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**Autor:** Borges, Júlio Ferry
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# Cracking and Deformability of Reinforced Concrete Beams

*La fissuration et la déformabilité des poutres en béton armé*

*Rißausbildung und Verformung von Stahlbeton-Trägern*

JÚLIO FERRY BORGES
Research Engineer, Head, Buildings and Bridges Dept., Laboratório Nacional de Engenharia Civil, Lisbon

## 1. Introduction

The need for completely describing the behaviour of reinforced concrete elements under increasing loads is generally recognised.

The hypotheses required for this description should be as simple as possible but they must take the principal parameters into consideration in order to represent reality sufficiently well. It is also important not to forget their possible generalisation to analogous problems, particularly to prestressed concrete.

Cracking, deformation and redistribution of moments being closely inter-related, they must be studied using the same or at least compatible hypotheses. This also applies to rupture, which may be considered as the last stage in the evolution of the behaviour of the structure.

Particular emphasis is laid on non-linear behaviour, due both to cracking and to the anelastic deformations of concrete and steel.

Results concerning bi-linear behaviour are used to derive simple design rules for computing displacements and for limiting the redistribution of bending moments in statically indeterminate beams.

The present paper applies a general method and results on the non-linear analysis of structures that were previously published (1 to 3) as well as results obtained in two theses recently prepared at the Laboratório Nacional de Engenharia Civil (4 and 5).

## 2. Cracking

### *2.1. Position of the Problem*

This paper deals only with transverse cracking at the level of reinforcement in beams. Other problems such as flexural cracking at middle depth of the web and cracking due to shear forces are not considered. The initial state of cracking is also disregarded, it being assumed that sufficiently high stresses are attained for the number of cracks not to increase for increasing loads.

The mean width of cracks is computed simply by assuming that it equals the product of the mean distance between cracks by the difference between the mean strains in steel and concrete at the reinforcement level. The way the problem is dealt with is not basically different from that used by other authors (6 and 7). The two main differences lie in the explicit consideration of the influences of the cover on the mean distance between cracks and of the percentage of reinforcement on the mean steel strain.

### *2.2. Mean Distance between Cracks*

The mean distance between cracks is considered to be expressed by a linear function of the cover of the bars and of the ratio between the diameter of the bars and the percentage of reinforcement.

The need to consider the influence of the thickness of the cover, $c$, is easy to understand. In fact, even if perfect bonding between concrete and steel existed, the mean distance between cracks would not be zero but proportional to $c$. In fact, only at a distance from $A$ proportional to $c$, does the tensile strain at the face $\overline{AB}$, fig. 1, reach the uniform strain considered for both concrete and steel.

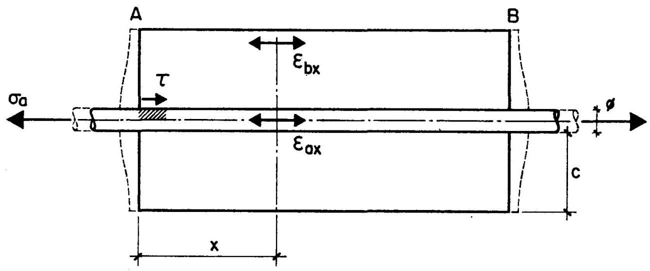



Fig. 1.

It is also easy to justify the proportionality of the mean distance between cracks to the ratio between the diameter of the bars and the percentage of reinforcement which is equivalent to the ratio of the area of concrete under tensile stress $B_t$ to the perimeter of the bars $p$.

In fact, assuming that the bond stress between steel and concrete is only a function of the length measured along the bars and disregarding the nonuniformity of the state of stress, the distance between cracks results to be proportional to $B_t/p$ (6).

A complete analysis of the state of stress by superimposing the two effects would show that they are additive. The mean distance between cracks, $\Delta 1$, is thus given by

$$\Delta 1 = k_1 c + k_2 \frac{\phi}{\tilde{\omega}_0}. \tag{1}$$

In the case of a uniform tensile stress the percentage $\tilde{\omega}_0$ refers to the total cross section of the bar. In the case of bending it seems preferable to refer the percentage $\tilde{\omega}_0$ to the total area of the web. If bending is combined with compression, the percentage should be referred to the area of concrete under tension. The way the percentage $\tilde{\omega}_0$ is considered obviously influences the value of the coefficient $k_2$.

Experimental studies on the similitude of reinforced concrete beams in which the thickness of cover $c$ was varied in a wide range (8), showed that $k_1$ can be taken equal to 1.5.

The factor $k_2$ would also depend on the ratio of the ultimate tensile strength of the concrete, $\sigma_{bt}$, to the ultimate bond stress, $\tau$. For different qualities of concrete both values vary in the same way so that their influence can be disregarded. Likewise the influence on $k_2$ of the shape of the surface of the bars, for the deformed bars currently used, is not as important as it could be

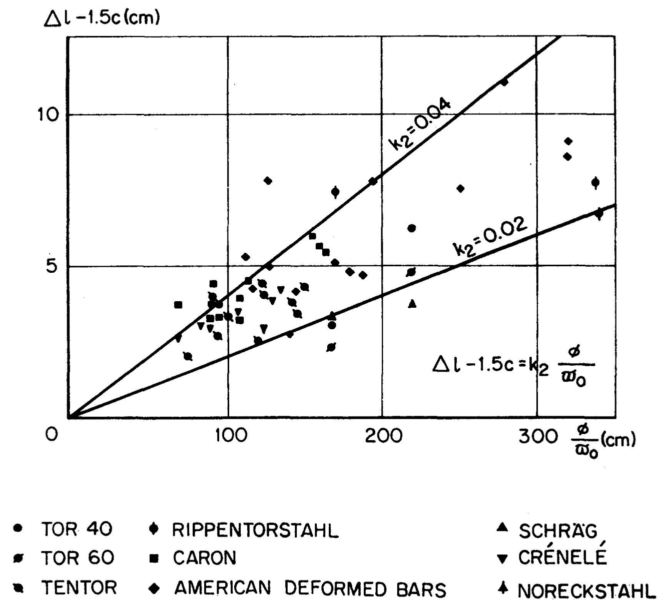



Fig. 2.

supposed. In fact the experimental results available (Annex I) lie between the lines corresponding to $k_2 = 0.02$ and 0.04, fig. 2.

It must be emphasized that if the influence of the cover were ommitted, a very weak correlation between $\Delta 1$ and $\phi/\tilde{\omega}_0$ would be obtained.

### *2.3. Mean Steel Strain*

The usual reinforced concrete design hypotheses enable a sufficiently accurate determination of the steel stress at a section where a crack exists. Due to the bond between steel and concrete, stresses vary along the bars and this variation has to be considered to compute the mean steel strain.

Tensile tests of bars surrounded by concrete and of beams (9) showed that the mean steel strain in the bars, $\bar{\epsilon}_a$, can be well represented by the simple expression

$$\bar{\epsilon}_a = \frac{1}{E_a}\left(\sigma_a - \frac{k_3}{\tilde{\omega}_0}\right), \tag{2}$$

where $E_a$ = modulus of elasticity of steel,
$\sigma_a$ = stress in steel in a cracked section,
$\tilde{\omega}_0$ = percentage of reinforcement referred to the web section.

This expression duly takes into consideration the reduction of the mean steel strain due to the concrete around the bars. The coefficient $k_3$ depends on the bond between steel and concrete but, for the usual types of deformed bars, it can be considered as constant.

Tests performed (4, 9) show that for the usual types of deformed bars the value of $k_3 = 7.5$ kg/cm² can be adopted. For this value of $k_3$ the family of stress-strain diagrams indicated in fig. 3 is obtained.

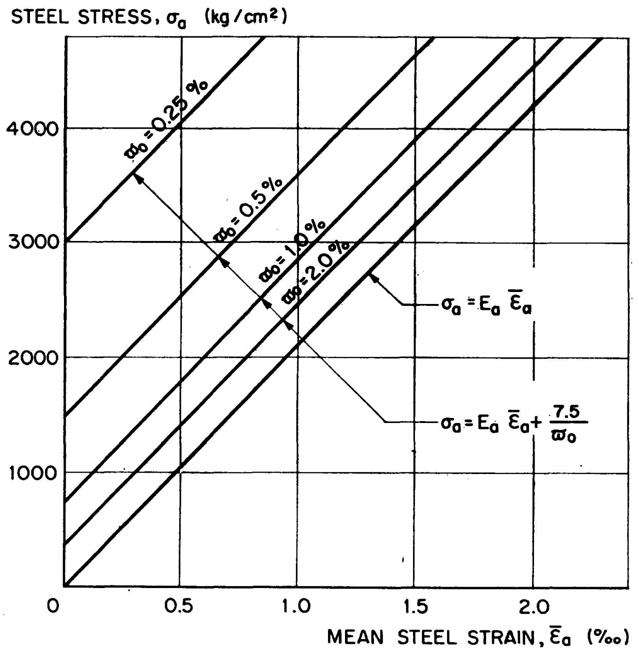



Fig. 3.

### *2.4. Mean and Maximum Crack Width*

As indicated in 2.1, the mean crack width, $\bar{w}$, can be computed by

$$\bar{w} = \Delta l\,(\bar{\epsilon}_a - \bar{\epsilon}_b), \tag{3}$$

where $\Delta 1$ = mean distance between cracks,
$\bar{\epsilon}_a$ = mean strain in steel,
$\bar{\epsilon}_b$ = mean strain in concrete.

In general, for practical purposes, the concrete strain can be disregarded in comparison with that of steel, except when shrinkage is important and the resulting strains are avoided by a high percentage of reinforcement or external connections.

Disregarding the concrete strain, the mean crack width can be computed by combining expressions (1) and (2). The following expression is then obtained

$$\overline{w} = \frac{1}{E_a}\left(k_1 c + k_2 \frac{\phi}{\tilde{\omega}_0}\right)\left(\sigma_a - \frac{k_3}{\tilde{\omega}_0}\right). \tag{4}$$

The analysis of about 150 tests on beams reinforced with deformed steel (Annex I) showed that agreement between theoretical and experimental results is good and considering the percentage $\tilde{\omega}_0$ referred to the section of the web and taking:

$$E_a = 2.1 \times 10^6 \text{ kg/cm}^2, \quad k_1 = 1.5, \quad k_2 = 0.04 \quad \text{and} \quad k_3 = 7.5 \text{ kg/cm}^2$$

the following expression then results

$$\overline{w} = \frac{1}{2.1 \times 10^6}\left(1.5\, c + 0.04 \frac{\phi}{\tilde{\omega}_0}\right)\left(\sigma_a - \frac{7.5}{\tilde{\omega}_0}\right). \tag{5}$$

Fig. 4 makes possible to compare the mean crack widths as measured in the tests and as computed according to expression (5). Since for each beam this comparison is possible for different values of the steel stress, each beam test corresponds to several points in fig. 4. As many points lie in area A, not all of them could be represented.

Table I contains the mean values and the coefficients of variation of the ratio of computed to measured crack widths for different ranges of the steel stress, $\sigma_a$.

*Table I. Mean value and coefficient of variation of the ratio of computed to measured mean crack widths*

| Range of steel stress, $\sigma_a$ kg/cm² | Mean value | Coefficient of variation |
|---|---|---|
| $2100 < \sigma_a < 2700$ | 0.96 | 0.32 |
| $2700 \leqq \sigma_a < 3250$ | 1.06 | 0.31 |
| $3250 \leqq \sigma_a < 3750$ | 1.10 | 0.25 |
| $3750 \leqq \sigma_a < 4000$ | 0.99 | 0.23 |
| $2100 < \sigma_a < 4000$ | 1.03 | 0.32 |

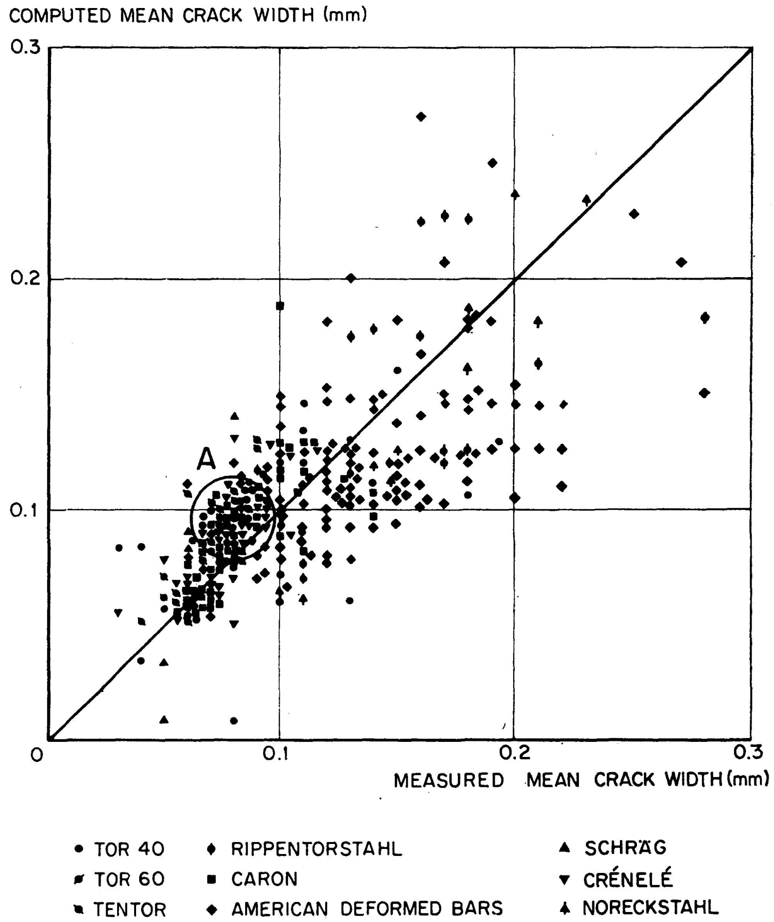



Fig. 4.

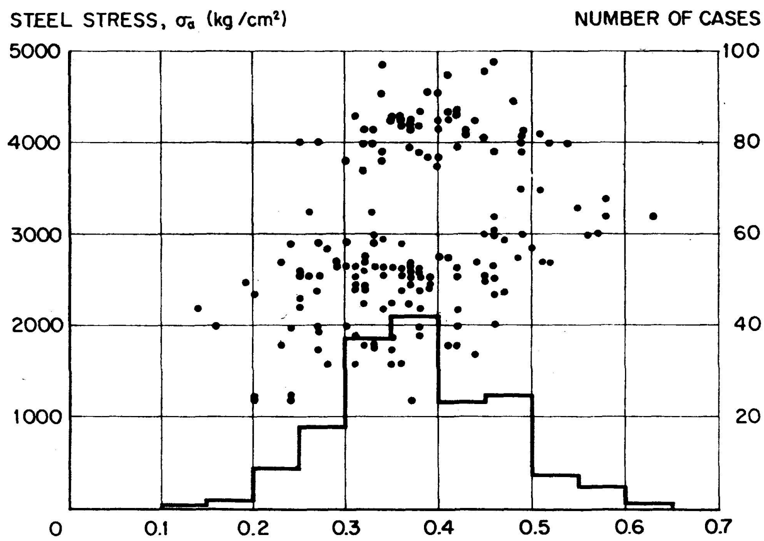



Fig. 5.

The mean value of 1.03 was obtained for all the results considered and no significant change of the mean values was noticed for increasing values of the steel stress. The coefficient of variation decreases from 0.32 to 0.23 as stresses increase from 2100 to 4000 kg/cm$^2$.

In practice it is usual to limit not the mean but the maximum width of cracks. Consequently it is of interest to analyse the relashionship between mean and maximum values. For this purpose the coefficient of variation of the crack widths measured in the region of constant bending moment was also computed from the test results available. The values obtained, fig. 5, show that the coefficient of variation is pratically independent of the stresses in steel in the interval 1500—4000 kg/cm$^2$. The coefficients of variation are distributed according to the frequency diagram also presented in fig. 5, their values ranging from 0.3 to 0.5 in 80% of the cases. The value 0.4 can be taken as the most representative.

Assuming as maximum a width that is not surpassed in more then 5% of the cases, and considering the distribution of the crack width to be normal with the referred coefficient of variation of 0.4, it results that

$$w_{max} = (1+1.65\times 0.4)\,\overline{w} = 1.66\,\overline{w}. \tag{6}$$

According to this expression, to maximum crack widths of 0.2 and 0.3 mm correspond mean values of 0.12 and 0.18 mm, respectively.

A computation of the mean value of the ratio of maximum to mean crack widths for the test results available yields a value of 1.72, which agrees well with the precedingly obtained value of 1.66.

### *2.5. Design Rules*

By means of expression (5), simple design rules can be derived which enable a satisfactory design of beams from the point of view of cracking.

In fact if the values of the cover, $c$, of the mean crack width, $\overline{w}$ and the steel stress, $\sigma_a$, are fixed, it is possible by expression (5) to establish a relation between the diameter $\phi$ and the percentage of reinforcement $\tilde{\omega}_0$. For a given percentage of reinforcement the use of bars with diameters exceeding the values supplied by expression (5) will correspond to a mean width of cracks above the values considered. By taking $c=3$ cm, $\overline{w}=0.12$ mm and $\sigma_a=2400$ and 3000 kg/cm$^2$, the relation between $\phi$ and $\tilde{\omega}_0$ is expressed by the lines indicated in fig. 6 and 7.

The results available were also used to supply an experimental confirmation of the relations thus derived. For the stresses of 2400 and 3000 kg/cm$^2$ indicated above, the diameters corresponding to a mean crack width of 0.12 mm were plotted in function of the percentage of reinforcement. The number of results in these conditions being small, advantage was taken of the similitude conditions (8) for using the test results in which a mean crack width $\overline{w}$ different from 0.12 mm had been obtained. In these cases it was not the diameter $\phi$ but the diameter $\phi_0=\frac{\overline{w}}{0.12}\,\phi$ that was represented.

The ratio $\frac{\overline{w}}{0.12}$ has the meaning of a change of scale. The objection that can be raised against this procedure is that the same ratio will also apply to the cover. That is why only ratios between 0.6 and 2.0 were considered.

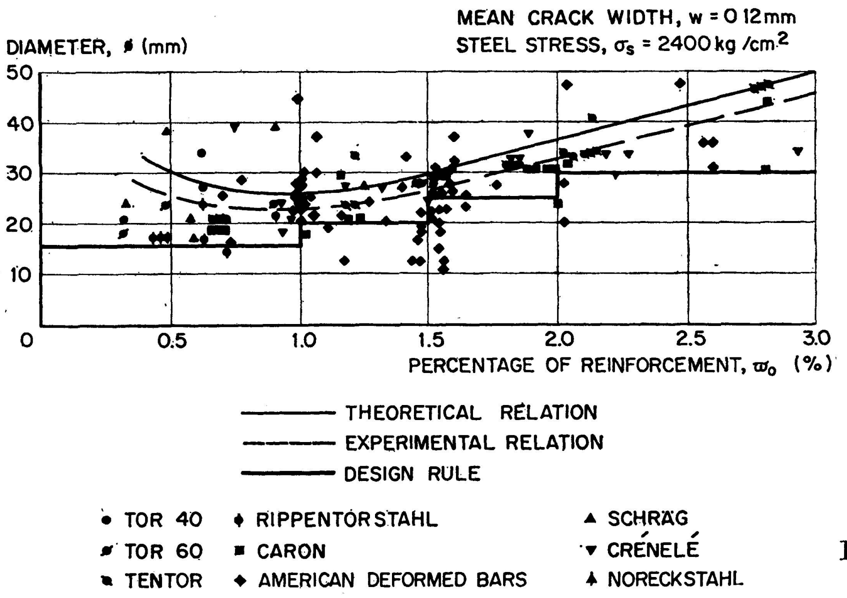



Fig. 6.

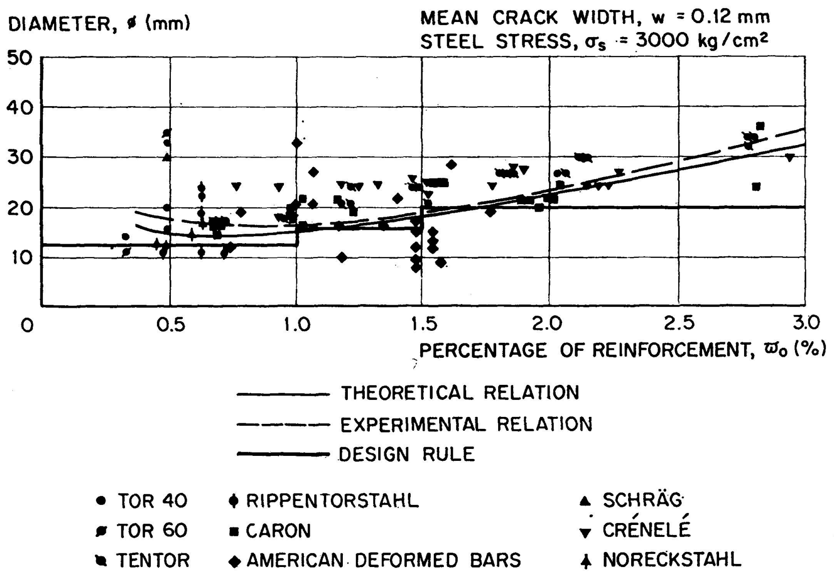



Fig. 7.

The dotted lines representing the mean of the experimental values obtained agree well with the theoretical lines. As the scatter of the individual results is considerable, it is advisable for practical purposes to limit the diameters to values below the thick line also indicated in fig. 6 and 7. It should be noted that the experimental results which do not satisfy this limit correspond to tests in which the cover considerably exceeds the usual values.

The relation presented between diameter and percentage of reinforcement allows the quantification of the well-known rule: the higher the stresses and the smaller the percentage of reinforcement, the smaller the diameter of the bars required to avoid excessive cracking.

The design rules presented in the C.E.B. Recommendations (10) are of this type.

## 3. Deformation

### *3.1. Position of the Problem*

The deformability of reinforced concrete elements is particularly important both to determine the displacements in service conditions and to study the redistribution of moments under ultimate loads.

By conveniently defining moment-curvature diagrams and by integrating the curvature, it is possible entirely to follow the deformability of beams under increasing loads. This way of dealing with the problem was presented in (1) to (3). In (4) it was shown that a very good agreement between analytical and experimental results is obtained.

Schematically three types of behaviour of reinforced concrete elements can be considered: (I) uncracked stage, (II) cracked stage in the elastic range, and (III) anelastic behaviour.

In order to derive rules to forecast displacements in service conditions, only the two first types of behaviour have to be considered. For studying the redistribution of moments near rupture the last type of behaviour is of paramount importance.

### *3.2. Moment-curvature Diagrams for the Cracked Stage*

Moment-curvature diagrams accurately representing the behaviour of the element during phases (I) and (II) can be obtained from the following simple hypotheses.

a) The stress-strain diagram for concrete is linear, corresponding to a modulus of elasticity, $E_b$.

b) In stage (I) the complete section of concrete is considered and for the usual percentages of reinforcement the steel section need not be taken in account.

c) In stage (II) the tensile strength of concrete is disregarded, stresses are computed assuming that deformed sections remain plane and that the mean steel strain is obtained from expression (2).

For rectangular sections, these hypotheses enable bi-linear moment-curvature diagrams to be very easily defined. For representing these diagrams, values of $m=\frac{M}{b\,h^2 E_b}$ are taken as ordinates and values of $\theta=\bar{\epsilon}_a+\bar{\epsilon}_b$ as abscissae. These values of $\theta$, of sum the mean steel and concrete strains, correspond to a reduced curvature. In fact, $r$ being the radius of curvature and $h$ the effective depth of the section, $1/r=\theta/h$.

For phase (I) the relation $M(\theta)$ is thus expressed by $\frac{M}{E_b I}=\frac{\theta}{h}$ and, disregarding the steel area when computing the moment of inertia I and assuming that the total depth is $h_t=1.06\,h$, the expression $m=\theta/10$ is obtained.

In phase (II) the relation between $m$ and $\theta$ according to the indicated hypotheses can be well represented by straight lines depending on $E_a/E_b$ and on the percentage of reinforcement, $\tilde{\omega}_0$.

The bi-linear diagrams thus obtained are presented in fig. 8 to 11. These diagrams consider the effect of cracking but cannot be used for studying the anelastic behaviour. In this latter case the yielding of steel and curved diagrams for concrete have to be considered, (1) to (4).

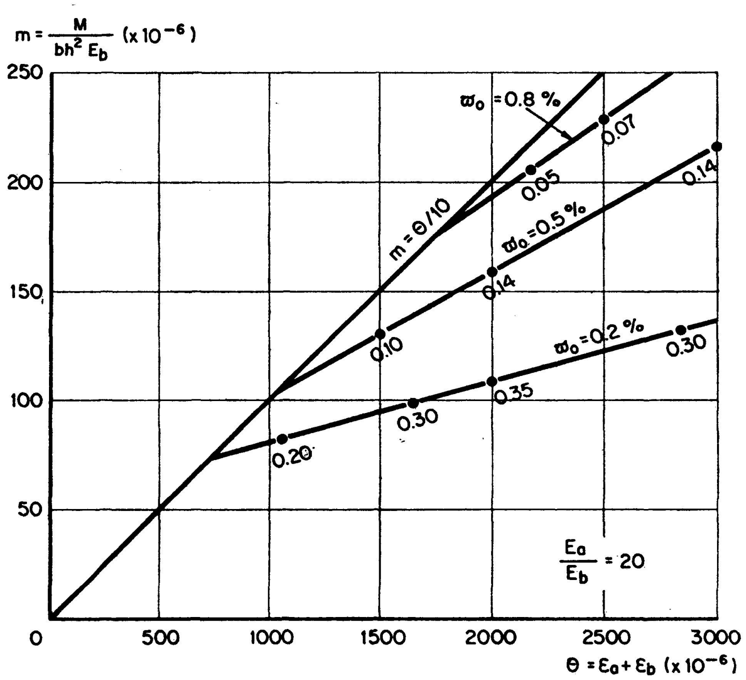



Fig. 8.

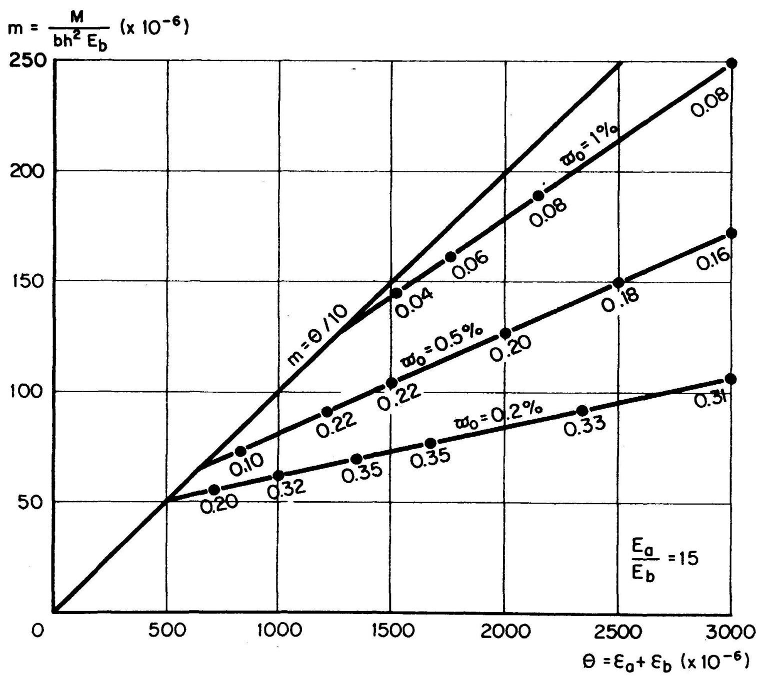



Fig. 9.

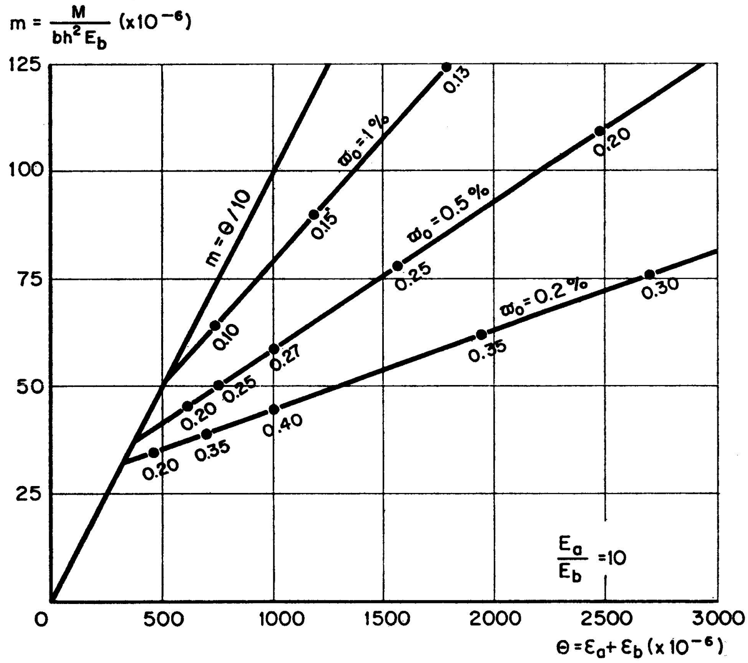



Fig. 10.

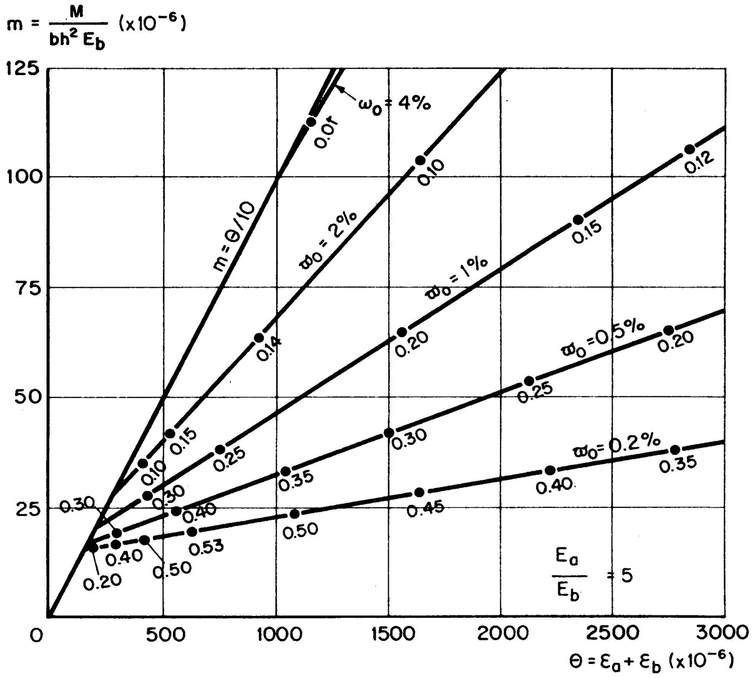



Fig. 11.

### *3.3. Force-displacement Diagrams*

It is well known that displacements can be computed by double integration of the curvature along the bars. From the moment-curvature diagrams the corresponding force-displacement diagrams can be easily obtained.

The study of force-displacement diagrams corresponding to the behaviour of typical cases of cantilever and simply supported beams with concentrated and distributed loads for different types of bi-linear moment-curvature diagrams is of particular interest.

The possible types of bi-linear diagrams can be defined by the position of their vertices referred to reduced coordinates obtained by dividing the bending moments and curvatures by their ultimate values. The 10 typical diagrams indicated in fig. 12 to 15 cover sufficiently well the different possible bi-linear diagrams.

The force-displacement diagrams corresponding to these 10 typical bi-linear diagrams are also indicated in fig. 12 to 15. The results obtained show that,

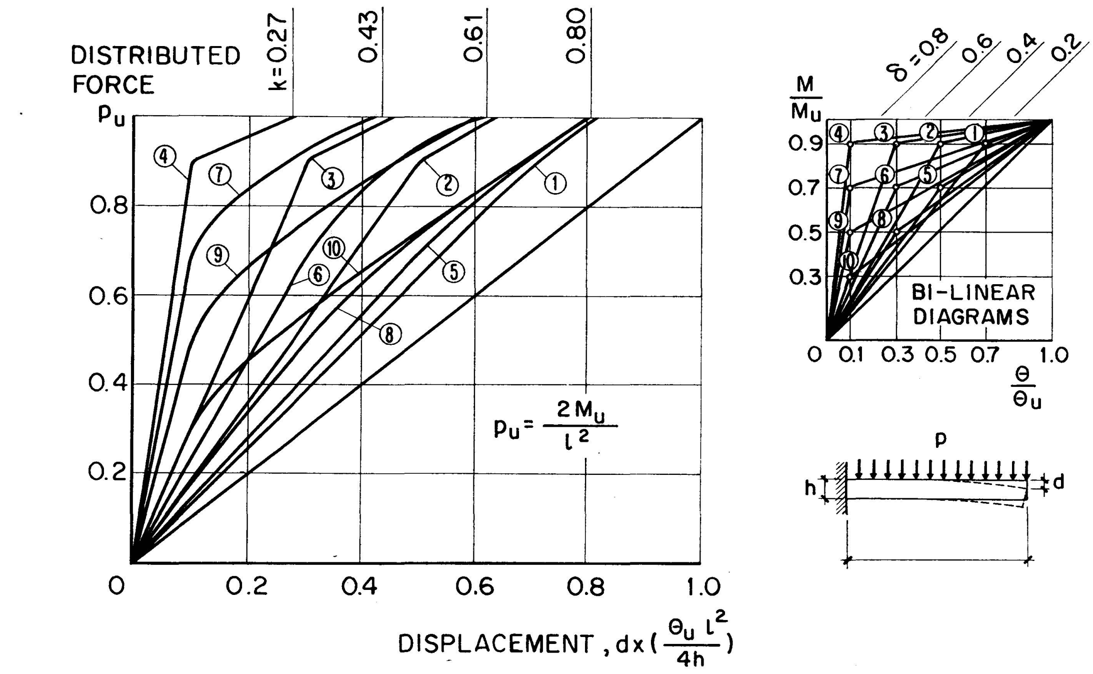



Fig. 12.

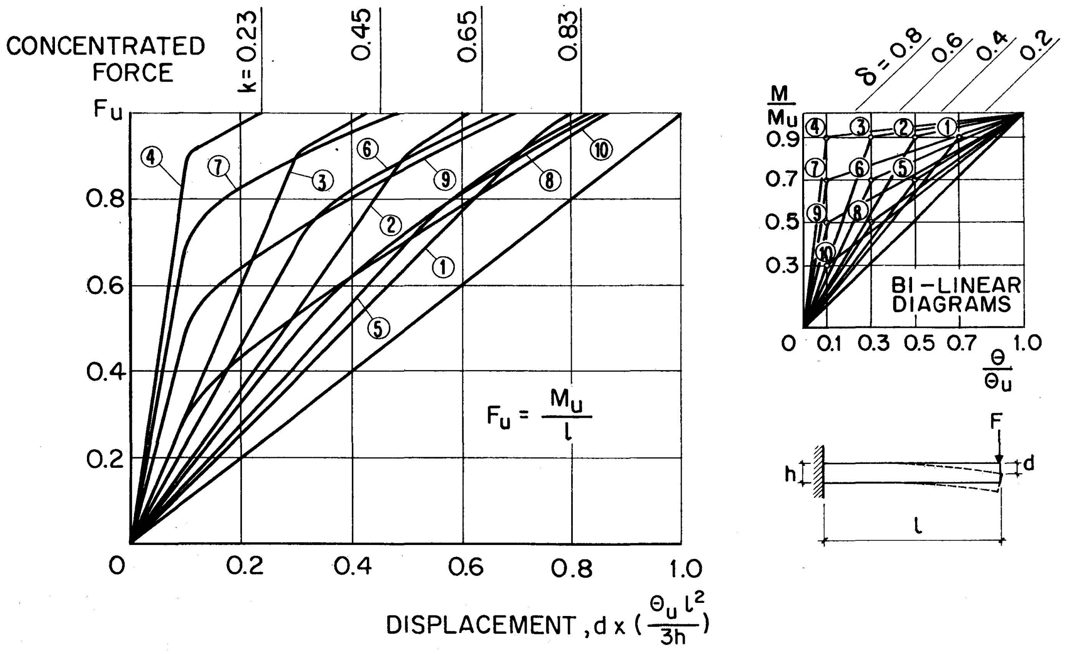



Fig. 13.

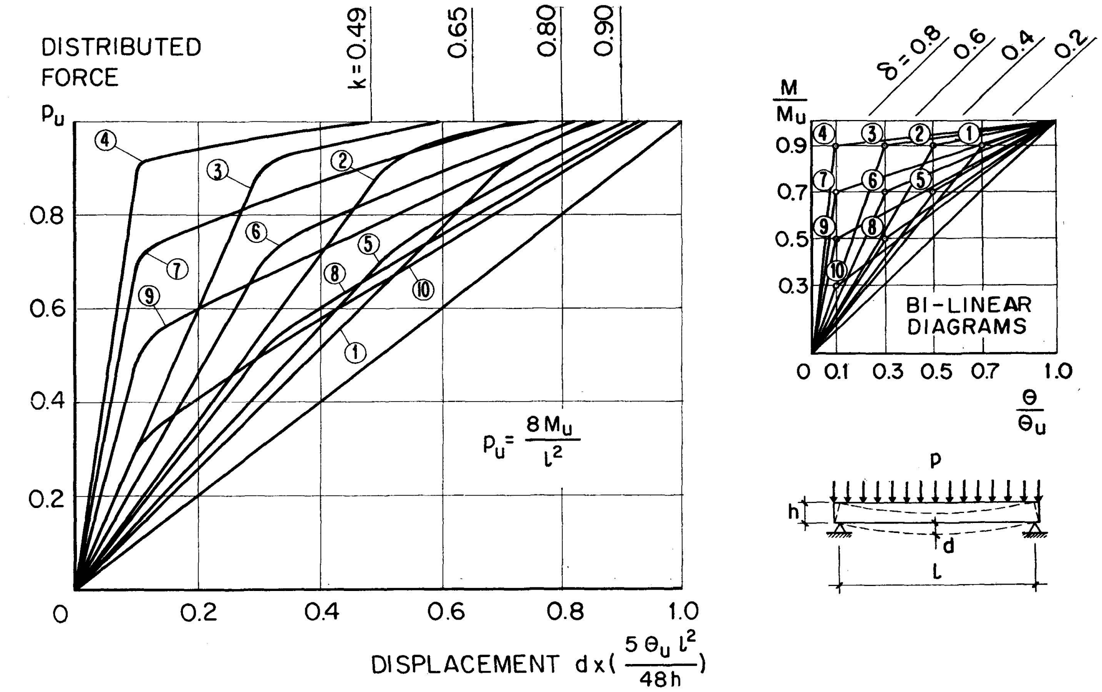



Fig. 14.

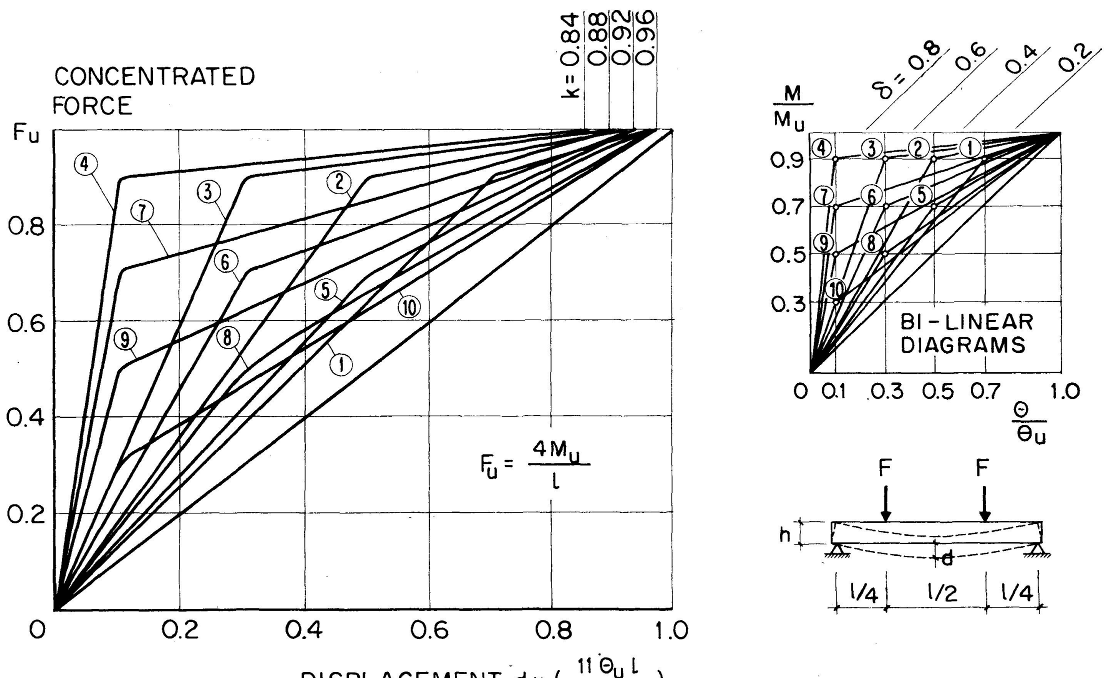



Fig. 15.

for bi-linear diagrams with the same area, force-displacement diagrams end at almost the same point.

It thus seems advisable to introduce the concept of ductility of a non-linear diagram, measured by $\delta = A_1/A_0$ (fig. 16), ratio of the area of the non-linear part of the moment curvature diagram to the area of the linear diagram with the same ultimate bending moment and curvature.

From fig. 12 to 15 it can be seen that the displacements corresponding to a bi-linear behaviour can be directly obtained from those corresponding to linear behaviour by multiplication by a correction factor, $k$, which way be considered as a function of the ductility factor, $\delta$, only.

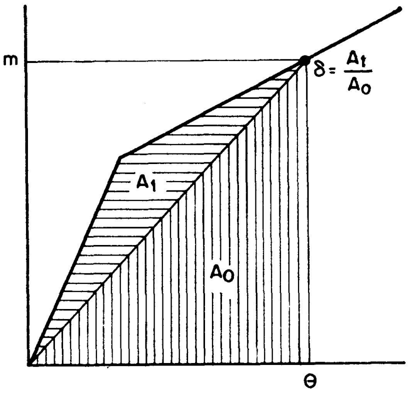



Fig. 16.

*Table II*

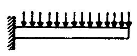
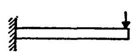
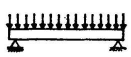
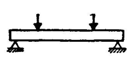
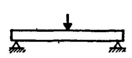
| |  |  |  |  |  |
|---|---|---|---|---|---|
| | $d = K_1 \frac{\theta l^2}{4h}$ | $d = K_2 \frac{\theta}{3} \frac{l^2}{h}$ | $d = K_3 \frac{5\theta}{48} \frac{l^2}{h}$ | $d = K_4 \frac{11\theta}{96} \frac{l^2}{h}$ | $d = K_5 \frac{\theta}{12} \frac{l^2}{h}$ |
| $\delta$ | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ |
| 0.2 | 0.80 | 0.83 | 0.90 | 0.96 | 0.86 |
| 0.4 | 0.61 | 0.65 | 0.80 | 0.92 | 0.65 |
| 0.6 | 0.43 | 0.45 | 0.65 | 0.88 | 0.45 |
| 0.8 | 0.27 | 0.23 | 0.49 | 0.84 | 0.23 |

Table II indicates the expressions of the displacements in function of the maximum values of $\theta$ for linear behaviour and the correction factors, $k$, by which these displacements have to be multiplied in order to obtain the displacements corresponding to bi-linear behaviour.

### *3.4. Design Rules*

The foregoing can be directly used to compute displacements in reinforced concrete beams.

In fact from the diagrams of fig. 8 to 11 it is possible to determine the reduced curvature, $\theta$, corresponding to a given bending moment. Additionally the values of the ductility factor, $\delta$, as defined in 3.3, are marked on the lines that represent the behaviour of the beam in the cracked stage.

The values of $\theta$ and $\delta$ being known, the displacements corresponding to the bi-linear behaviour considered can be computed from table II, merely by multiplying the displacements corresponding to a linear behaviour by the correction factor, function of the ductility factor.

The modulus of elasticity of the concrete to be used must duly take into consideration the effect of creep.

If permanent and accidental loads are combined, the equivalent modulus of elasticity $E$ to be used must suitably account for long term and short term deformations. It is easy to derive the following expression of the equivalent modulus of elasticity:

$$E = \frac{1+\alpha}{\frac{\varphi}{E_j} + \frac{\alpha}{E_k}}, \qquad (7)$$

where $\alpha$ = ratio of the accidental load applied at age $k$ to the permanent load applied at age $j$,

$E_j$ = modulus of elasticity at age $j$,

$E_k$ = modulus of elasticity at age $k$,

$\varphi$ = creep factor for the permanent loading conditions assumed.

## 4. Redistribution of Bending Moments

### *4.1. Position of the Problem*

In reinforced concrete the dilemma between elastic and limit designs cannot be solved without studying the non-linear behaviour of structures. Deformability under increasing forces has to be accurately followed and the consequent redistribution of bending moments analysed.

### *4.2. Moment-curvature Diagrams till Rupture*

In a previous paper, moment-curvature diagrams till rupture for reinforced concrete beams were computed considering only very general hypotheses (1), and in (4) it was shown that the agreement between theoretical diagrams thus obtained and experimental ones is very good.

If the behaviour under the ultimate load is to be studied, the distinction between the elastic behaviour before and after cracking can be disregarded and bi-linear diagrams can be used with a first stretch corresponding to elastic behaviour and a second one corresponding to anelastic behaviour. The error due to representing by a straight line the two initial phases corresponding to the uncracked stages is not important because curvature are considerably more marked in the third than in the two initial phases. That this is in fact so is exemplified by the moment-curvature diagrams presented in fig. 17 and 18 which correspond to reinforced concrete rectangular sections with steel 40

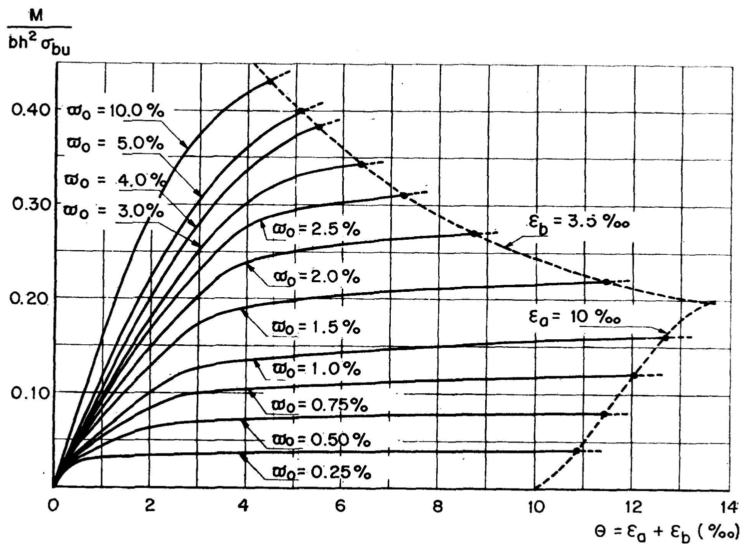



Fig. 17.

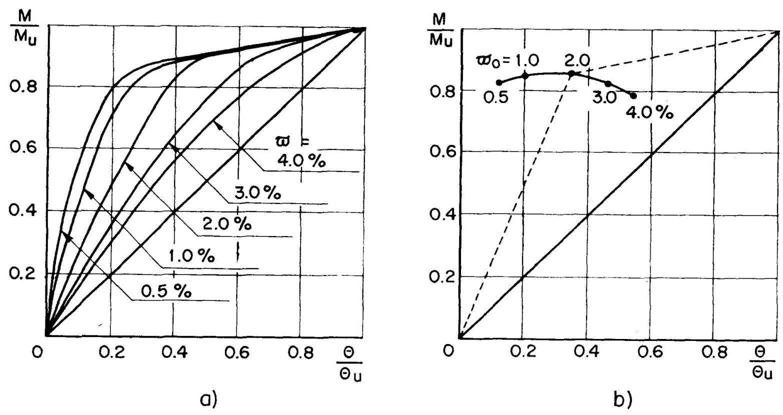



Fig. 18.

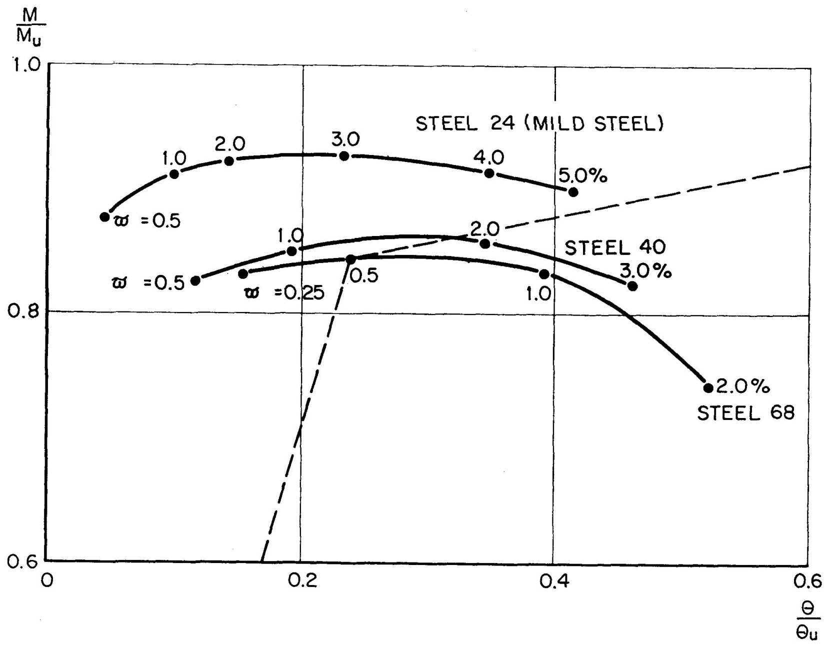



Fig. 19.

($\sigma_{0.2} = 4000$ kg/cm²) and concrete B 300 ($\sigma_{bu} = 300$ kg/cm²). The moment-curvature diagrams presented in fig. 17 if reduced to the same ultimate values become the diagrams presented in fig. 18a. These correspond to bi-linear diagrams having the vertices at the positions indicated in fig. 18b.

The position of the vertices corresponding to different qualities of steel and percentages of reinforcement (3) are indicated in fig. 19.

In the following two types, A and B, of bi-linear diagrams are considered, fig. 20a. Diagram A represents sufficiently well the behaviour of mild steel in

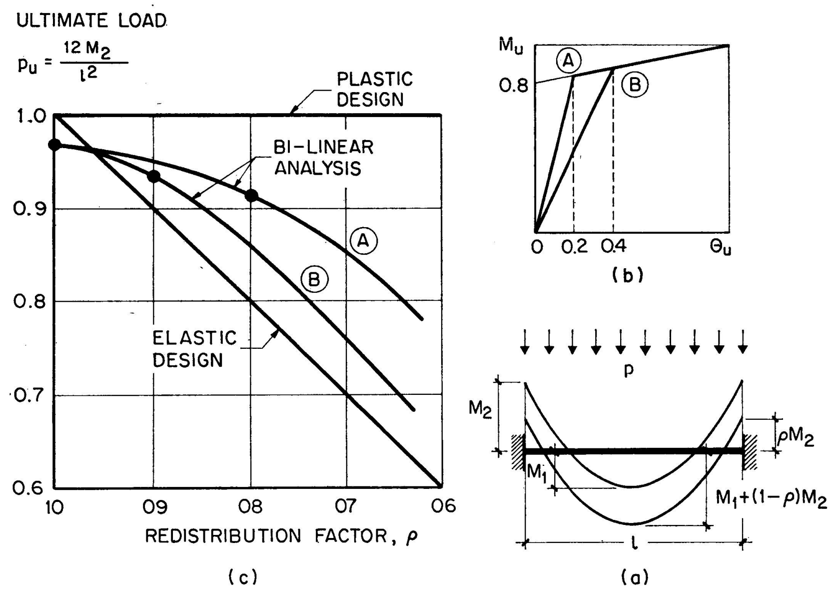



Fig. 20.

the usual percentages ($\tilde{\omega}_0 < 3\%$) and of steel 40 in percentages $\tilde{\omega}_0 < 1\%$. Diagram B is on the safe side for percentages $\tilde{\omega}_0 < 2.5\%$ of steel 40 and can be considered to correspond to the usual percentages of steel of higher quality.

### *4.3. Redistribution of Bending Moments*

The redistribution of bending moments in built-in and in continuous beams for different types of bi-linear diagrams was studied in (3). In that paper the same moment-curvature diagrams were considered for both positive and negative bending moments and the results of bi-linear analysis were compared with linear and perfectly plastic design.

For discussing the amount of redistribution of bending moments to be allowed in the design of beams, it is necessary to compute the ultimate load that can be reached when the ultimate bending moments vary and are not distributed according to elastic design assumption.

Let us consider a built-in beam of uniform section with a uniformly distributed load (fig. 20a) and let us take for ultimate moments at the middle and at the built-in sections, $M_1$ and $M_2$, respectively. According to elastic design $M_2 = 2\,M_1$.

A redistribution factor $\rho$ is defined in such a way that for it, the ultimate negative moment is equal to $\rho M_2$ and the ultimate positive moment is equal to $M_1 + (1-\rho) M_2$. This redistribution is such that, according to the hipotheses of plastic design, it will not affect the ultimate load. In elastic design conditions the ultimate load would decrease in proportion to the value of $\rho$.

Taking for reference the bi-linear diagrams of the types A and B (fig. 20b), maintaining the same ultimate value of $\theta$ and varying the ultimate bending moments as indicated, the ultimate load corresponding to bi-linear analysis are indicated in fig. 20c. It is seen that, as $\rho$ decreases, the ultimate load also decreases. As could be expected, this reduction is more rapid for diagrams of type B than for diagrams of type A.

It is to be noticed that, even if ultimate moments were distributed according to elastic design assumptions, the ultimate load corresponding to bi-linear design is 97% of that corresponding to both elastic and plastic design. If for a bi-linear behaviour the ultimate load is not to decrease by more then about 5%, the redistribution factor cannot exceed 0.8 for diagram A and 0.9 for diagram B.

### *4.4. Design Rules*

Programs now available for the electronic computer (1) enable a complete non-linear analysis of plane structures. The use of these programs is not practical in the usual cases and so it seems convenient to define simple design rules that indicate for the different conditions the amount of redistribution of bending moments than can be adopted.

It must be emphasized that it is not illogical to design the sections according to their ultimate capacity (computed by considering non-linear stress-strain diagrams for steel and concrete) and to assume that bending moments are distributed according to the hypothesis of perfect elasticity. In most cases this corresponds to the best use of the ultimate strength of the sections, although, as fig. 20 shows, if not too small values of the coefficient of redistribution are adopted, the ultimate capacity of the structure is not considerably affected.

Performing a bi-linear analysis for different types of continuous beams in the same lines as described for the perfect built-in beam with uniform load, the decrease of ultimate load indicated in fig. 21 is obtained. Limiting the decrease of the ultimate load to about 5%, as before, the limit values of the redistribution factors indicated in table III are obtained. The values corresponding to the diagrams of type A can be used in general in beams reinforced with mild steel or with a percentage of steel 40 of less than 1%. The values corresponding to the diagrams of type B must be adopted in beams reinforced with steel 40 in percentages higher than 1% or, in general, with steel of higher quality.

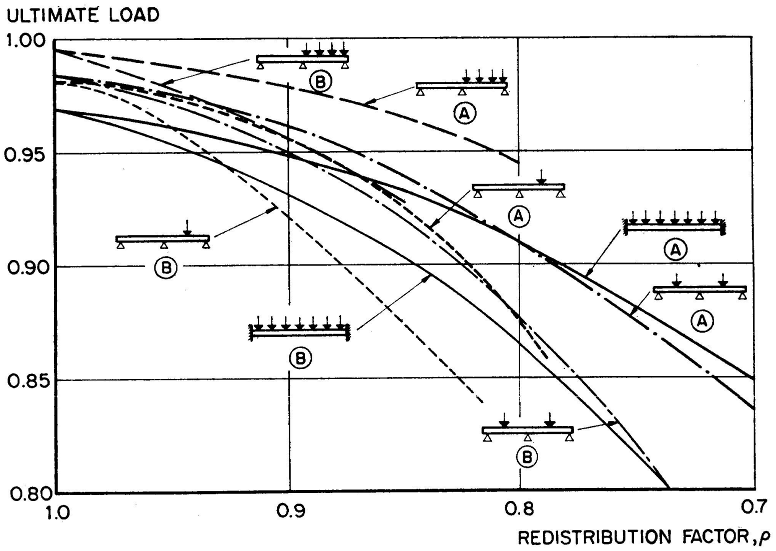



Fig. 21.

*Table III*

| TYPE OF STRUCTURE | REDISTRIBUTION FACTOR, $\rho$ (AT SECTION ②) | |
|---|---|---|
| | Ⓐ MILD STEEL STEEL 40 $\varpi_0 < 1\%$ | Ⓑ STEEL 40 $\varpi_0 > 1\%$ STEEL 68 |
| ② ① ② ② ① ② ① ② | 0.80 | 0.90 |
| ② ① ① ② ① | 0.90 | — |
| ① ② | 0.80 | 0.85 |
| ② ① ① ② ① | 0.85 | 0.90 |
| ① ② | 0.85 | 0.90 |

The results presented enable a quantification of the rules on the redistribution of moments included in C.E.B. Recommendations (8) and in several other national codes.

## 5. Conclusions

By only considering the principal variables that influence the phenomena of cracking, deformation and redistribution of moments and interrelating them by means of simple and compatible hypotheses, theoretical results were obtained which agree well with the experimental results available and which allow to establish simple design rules.

## Acknowledgment

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## Summary

The problems of cracking, deformation and redistribution of bending moments in reinforced concrete beams are treated by means of general and mutually compatible hypotheses. Theoretical results thus obtained are compared with experimental results available.

Simple design rules are derived for avoiding excessive cracking, for computing the displacements in service and for considering the redistribution of bending moments due to non-linear behaviour.

## Résumé

Au moyen d'hypothèses générales et compatibles entre elles on traite les problèmes de la fissuration, de la déformation et de la redistribution des moments fléchissants dans les poutres en béton armé. Les résultats théoriques ainsi obtenus sont comparés avec les résultats expérimentaux dont on dispose.

On établit des règles de calcul simples en vue d'éviter les fissurations excessives, de calculer les déplacements sous les charges de service et de tenir en compte la redistribution des moments flechissants due au comportement non-linéaire.

## Zusammenfassung

Die Probleme der Rißausbildung, Verformung und des Biegemomentenausgleichs von Stahlbetonträgern werden aufgrund allgemeiner und miteinander verträglicher Bedingungen behandelt. Die so erhaltenen theoretischen Ergebnisse werden mit den zur Verfügung stehenden Versuchsergebnissen verglichen.

Einfache Konstruktionsregeln werden aufgestellt für die Vermeidung unzulässiger Rißausbildung, für die Berechnung der Verschiebungen unter Gebrauchslast und für die Berücksichtigung des Biegemomentenausgleichs infolge des nichtlinearen Verhaltens.

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