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Analysis of Three-Dimensional Building Frames

Calcul d'ossatures d'immeubles à trois dimensions

Berechnung dreidimensionaler Rahmen

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Introduction

The structural framework of a modern, multi-story building consists in a three-dimensional rigid frame of an extremely high degree of statical indeterminacy, and having enormous numbers of degrees of joint-displacement freedom. Before the advent of automatic digital computers, the complexity of such structures precluded any attempt at exact analyses. Even with the aid of powerful computers, the computational problem associated with the direct analysis of a multi-story building is very great. For example, the upper stories of a relatively slender, high-rise structure might appear as shown in Fig. 1. This framework has 18 joints in each story or, considering 6 degrees of freedom per joint, 108 degrees of freedom per story. The direct displacement method analysis of a 30 story building of this configuration would thus require the solution of 3240 simultaneous equations, a formidable task even with the best of modern computational equipment. It is the purpose of this paper to present a simpler and more practical approach to the computer analysis of multi-story building frames such as this, taking account of their actual three-dimensional character.

Because of the magnitude of the computational task involved in the analysis of large three-dimensional frames, and also because of the special characteristics of typical multi-story building frameworks, it is customary and permissible to make various simplifying assumptions and approximations in their analysis (in addition to the ordinary assumptions of linear structural analysis). The rectangular arrangement of the framing members leads naturally to the treatment of the system as two sets of parallel plane frames, designated x-frames and y-frames in Fig. 1. These frames are all interconnected by floor slabs (diaphragms) at each floor level. Except for this diaphragm coupling, however, it is assumed that each frame acts in its own plane, independently of all others.

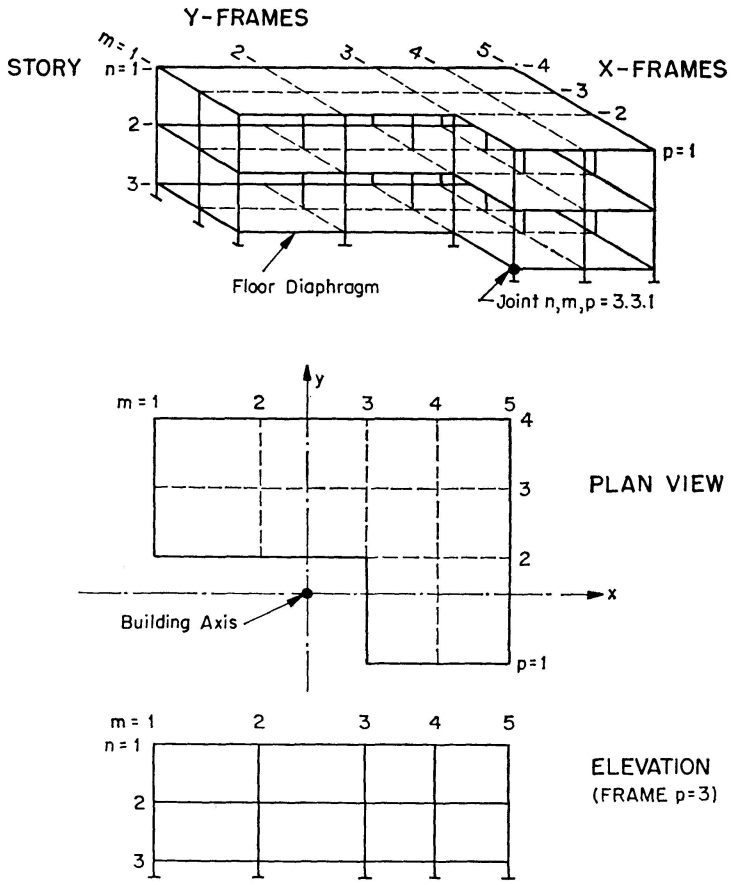


Fig. 1. Three dimensional building frame.

The floor diaphragms are assumed to be infinitely rigid in their own plane (but with no rigidity out of this plane). As a result of this assumption, the building has only three degrees of translational freedom per story, and all girders are constrained against axial deformations. Two additional assumptions are sufficient to decouple the various plane frames in all respects except for the diaphragm action:

1. Torsional rigidity of all girders is neglected. Thus the y-frame girders do not carry over joint rotation effects from one x-frame to another, or vice versa. It is also customary (but not essential) to neglect the torsional rigidity of the columns. In most structures, the torsional rigidity of all members is small enough that this assumption has little effect on the results.

2. Joints are constrained against vertical displacement. This assumption is

essential to the independence of frame action, because any vertical joint displacement obviously will cause a transfer of forces through the girders to adjacent frames in both the x and y directions. Such vertical displacements might result from axial column deformations as well as from rotations in a vertical plane of columns having finite width (generally known as "shear walls"). In a tall slender frame, vertical joint displacements can have a significant effect on the distribution of forces throughout the frame; thus this assumption limits the validity of the analysis. An approach to eliminating this assumption is discussed at the end of the paper.

Within the limitations of these assumptions, the method presented herein provides an "exact" analysis of any three-dimensional building frame. In contrast with most other building frame analysis procedures, this method considers the complete assemblage of plane frames rather than each frame or each axis of the building separately. It takes account of the twisting of the building about a vertical axis, as well as translational displacements along both axes, and thus is suited to the analysis of unsymmetrical structures as well as to symmetrical frameworks subjected to eccentric loadings.

Because the complete building is considered to be an assemblage of independent plane frames, the basic operation of this method of analysis is the evaluation of the lateral stiffnesses of the individual x- and y-frames. The lateral stiffness of any given frame is found by use of recursion equations applied successively to each story level from the top downwards. The computational effort increases only linearly with the height of the building and with the number of frames, thus the method is ideally suited to the analysis of tall buildings having many different types of frames.

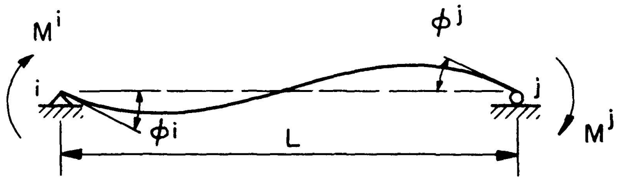
For convenience in identifying the numerous members and joints contained in a multi-story building, a triple subscripting system has been adopted, as shown in Fig. 1. The successive subscripts refer respectively to the story level, the y-frame number, and the x-frame number, and are designated n, m, and p. It should be noted that the story levels are numbered from the top downwards, in accordance with the sequence of the recursion solution.

Lateral Stiffness of a Single Frame

The procedure which is used for determining the lateral stiffness of an individual frame is based upon treating it as an assemblage of individual story segments. It is a modification of the "tri-diagonal" solution technique which has been applied to building frame analyses previously [1]. The modified lateral stiffness analysis technique also has been described before [2], but only with regard to deformations in a single plane. The general treatment of a three-dimensional building frame by this approach is presented here for the first time.

Stiffness of a Story Segment

The first step in the analysis procedure is the evaluation of the stiffness of each single story segment of the frame. This may be accomplished by standard matrix analysis techniques which have been described many times before [1, 2, 3, 4], so the technique will be outlined only briefly here. First, the stiffness, k_g, of each column and girder within the segment is defined by means of the well-known matrix expression shown in Fig. 2. Then, using the displace-



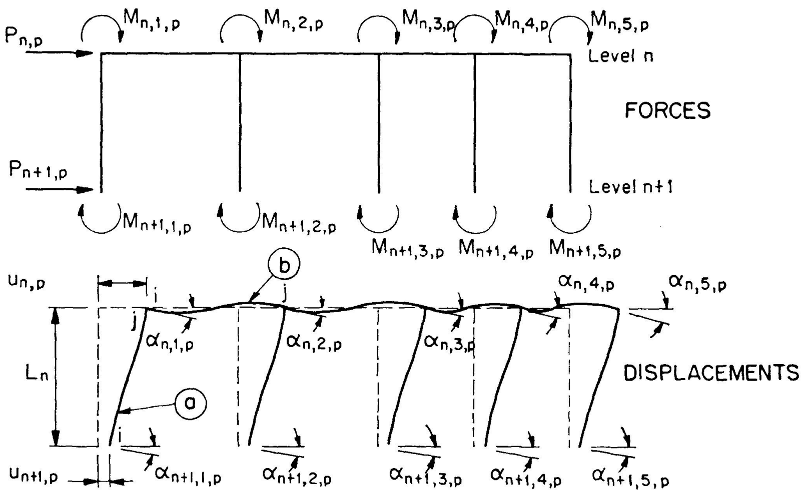
Young's modulus $= E$, moment of inertia $= I$, shear area $= \bar{A}$, shear modulus $= G$.
Stiffnes matrix

$$\begin{Bmatrix} M^i \\ M^j \end{Bmatrix} = \frac{2EI}{L(1+2\gamma)} \begin{bmatrix} (2+\gamma) & (1-\gamma) \\ (1-\gamma) & (2+\gamma) \end{bmatrix} \begin{Bmatrix} \phi^i \\ \phi^j \end{Bmatrix},$$

where the shear flexibility factor γ is:

$$\gamma = \frac{6EI}{L^2 \bar{A} G}.$$

Fig. 2. Typical frame member.



Typical member-segment deformation relationships:

$$\text{member "}a\text{": } \phi_a^i = \alpha_{n+1,1,p} + \frac{u_{n+1,p} - u_{n,p}}{L_n},$$

$$\phi_a^j = \alpha_{n,1,p} + \frac{u_{n+1,p} - u_{n,p}}{L_n},$$

$$\text{member "}b\text{": } \phi_b^i = \alpha_{n,1,p},$$

$$\phi_b^j = \alpha_{n,2,p}.$$

Fig. 3. Story segment "n" of frame "p".

ment transformation matrix, a_g, which expresses the relationship between the member deformations and the joint displacements of the structure (as shown for typical members in Fig. 3), each member stiffness may be expressed in terms of the segment displacement coordinates, as follows:

$$k_g^0 = a_g^T k_g a_g, \tag{1}$$

where the superscript "zero" shows that the term is related to the segment displacement coordinates. Finally, the complete segment stiffness, expressed in the segment coordinates, is obtained by merely adding together the member stiffnesses, k_g^0, which contribute to each joint. The result of this operation for story segment "n" may be expressed:

$$\begin{Bmatrix} M_n^t \\ M_n^b \\ P \end{Bmatrix} = \begin{bmatrix} k_n^t & c_n & e_n^t \\ (c_n)^T & k_n^b & e_n^b \\ (e_n^t)^T & (e_n^b)^T & \tilde{k}_n \end{bmatrix} \begin{Bmatrix} \alpha_n^t \\ \alpha_n^b \\ u \end{Bmatrix}, \tag{2}$$

in which

M_n^t = vector of all joint moments applied at the top of segment "n".
M_n^b = vector of all joint moments applied at the bottom of segment "n".
α_n^t = vector of joint rotations at top of segment.
α_n^b = vector of joint rotations at bottom of segment.
P = vector of lateral forces applied at all levels of frame.
u = vector of corresponding displacements.

It will be noted that the force and displacement vectors acting in the story segment have been partitioned so as to separate the rotative effects at the top, the rotative effects at the bottom, and all lateral displacements. Each of the nine stiffness submatrices of Eq. (2) relates force and displacement effects of these three types; thus c_n represents the moments at the top due to rotations at the bottom, k_n represents lateral forces at *all* levels (contributed by story segment "n") due to lateral displacements at all levels, etc. It is important to note that e_n and $\tilde{k}_n$ have dimensions relating to the number of stories in the complete structure, but have non-zero elements only in positions corresponding with the location of element "n".

The moments M_n^t in Eq. 2 represent moments applied externally at the top joints of the segment. Generally these result from vertical loads acting on the segment girders; i.e., they are the "fixed end moments" of the well-known moment distribution procedure. Because no girders are included at the bottom of the segment, the moments M_n^b generally are not present. The forces P represent the lateral loads applied externally to the given frame. However, because the lateral loads are usually assumed to be applied to the building as a whole, rather than to an individual frame, these loads also generally are zero.

Stiffness of the Complete Frame

When the individual story segment stiffnesses have been evaluated, the stiffness of the complete frame may be obtained merely by summing appropriate submatrices of the segment stiffnesses. The result of this process may be expressed as follows:

$$\begin{array}{c} (a) \\ (b) \\ \\ \\ \\ \\ \\ \\ (c) \end{array} \left\{\begin{array}{l} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_n \\ M_{n+1} \\ \vdots \\ M_N \\ P \end{array}\right\} = \begin{bmatrix} k_1 & c_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & e_1 \\ c_1^T & k_2 & c_2 & \cdots & 0 & 0 & \cdots & 0 & e_2 \\ 0 & c_2^T & k_3 & \cdots & 0 & 0 & \cdots & 0 & e_3 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & k_n & c_n & \cdots & 0 & e_n \\ 0 & 0 & 0 & \cdots & c_n^T & k_{n+1} & \cdots & 0 & e_{n+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & e_N \\ e_1^T & e_2^T & e_3^T & \cdots & e_n^T & e_{n+1}^T & \cdots & e_N^T & \bar{K} \end{bmatrix} \left\{\begin{array}{l} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_n \\ \alpha_{n+1} \\ \vdots \\ \alpha_N \\ u \end{array}\right\} \tag{3}$$

in which

$$\begin{array}{ll} M_1 = M_1^t & \alpha_1 = \alpha_1^t \\ M_2 = M_1^b + M_2^t & \alpha_2 = \alpha_1^b = \alpha_2^t \\ \text{etc.} & \text{etc.} \\ k_1 = k_1^t & e_1 = e_1^t \\ k_2 = k_1^b + k_2^t & e_2 = e_1^b + e_2^t \\ \text{etc.} & \text{etc.} \end{array}$$

$$\bar{K} = \sum_{n=1}^{N} \tilde{k}_n$$

and where the subscript N represents the total number of stories in the frame. The physical significance of each of the terms in the stiffness matrix of Eq. (3) is obvious. The tri-diagonal relationship between M and α results from the fact that each story segment is connected only to segments above and below, thus forces are carried over only from one level to the next.

Reduction to Lateral Stiffness

The recursive reduction process which may be used to evaluate the lateral frame stiffness from Eq. (3) is a direct consequence of this tri-diagonal arrangement of the stiffness submatrices relating M and α. Considering first Eq. (3a), which may be written as follows:

$$M_1 = k_1 \alpha_1 + c_1 \alpha_2 + e_1 u \tag{3a}$$

the rotations at the top of the frame may be expressed

$$\alpha_1 = k_1^{-1} M_1 - k_1^{-1} c_1 \alpha_2 - k_1^{-1} e_1 u. \tag{4}$$

Rewriting Eq. (3b) as follows:

$$M_2 = c_1^T \alpha_1 + k_2 \alpha_2 + c_2 \alpha_3 + e_2 u \tag{3b}$$

and substituting Eq. (4) for α_1 yields:

$$M_2 = c_1^T k_1^{-1} M_1 - c_1^T k_1^{-1} c_1 \alpha_2 - c_1^T k_1^{-1} e_1 u + k_2 \alpha_2 + c_2 \alpha_3 + e_2 u .$$

Introducing the new symbols

$$\begin{aligned} \overline{M}_2 &= M_2 - c_1^T k_1^{-1} M_1, \\ \bar{k}_2 &= k_2 - c_1^T k_1^{-1} c_1, \\ \bar{e}_2 &= e_2 - c_1^T k_1^{-1} e_1, \end{aligned}$$

this equation becomes

$$\overline{M}_2 = \bar{k}_2 \alpha_2 + c_2 \alpha_3 + \bar{e}_2 u ,$$

which has the same form as Eq. (3a), but with all subscripts increased by one.

Following the same procedure, a general recursive relationship may be established for step "$n-1$" of the reduction process:

$$\overline{M}_n = \bar{k}_n \alpha_n + c_n \alpha_{n+1} + \bar{e}_n u , \tag{5}$$

in which

$$\begin{aligned} \overline{M}_n &= M_n - c_{n-1}^T \bar{k}_{n-1}^{-1} \overline{M}_{n-1}, \\ \bar{k}_n &= k_n - c_{n-1}^T \bar{k}_{n-1}^{-1} c_{n-1}, \\ \bar{e}_n &= e_n - c_{n-1}^T \bar{k}_{n-1}^{-1} \bar{e}_{n-1}. \end{aligned} \tag{6}$$

A similar reduction process must also be applied to Eq. (3c), which in its original form may be written:

$$P = e_1^T \alpha_1 + e_2^T \alpha_2 + \cdots + e_N^T \alpha_N + \overline{K} u . \tag{3c}$$

Substituting α_1 from Eq. (4), and introducing the new symbols

$$\begin{aligned} \overline{P}_1 &= P - e_1^T k_1^{-1} M_1, \\ \bar{e}_2^T &= e_2^T - e_1^T k_1^{-1} c_1, \qquad (= \text{transpose of } \bar{e}_2, \text{ above}) \\ \overline{K}_1 &= \overline{K} - e_1^T k_1^{-1} e_1, \end{aligned}$$

this becomes:

$$\overline{P}_1 = \bar{e}_2^T \alpha_2 + e_3^T \alpha_3 + \cdots + e_N^T \alpha_N + \overline{K}_1 u ,$$

which has the form of Eq. (3c), but with pertinent subscripts increased by one. The second general recursion relationship for step "$n-1$" thus becomes:

$$\overline{P}_{n-1} = \bar{e}_n^T \alpha_n + e_{n+1}^T \alpha_{n+1} + \cdots + e_N^T \alpha_N + \overline{K}_{n-1} u , \tag{7}$$

in which

$$\begin{aligned} \overline{P}_{n-1} &= \overline{P}_{n-2} - \bar{e}_{n-1}^T \bar{k}_{n-1}^{-1} \overline{M}_{n-1}, \\ \bar{e}_n^T &= e_n^T - \bar{e}_{n-1}^T \bar{k}_{n-1}^{-1} c_{n-1}, \\ \overline{K}_{n-1} &= \overline{K}_{n-2} - \bar{e}_{n-1}^T \bar{k}_{n-1}^{-1} \bar{e}_{n-1}. \end{aligned} \tag{8}$$

Applying these recursive relationships (Eqs. (5), (6), (7), (8)) successively from the top of the frame downward, the rotational displacements are eliminated consecutively from the equations of equilibrium, so that at the next to the last step of the process Eq. (3) has been reduced to the form:

$$\begin{Bmatrix} \bar{M}_N \\ \bar{P}_{N-1} \end{Bmatrix} = \begin{bmatrix} \bar{k}_N & \bar{e}_N \\ \bar{e}_N^T & \bar{K}_{N-1} \end{bmatrix} \begin{Bmatrix} \alpha_N \\ u \end{Bmatrix}. \tag{9}$$

The final reduction then leads to

$$\bar{P}_N = \bar{K}_N u, \tag{10}$$

in which $\bar{K}_N$ represents the lateral stiffness matrix of the frame, relating the translational displacements u to the effective lateral forces $\bar{P}_N$.

This reduction process has physical significance which is worthy of note. The original stiffness matrix of Eq. (3) represents the complete stiffness of the structure, in which each of the joints is restrained against rotation and each floor level against translation. The reduction process represents the relaxation of rotational constraints at the successive levels from the top downward. The modified force and stiffness matrices of Eqs. (6) and (8) represent the changes in these quantities at level "n" resulting from relaxation of the rotational constraints at level "$n-1$". Thus $\bar{K}_N$ represents the lateral stiffness of the frame when all rotational joint constraints have been relaxed. Similarly, $\bar{P}_N$ represents the effective lateral forces when the rotational constraints are relaxed. They result directly from the vertical load fixed-end moments in the girders.

Analysis of the Complete Building

A plan view depicting the arrangement of the x- and y-frames, as well as the lateral loads applied to a typical building is shown in Fig. 4a. In order to identify the various frame stiffnesses and loadings in this figure, the notation of Eq. (10) has been modified and amplified as follows:

	Equation (10)	New notation	
		x-frames	y-frames
Lateral Frame Stiffness:	$\bar{K}_N$	$\bar{K}_p^x$	$\bar{K}_m^y$
Lateral Frame Loading:	$\bar{P}_N$	$\bar{P}_p$	$\bar{Q}_m$

The vertical axis of the building, representing the origin of the $x-y$ coordinate system shown in the figure, is located arbitrarily. It is necessary however, that it have the same plan location for each story, i. e. that it be represented by a vertical line.

The rigid body displacements (translation and rotation) of story "n" of the building are indicated in Fig. 4b. The relationship between the translational displacements of arbitrary x- and y-frames at this level and the story displacement components may be expressed as follows:

$$\begin{aligned} u_{np} &= U_n + y_p \Theta_n, \\ v_{nm} &= V_n - x_m \Theta_n, \end{aligned} \tag{11}$$

where each of the symbols is defined in the figure. The resultant loads acting in the nth level floor diaphragm, which include the lateral frame loads as well as the externally applied forces $\bar{X}_n$ and $\bar{Y}_n$, are given similarly by the relationships:

$$\begin{aligned} X_n &= \bar{X}_n + \sum_p \bar{P}_{np}, \\ Y_n &= \bar{Y}_n + \sum_m \bar{Q}_{nm}, \\ T_n &= \bar{X}_n \bar{y}_n - \bar{Y}_n \bar{x}_n + \sum_p \bar{P}_{np} y_p - \sum_m \bar{Q}_{nm} x_m, \end{aligned} \tag{12}$$

in which the symbols in the right hand members of the equations are defined in Fig. 4a and those to the left represent the story force resultants.

The relationship between the complete vector of story resultants (of dimensions $3N$) and the corresponding displacements, may then be written:

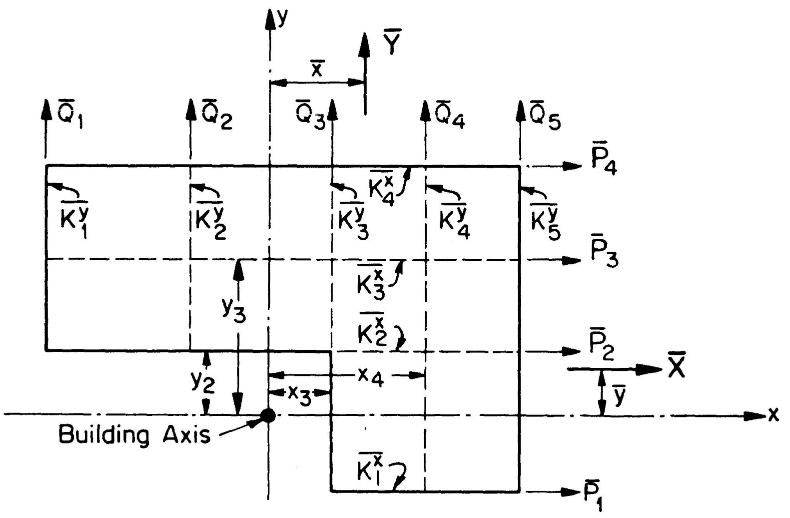


Fig. 4a. Frame arrangement and applied loads.

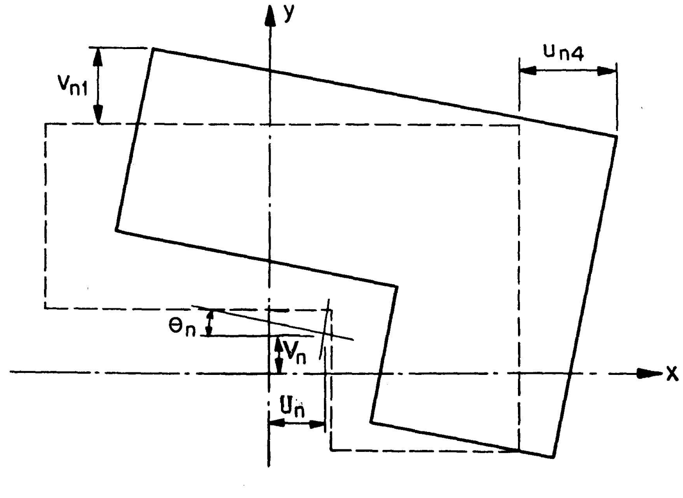


Fig. 4b. Story and frame displacements.

$$\begin{Bmatrix} X \\ Y \\ T \end{Bmatrix} = \begin{bmatrix} K_{XX} & 0 & K_{XT} \\ 0 & K_{YY} & -K_{YT} \\ K_{TX} & -K_{TY} & K_{TT} \end{bmatrix} \begin{Bmatrix} U \\ V \\ \Theta \end{Bmatrix}, \tag{13}$$

in which $X = \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{Bmatrix}; \quad Y = \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{Bmatrix}; \quad T = \begin{Bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{Bmatrix}.$

The stiffness matrix of Eq. (13) is made up of submatrices each having the dimensions (N) of the number of stories in the building. These submatrices are defined as follows:

$$\begin{aligned} K_{XX} &= \sum_p \bar{K}_p^x, \\ K_{XT} &= \sum_p \bar{K}_p^x\, y_p = (K_{TX})^T, \\ K_{YY} &= \sum_m \bar{K}_m^y, \\ K_{YT} &= \sum_m \bar{K}_m^y\, x_m = (K_{TY})^T, \\ K_{TT} &= \sum_p \bar{K}_p^x\, y_p^2 + \sum_m \bar{K}_m^y\, x_m^2, \end{aligned} \tag{14}$$

in which the distances of the frames from the building axis, given by y_p and x_m, are scalar multipliers of the lateral frame stiffness matrices.

After the stiffness matrix of Eq. (13), defining the stiffness properties of the complete building, has been established and the applied loads of Eq. (12) have been determined, the building story displacements U, V and θ may be obtained from Eq. (13) by standard simultaneous equation solution techniques. Although a large system of equations may still be involved (for example, for a 30 story building there will be 90 equations) the size of this computational problem generally will be orders of magnitude less than would be required for the direct analysis of the complete building. For example, in the building of Fig. 1, there are only $^1/_{36}$ as many equations to be solved simultaneously. Moreover, Eq. (13) may be partitioned readily to take advantage of its zero submatrices, thus reducing the inversion problem to matrices of order N.

When the displacement components of the floor diaphragms have been computed from Eq. (13), the displacements of each of the components of the building may be determined by a "back substitution" process. First, the displacements of the individual frames are found from a matrix equation analogous to Eq. (11). Then for each frame, the first of the two equations given by matrix equation (9):

$$\bar{M}_N = \bar{k}_N\, \alpha_N + \bar{e}_N\, u$$

is solved for the joint rotations, α_N, at the lowest story level:

$$\alpha_N = \bar{k}_N^{-1}\, \bar{M}_N - \bar{k}_N^{-1}\, e_N\, u,$$

Rotations of joints at successive levels above this may then be found by successive application of equations of the form:

$$\alpha_{N-1} = \bar{k}_{N-1}^{-1} \bar{M}_{N-1} - \bar{k}_{N-1}^{-1} C_{N-1} \alpha_N - \bar{k}_{N-1}^{-1} \bar{e}_{N-1} u, \tag{15}$$

which is equivalent to Eq. (4).

After all joint displacements in the frame have been determined, the deformations of all members can be found from expressions equivalent to those presented in Fig. 3, and finally the member forces are found from the member stiffness expressions of Fig. 2. Only the member moments will be obtained directly by this process. However, the member shears can be computed easily from their end moments, and column axial forces can be obtained by simple summations of the pertinent girder shears and the axial force in the column above.

Automatic Computer Analysis

Although the analysis procedure described herein is simple in principle, it is clear that the volume of computations required for treatment of even a rather small building makes the method practicable only for use with automatic digital computers. Fortunately, the highly systematized procedure (and its matrix formulation) is ideally suited to computer programming, and it is not difficult to write a standard program capable of analyzing any building which fits within the size limitations of the particular computer hardware to be used.

One such standard program (Program "HIRISE") has been written for the IBM 7090 operated by the University of California Computer Centre. This program will analyze any structure of not more than 50 stories, made up of not more than 10 *different* frame configurations. There may be as many as ten identical frames of each type, and each frame may have up to 11 column lines[1]). Thus the largest building which can be treated would have 121 joints per story for 50 stories, which totals up to 36,300 degrees of freedom (or 12,250 according to the assumed diaphragm behaviour). Although treatment of a structure of this size has not yet been attempted by this program, its analysis is completely practicable and would not require an exhorbitant amount of computer time.

The computer program generally follows the sequence of operations outlined above: the lateral stiffness of each frame is computed first (and stored on magnetic tape) and then the stiffness of the complete structure is formed by means of Eq. (14). The input data provides a complete description of the structure and the applied loads (three different loading conditions may be

[1]) It should be noted that these size limitations were arbitrary, and could have been doubled without difficulty.

treated in a single analysis operation). This descriptive data is punched on cards in the following sequence: General information cards, identifying the problem, specifying numbers of frames in the x and y directions, etc.; story cards, listing story heights, applied lateral story loads $\bar{X}_n$, $\bar{Y}_n$, etc.; and frame description cards, specifying the bay widths, the size of each column and girder in each frame, as well as the vertical loads applied to the girders.

The output of the program includes a listing of the building displacements U_n, V_n, and θ_n for each story; then for each frame, story by story, a listing of the story displacements, the girder end moments, and the column end moments, shear and axial force.

Partial results of the analysis of an example building are presented in Figs. 5 and 6. The general arrangment of the six story, L-shaped building is

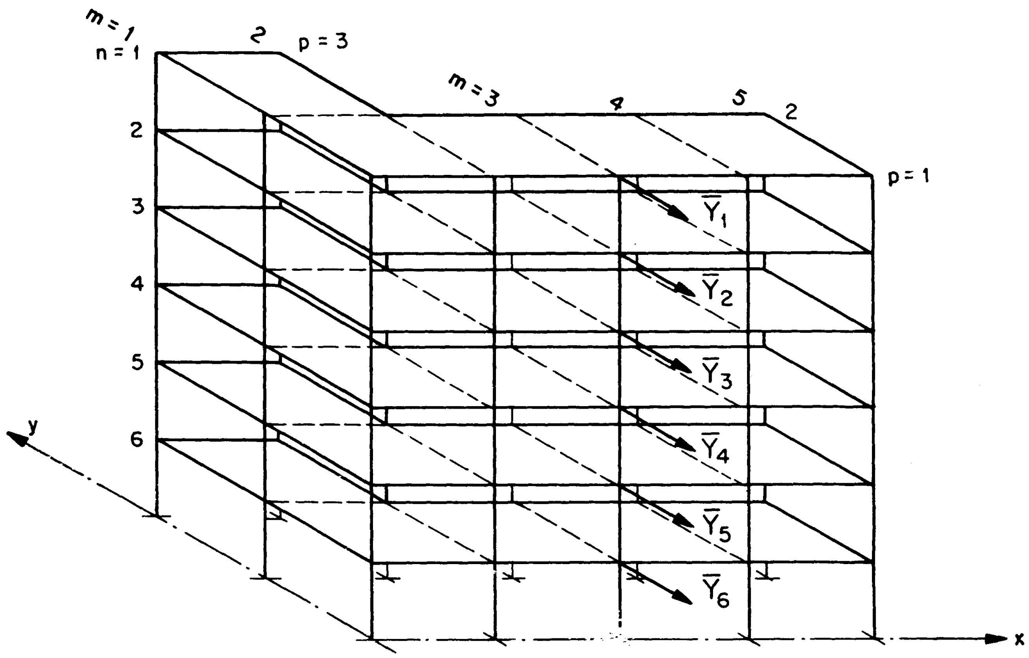


Fig. 5a. Examples building with applied loads.

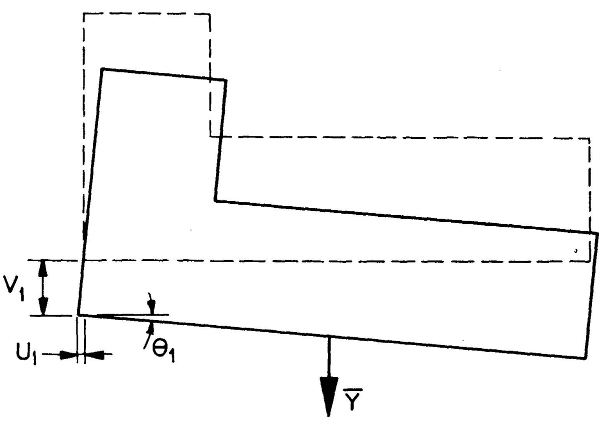


Fig. 5b. Displacement of top story.

shown in Fig. 5a. The applied load is assumed to be a wind pressure acting parallel with the y-axis. The computed displacement of the top story is indicated (to an exaggerated scale) in Fig. 5b. The twisting deformation which results from this symmetrical loading acting on the unsymmetrical framework is quite apparent. Additional representations of the deflected shape of the structure are shown in Fig. 6.

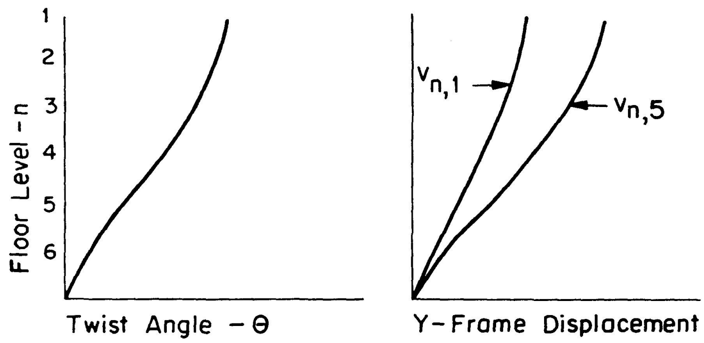


Fig. 6a.

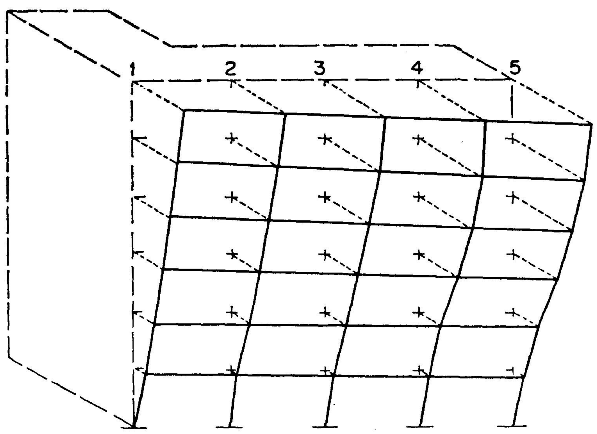


Fig. 6b. Displacements of front face of building.

Discussion

The method of analysis presented in this paper will provide an exact solution of any multi-story building framework, within the limitations of the assumptions which were defined at the outset. However it is clear that the reliability of the results is dependent directly upon the validity of these assumptions, thus it is advisable to consider them again. For modern, high-rise buildings, the assumption that the floor diaphragm is rigid in its own plane is quite reasonable and should lead to no impotant discrepancies. It is evident, however, that the floor slab rigidity in the vertical direction is not negligible: it may contribute significantly to the flexural stiffness of the girders and its effect should be considered in assessing the girder properties. On the other hand, the vertical coupling between adjacent frames provided by the slab probably can be neglected without significant error, as can the torsional rigidity of the girders. Thus the majority of the assumptions made in the analysis are not expected to lead to significant errors.

The one remaining assumption, however, that there are no vertical displacements of the joints, can be an important source of error. In the analysis of a single plane frame of relatively slender proportions (for which vertical joint displacement effects can be included without difficulty) [1, 2], axial column deformations can be shown to cause significant changes in the member moments and axial forces as well as a reduction of the lateral stiffness of the frame. In addition, the vertical joint displacement effects associated with the flexural deformation of vertical shear walls certainly cannot be neglected. Thus, it is clear that the method of analysis presented here , although it provides a first step toward the analysis of complete three-dimensional building frames, cannot be considered the final answer to the problem.

At present, the most effective method for including vertical joint displacement effects in a three-dimensional building analysis appears to be by means of an iteration process. The analysis described herein, which neglects vertical joint displacements, provides the first step in the iteration and yields a first estimate of the axial column forces. Then, knowing these forces and having derived the vertical joint stiffnesses (which include both axial column stiffness and girder shear stiffness) the vertical joint displacements are computed using an iterative process similar to that described in Reference 1, while constraining the structure against other forms of displacement. As a result of these vertical displacements, additional girder moments are developed in both x- and y-frames. Calculation of the response of the structure to these new fixed-end moments represents the next stage of the iteration process and is carried out in exactly the same way as was the original analysis (i. e. without vertical joint motion). This is followed by another vertical joint displacement iteration, etc., until satisfactory convergence of the process is achieved. Because the vertical joint displacement effects are of a secondary nature (at least in buildings of moderate height), it is likely that the iteration would converge very quickly, probably requiring no more than two or three vertical adjustment cycles.

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Summary

A typical multistory building may be assumed to behave as an assemblage of independent frames, oriented in the x and y directions and coupled at each floor level by a diaphragm which is rigid in its own plane. The paper presents a method of analysis for such structures which takes account of twisting of the building about its vertical axis, as well as translations in the x and y directions. The method involves evaluation of the lateral stiffness of each individual frame, making use of recursion equations derived from the "tri-diagonal" character of the frame stiffness matrix. The complete building stiffness is then obtained by appropriate combination of the frame stiffnesses.

The method of analysis is intended to be programmed for solution by digital computer, and the basic features of a program written for the IBM 7090 are described. Displacements and member forces produced by arbitrary combinations of vertical and lateral loads may be evaluated. Partial results of the analysis of an example building are presented.

Résumé

On peut considérer que l'ossature d'un immeuble à plusieurs étages se comporte comme un assemblage de portiques indépendants, orientés dans les directions x et y et solidarisés à chaque niveau par un diaphragme rigide dans son propre plan. On présente une méthode de calcul de ces ossatures qui tient compte de la torsion autour de l'axe vertical ainsi que des translations intervenant dans les directions x et y. On évalue d'abord la rigidité latérale de chaque portique au moyen des équations de récurrence déduites en tirant parti du caractère «tri-diagonal» de la matrice de rigidité. On obtient ensuite la rigidité de l'ensemble en combinant convenablement les rigidités des portiques.

Cette méthode de calcul est destinée à être programmée sur un calculateur digital et l'on décrit les caractéristiques fondamentales d'un programme destiné à l'IBM 7090. On peut calculer les déplacements et les sollicitations dans les éléments dus à des combinaisons arbitraires de charges verticales et latérales. On présente les résultats partiels du calcul d'un bâtiment type.

Zusammenfassung

Die Tragkonstruktion eines normalen, mehrstöckigen Gebäudes kann als Zusammensetzung unabhängiger, in jedem Stockwerk durch in ihrer Ebene starre Decken verbundener Rahmen betrachtet werden, deren Achsen parallel zur x-, bzw. y-Richtung verlaufen. Es wird eine Methode zur Berechnung solcher Tragwerke beschrieben, welche die Verdrehung des Gebäudes um seine vertikale Achse und dessen Verschiebungen in x- und y-Richtung berücksichtigt. Zuerst wird die Quersteifigkeit jedes einzelnen Rahmens mit Hilfe von Rekursionsgleichungen ermittelt, die aus der besonderen Form der Steifigkeitsmatrix (dreigliedrig) hergeleitet werden. Die Gesamtsteifigkeit des Gebäudes wird durch geeignete Zusammensetzung der einzelnen Rahmensteifigkeiten ermittelt.

Die Berechnungsmethode soll für einen Digitalrechner programmiert werden. Die Grundlagen eines Programmes für die IBM 7090 werden beschrieben. Die Verschiebungen und die Beanspruchungen, die von einer beliebigen Kombination vertikaler und seitlicher Kräfte hervorgerufen werden, können ermittelt werden. Teilergebnisse der Berechnung eines Mustergebäudes werden angegeben.