Buckling strength of plates in the plastic range

Autor(en): **Kato, Ben**

Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **25 (1965)**

PDF erstellt am: **02.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-20352

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.
Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.
Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Ein Dienst der *ETH-Bibliothek*
ETH Zürich, Rämistrasse 101, 8092 Zürich, Schweiz, www.library.ethz.ch

http://www.e-periodica.ch

Buckling Strength of Plates in the Plastic Range

Résistance au voilement dans le domaine plastique

Ausbeulen von Platten im plastischen Bereich

BEN KATO
Dr. Eng., Associate Professor, University of Tokyo

Introduction

In the plastic design of steel structures or in the design of tall buildings based on non-linear dynamic response to severe earthquakes, beams and columns are required to have adequate rotation capacity at the positions of the plastic hinges, that is to say, it is necessary that the plate members of the columns and beams should undergo sufficient plastic deformation without loss of strength due to the occurrence of local buckling when they are subjected to compression.

The problem is to determine the width-thickness ratio of plates which are subjected to compression in order that their yield strength should not be decreased by the occurrence of local buckling, during the performance of the necessary plastic deformation.

Several contributions dealing with this problem have been made [1, 2], they are all attempts to obtain the solution to this problem by treating it as an eigenvalue or as a problem of stability.

In the analysis of the buckling of plates in the plastic range, however, the critical value always coincides with the yield point of the material in so far as an elasto-plastic pattern is assumed as the stress-strain relation of the material, and this cannot afford an answer to the given problem.

In an actual plate, if the width-thickness ratio is sufficiently small, a slight warping of the plate is observed as soon as the plate yields. At this point, however, the plate does not lose its carrying capacity, but continues to deform to some extent while supporting its yield load, and the amount of deformation in the period during which the yield load is retained it depends upon the width-thickness ratio of the plate and upon the yield ratio of the material.

This behaviour of the plate may be explained as follows; when the stress in the compressed plate reaches the yield point of the material, the strain in the section can assume an arbitrary distribution without any external disturbance, because the strain does not correspond to the stress in the plastic flow range. This means that the plate can assume indifferent warping at the yield load.

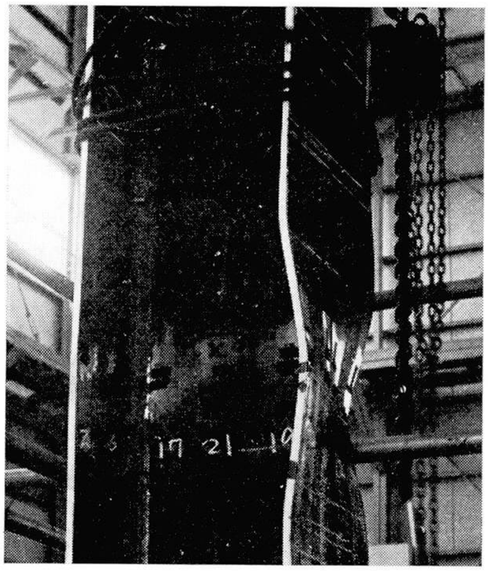


Photo 1. Column.

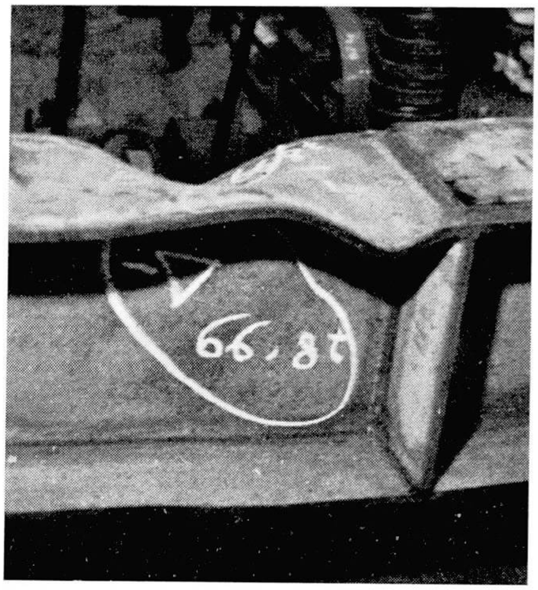


Photo 2. Beam.

Along the ridge of the warped plate, the strain would pass into the strain-hardening range in a moment, and the plate would then obtain the resistance against the bending moment which is introduced by the axial force.

The maximum strength of the plate will be reached when the stress in the whole section reaches its maximum strength (tensile strength) and produces a kind of plastic hinge, and until this stage is reached the plate can deform without losing its load-carrying capacity after the initiation of buckling.

When the determination of the deformation capacity in the plastic range of the plate is required, it would seem to be more suitable, from the above consideration, to treat this problem as a problem of the bearing capacity of the plate rather than to treat it as that of the eigenvalue.

From this point of view, the load-carrying capacity of the plate, after it has reached the yield load, is investigated in this paper on the basis of the actual stress-strain relation of the material including the strain-hardening range.

In the analysis, an appropriate mode of collapse of the plate is assumed and, furthermore, the elastic strain of the material is neglected, for the sake of simplicity, the error caused by this simplification would appear to be small, because the amount of the deformation in question is much greater than that induced by the elastic strain.

Analysis

The results of compression tests on plates in which the width-thickness ratios are small, show that, although warping is observed when the yield load of each plate is reached, this does not mean that the limit of the capacity has been attained. The plate can continue to deform to some extent while retaining its yield load or, in some cases, it may show a further increase of load (Fig. 1).

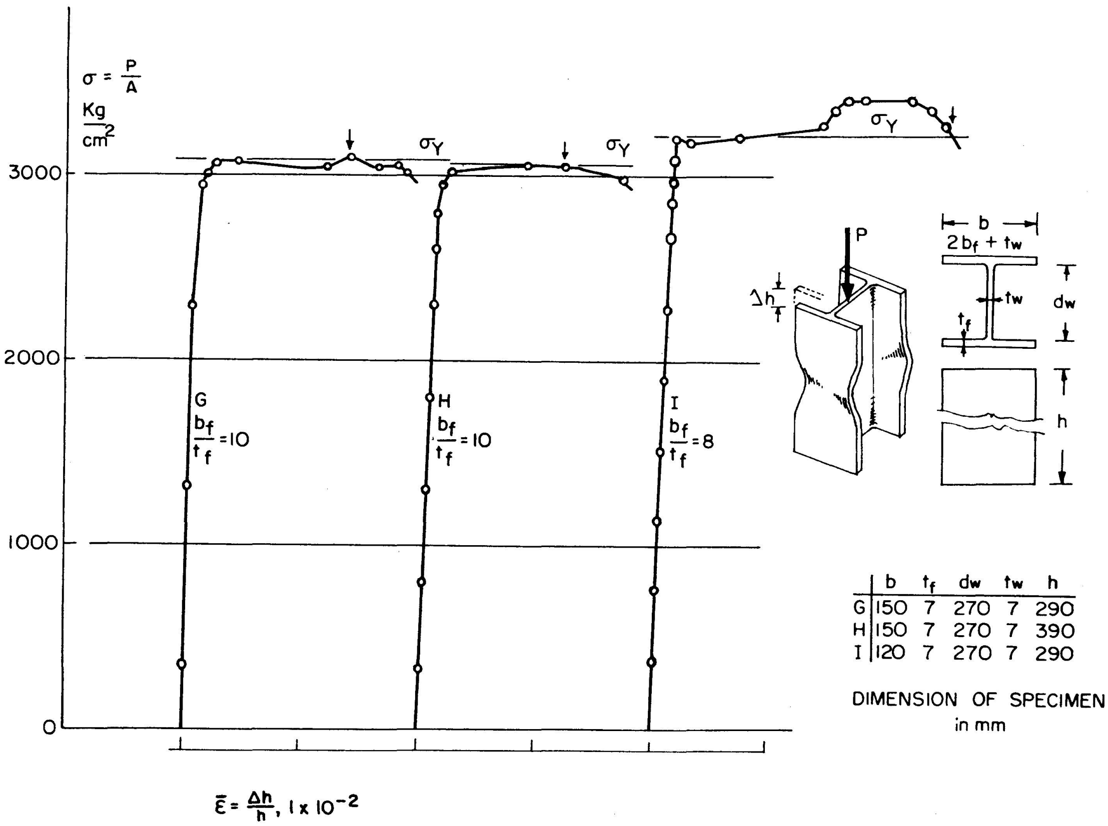


Fig. 1. Deformation Capacity of Buckled-Plate.

On the basis of these experimental results, an analysis of the strength of a plate subjected to compression in its plane is attempted, using the equation of work under the following assumption: "a plate which has buckled in the plastic range can continue to deform to some extent while retaining its yield load, and the extent of the deformation depends upon the width-thickness ratio of the plate and upon the yield ratio of the material."

Case 1. Rectangular Plate Supported on one Edge and Free at the Other Edge

These are cases the flanges of wide-flange shapes of the legs of angles subjected to compression by axial force or by bending. Warping of a plate which has initial curvature will assume a shape such a that shown in Fig. 2 or in photo 1 after buckling had occurred. So that the equilibrium of this distorted plate may be considered, it is reduced to a simplified model, as shown in Fig. 3.

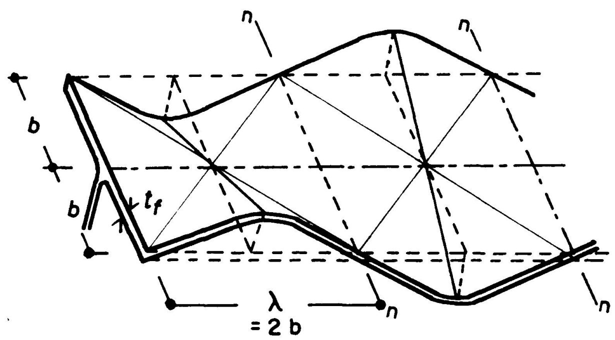


Fig. 2.

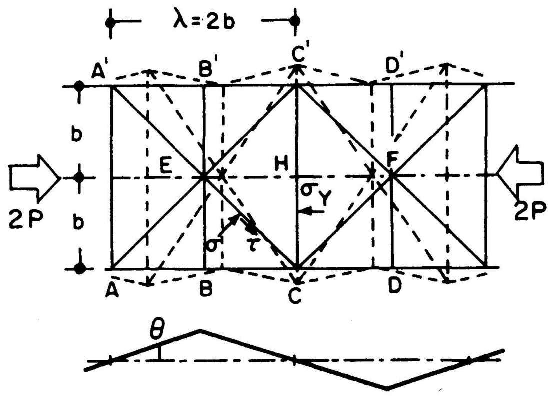


Fig. 3.

The analysis is put forward on the basis of the following assumptions:

1. $n-n$ in Fig. 2 are nodal lines, and do not rotate;
2. at or near the collapse stage, the half-wave length is equal to 2b;
3. the rectangle $CEC'F$ (in Fig. 3) is maintained plane, and will be subjected to shear deformation only, and each side of the rectangle remains unchanged in length;
4. plastic hinges are formed along the CE, $C'E$, CF, $C'F$, BEB' and DED' lines. A half-wave sub-rectangle $BB'D'D$ is considered and analysed.

Work Done by the Shear Stress

Fig. 4 shows the shape of the rectangle $CEC'F$ as deformed by the shear stress τ acting along its sides.

Displacement Δ of E and F is nearly equal to that of C and C', when Δ is very small and the plate is square; the work then done by the shear stress is described by:

$$_{\mathrm{I}}W_s = \sqrt{2}\, b\, \tau\, t\, 2\sqrt{2}\, \Delta = 4\, b\, t\, \Delta\, \tau. \tag{1}$$

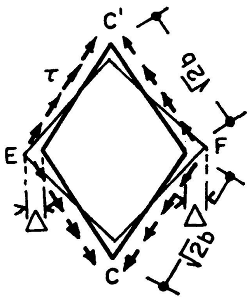


Fig. 4.

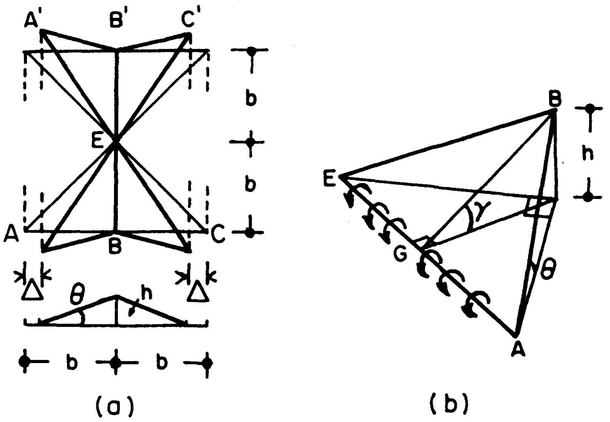


Fig. 5.

Work Done by the Bending Moment

On the assumption that AE, CE do not change in length, the deflection h of the point B can be expressed by the displacement Δ.

Referring to Fig. 5a, we have:

$$h = \sqrt{b^2 - (b-\Delta)^2} \doteqdot \sqrt{2\, b\, \Delta}\left(1 - \frac{1}{4}\frac{\Delta}{b}\right), \qquad \theta \doteqdot \frac{h}{b} = \sqrt{2}\sqrt{\frac{\Delta}{b}}\left(1 - \frac{1}{4}\frac{\Delta}{b}\right).$$

Referring to Fig. 5b, the rotation γ of the plate ABE, with respect to the axis AE, is expressed as follows:

$$BG = \frac{b}{\sqrt{2}}, \qquad \gamma \doteqdot h \div \frac{b}{\sqrt{2}} = 2\sqrt{\frac{\Delta}{b}}\left(1 - \frac{1}{4}\frac{\Delta}{b}\right). \tag{a}$$

The angle of plate ABE and plate CBE is nearly equal to 2θ.

Now let us consider the plastic bending moment of the plate in the strain-hardening range. The plate is in the plastic state under the compressive load $2P$, and the stress in the direction of the applied force is σ_y (the yield point of the material), and it is assumed that strain-hardening would occur as soon as the plate is subjected to bending by buckling in this compressed state.

The material is obliged to obey von Mises's yield condition, but it is assumed that the octahedral stress σ_{oct} may be increased to the tensile strength of the material σ_T in this locally bent zone.

Taking an arbitrary axis of bending on the plate which meets the longitudinal edge of the plate at an angle ψ, the equilibrium of stress at a point on this axis will be as follows;

$$\sigma_1 = \sigma_y \sin^2\psi, \qquad \sigma_2 = \sigma_y \cos^2\psi, \qquad \tau = \tfrac{1}{2}\sigma_y \sin 2\psi, \tag{b}$$

where σ_1 is acting normally to the axis of bending, and σ_2 is acting along the axis of bending.

The maximum possible value of σ_1 is written as σ_{1m}, and is determined by the following condition.

$$\sigma_{oct}^2 = \sigma_T^2 = \sigma_{1m}^2 + \sigma_2^2 - \sigma_{1m}\sigma_2 + 3\tau^2$$

or

$$\sigma_{1m} = \tfrac{1}{2}\{\sigma_2 \pm \sqrt{\sigma_2^2 + 4(\sigma_T^2 - \sigma_2^2 - 3\tau^2)}\}. \tag{c}$$

Introducing Eq. (b) into Eq. (c), we have:

$$\sigma_{1m} = \tfrac{1}{2}\{\cos^2\psi \pm \sqrt{\cos^4\psi + 4[\alpha^2 - \cos^2\psi(1 + 2\sin^2\psi)]}\}\,\sigma_y, \tag{d}$$

$\alpha = \sigma_T/\sigma_y$: the inverse of the yield ratio of the material.

For unit width of the plate (thickness: t), the interaction between the axial force N and the bending moment M in the full plastic state is expressed as follows,

$$\frac{M}{M_p} = 1 - \left(\frac{N}{N_p}\right)^2.$$

M_p = full plastic moment in pure bending, and is equal to $\frac{1}{4}t^2\sigma_{1m}$ with the bending axis we are now considering.

N_p = maximum compressive force normal to the axis of bending, and is equal to $t\sigma_{1m}$.

N = effective compressive force normal to the axis of bending, and is equal to $t\sigma_1$ in this case.

Then the maximum possible bending moment M may be written as follows by introducing the above expression,

$$M = \left\{1 - \left(\frac{\sigma_1}{\sigma_{1m}}\right)^2\right\} \times \frac{1}{4}t^2\sigma_{1m} = \frac{1}{4}\,\frac{\sigma_{1m}^2 - \sigma_1^2}{\sigma_{1m}}\,t^2. \tag{e}$$

Introducing σ_{1m} of Eq. (d) and σ_1 of Eq. (b) into Eq. (e),

$$\left.\begin{aligned} M &= \frac{1}{4}\left\{\frac{\cos^2\psi(3\cos^2\psi - 2 \pm \beta) + 2(\alpha^2 - 1)}{\cos^2\psi \pm \beta}\right\} t^2\sigma_y, \\ \beta^2 &= \cos^4\psi + 4[\alpha^2 - \cos^2\psi(1 + 2\sin^2\psi)], \end{aligned}\right. \tag{f}$$

The maximum possible bending moment with the axis CE will be obtained by introducing $\psi = \pi/4$ into Eq. (f),

$$M_{CE} = M_1 = \frac{1}{4}\left\{\frac{4(\alpha^2 - 1) + (\beta_1 - \frac{1}{2})}{2\beta_1 + 1}\right\} t^2\sigma_y, \qquad \beta_1^2 = \frac{1}{4} + 4(\alpha^2 - 1).$$

The maximum possible bending moment with the axis BE will be obtained by introducing $\psi = \pi/2$ into Eq. (f),

$$M_{BE} = M_2 = \frac{1}{4}\left(\frac{\alpha^2 - 1}{\alpha}\right) t^2\sigma_y.$$

Now the work done by bending until the moment when each portion of the plate approaches its maximum value can be calculated as follows:

$$
\begin{aligned}
{}_{\mathrm{I}}W_b &= 4\sqrt{2}\, b \frac{1}{2} M_1 \gamma + 2 b \frac{1}{2} M_2\, 2\theta \\
&= \left(\sqrt{2}\,\Phi_1 + \frac{1}{\sqrt{2}}\Phi_2\right)\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{4}\frac{\Delta}{b}\right) b\, t^2 \sigma_y ,
\end{aligned}
\tag{2}
$$

$$
\Phi_1 = \frac{4(\alpha^2-1)+(\beta_1-\frac{1}{2})}{2\beta_1+1}, \qquad \Phi_2 = \frac{\alpha^2-1}{\alpha}.
$$

It is to be expected that the plate would retain its yield load $2P = 2P_y = 2bt\sigma_y$ until a critical deformation 2Δ which is as yet unknown, is attained, and the work done by the external force can be calculated as follows:

$$
{}_0W = 2P_y\, 2\Delta = 4bt\Delta\,\sigma_y . \tag{3}
$$

The work done by the external force is equal to the work done by internal forces,

$$
{}_0W = {}_{\mathrm{I}}W_s + {}_{\mathrm{I}}W_b .
$$

This is written as follows by means of Eqs. (1), (2) and (3) and by introducing the expression for τ in Eq. (b),

$$
4bt\Delta\,\sigma_y = 2bt\Delta\,\sigma_y + \left(\sqrt{2}\,\Phi_1 + \frac{1}{\sqrt{2}}\Phi_2\right)\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{4}\frac{\Delta}{b}\right) b\, t^2 \sigma_y
$$

which is arranged as,

$$
\frac{2\sqrt{2}}{2\Phi_1+\Phi_2} = \left(\frac{t}{b}\right)\left(\sqrt{\frac{b}{\Delta}} - \frac{1}{4}\sqrt{\frac{\Delta}{b}}\right).
$$

Δ/b denotes the critical strain of the plate as is shown in Fig. (3), and by substituting the expression $\Delta/b = \epsilon$ in the above equation, the following result is obtained.

$$
\frac{2\sqrt{2}}{2\Phi_1+\Phi_2} = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right). \tag{4}
$$

Eq. (4) is the relation between the width-thickness ratio of a plate and the maximum compressive strain at which the plate would retain its yield strength, and it also shows that the relation is concerned with the yield ratio of the material.

Case 2. Rectangular Plate Simply Supported at Both Edges

This is the case for web plates of I-shapes or flanges of box-sections. In this case, the model of the distorted plate compressed to the plastic range is assumed to be as shown in Fig. 6.

The following assumptions were made in analysing this plate (see Fig. 7).

1. The length of the half-waves is equal to b.
2. The nodal lines AA', BB' do not change their length, and AC, AD, $A'C$ and $A'D$ also do not change their length. Then the squares $ACA'D$ and $BDB'E$ do not deform at all, but only rotate with the axes AA' and BB'.
3. The triangles ABD and $A'B'D$ may be subjected to shear deformation, and their deformed shapes are shown by the dotted line in Fig. 7.
4. Plastic hinges are formed along the AC, $A'C$, AD and $A'D$ lines.

A half wave subrectangle $AA'BB'$ is considered and analysed.

Work Done by the Shear Stress

The triangles ABD and $A'B'D$ as deformed by shear are shown in Fig. 7 by dotted lines, and these triangles are put together as shown in Fig. 8. This

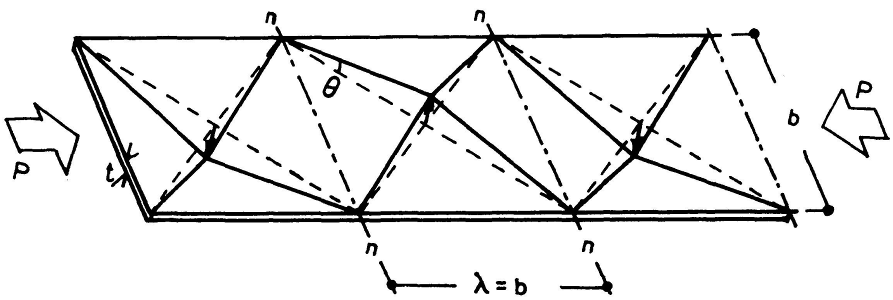


Fig. 6.

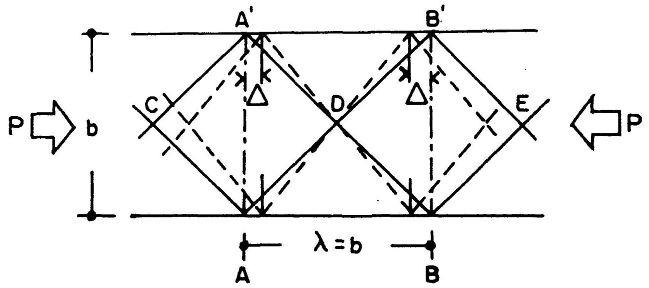
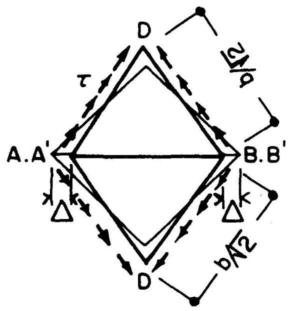


Fig. 7.



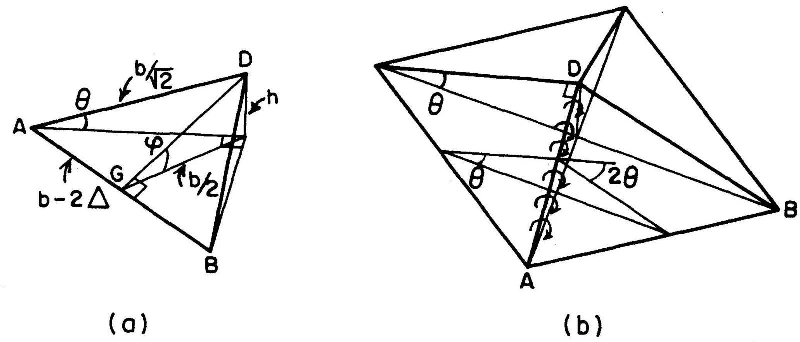


Fig. 8. Fig. 9.

shows the square as deformed by shear and diagonals of it change their length by 2Δ, then this is the same case as in *Case 1*, so the work done by the shear stress be written as follows,

$$_{\mathrm{I}}W_s = \frac{b}{\sqrt{2}}\tau t\, 2\sqrt{2}\Delta = 2bt\Delta\tau. \tag{5}$$

Work Done by the Bending Moment

Making the assumption (2) that AD, BD and the plate width b (AA', BB') do not change in length, and that AB changes its length by 2Δ, the deflection of the point D may be written as follows (see Fig. 9a):

$$GD = h^2+\left(\frac{b}{2}\right)^2 = \left(\frac{b}{\sqrt{2}}\right)^2 - \left(\frac{b}{2}-\Delta\right)^2, \qquad h \doteqdot \sqrt{b\Delta}\left(1-\frac{1}{2}\frac{\Delta}{b}\right),$$

$$\therefore\ \theta \doteqdot h \div \frac{b}{\sqrt{2}} = 2\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right). \tag{g}$$

Referring to Fig. 9b, it is clear that the angle between the plane $AA'D$ and the plane ABD is equal to 2θ.

The edges AB and $A'B'$ are simply supported, so that they are not concerned in the work done by bending. Work is done by bending along sides AD, DB', $A'D$ and DB only, and work done on each side has the same value.

Since the inclination of each side to the direction of the compressive force ψ is $\pi/4$, the maximum possible bending moment of the plate for unit width is equal to M_1 which was derived in *Case 1*.

Thus the work done by bending until the moment when each portion of the plate approaches its maximum value may be written as follows:

$$\begin{aligned} _{\mathrm{I}}W_b &= 4\frac{b}{\sqrt{2}}\frac{1}{2}M_1 2\theta, \\ &= \Phi_1\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right)bt^2\sigma_y. \end{aligned} \tag{6}$$

Work done by the external force P_y is,

$$_{0}W = P_y 2\Delta = 2bt\Delta\sigma_y.$$

The work done by the external force is equal to the work done by the internal forces:

$$_{0}W = {}_{\mathrm{I}}W_s + {}_{\mathrm{I}}W_b.$$

That is to say:

$$2bt\Delta\sigma_y = bt\Delta\sigma_y + \Phi_1\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right)bt^2\sigma_y$$

which is arranged as,

$$\frac{1}{\sqrt{2}\Phi_1} = \left(\frac{t}{b}\right)\left(\sqrt{\frac{b}{2\Delta}} - \frac{1}{4}\sqrt{\frac{2\Delta}{b}}\right).$$

In this equation, $2\Delta/b$ denotes the axial strain, and by substituting the expression $2\Delta/b=\epsilon$, the following result is obtained.

$$\frac{1}{\sqrt{2}\Phi_1} = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right). \tag{7}$$

Case 3. Rectangular Plate Clamped at Both Edges

Since the edge AB in Fig. 9a is fixed in this case, the plate performs the additional work done by bending due to the rotation φ of the plate.

Referring to Fig. 9a, φ is calculated as follows:

$$\frac{b}{2}\varphi \doteqdot h \doteqdot \frac{b}{\sqrt{2}}\theta, \qquad \therefore\ \varphi = \sqrt{2}\,\theta = 2\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right).$$

The maximum possible bending moment, with the axis AB, for unit width, may be obtained by introducing $\psi=0$ into Eq. (f).

$$M_{AB} = M_3 = -\frac{1}{4}\left\{\frac{2(\alpha^2-1)-(\beta_3-1)}{\beta_3-1}\right\} t^2\sigma_y,$$

$$\beta_3^2 = 1+4(\alpha^2-1).$$

So that the work done by bending along these edges is:

$$\begin{aligned} {}_{\mathrm{I}}W_b' &= 2b\frac{1}{2}M_3\varphi = \frac{1}{2}\Phi_3\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right)b\,t^2\sigma_y, \\ \Phi_3 &= \frac{2(\alpha^2-1)-(\beta_3-1)}{\beta_3-1}. \end{aligned} \tag{8}$$

The equilibrium of the work done by the external and the internal force is,

$${}_0W = {}_{\mathrm{I}}W_s + {}_{\mathrm{I}}W_b + {}_{\mathrm{I}}W_b'$$

or

$$2bt\Delta\sigma_y = bt\Delta\sigma_y + \left(\Phi_1+\frac{1}{2}\Phi_3\right)\sqrt{\frac{\Delta}{b}}\left(1-\frac{1}{2}\frac{\Delta}{b}\right)b\,t^2\sigma_y.$$

By the same operation as was performed in *Case 2*, the following result is obtained.

$$\frac{1}{\sqrt{2}\,(\Phi_1+\frac{1}{2}\Phi_3)} = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right). \tag{9}$$

Experimental Correlation

The theoretical results obtained above have been compared with the experimental results. The results of compression tests on plates in the plastic range that are now available are summarised in Table 1.

Table 1. Test results

Reporter		*	Shape	b/t_f	d/t_w	$\epsilon_{cr} \times 10^{-3}$ A**	$\epsilon_{cr} \times 10^{-3}$ B**	σ_y t/cm^2	Remark
Flange Test									
KATO	3	H	H 400.400.11.12,5	15,6	34,1	2,80	—	3,70	SM 50 (High Tensile Steel) As Delivered
		H	H 400.400.11.12,5	15,6	34,1	3,13	—	3,70	
		H	H 400.400.11.12,5	15,6	34,1	2,57	—	3,70	
		H	H 400.400.11.12,5	15,6	34,1	3,15	—	3,70	
	4	L	L 235.235.25	8,4	—	9,0	—	2,72	SS 50 (Mild Steel) As Delivered
		L	L 235.235.25	8,4	—	12,20	—	3,38	
		L	L 235.235.25	8,4	—	6,87	—	3,38	
	5	H	H 300.150.6.10	7,5	48,5	—	14,70	3,60	SM 50 (High Tensile Steel) As Delivered
		H	H 240.180.6.10	9,8	39,8	—	11,20	3,60	
		H	H 270.180.6.10	9,7	44,2	—	11,40	3,60	
		H	H 300.230.6.11	10,5	47,6	—	4,13	3,60	
HAAIJER	1	H	10 WF 33	9,2	31,9	8,5	7,0	2,41	A 7 (Mild Steel) As Delivered
		H	8 WF 24	8,6	32,3	13,5	23,0	2,38	
		H	10 WF 39	7,8	28,6	19,0	22,5	2,46	
		H	8 WF 35	8,5	24,8	17,0	22,0	2,56	
		L	A 22	12,6	—	3,0	—	—	A 7 (Mild Steel) Annealed
		L	A 31	8,85	—	16,5	—	2,44	
		L	A 32	8,79	—	16,5	—	2,42	
		L	A 33	8,73	—	16,0	—	2,89	A 7. As Delivered
		L	A 51	10,7	—	6,0	—	2,87	
OKUMURA	6	H	H 260.150.7.10	7,15	37,2	25,0	—	3,0	SS 41 (Mild Steel) As Weld
		H	H 260.180.7.10	8,65	37,2	14,0	—	3,0	
		H	H 260.180.7.10	8,65	37,2	20,0	—	3,0	
		H	H 260.180.7.10	8,65	37,2	23,0	—	2,50	SS 41. Annealed
		H	H 260.180.7.10	8,65	37,2	22,0	—	2,50	
		H	H 260.180.7.10	8,65	37,2	22,0	—	2,50	
		H	H 260.120.7.10	5,65	37,2	29,0	—	3,0	SS 41. As Welded
		H	H 260.120.7.10	5,65	37,2	31,0	—	3,0	
		H	H 272.150.7.7	10,2	38,9	14,0	—	2,5	SS 41. Annealed
		H	H 272.150.7.7	10,2	38,9	11,0	—	2,5	
		H	H 272.120.7.7	8,1	38,9	21,0	—	3,0	SS 41. As Welded
		H	H 272.120 7.7.	8,1	38,9	12,0	—	3,0	
		H	H 272.120.7.7	8,1	38,9	22,0	—	2,5	SS 41. Annealed
		H	H 272.120.7.7	8,1	38,9	12,0	—	2,5	
Web Test									
KATO	3	H	H 400.400.11.12,5	15,6	34,1	2,23	—	3,70	SM 50 As Delivered
		H	H 400.400.11.12,5	15,6	34,1	2,80	—	3,70	
		H	H 400.400.11.12,5	15,6	34,1	4,70	—	3,70	
HAAIJER	1	H	8 WF 24	8,6	32,3	12,70	—	2,38	A 7. As Delivered
		H	12 WF 50	6,6	33,0	5,0	—	2,45	
		H	10 WF 21	9,1	40,9	1,6	—	2,66	

Note: * H = Wide Flange Shape. L = Angle Shape.
** A = Axial Compression Test. B = Bending Test.

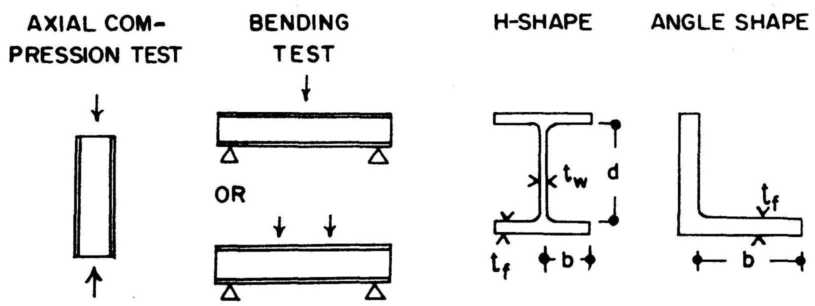


Fig. 10.

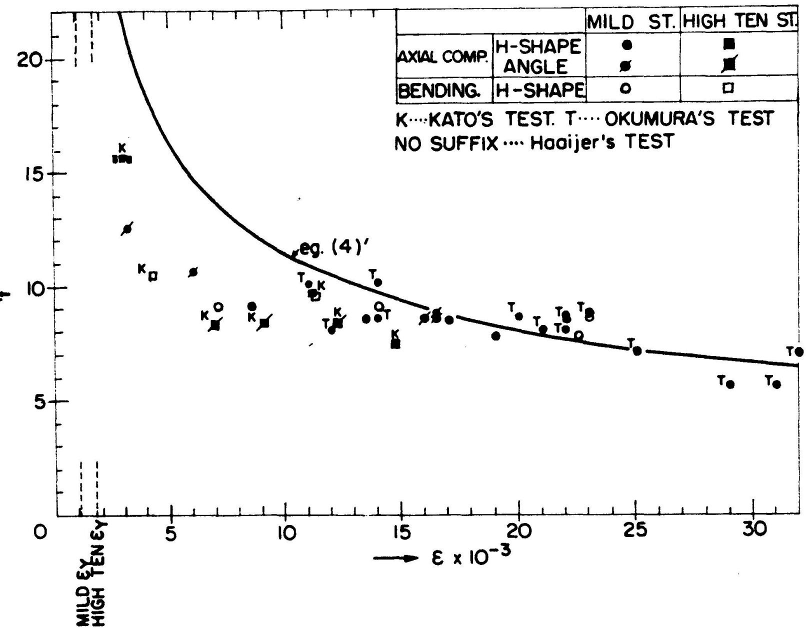


Fig. 11. $b/t_f - \epsilon$ Relation of Out-Standing Flange.

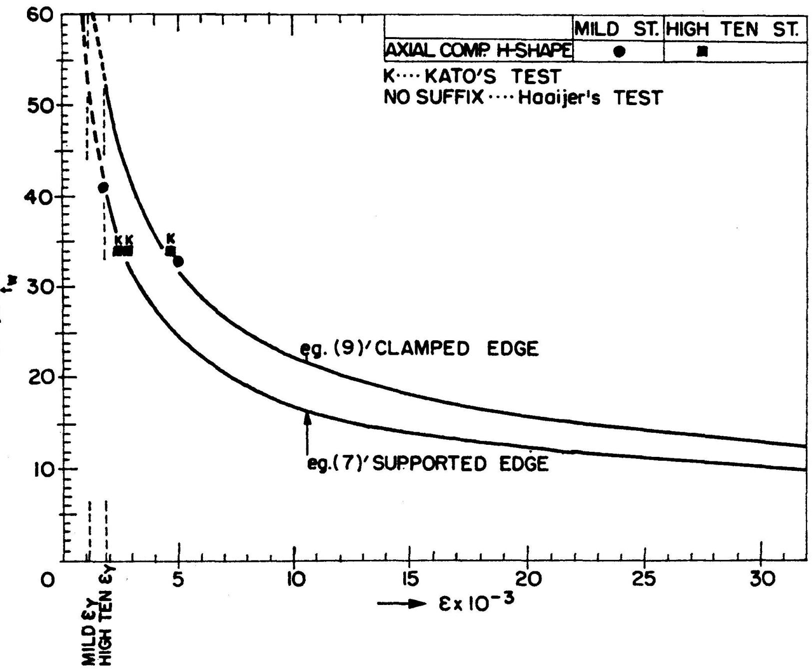


Fig. 12. $d/t_w - \epsilon$ Relation of Webs.

In this table, "*Flange Test*" means the tests performed in order to obtain the relation between the critical strain and the width-thickness ratio of wide flanges by means of axial compression tests or of bending tests as illustrated in Fig. 10, and "*Web Test*" means the axial compression tests of webs of I-shapes undertaken for the same purpose.

The yield ratios of steel plates used in the author's tests varied from 0.63 to 0.685 (expressed as α, $\alpha = 1.46 \sim 1.59$). If the mean value, $\alpha = 1.5$, is adopted for the sake of simplicity, then the following equations would result, from the introduction of $\alpha = 1{,}5$ into Eqs. (4), (7) and (9), respectively.

$$0.865 = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right), \tag{4'}$$

$$0.579 = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right), \tag{7'}$$

$$0.446 = \left(\frac{t}{b}\right)\left(\frac{1}{\sqrt{\epsilon}} - \frac{\sqrt{\epsilon}}{4}\right). \tag{9'}$$

These estimations are compared with the test results in Fig. 11 and Fig. 12. In this comparison it should be noted that the yield ratios of the materials used in Haaijer's and Okumura's tests were not then known.

It is presumed, however, that they would not be greatly different from the yield ratio used the in author's tests, because the steels used are all of similar grade.

With regard to the behaviour of wide flanges (Fig. 11), the theoretical prediction had shows fairly good agreement with the experimental results, but as far as the behaviour of webs is concerned, the test results are too few to enable the theory to be evaluated.

Related Remarks and Conclusions

1. In the specifications given in "Plastic design in steel" [7] and "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, Part 2" [8] of the AISC, the following limitations are specified for A-7 type steel.

For flanges of rolled shapes and flange plates of similar built-up shapes	$b/t_f \leqq 8.5$.
For flange plates in box-sections	$b/t_f \leqq 32$.

The limitation of width-thickness ratio for wide flanges is based on the condition that a section under uniform compression can be strained to the point of strain-hardening before local buckling influences its carrying capacity.

The standard strain of A-7 type steel at the point of strain-hardening is

$\epsilon_{st} = 14 \times 10^{-3}$, and by substituting this value in Eq. (4′), the following result is obtained,

$$b/t = 9.77.$$

This shows fairly good agreement with the specified value of 8.5.

In the case of box-sections and cover plates, the critical strain is required to be 4 times the yield strain ϵ_y. Taking the yield strain $\epsilon_y = 1.2 \times 10^{-3}$ for A-7 type steel, $4\,\epsilon_y$ is introduced into Eq. (7′) and Eq. (9′), and we obtain,

For a simply supported edge $b/t = 25.$

For a clamped edge $b/t = 32{,}4.$

It would seem that, in the specifications of AISC, some clamping effect is taken into consideration in actual sections.

2. It has been shown, in this analysis, that the critical strain depends not only upon the width-thickness ratio of the plate, but also upon the yield ratio of the material.

It is deduced from this theory that the higher yield ratio of the material corresponds to the smaller critical strain, and when steels which have extremely high yield ratio would come into practise, this point should be examined carefully.

When the necessary value of the critical strain be specified by the term of the ductility factor (the ratio of the critical strain to yield strain, for example $\epsilon_{cr}/\epsilon_y = 4$), it will result that the higher the yield point of the material the smaller the width-thickness ratio of the plate.

Bibliography

1. Haaijer, G.: Plate Buckling in the Strain-Hardening Range. Journal of Engineering Mechanics Division, Proc. ASCE, 83 (EM-2), 1957.
   Haaijer, G. and Thürlimann, B.: On Inelastic Buckling in Steel. Journal of Engineering Mechanics Division, Proc. ASCE, EM-2, 1958.
2. Tanaka, H. and Takanashi, K.: Study on the Limitation of Width-thickness Ratio in the Plastic Hinges (1), (2). Trans. of the Architectural Institute of Japan. No. 96, 1964 and No. 99, 1964 (in Japanese).
3. Naka, T. and Kato, B.: Buckling Tests on Columns with a H-Profile built-up by welding. Sogo-shikenjo-nenpo, vol. 20, No. 1, 1961, Engineering Research Institute, Faculty of Engineering, University of Tokyo (in Japanese).
4. Kato, B.: Tests on Welded Wide-Angles. Trans. of the Architectural Institute of Japan. No. 89, 1963 (in Japanese).
5. Naka, T. and Kato, B.: Local Buckling of Compression Flanges of Wide-Flange Beams. Kanto-shibu, Architectural Institute of Japan, No. 35, 1964 (in Japanese).
6. Okumura, T.: Tests on Local Buckling of Flanges of H-shapes. Dept. of Civil Engineering, University of Tokyo, June 1963 (unpublished).
7. Plastic Design in Steel. AISC, New York, N. Y., 1963.
8. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, 1963. AISC, New York, N. Y.

Summary

The ultimate strength of buckled plates in the plastic range is analysed. The role of strain-hardening is considered to be important for the evaluation of the ultimate strength of the plates after inelastic buckling.

A simple mode of collapse of the plate is assumed in the analysis.

The results obtained in this paper were compared with experimental results and with recommended formulas in current specifications.

Résumé

L'auteur étudie la résistance limite de plaques voilées dans le domaine plastique. Il montre que, dans ce cas, l'écrouissage joue un rôle important pour la détermination de la résistance limite dans le domaine post-critique. Pour cette étude, on admet un mode de ruine simple. On compare les résultats théoriques à des valeurs expérimentales et aux formules données dans les réglementations existantes.

Zusammenfassung

Die Traglast von im plastischen Bereich ausgebeulten Platten wird untersucht. Dabei zeigt sich, daß die Materialverfestigung einen großen Einfluß auf die Bestimmung der Traglast im plastischen Bereich ausgebeulter Platten ausübt. Für die Bestimmung dieser Traglast wird ein einfacher Gelenkmechanismus angenommen. Die so ermittelten Werte werden mit Versuchswerten sowie mit den nach bestehenden Vorschriften empfohlenen Werten verglichen.

Leere Seite
Blank page
Page vide