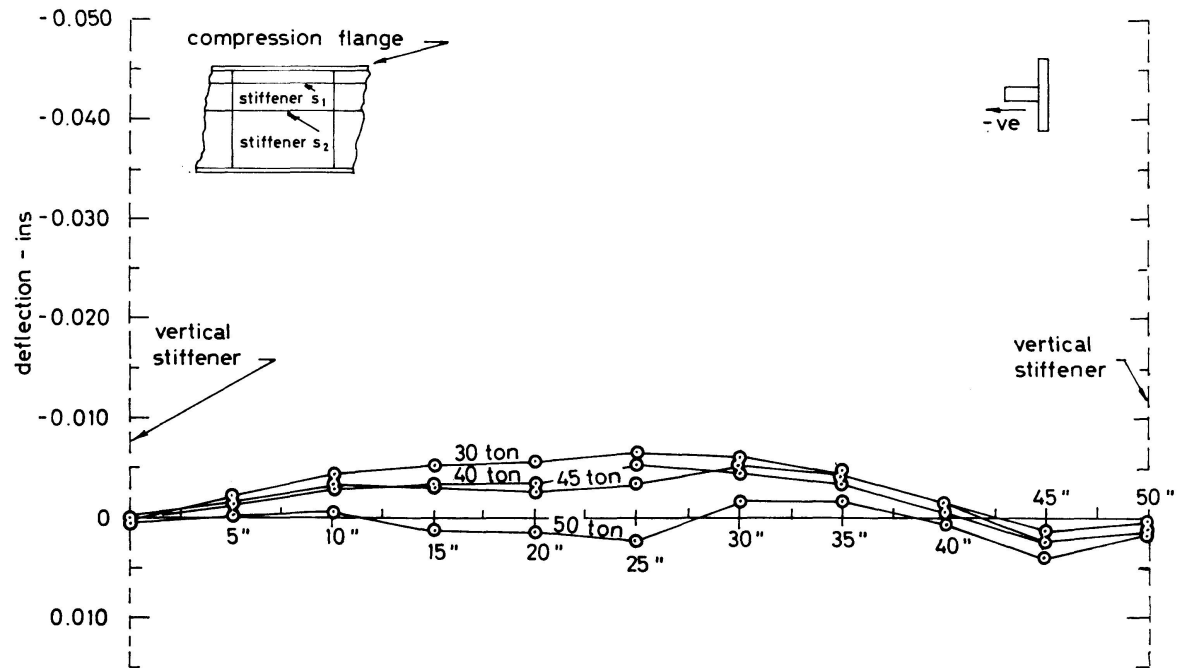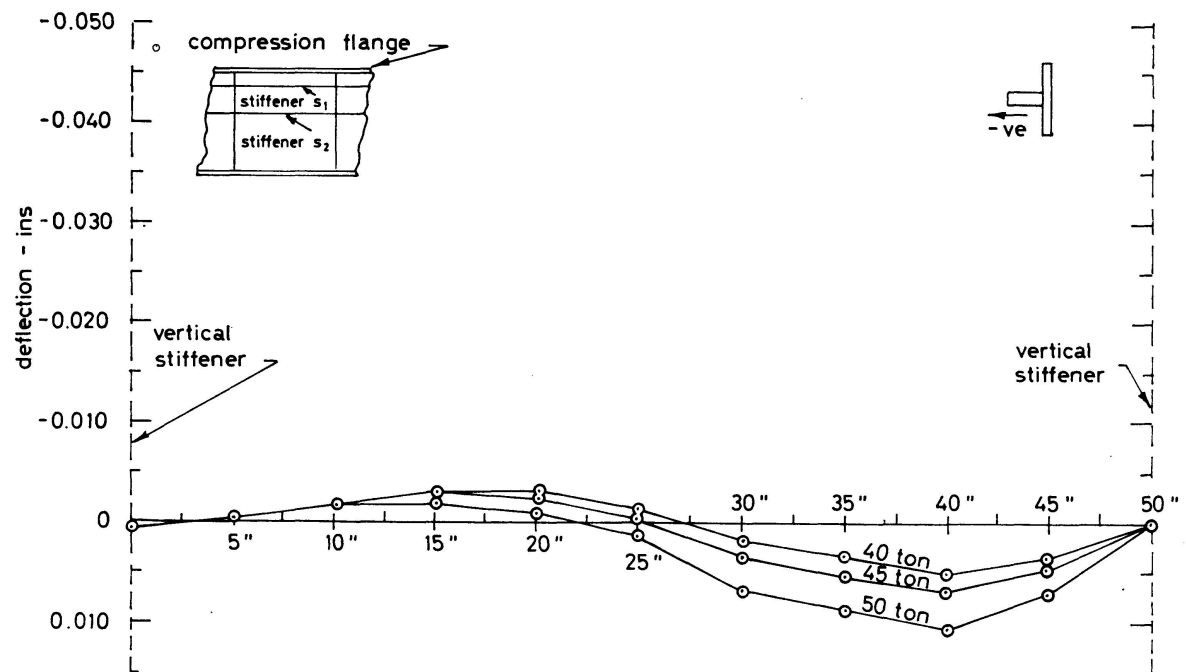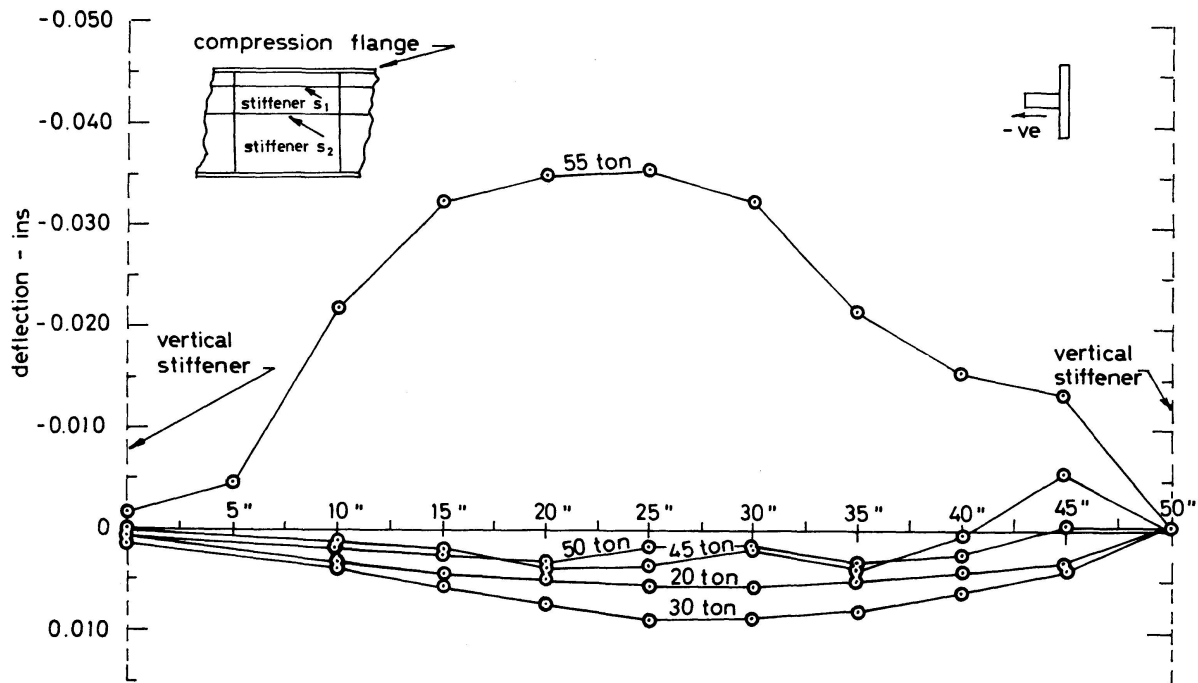Fig. 17b. Lateral displacement of longitudinal stiffener S 2, in central panel of girder TG 2-2.

Fig. 18a. Lateral displacement of longitudinal stiffener S 1, in central panel of girder TG 3-2.

Fig. 18b. Lateral displacement of longitudinal stiffener S 2, in central panel of girder TG 3-2.

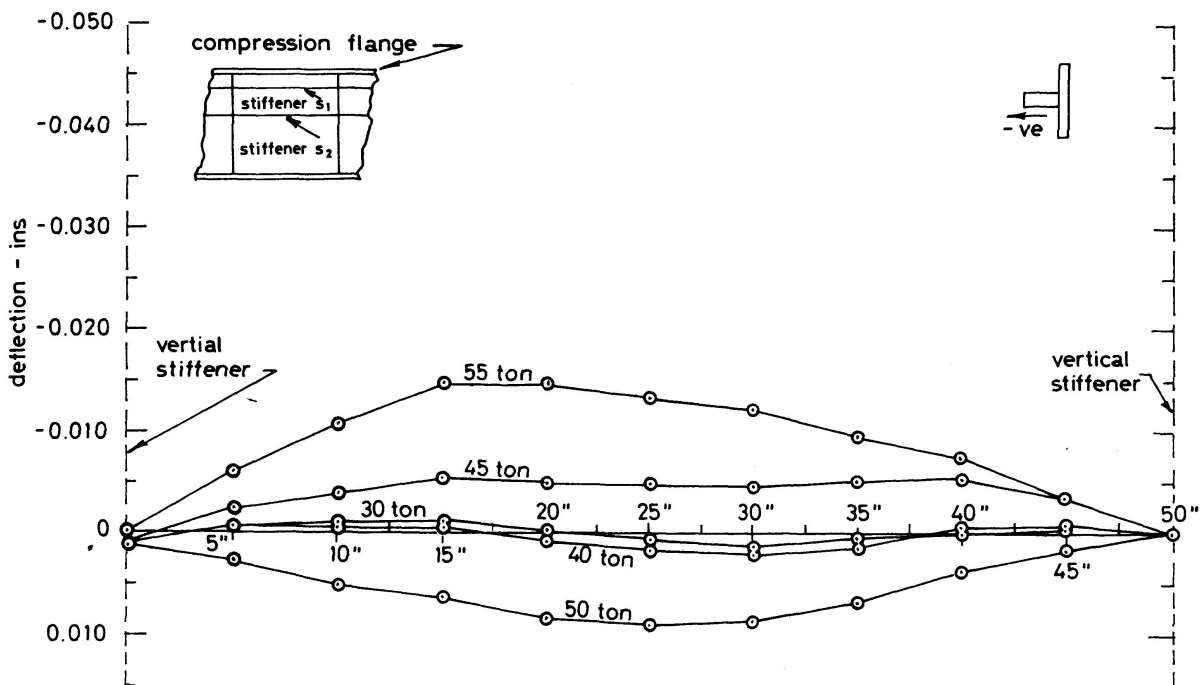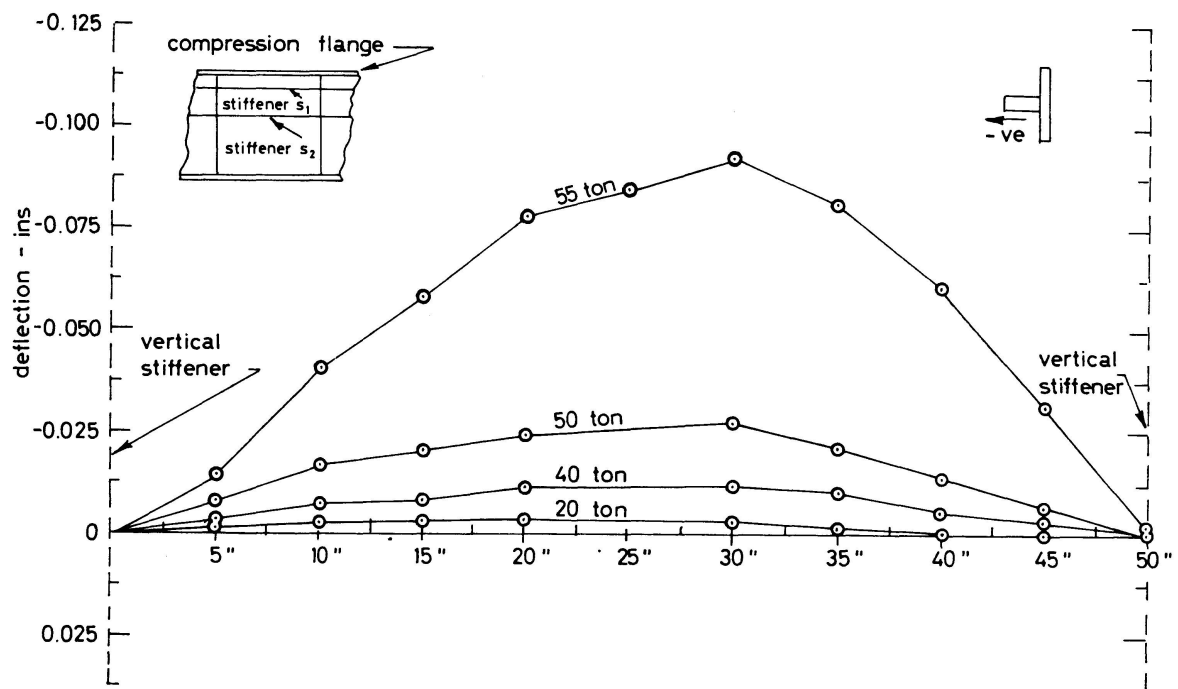Fig. 19a. Lateral displacement of longitudinal stiffener S 1, in central panel of girder TG 4-2.
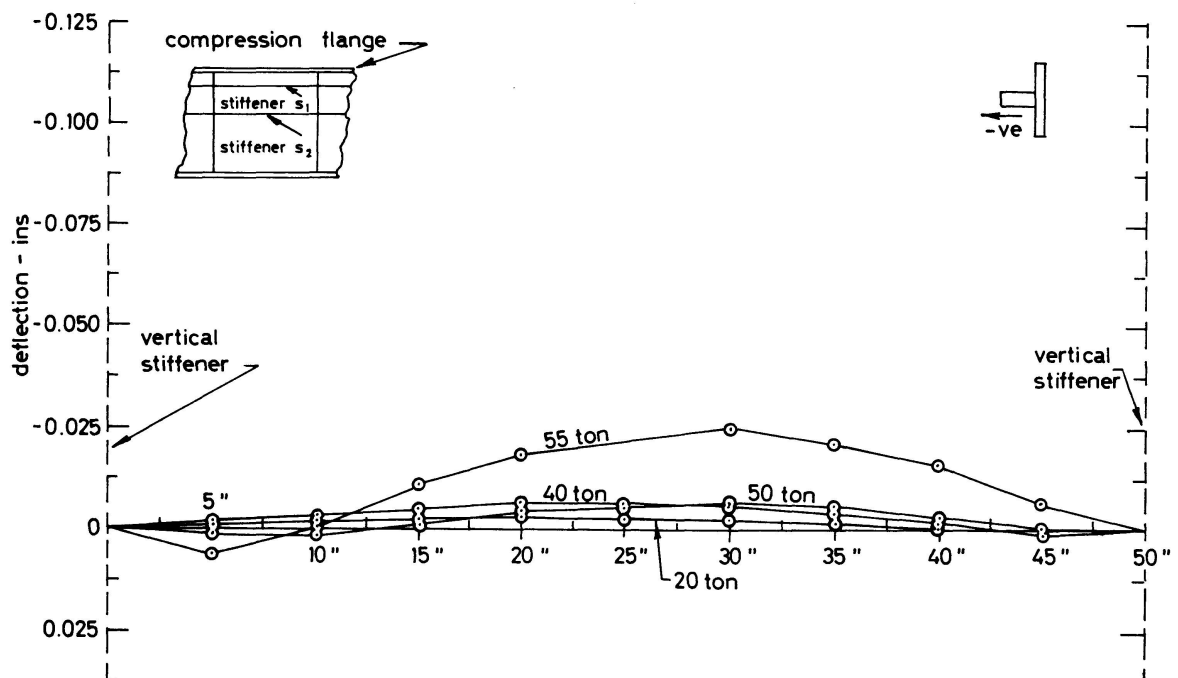

Fig. 19b. Lateral displacement of longitudinal stiffener S 2, in central panel of girder TG 4-2.

Fig. 20 a. Lateral displacement of longitudinal stiffener S 1, in central panel of girder TG 5-2.

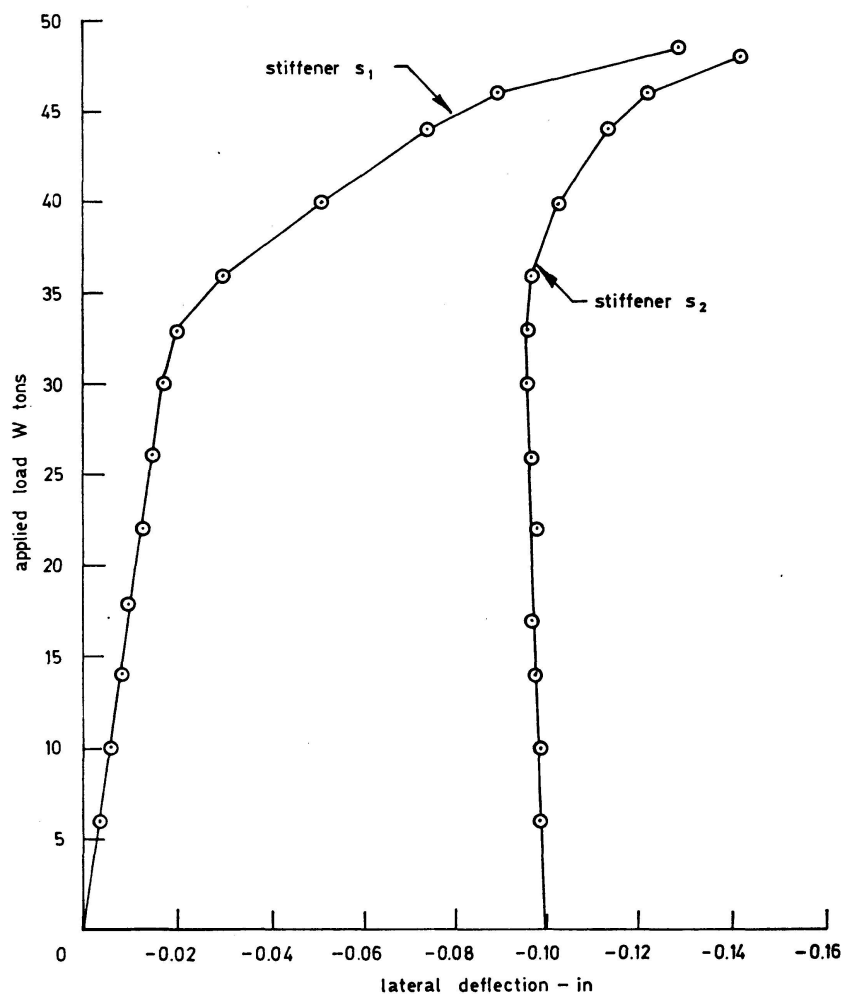Fig. 20 b. Lateral displacement of longitudinal stiffener S 2, in central panel of girder TG 5-2.

Fig. 21. Maximum deflection occuring in stiffeners S 1 and S 2 on girder TG 1-2.

serious consideration. Fig. 16–20 give the lateral deflection of the stiffeners at different loads. It should be noted that the longitudinal stiffener closest to the neutral axis did not deflect nearly so much as the longitudinal stiffener adjacent to the compression flange, as is well illustrated in Fig. 21. Since both stiffeners on a given girder had the same cross section, this finding is in agreement with Massonnet's conclusion which was derived from tests on girders reinforced by a single line of double sided stiffeners.

It will also be noted from Figs. 16–20 that with an increase in the γ/γ^* ratio the lateral deflections tended to decrease.

The influences of web buckling are well demonstrated in Fig. 22 which gives the surface and mean web strains occurring along the central section of girder TG 5-2. It will be noted that at loads as low as 10 tons considerable bending of the webplate has occurred. Even at this low load some web load shedding is evident. With an increase in load, the bending strains are seen to increase and significant load shedding to occur. The effect of this load shedding is for the neutral axis to move towards the tension flange, this movement is clearly shown by the mean strain relationship given in Fig. 22 and also in
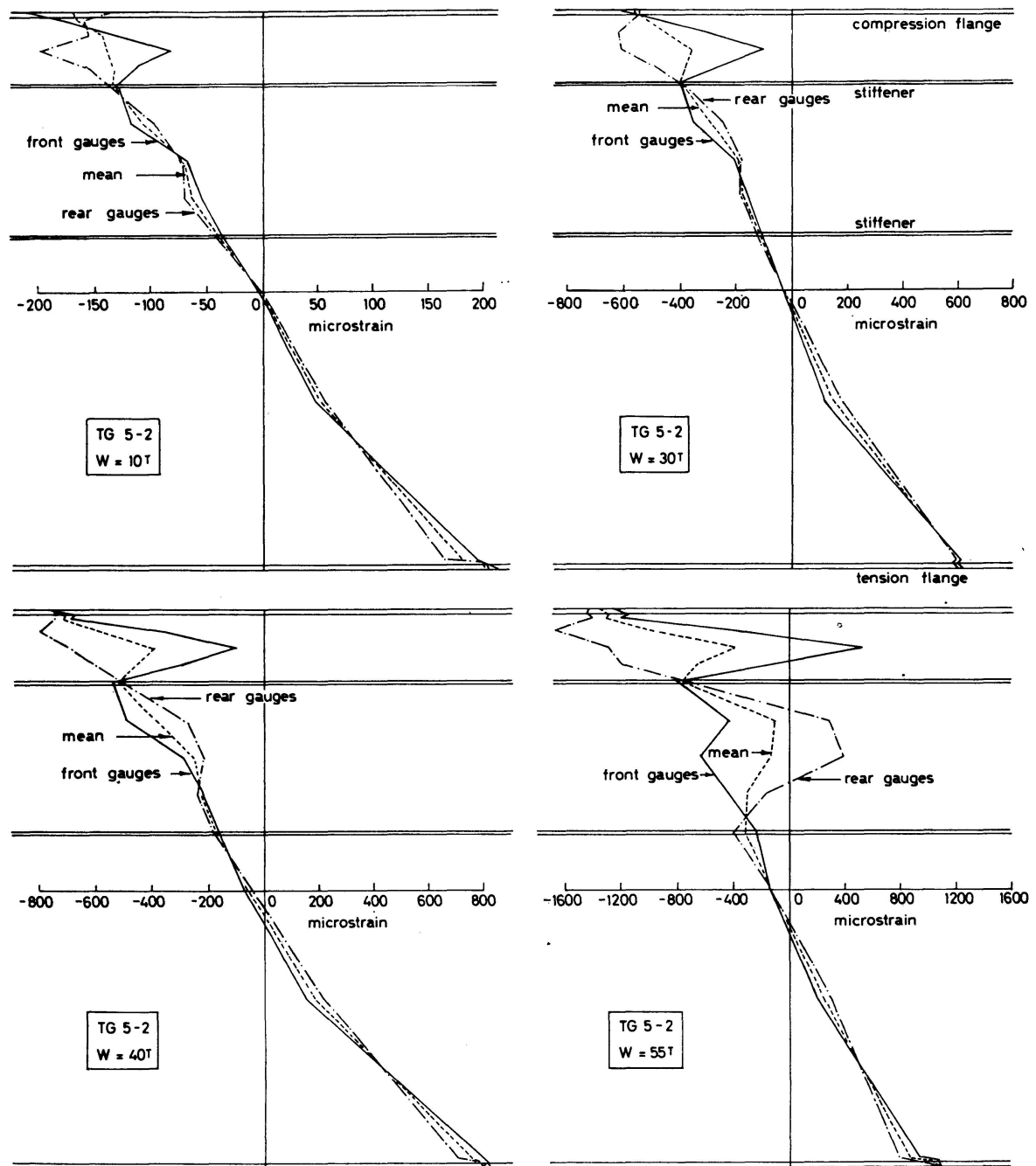
Fig. 22. Strain distribution along centre section of girder TG 5-2.

Fig. 23 which gives the mean web strains along the centre line of girder TG 3-2. This figure is of interest both because of the well defined web load shedding and the accompanying shift of the neutral axis and also for the demonstration of the influence of residual stresses upon the strains occurring in the web and the flange. Because of the high residual tensile strains occurring along the junction of the web with the flange, the web goes plastic much earlier than the edge of the flanges which would be in residual compression.
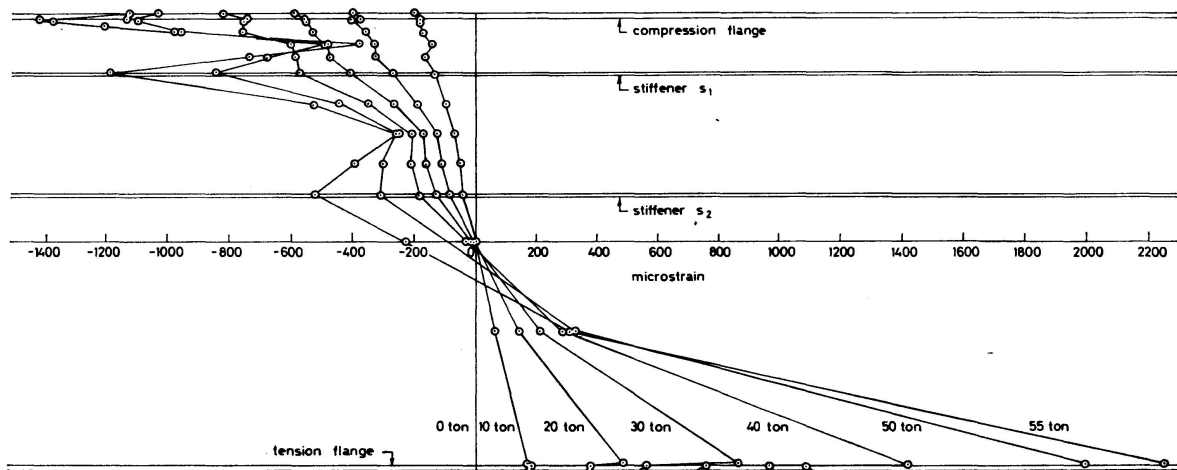
Fig. 23. Mean longitudinal strain distribution along central section of girder TG 3-2.
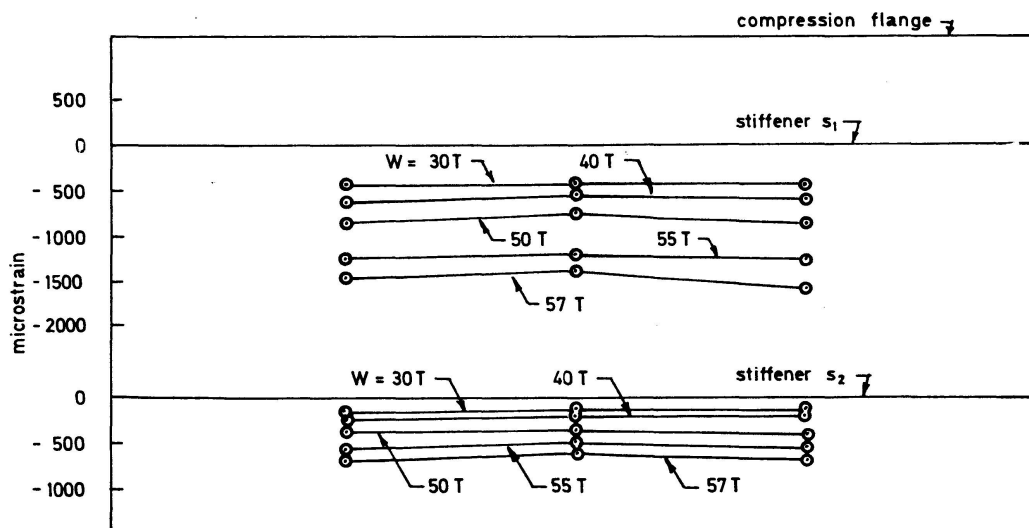

Fig. 24a. Distribution of axial strains in longitudinal stiffeners on girder TG 5-2.
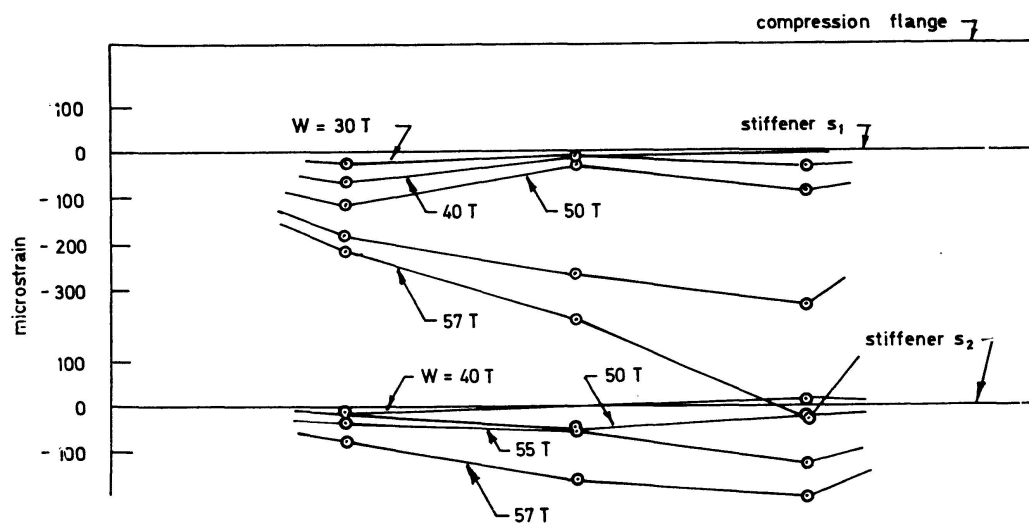

Fig. 24b. Distribution of bending strains in longitudinal stiffeners on girder TG 5-2.

Fig. 24 shows the axial and bending strains which occur in the stiffeners attached to girder TG 5-2. Note that the axial strains are uniformly distibuted along the stiffener and that the strains occurring in stiffener S 1 are greater than those occurring in stiffener S 2. The bending strains are also seen to be much greater in stiffener S 1 than stiffener S 2.

Figs. 25–28 show various views of girders TG 1-2, 4-2 and 5-2. Fig. 26 which shows a view of the compression flange of girder TG 4-2 after failure, is of
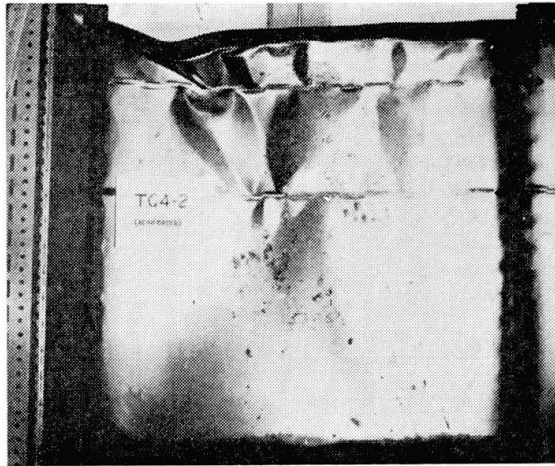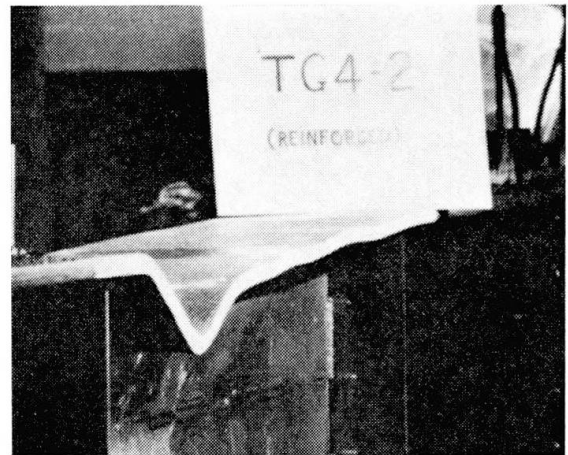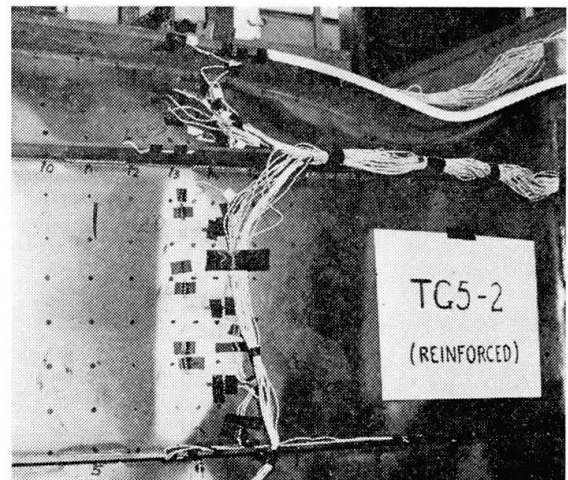

Fig. 25.

Fig. 26.

Fig. 27.

Fig. 28.

particular interest because it shows that the whole flange is waving and that failure could have occurred at any point along the flange. Since the lateral membrane loading imposed by the web is greatest in the region of the largest buckles, the inward collapse of the flange generally coincides with the region of maximum web deflections.

During the final stages of testing girder TG 2-2 slight lateral buckling of the compression flange occurred, checks on the stabilizing trusses indicated that two had not been fully secured. For this reason girder TG 2-2 failed slightly prematurely.

General Discussion

The tests have shown that there is a distinct increase in ultimate load with an increase in the rigidity of the longitudinal stiffeners. In this present section the variation of the web, stiffener and flange deflections and strains with the increase in stiffener rigidity will be examined.

Fig. 29 shows how the mean compressive strain in the flange at the central section varies with the γ/γ^* ratio for the girders fitted with the two lines of longitudinal stiffener. It will be noted that the compression flange of girder TG 0 was strained much greater than the flanges of the reinforced girders.
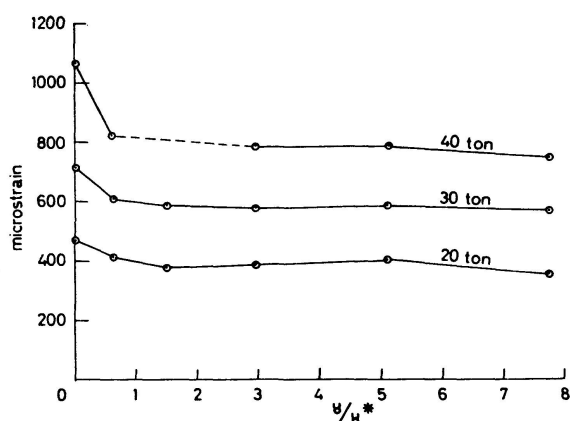

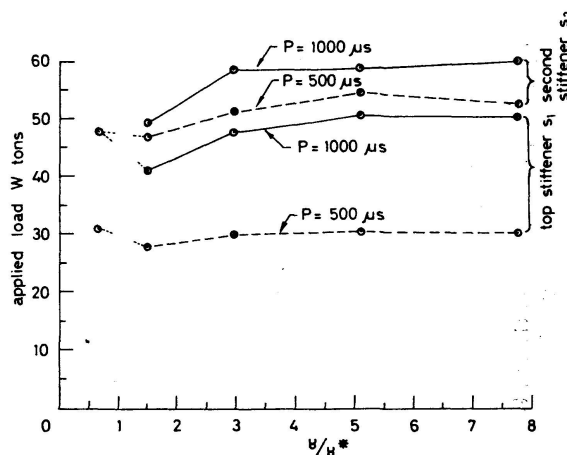Fig. 29. Variation of mean compression flange stress with γ/γ^*.

Fig. 30. Applied loads producing strains of 500 and 1000 micro strain in stiffeners.

Fig. 30 gives the loads required to produce strains of 500 and 1000 micro strains in the longitudinal stiffeners. This graph shows how with an increase in the γ/γ^* ratio the effectiveness of the stiffener increases.

In Figs. 22 and 23 the effect of load shedding was well defined. When a plate buckles under compression, the central section loses its capacity to carry additional load, any further application of load having to be carried by the material adjacent to the edges, see Fig. 31; failure occurring when the stress in this edge material reaches the yield stress. Fig. 32 gives the percentage moment loss, as determined from the strain gauge readings, in the panels of girder TG 5-2 due to load shedding. Although these losses are seen to be quite significant in respect of the web contribution, the % overall loss in moment for the complete girder is of the order of 2%.

Fig. 33 gives the % gain in ultimate strength with the γ/γ^* ratio for the
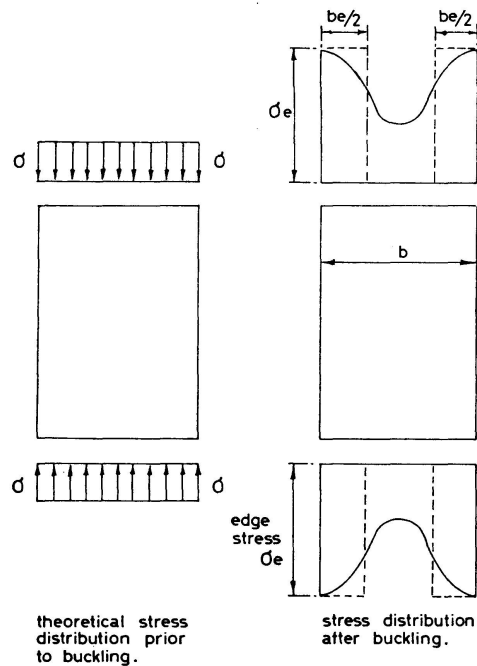
Fig. 31. Effective width concept.
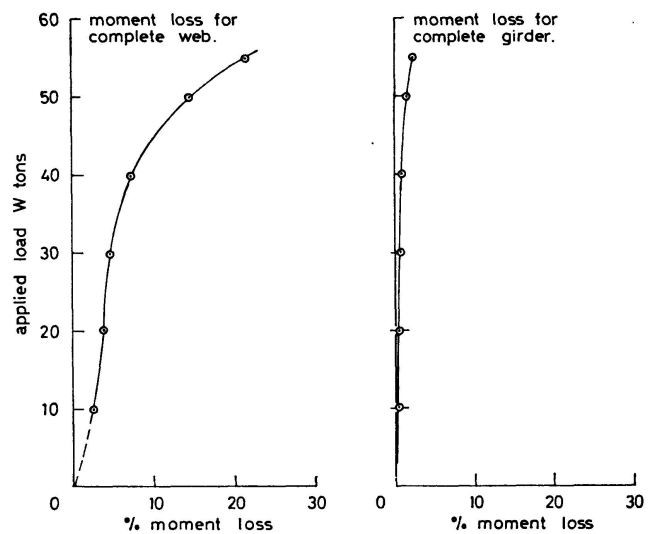
Fig. 32. Effect of load shedding as recorded on girder TG 5-2.
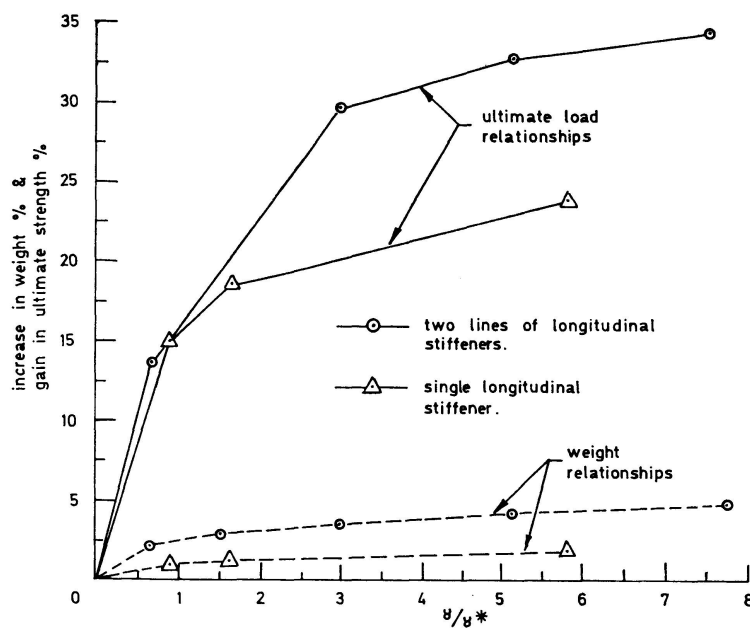
Fig. 33. Percentage increase in strength with γ/γ^* ratio together with corresponding percentage increage in overall weight.
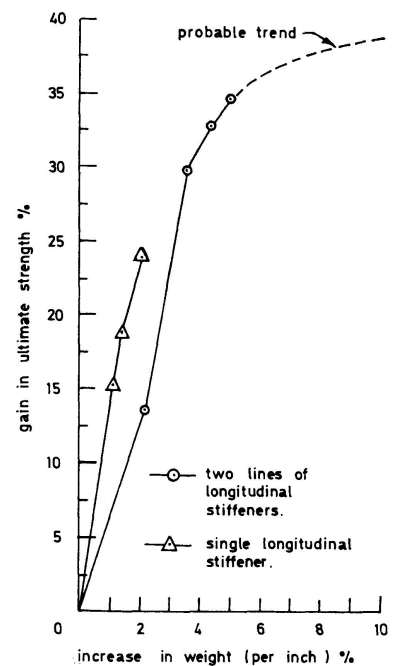
Fig. 34. Relationship between gaind in ultimate strength an corresponding increase in weight.

girders reinforced by one and two longitudinal stiffeners. The gain in ultimate strength in the case of the double line of stiffeners is seen to be quite considerable amounting to $34\frac{1}{2}\%$ in the case of girder TG 5-2. Also shown plotted is the corresponding % increase in weight. Associated with the $34\frac{1}{2}\%$ increase

in the ultimate load of girder TG 5-2 over that of girder TG 0 is the 5% increase in weight due to the stiffeners. Fig. 34 presents the relationship between the % gain in strength with % increase in weight, which again demonstrates how a small amount of reinforcement correctly placed can significantly increase the strength of a member.

Although in all cases failure was due to inward collapse of the compression flange, this occurred after the flanges and web had become plastic; CARSKADDAN [24] having noted this same fact from his tests on unreinforced girders.

a) No longitudinal stiffener $k=27.1$
(Reference 26).

b) Longitudinal stiffener $k=129$
(Reference 26).

c) Approximate wave patterns in panel No 1 when two longitudinal stiffeners are placed as shown. $k=313$.
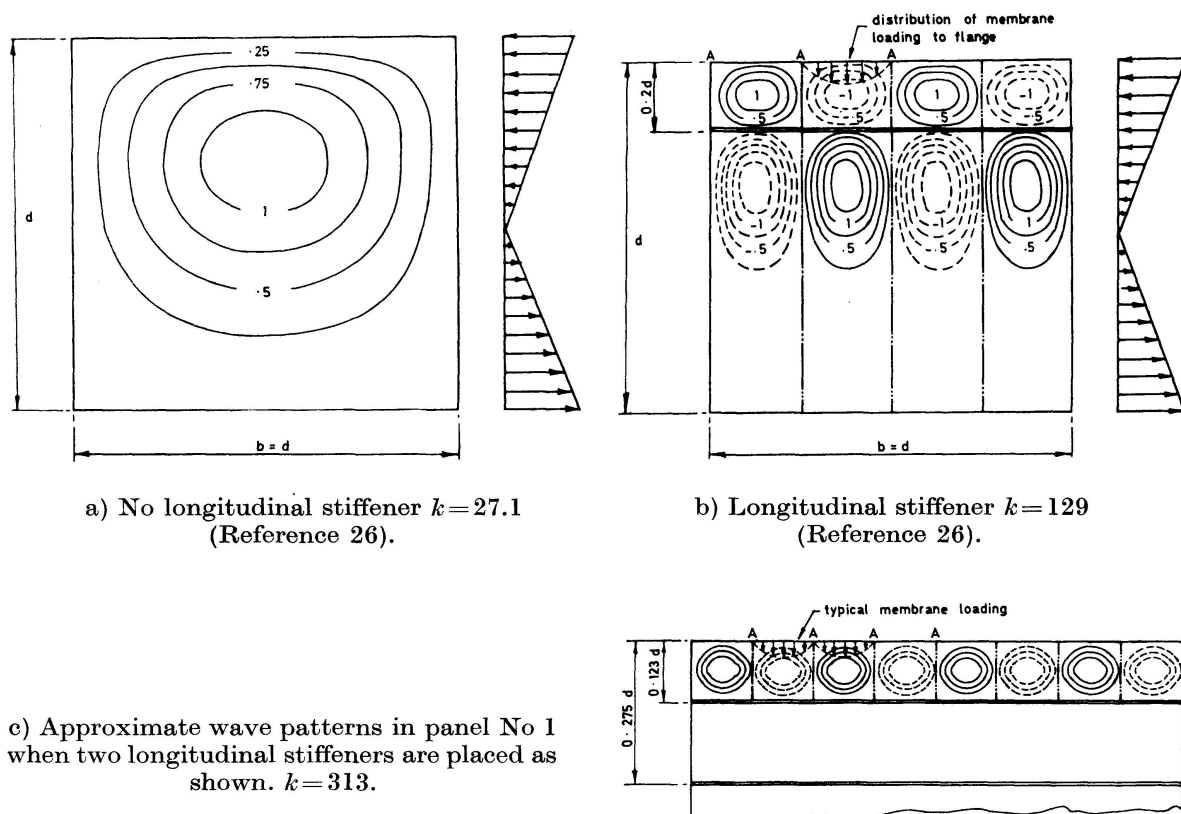
Fig. 35. Buckle patterns in simply supported web plates.

Fig. 35 illustrates the web buckles which will form in the panel adjacent to compression flange for a girder having no longitudinal stiffener, one longitudinal stiffener and two longitudinal stiffeners. Corney and one of the present Authors have shown that the flange receives a lateral loading as indicated in the diagrams; the web supporting the flange at positions such as A. If a stiffener deflects then the wave form in the panel will be modified, increasing in length. As a result the flange will be less effectively supported by the web, since the points A will be further apart. It is this factor which influences the collapse load, it having been observed that the length of the inward collapse buckle of the flange decreases with the number of stiffeners and the effectiveness of the stiffeners.

WINTER [25] has proposed the following relationship between the effective width be of a plate element in pure compression and the ratio $\frac{\sigma_{cr}}{\sigma_{max}}$; see Fig. 36.

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \left[1 - 0.22 \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \right].$$ (2)

Applying this effective width formula to panels W 1, W 2 and W 3 as indicated in Fig. 36, the authors have calculated the collapse load for girders which are reinforced by either a single line or a double line of longitudinal stiffeners placed at the optimum position as was the case with the experimental girders.

The collapse loads were determined for a fully plastic cross section, the yield stress for the web and flange materials being used. The effects of work hardening, which did occur, being neglected.

The collapse loads were calculated for sections corresponding to the experimental girders, and Table 4 gives the calculated values together with the experimental values. It will be noted the ratio of experimental collapse load / theoretical collapse load (load on reduced section) increases with the γ/γ^* ratio, see Fig. 37. Thus it is seen that by the application of the effective width concept to the longitudinally reinforced girders, one is able to obtain solutions to within 8% of the experimental values for girders having adequate stiffeners. These calculations have also shown that if a γ/γ^* value of 8 is chosen for the case of two lines of stiffener placed at their optimum position and 6 for the case of a single line stiffener placed at $0.2\,d$ then collapse values close to the load corresponding to the reduced fully plastic modulus will be obtained.
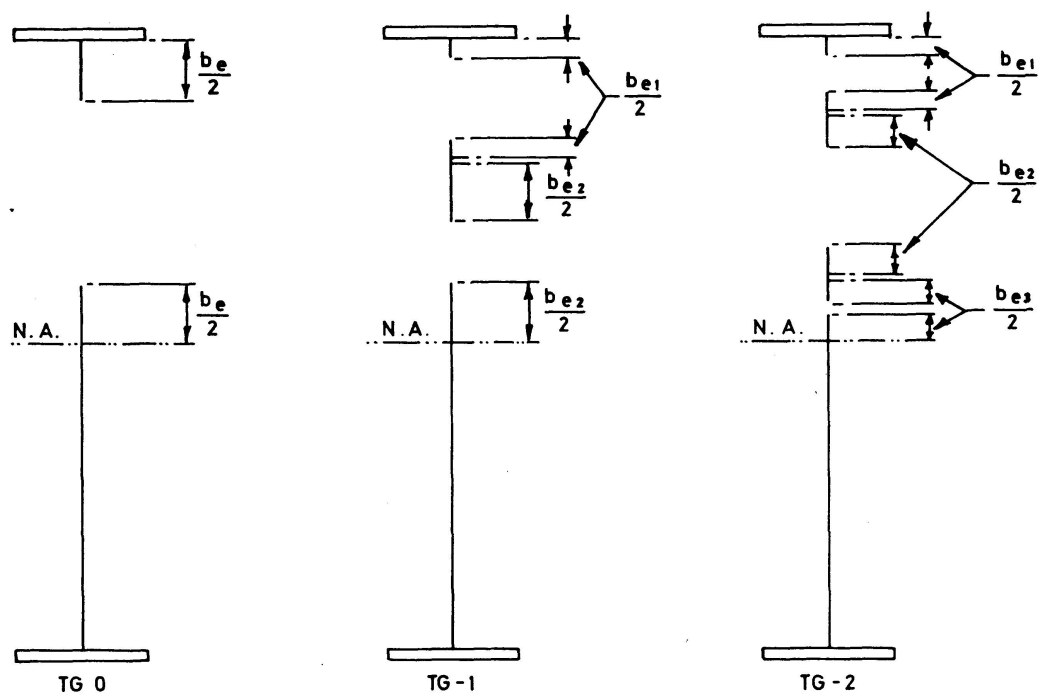

Fig. 36. Effective sections used in calculations of collapse load of girders.
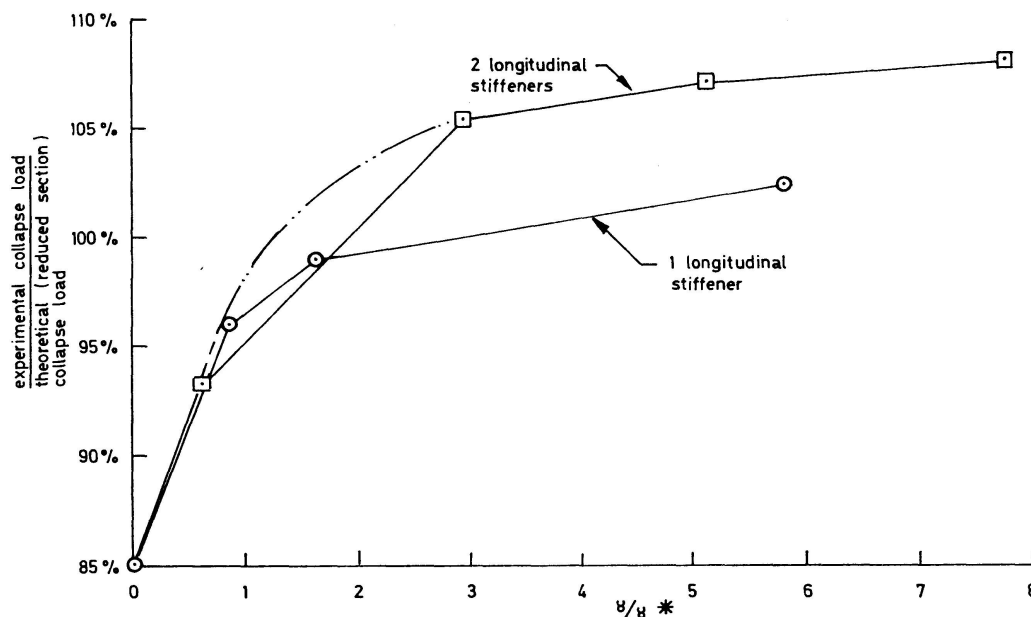
Fig. 37. Variation of ratio, experimental collapse load / theoretical collapse load based on reduced section with the stiffeners' rigidity ratio γ/γ^*.
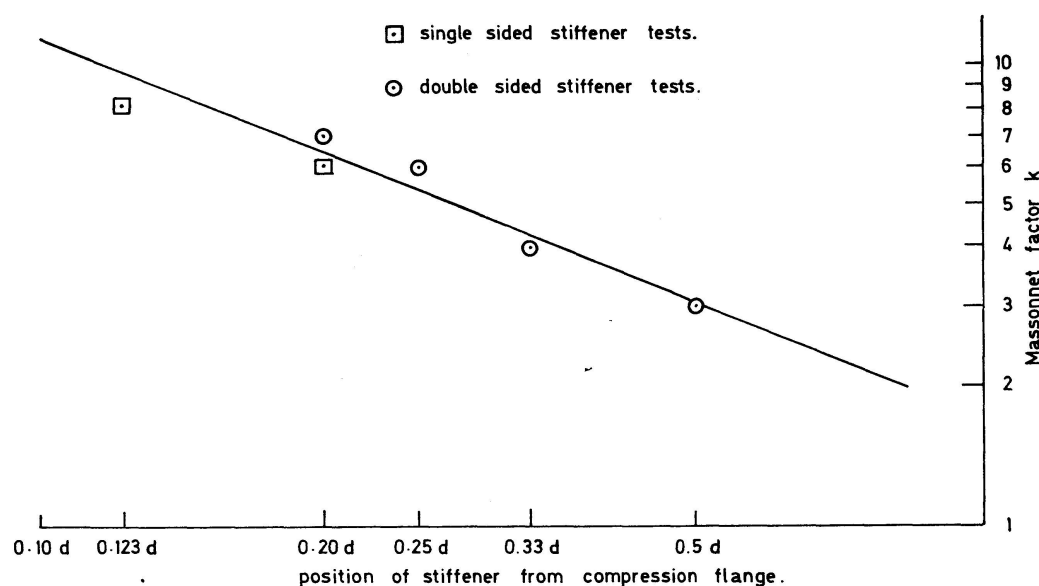
Fig. 38.

In Fig. 38 the k values recommended by MASSONNET for double sided stiffeners has been plotted together with the k values resulting from the present investigation. Here the k values are for equal size stiffeners and for the stiffener closest to the neutral axis is clearly conservative, but since the behaviour of the stiffener closest to the compression flange has a critical effect upon the behaviour of the girder and any variation in the size of the second stiffener will affect its behaviour it is considered reasonable to use the value. From Fig. 38 it will be seen that the degree of agreement between the values

recommended by MASSONNET and those recommended as a result of the present study is quite good.

The mere fact that load shedding occurs indicates that to allow a webplate to buckle under pure bending is not economical. To avoid a theoretical web buckling before yielding occurs it would be necessary to comply with the facts given in Table 5. These figures have been derived assuming that the stiffeners are placed at the optimum position as given in references [16] and [17].

Table 5. Depth to thickness ratios of simply supported steel web plates having pure bending buckling stress equal to yield stress

No. of stiffeners	Position of stiffeners from compression flange	Yield stress Tons/in.^{2}	
		B.S. 15 15.25 Tons/in.^{2}	B.S. 968 22.0 Tons/in.^{2}
0	—	138	115
1	0.2 d	320	266
2	0.123 d, 0.275 d.	498	415
3	0.093 d, 0.198 d, 0.323 d.	650	540
4	0.073 d, 0.152 d, 0.242 d, 0.349 d.	817	680
5	0.06 d, 0.124 d, 0.194 d, 0.273 d, 0.367 d.	983	819

Conclusion

The tests have shown that the collapse behaviour of a longitudinally reinforced girder is significantly influenced by the stiffness of the longitudinal stiffener. If the stiffeners have insufficient stiffness they will not give the web adequate support and this will reduce the ultimate load carrying capacity.

The tests have provided efficiency values k for single sided stiffeners for the case of webs reinforced by either one or two longitudinal stiffeners.

It has also been shown that providing the stiffeners are designed according to the foregoing recommendation, it is possible to calculate the collapse load of girders having webs reinforced by longitudinal stiffeners.

Acknowledgement

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Summary

The paper examines the influence of the Flexural Rigidity of Longitudinal Stiffeners upon the Post Buckled behaviour of deep Plate Girders.

Tests on 10 girders having a web depth of 50 ins. (1.27 metres) and reinforced by either a single or two longitudinal stiffeners have shown that the ultimate load of a girder can be significantly reduced if the longitudinal stiffeners have insufficient rigidity and have provided Massonnet type efficiency values for single sided longitudinal stiffeners. Provided longitudinal stiffeners are designed according to these relationships then girders should develop full plasticity prior to collapse.

Résumé

On examine dans cet article l'influence de la résistance à la flexion de raidisseurs longitudinaux sur les propriétés après voilement de profils à âme haute. Des essais effectués sur 10 profils ayant une âme de 50 pouces (1,27 mètre) et renforcés par 1 ou par 2 raidisseurs longitudinaux ont démontré que la charge limit peut être réduite de manière significative si les raidisseurs longitudinaux ont une rigidité insuffisante. Ces essais ont prouvé l'efficacité des pièces de type Massonnet utilisés comme raidisseurs longitudinaux simples. Les raidisseurs longitudinaux stipulés sont calculés en fonction de ces relations car les profils doivent présenter une plasticité totale avant la rupture.

Zusammenfassung

Der Beitrag untersucht den Einfluß der Biegesteifigkeit der Längssteifen auf das Traglastverhalten hoher Blechträger.

Versuche an 10 Trägern mit einer Steghöhe von 50 Zoll (1,27 Meter) und entweder einer oder zwei Längssteifen haben gezeigt, daß die Traglast erheblich vermindert werden kann, wenn die Längssteifen ungenügende Steifigkeit haben und ergaben wirksame Werte nach Massonnet für eine einzige Steife. Die verwendeten Längssteifen sind gemäß dieser Beziehungen berechnet worden, denn die Träger sollten für volle Plastizität vor dem Bruch entwickelt werden.