Long Reinforced Concrete Columns in Biaxial Bending

Flexion biaxiale de colonnes élancées en béton armé

Lange Stahlbetonsäulen unter zweiaxialer Biegung

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1. Introduction

Various methods are available for computing the ultimate strength in combined compression and biaxial bending of rectangular [1, 2, 3] and irregularly shaped [4] reinforced concrete sections. The strength of long reinforced concrete compression members has also been studied to some extent for the specific case of bending about one principal axis [5, 6, 7]. The general case of long reinforced concrete column behaviour in biaxial bending has not, to the knowledge of the author, been considered in the literature, despite the fact that columns in many structures are subjected to significant biaxial bending action.

A method for determining the load-deformation characteristics of reinforced concrete column sections in biaxial bending is briefly described in this paper. The method is then used in a study of the behaviour of pin-ended columns subjected to loads with biaxial end eccentricities.

Notation

a depth of section
b width of section
e_r eccentricity of load measured from centroid of section
e_x eccentricity of load in x–y axis system; see Fig. 1a
e_y eccentricity of load in x–y axis system; see Fig. 1a
e_α e_x/a

e_β e_y/b
e_{r0} eccentricity of load on long column; see Fig. 3
f_c concrete stress
f'_c concrete control cylinder strength
f_s steel stress
f_{sy} steel yield stress
k_1 ratio of strength of concrete in column to control cylinder strength
l column length
p A_s/A_c; proportion of steel reinforcement
q $\frac{p f_{sy}}{k_1 f'_c}$; reinforcement ratio
A_c concrete area in section
A_s steel area in section
E_c ϵ_c/ϵ'_c; concrete strain ratio
E_{ij} value of E_c in the (i,j)th concrete elemental area
E_k value of E_s in the k-th steel elemental area
E_s ϵ_s/ϵ_{sy}; steel strain ratio
E_0 ϵ_0/ϵ'_c
E_1 ϵ_1/ϵ'_c
N_a number of rows of elemental concrete areas
N_b number of columns of elemental concrete areas
N_c total number of elemental concrete areas
N_s number of elemental steel areas
P compressive force in section
P_{max} strength of long column; see Eq. (33)
P_0 strength of column of zero length
P_u ultimate strength of section for zero eccentricity
S_c $f_c/k_1 f'_c$; concrete stress ratio
S_{ij} value of S_c in the (i,j)th concrete element
S_k value of S_s in the k-th steel element
S'_k equivalent concrete stress ratio in the k-th steel element
S_s f_s/f_{sy}; steel stress ratio
α x/a; non-dimensional co-ordinate
β y/b; non-dimensional co-ordinate
γ_1 parameter affecting the shape of the loading portion of the concrete stress-strain relation
γ_2 parameter affecting the shape of the unloading portion of the concrete stress-strain relation
ϵ_c concrete strain
ϵ'_c concrete strain when stress is $k_1 f'_c$
ϵ_s steel strain
ϵ_{sy} steel yield strain
ϵ_0 strain at corner 0; maximum compressive strain in section

ϵ_1 strain at corner 1; minimum strain in section
η a/b; depth to width ratio of section
θ inclination of neutral axis of strain to y axis. Also inclination of plane of assumed deflected shape to the x axis
ϕ curvature
ψ angle defining biaxial eccentricity of force P acting on a section
ψ_0 angle defining biaxial eccentricity of load on a long pin ended column

2. Load Deformation Characteristics of Cross-Section

A symmetrically reinforced rectangular cross-section is considered and an x–y axis system is taken with origin at the corner 0, as shown in Fig. 1a.

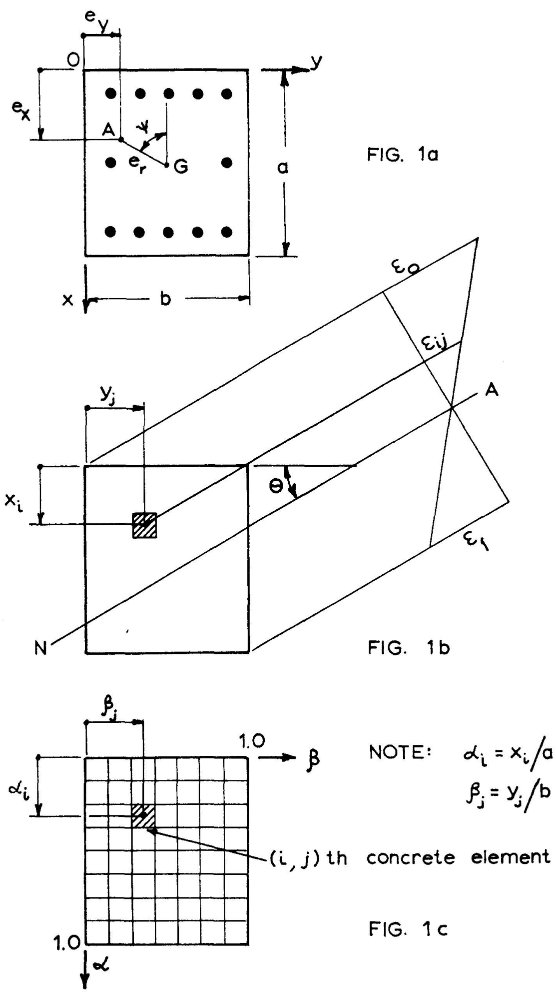


Fig. 1. Cross section details.

A force P is assumed to act at point A, defined in the x–y axes by the eccentricities e_x and e_y. The eccentricity of the load with respect to the centroid G of the section is defined by the distance e_r and the angle ψ, where

$$e_r = \sqrt{(b/2 - e_y)^2 + (a/2 - e_x)^2}, \tag{1}$$

$$\psi = \tan^{-1}\left(\frac{b/2 - e_y}{a/2 - e_x}\right). \tag{2}$$

If the usual assumptions of perfect bonding and plane distribution of strains are made, the strain distribution in the section can be defined by the three quantities ϵ_0, ϵ_1 and θ. The strain ϵ_0 occurs in corner 0. For convenience it is assumed that e_0 is the maximum positive (i.e. compressive) strain in the section. The strain ϵ_1, which may be positive or negative, occurs in the corner diagonally opposite 0 and is the minimum strain in the section. The angle θ, which lies in the range $0 \leqq \theta \leqq \pi/2$, defines the inclination of the neutral axis of strain to the y axis, as indicated in Fig. 1b.

For various particular cases, e.g. uniaxial bending ($\theta = 0$) and diagonal bending of square sections ($a = b$, $\theta = 45°$), it is possible to derive simple analytic expressions for the compressive force and moments which exist in a section with a given strain distribution. However, an analytic approach is not feasible in the general case of biaxial bending. In the present study, an approximate method is adopted in which the steel and concrete in the cross-section are partitioned into many small elemental areas. The total force in the section is thus evaluated by summation of the elemental forces acting on the elemental areas, while the moments are found by summation of the moments of the elemental forces. This method is essentially an extension of a previous study of the ultimate strength of reinforced concrete sections in combined compression and biaxial bending [4].

The rectangular concrete section is thus considered to be made up of N_c rectangular elements, each of width Δb and depth Δa and area $\Delta A_c = \Delta b \Delta a$.

The numbers of rows and colums of elements are $N_a = a/\Delta a$ and $N_b = b/\Delta b$, respectively. The position of the (i, j)th element is defined by the co-ordinates of its centroid, x_i and y_j, as shown in Fig. 1c. The strain at this point is given by the following expression

$$\epsilon_{ij} = \epsilon_1 + (\epsilon_0 - \epsilon_1)\left(1 - \frac{x_i \cos\theta + y_j \sin\theta}{a\cos\theta + b\sin\theta}\right). \tag{3}$$

The concrete stress-strain relation is assumed to be of the shape indicated by Fig. 2a, and is expressed non-dimensionally in terms of stress ratios and strain ratios as

$$E_c \leqq 0: \qquad S_c = 0, \tag{4a}$$

$$0 \leqq E_c \leqq 1.0: \qquad S_c = \gamma_1 E_c + (3 - 2\gamma_1) E_c^2 + (\gamma_1 - 2) E_c^3, \tag{4b}$$

$$1.0 \leqq E_c \leqq \gamma_2: \qquad S_c = 1 - \frac{1 - 2E_c + E_c^2}{1 - 2\gamma_2 + \gamma_2^2}, \tag{4c}$$

$$E_c \geqq \gamma_2: \qquad S_c = 0, \tag{4d}$$

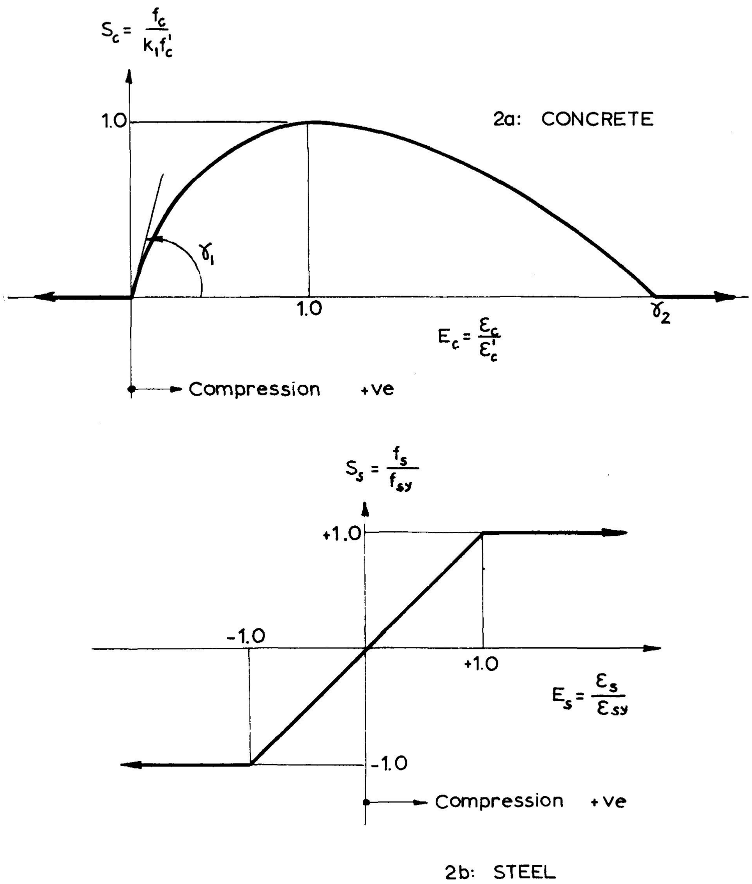


Fig. 2. Stress strain relations.

where the quantity S_c is stress, non-dimensionalised by dividing by the concrete compressive strength,

$$S_c = \frac{f_c}{k_1 f_c'}, \tag{5}$$

and E_c is strain, normalised by dividing by the strain ϵ_c' which occurs at the stress $k_1 f_c'$,

$$E_c = \frac{\epsilon_c}{\epsilon_c'}. \tag{6}$$

The open parameter γ_1 allows the shape of the loading portion of the curve to be adjusted to suit concretes of various stiffness E_m,

$$\gamma_1 = \frac{E_m \epsilon_c'}{k_1 f_c'}, \tag{7}$$

while the parameter γ_2 allows the shape of the descending portion of the curve to be varied. It will be noted also that the unloading region, $E_c > 1.0$, extends indefinitely and so caters for large compressive strains which can occur in restrained members under certain conditions [8].

It is convenient to rewrite Eq. (3) in terms of strain ratios,

$$E_{ij} = E_1 + (E_0 - E_1)\left(1 - \frac{\eta\,\alpha_i \cos\theta + \beta_j \sin\theta}{\eta\cos\theta + \sin\theta}\right), \tag{8}$$

in which α_i and β_j are non-dimensionalised coordinates,

$$\alpha_i = \frac{x_i}{a}, \qquad \beta_j = \frac{y_j}{b},$$

and η is the depth-to-width ratio of the section,

$$\eta = \frac{a}{b}.$$

Eqs. (4) and (8) together allow the stress ratio S_{ij} and hence the elemental force in the (i,j)th concrete element,

$$\Delta P = k_1 f_c' \Delta A_c S_{ij}, \tag{9}$$

to be determined for any assumed strain distribution. The moments of this elemental force about the x and y axes are

$$\Delta M_x = b\,k_1 f_c' \Delta A_c \beta_j S_{ij}, \tag{10}$$

$$\Delta M_y = a\,k_1 f_c' \Delta A_c \alpha_i S_{ij}. \tag{11}$$

The steel reinforcement in the cross-section is assumed to be distributed in N_s elements, each of area ΔA_s. The position of the k-th steel element is defined by its co-ordinates $\alpha_k = x_k/a$ and $\beta_k = y_k/b$, so that the strain ϵ_k in this element can be expressed, using a strain ratio $E_k = \epsilon_k/\epsilon_{sy}$, as

$$E_k = \left\{E_1 + (E_0 - E_1)\left(1 - \frac{\eta\,\alpha_k \cos\theta + \beta_k \sin\theta}{\eta\cos\theta + \sin\theta}\right)\right\}\frac{\epsilon_c'}{\epsilon_{sy}}. \tag{12}$$

The elasto-plastic stress-strain relation represented in Fig. 26 is taken to apply to all steel elements;

$$E_s \leqq -1.0 : S_s = -1.0, \tag{13a}$$

$$-1.0 \leqq E_s \leqq +1.0 : S_s = E_s, \tag{13b}$$

$$E_s \geqq +1.0 : S_s = +1.0. \tag{13c}$$

In Eqs. (13) the stress and strain ratios are obtained by dividing by yield stress and yield strain, respectively. Eqs. (12) and (13) define the stress ratio S_k for the k-th steel element, and hence the elemental force,

$$\Delta P = f_{sy} \Delta A_s S_k, \tag{14}$$

can be determined together with the elemental moments,

$$\Delta M_x = b\,f_{sy} \Delta A_s \beta_k S_k, \tag{15}$$

$$\Delta M_y = a\,f_{sy} \Delta A_s \alpha_k S_k. \tag{16}$$

Since each elemental steel area excludes an equal area of concrete from the section, the above expressions are modified as follows:

$$\Delta P \;\; = \Delta A_s (f_{sy} S_k - k_1 f'_c S'_k), \tag{14a}$$

$$\Delta M_x = b \Delta A_s \beta_k (f_{sy} S_k - k_1 f'_c S'_k), \tag{15a}$$

$$\Delta M_y = a \Delta A_s \alpha_k (f_{sy} S_k - k_1 f'_c S'_k). \tag{16a}$$

In the above equations, the double subscript ij is used for concrete stresses and strains, while the single subscript k applies to steel stresses and strains. The only term which does not follow this pattern is S'_k, which is an "equivalent" concrete stress ratio in the k-th steel element. This quantity is the concrete stress ratio obtained from Eqs. (13) for the strain in the k-th steel element.

The total force P and the total moments M_x and M_y are now obtained by summation over the cross-section. This yields the expressions

$$P \;\; = k_1 f'_c \Delta A_c \{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} S_{ij} + q' \sum_{k=1}^{N_s} S_k - p' \sum_{k=1}^{N_s} S'_k\}, \tag{17}$$

$$M_x = P e_y = b k_1 f'_c \Delta A_c \{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \alpha_i S_{ij} + q' \sum_{k=1}^{N_s} \alpha_k S_k - p' \sum_{k=1}^{N_s} \alpha_k S'_k\}, \tag{18}$$

$$M_y = P e_x = a k_1 f'_c \Delta A_c \{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \beta_j S_{ij} + q' \sum_{k=0}^{N_s} \beta_k S_k - p' \sum_{k=0}^{N_s} \beta_k S'_k\}, \tag{19}$$

in which the following terms are introduced,

$$p' = \frac{N_c}{N_s} p, \qquad q' = \frac{N_c}{N_s} q, \qquad p = \frac{A_s}{A_c}, \qquad q = \frac{p f_{sy}}{k_1 f'_c}.$$

In the special case of zero eccentricity, i.e. for constant strain over the cross-section, the ultimate strength is given by

$$P_u = k_1 f'_c \Delta A_c N_c (1 - p + q S_0), \tag{20}$$

where S_0 is the steel stress ratio corresponding to the strain ratio $\epsilon'_c/\epsilon_{sy}$. Eq. (17) may be non-dimensionalised by dividing by Eq. (20);

$$\frac{P}{P_u} = \frac{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} S_{ij} + q' \sum_{k=1}^{N_s} S_k - p' \sum_{k=1}^{N_s} S'_k}{N_c (1 - p + q S_0)}. \tag{21}$$

The equations for moments M_x and M_y can likewise be non-dimensionalised by dividing by Eq. (17). This yields expressions for the eccentricities $e_\alpha = e_x/a$ and $e_\beta = e_y/b$;

$$e_\alpha = \frac{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \alpha_i S_{ij} + q' \sum_{k=1}^{N_s} \alpha_k S_k - p' \sum_{k=1}^{N_s} \alpha_k S'_k}{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} S_{ij} + q' \sum_{k=1}^{N_s} S_k - p' \sum_{k=1}^{N_s} S'_k}, \tag{22}$$

$$e_\beta = \frac{\sum_{i=1}^{N_a}\sum_{j=1}^{N_b}\beta_j\,S_{ij} + q'\sum_{k=1}^{N_s}\beta_k\,S_k - p'\sum_{k=1}^{N_s}\beta_k\,S'_k}{\sum_{i=1}^{N_a}\sum_{j=1}^{N_b}S_{ij} + q'\sum_{k=1}^{N_s}S_k - p'\sum_{k=1}^{N_s}S'_k}. \tag{23}$$

Finally, Eqs. (1) and (2) can be rewritten in terms of e_α and e_β;

$$\frac{\epsilon_r}{b} = \sqrt{(0.5-e_\beta)^2 + \eta^2\,(0.5-e_\alpha)^2}, \tag{24}$$

$$\psi = \tan^{-1}\left(\frac{1}{\eta}\,\frac{0.5-e_\beta}{0.5-e_\alpha}\right). \tag{25}$$

The above equations allow the force and moments to be calculated in a rectangular section with a known strain distribution. The large number of arithmetic calculations required can be readily and speedily executed with a digital computer. Indeed, this analysis has been developed specifically for computer use, and the equations have been incorporated in several programs. These programs use iterative type techniques which search out and find the deformation condition in a section for a given loading condition. One such program has been used in a study of the general moment curvature relation for column sections in biaxial bending [9].

For the study of long column behaviour described in the following section of this paper a simple non-iterative subroutine was written in FORTRAN IV language which evaluates the force P/P_u together with the biaxial eccentricity (i.e. e_r/b and ψ) for any given set of input values E_0, E_1 and θ. Other input data for this sub-program include partitioning details (N_a, N_b, N_s) and section properties (η, p, q). The co-ordinates of the steel elements, α_k and β_k, are generated in the subroutine, but details such as reinforcement cover, number of bars (elemental areas) in top, bottom and side faces, are read in as data.

Although computations carried out using the method described in this section are necessarily approximate, any desired accuracy can be obtained by a suitable choice of the fineness of the partitioning of the section. Furthermore, several important advantages are associated with this finite approach and have been discussed in some detail in References [4, 9]. Briefly, sections with irregular shape and non-uniform distribution of reinforcement can be treated in a simple manner, while effects such as strain hardening in the steel reinforcement and finite concrete tensile strength can be taken into account with only minimal changes to the computer programs and very slight increases in required computer computation times.

3. Long Column Behaviour

If the equations derived in the previous section of this paper are used to evaluate the actions in a section for various assumed strain distributions it becomes clear that the neutral axis of strain is not usually perpendicular to the line $A\,G$ (Fig. 1a) joining the point of load application A to the centroid G of the section. For a square, symmetrically reinforced cross-section the difference between the angles θ and ψ can be as large as ten degrees. For a constant load P, the angle difference $\delta = \theta - \psi$ tends to increase with increasing moment [9]. For rectangular sections, the difference δ can of course become much larger than ten degrees.

It therefore follows that the deflected longitudinal axis of a reinforced concrete column in biaxial bending would be non-planar, following a complicated path in three dimensions. It further follows that a typical cross-section in such a column would be subjected to small torsions and shears, in addition to the axial force and bending moments. Nevertheless, at least in the case of members with solid sections, it is doubtful if these secondary actions would be large enough to affect significantly the deflected shape and strength of the column. In this present preliminary study the section is assumed to be torsionally rigid and the torsions and shears are ignored. Furthermore, the non-planar deflected shape of the column is approximated by a planar curve.

The behaviour of a pin ended reinforced concrete column is considered for the special case of longitudinal loads applied with equal biaxial eccentricities at each end. This load condition is shown in Fig. 3, with a force P acting at an eccentricity defined by the distance e_{r0} from the geometric centre and the angle ψ_0.

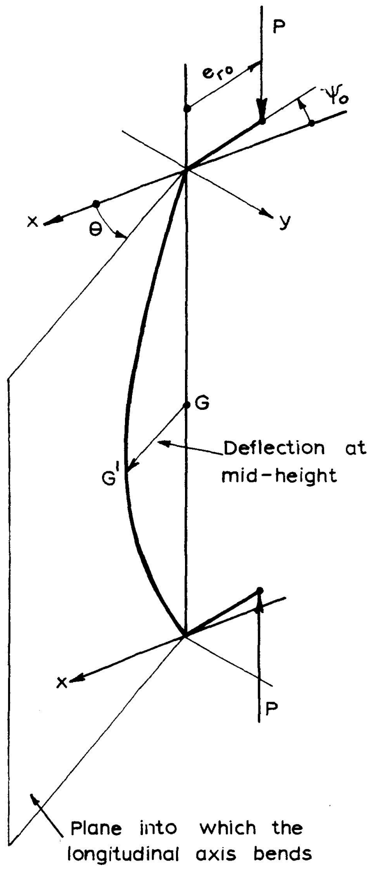


Fig. 3. Assumed deflected shape.

The deflected shape is approximated by a cosine curve. The two parameters defining the assumed curve are w, the maximum deflection in the column at mid-height, and θ, the angle between the x axis and the plane into which the centre line is assumed to have deflected. Note that the angle θ is not equal to the angle ψ_0. The deflection is therefore represented by θ and the equation

$$s = w \cos \frac{\pi z}{l}.$$

The curvature in the section at mid-height is therefore

$$\phi = w\left(\frac{\pi}{l}\right)^2.$$

A second equation for this curvature can be written in terms of the strains in the section,

$$\phi = (\epsilon_0 - \epsilon_1)(a \cos\theta + b \sin\theta)^{-1},$$

hence the following expression for the deflection w is obtained,

$$\frac{w}{b} = \frac{\epsilon_c'}{\pi^2} \frac{E_0 - E_1}{(\eta \cos\theta + \sin\theta)} \left(\frac{l}{b}\right)^2. \tag{26}$$

Consideration of the deformations in the section at mid-height shows that the neutral axis of strain for this section is perpendicular to the deflection plane. Thus, the neutral axis is inclined at the angle θ to the y axis. In Fig. 4 the original and the deflected positions of the section are represented. The eccentricity of the force P relative to the geometric centre in its deflected position, G', is made up of the initial eccentricity e_{r0} at inclination ψ_0, plus

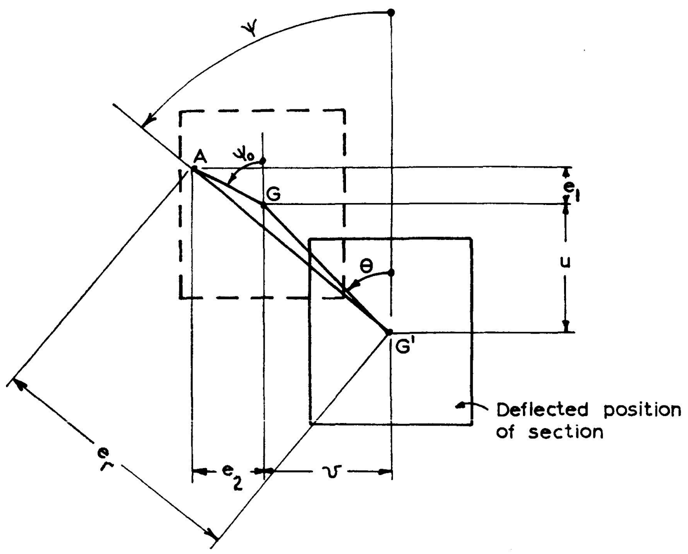


Fig. 4. Deflection of section at mid-height.

the deflection w at inclination θ. The components of w in the x and y directions are

$$u = w\cos\theta, \tag{27}$$

$$v = w\sin\theta, \tag{28}$$

respectively, and those of e_{r0} are

$$e_1 = e_{r0}\cos\psi_0, \tag{29}$$

$$e_2 = e_{r0}\sin\psi_0. \tag{30}$$

The total eccentricity of P is thus given by

$$\frac{e_r}{b} = \sqrt{\left(\frac{e_1}{b}+\frac{u}{b}\right)^2+\left(\frac{e_2}{b}+\frac{v}{b}\right)^2}, \tag{31}$$

$$\psi = \tan^{-1}\left(\frac{e_2/b+v/b}{e_1/b+u/b}\right). \tag{32}$$

Eqs. (26) through (32) allow the eccentricity of P to be calculated for any given set of values E_0, E_1 and θ in the mid-height cross-section. An independent calculation of e_r/b and ψ for this same strain distribution can also be made by considering equilibrium of internal moments in this section, i. e. by using the equations derived in Section 2 of this paper.

In general, the two sets of values of e_r/b and ψ will not agree if computed for an arbitrarily chosen set of input values E_0, E_1 and θ. This non-agreement indicates that the assumed strain distribution cannot occur at any stage of loading of the particular column under consideration. Conversely, only when the separate calculations give the same values for e_r/b and ψ does the strain distribution represent a possible stage in the loading of the column.

It is also clear that the maximum compressive strain E_0 increases monotonically from zero as the column deflects. This is not true, necessarily, for either E_1 or θ. Nevertheless, for a chosen E_0 value, a search technique can be employed to determine a set of values of E_1 and θ which equates, within a specified allowable error, the two sets of values of e_r/b and ψ as calculated on the one hand from the deflection requirement (Eqs. (31) and (32)), and on the other hand from the equilibrium requirement (Eqs. (24) and (25)).

Furthermore, by choosing successive, increasing values of E_0 and searching each time to obtain the corresponding E_1 and θ, a sequence of points on the load deflection curve of the column can be obtained. The strength of the column can then be found as the load at which the deflection curve becomes horizontal, i. e. when

$$\frac{dP}{dw} = 0. \tag{33}$$

A general computer program has been written which can be used to determine the load-deflection curve (P/P_u versus w/b and θ) for a wide range of possible column shapes and sizes. Computations based on the equilibrium requirement in this program are made using the subroutine described in Section 2 of this paper.

Development of the necessary search technique was simplified to some extent by the fact that the values of ψ and θ change only slowly as E_0 increases, while changes in E_1 are usually of the same order of magnitude as the changes in E_0. Thus, for the n-th E_0 value, the initial trial values of both θ and E_1 can be taken equal to their final values for the previous, $(n-1)$-st, step. However, an exhaustive and time consuming computer "search" is sometimes required to find the values of θ and E_1 which correspond to the first (smallest) E_0 value.

Additional input data required in the main program include the errors in e_r/b and ψ which will be tolerated in the search procedure, and magnitude of the increment of E_0. To maintain maximum generality, all input and output data are non-dimensionalised while all stresses and strains used in the computations are evaluated in the ratio forms of Eqs. (5) and (6). The program has been written in FORTRAN IV language and has been tested and run on an IBM System 360/50 computer. The computation time required to obtain one complete load deflection curve is somewhat less than one minute.

In Fig. 5 typical load deflection curves are plotted from computations made

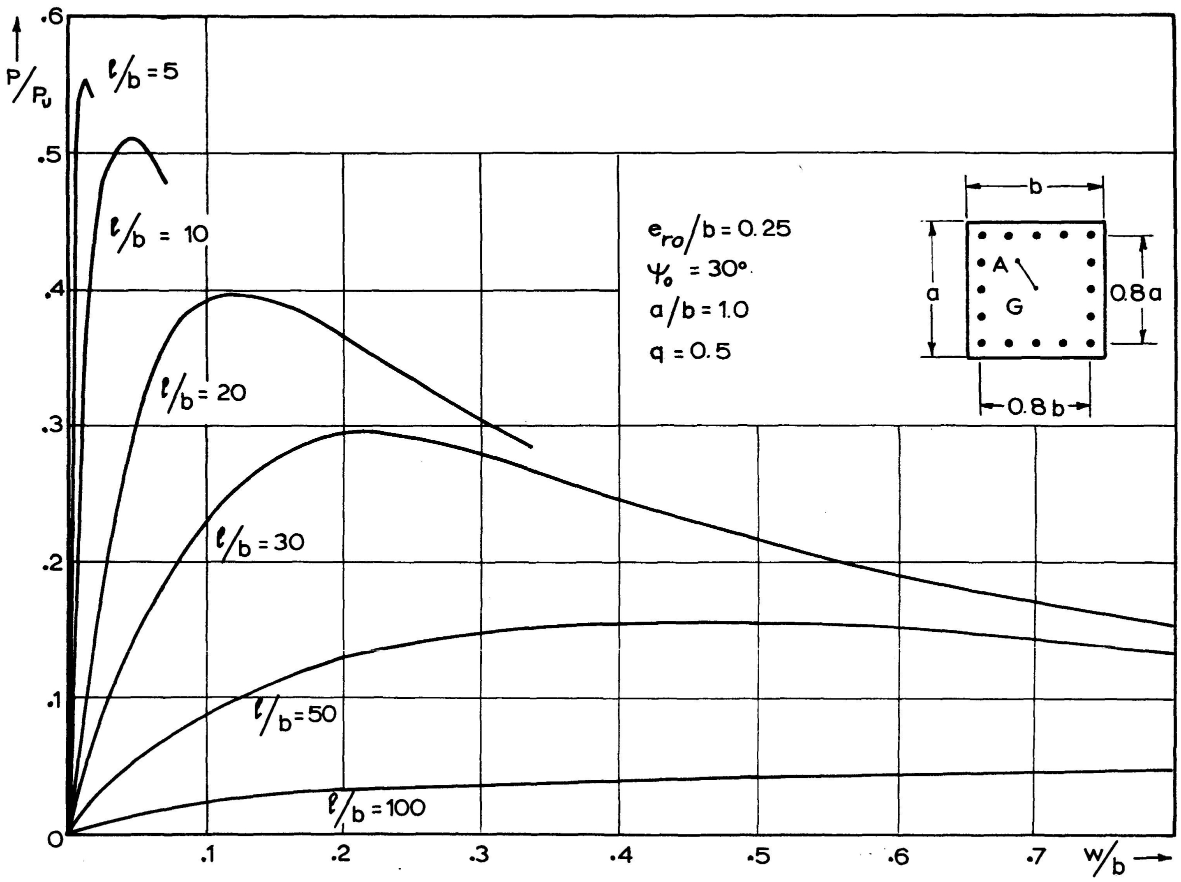


Fig. 5. Load deflection curves; various column lengths.

for a column section with the length-to-width ratio varying between zero and 100. The computations were carried out for the following data:

Material Properties

$\gamma_1 = 2.2$; $\gamma_2 = 4.0$; $\epsilon_{sy}/\epsilon_c' = 0.5$;

Column Details

$a/b = 1.0$; $q = 0.5$; $e_{r0}/b = 0.25$; $\psi_0 = 30°$;
Reinforcing bars distributed as shown in Fig. 5.

Partitioning Details

$N_a = N_b = 10$; $N_s = 16$;

Computation Details

Allowable error in $e_r/b = 0.005$;
Allowable error in $\psi = 0.01$ radians;
Increment in $E_0 : \Delta E_0 = 0.05$.

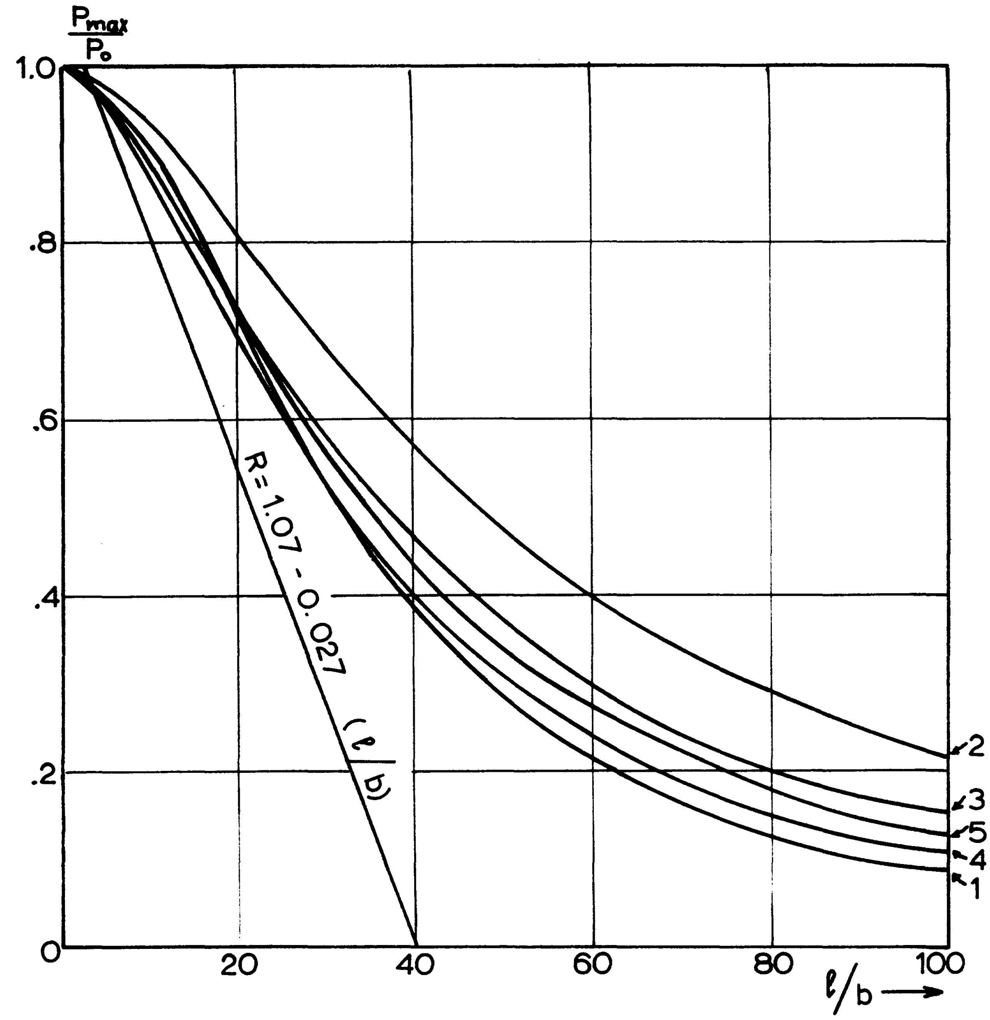
The unloading portions of the curves in Fig. 5 were obtained by incrementing the strain ratio E_0 until it reached a value of 3.0, when computations were arbitrarily discontinued. No physical significance should therefore be associated with the points of termination of the plotted curves.

The variation in column strength is plotted against length-to-width ratio in Fig. 6 for several sections and load eccentricities. To obtain uniformity in the load scale, column strengths have been divided by the strength of a similar column of zero length, P_0. For purposes of comparison the ACI strength reduction factor, R, for use in the design of long columns in single curvature uniaxial bending, is also plotted in Fig. 6 against l/b. The value of R is given [10] as

$$R = 1.07 - 0.008\,(l/r) \leqq 1.0, \tag{34}$$

where r is the radius of gyration of the section, which is here taken as $0.3\,b$. Although the line representing Eq. (34) lies on the safe side of all of the column curves plotted in Fig. 6, it must be remembered that a considerable additional safety margin is required in the design interaction equation to account for the effects of creep deflection in columns under sustained loading. Indeed, the closeness of several of the curves to this interaction line for relatively small length-to-width ratios might well indicate that a design equation more conservative than Eq. (34) would be appropriate for columns in biaxial bending under sustained loading.

The computer program described in this paper is at present (1968) being run for a wide range of input data to obtain further information for the development of simple interaction equations for the design of columns in biaxial bending.



Curve	q	e_{r0}	ψ_0	P_0/P_u
1	0.50	0.25	30°	0.562
2	0.50	1.00	30°	0.159
3	0.25	0.75	15°	0.177
4	0.25	0.50	45°	0.281
5	0.50	0.50	15°	0.349

Fig. 6. Typical curves of strength vs. slenderness for columns with square sections.

4. Concluding Remarks

Various simplifying assumptions have been made in this study of reinforced concrete column behaviour in biaxial bending. In order to emphasise the preliminary nature of the work, the more important of these assumptions are discussed briefly.

In the analysis of the load-deformation characteristics of the cross-section, the two basic assumptions of perfect bonding and plane distribution of strains are made. These assumptions are probably accurate provided compressive stress exists over the entire section preventing the formation of cracks. When the section is cracked the assumption of perfect bonding is highly idealised and may well lead to significant error in computed curvatures. Nevertheless, most previous studies of the uniaxial bending of reinforced concrete columns have also been based on this simplification.

A two parameter cosine curve has been used to approximate the deflected column shape. It has been indicated that the actual shape must be non planar. Although an over-estimation of column strength will result from this approximation, the magnitude of the error is unknown and can be determined only when a more accurate analysis has been undertaken. However, in the special case of uniaxial bending it has been determined that column strength is relatively insensitive to variations in the assumed column shape. This may well be true also in the general case of biaxial bending.

Long column behaviour has been studied in this paper only for short-time loadings. Significant reductions in the strength of actual in-service columns will occur as a result of time dependent effects such as concrete creep.

Despite the limitations implied by the simplifying assumptions, the analysis presented should prove to be a useful tool in the study of reinforced concrete column behaviour. In particular, the method of determining the load-deformation characteristics of a section is convenient when programmed for computer computation. It represents the only feasible method of analysing sections of non-symmetric or irregular shape,

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Summary

A study is made of the behaviour of long reinforced concrete columns in biaxial bending. A method is first developed for determining the deformations in a rectangular reinforced concrete section subjected to a biaxially eccentric force. The method involves a large number of elementary arithmetical calculations and has been developed for computer computation. On the basis of several simplifying assumptions, notably concerning deflected shape, a study is made of the behaviour and strength under short-time loadings of pin-ended columns under biaxially eccentric loading.

Résumé

L'auteur développe une méthode pour déterminer les déformations d'une section rectangulaire de béton armé soumise à une force excentrique par rapport aux deux axes. La méthode, qui comprend un grand nombre d'opérations arithmétiques élémentaires, à été développée pour le calcul électronique. Sur la base de certaines hypothèses de simplification, notamment pour la forme de la section déformée, on étudie le comportement et les contraintes de colonnes articulées soumises à une force biaxialement excentrique de courte durée.

Zusammenfassung

Untersucht worden ist das Verhalten langer Stahlbetonsäulen bei Biegung um beide Hauptachsen. Zuerst wird ein Verfahren zur Bestimmung der Verformungen in einem rechteckigen Stahlbetonquerschnitt unter zweiaxial ausmittiger Last entwickelt. Dieses Verfahren zieht eine große Anzahl einfacher Rechenoperationen nach sich, so daß der Elektronenrechner zu Hilfe gezogen wurde. Auf Grund verschiedener vereinfachender Annahmen, unter anderem bezüglich der Biegelinie, wurde das Verhalten und die Beanspruchung unter Kurzzeitlasten für gelenkig gelagerte Stützen untersucht.