Ultimate load design of reinforced concrete frames: a recapitulation and appraisal. March, 1963

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Ultimate Load Design of Reinforced Concrete Frames: A Recapitulation and Appraisal. March, 1963

Calcul à la rupture des ossatures en béton armé: Récapitulation et commentaires

Bemessung von Stahlbetonrahmen nach dem Traglastverfahren Zusammenfassende und kritische Übersicht

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Notation

Space frame

X_k^x, X_k^y, X_k^z unknown forces acting in directions X, Y and Z respectively at section k.

$X_k^{(x)}\, X_k^{(y)}\, X_k^{(z)}$ unknown moments in plane normal to $0x$, $0y$, $0z$, X_k^x typifies moment or force in general equations (ref. fig. 1).

$\delta_{ki}^{xx} \ldots \delta_{ki}^{(z)(z)}$ total displacement at a release section due to elastic deformation of the frame members only, the indices being as follows. In the case of δ_{ki}^{xx} the first upper and lower indices indicate displacement is at section k in direction of action of X_k^x, and the second upper and lower indices indicate displacement is caused by $(X_i^x = 1)$ acting.

$\delta_k^{x\,0} \ldots \delta_k^{(z)\,0}$ total displacement at a release section due to elastic deformation of the frame members only, the indices being as follows. In the case of $\delta_k^{x\,0}$, the first upper and lower indices indicate displacement is at section k in direction of action X_k^x, and the upper index 0 indicates displacement is caused by external load.

$w_k^x \ldots w_k^{(z)}$ influence coefficients; that is, ordinates of distribution diagram of internal restraints acting in the direction indicated by the upper index and caused by $(X_k = 1)$ acting, but not necessarily in the same direction.

$\bar{s}_l^x \ldots \bar{s}_l^{(z)}$	total displacement over length $\bar{l}_p$ in directions $0X$, $0Y$ and $0Z$, due to inelasticity (ref. fig. 1).
s_{sc}^x	displacement per unit length in direction x due to shrinkage and creep.
s_T^x	displacement per unit length in direction x due to temperature change.
$\bar{a}_k^x \ldots \bar{a}_k^{(z)}$	short-term displacements of $X_k^x \ldots X_k^{(z)}$ in their directions of action due to short-term inelastic deformation assumed to be concentrated at k.
$a_{ksc}^x \ldots a_{ksc}^{(z)}$	long-term or high-temperature displacements of $X_k^x \ldots X_k^{(z)}$ in their directions of action due to shrinkage and creep, and assumed to be concentrated at k.
$a_{kd}^x \ldots a_{kd}^{(z)}$	displacement of $X_k^x \ldots X_k^{(z)}$ in their direction of action due to movement of an external support at k.
$e^x \ldots e^{(z)}$	displacement or rotation per unit length of a point or plane of action of a restraint in directions $w^x \ldots w^{(z)}$ due to elastic deformation for $w_k^x \ldots w_k^{(z)}$ assumed equal to unity.

Plane frame

X_k	unknown bending moment acting at hinge k.
M_k	ordinate of bending moment diagram for $X_k = 1$.
M_0	ordinate of bending moment diagram for external loads.
r	radius of curvature.
EI	Mr = flexural rigidity of section (based on $0L_1$).
$E'I'$	Mr = flexural rigidity of section (based on $0A$). Fig. 1.
$\psi_{\bar{n}}$	rotation at critical section $\bar{n}$ due to inelastic deformation at a section not assumed to be a hinge when the initial flexural rigidity is assumed for elastic calculations.
θ_i'	rotation at hinge i due to inelastic deformation when the initial flexural rigidity is used for elastic calculation.
θ_i	resultant rotation at hinge i (flexural rigidy based on $0A$, fig. 1).
N_k	axial force on a member due to $X_k = 1$ acting.
N_0	axial force on a member due to external load.
N_p	axial force on a member due to prestress force.
E'	value of elastic modulus of concrete at zero stress.
A	equivalent concrete section at zero restraint.
θ_p	permissible rotation at hinge i (on one side of critical section).
e_{cu}	strain in concrete at L_2.
e_{ce}	strain in concrete at L_1.
$n_u d$	depth of neutral axis at L_2.

d	effective depth of section, or overall depth when there is no tension.
$\bar{l}_p$	plastic length ($0\,L_1$ assumed elastic ref. fig. 1).
$\bar{\bar{l}}_p$	plastic length ($0\,A$ assumed elastic ref. fig. 1).
l_p	equivalent plastic length, ref. fig. 1.
z	distance from critical section to point of contraflexure.

Three View-points

The discussions [1, 3] reveal three different groups of protagonists. There are those who:

1. Prefer to analyse frames by conventional elastic theory, ignoring the influence of inelastic behaviour either at working, or at ultimate, load, but accepting a nominal limited transfer of bending moments between critical sections. Better design for serviceability is claimed. Flexural rigidity is generally based on uncracked sections, but an ultimate stress and strain condition may be assumed for calculating the bending strength of sections.
2. Those who accept the validity of design methods based on ultimate strength and compatibility equations which allow for inelastic behaviour, and who apply an approximate check to working load stresses, as recommended in [1].
3. Those who consider that designs may be based on ultimate strength calculations made as with structural steel frames, sometimes assuming the members to be rigid and without checking the extent of the inelastic deformations required for compatibility with the assumed bending moment values.

The criteria and methods of calculation accepted by group 2 include both ultimate and working load conditions, and need not involve unnecessary complications. In the case of highly statically indeterminate structures, calculations are simpler than those required by "correct" conventional elastic analysis. Design rules are being developed [1] which are no more complicated than the simple bending moment coefficients permitted by codes of practice which cannot always be considered correct. Optimum designs produced thereby can save steel. The distribution of bending moments and the inelastic rotations are calculated for ultimate load with the degree of accuracy warranted for the particular structure, and for working load the bending moments and stresses are checked approximately.

Designers of group 1 generally claim that a better safeguard against excessive cracking at working load is provided as compared with the methods of group 2. This may be so, if it happens that the relative stiffness of members assumed in the calculations is not greatly different from actuality or if there is no serious variation from the calculated bending moments due to the influence of cracking, differential creep, shrinkage or temperature. (Ref. example in [2].) If adjustments of 15%—25% have been made quite arbitrarily to the bending moments, calculated by elastic theory, working load stresses will be similarly varied from the calculated values. In the method of group 2, an approximate check of the influence of such adjustments is made, allowing for the influence of cracking on stiffness. This provides a reasonable safeguard against the occurrence of excessive stresses under working load.

The criteria of group 3 give no consideration to the influence of inelastic rotations on compatibility which, particularly in conjunction with an unfavourable disposition of continuous spans and distribution of stiffness and live load, may be excessive. The permissible inelastic rotations, calculated by formulae based on test results [1], clearly indicate that this is so. Moreover, an approximate check should be made of the stresses under working load, and for some structures, such as, for instance, bridges or water tanks, a precise calculation should be made.

The use of conventional elastic theory is often assumed to be a safeguard against cracking which causes dissatisfaction on the part of clients, and is preferred for that reason, although the Load Factor value for the structure may be less than for a design based on Ultimate Theory. The safeguard, of course, does exist, if it so happens that all factors in the calculations, such as relative stiffness of members, are correct, but most designers prefer to reduce the risk of cracking and high deflection at mid-span by transferring some calculated bending moments from the supports, although the risk of cracks at supports is thereby increased. The risk of cracking and excessive deflection has, of course, been increased in some modern codes, by permitting the use of higher concrete stresses and a parabolic instead of linear distribution of compressive stress. This influence on serviceability must, however, not be confused with the use of plastic hinges to redistribute bending moments at ultimate load.

It is therefore now contended that the general principles and methods of calculation accepted by group 2 provide a sounder basis of design than those of 1 and 3. The principles accepted by group 2, however, do not imply a single common approach for the design of all kinds of structures. Every case must be considered individually. In a design office, the supervisor must use his judgement and decide what different cases of load, temperature or other effects must be considered in the calculations, whether a precise or Limit strength calculation should be used, and whether an accurate or approximate calculation of working stresses is required. The decision will depend on many factors, in particular the nature of the structure. In an important continuous beam bridge, it may be necessary to calculate accurately the stresses at working load, allowing for inelastic deformations as well as calculating the ultimate strength. In a building frame, a Limit design calculation, using a simple design rule, may often be adequate.

Idealised Concept of Frames

Space frames, as regards calculation assumptions, may be classified, as follows:

1. Those having three-dimensional continuity of members in which compatibility of restraint and deformation due to bending, axial force, shear and torque are considered.
2. As for 1, but the frame is either uni-planar or only the influence of continuity on deformation in one plane is considered.
3. As for 2, but only bending moment and deformation compatibility is considered.

In each of the above classifications, individual frame members are assumed to obey a linear restraint deformation law between critical sections. Inelastic deformations are assumed to be concentrated at critical sections. This assumption is generally sufficiently accurate, but has been questioned in some cases [9].

At a critical section, the restraint-integrated deformation relationship is assumed to follow (ref. Fig. 1a case of bending) either

1. a tri-linear Limit line $0\,L_0, L_1, L_2$ or $0\,L_0, L_1', L_2'$

or 2. a bi-linear Lower Limit line $0\,L_1, L_2$,

where L_0, L_1 and L_2 are characteristic cracking, yield and failure points. Such lines are derived from measured total deformations in tests over sufficient

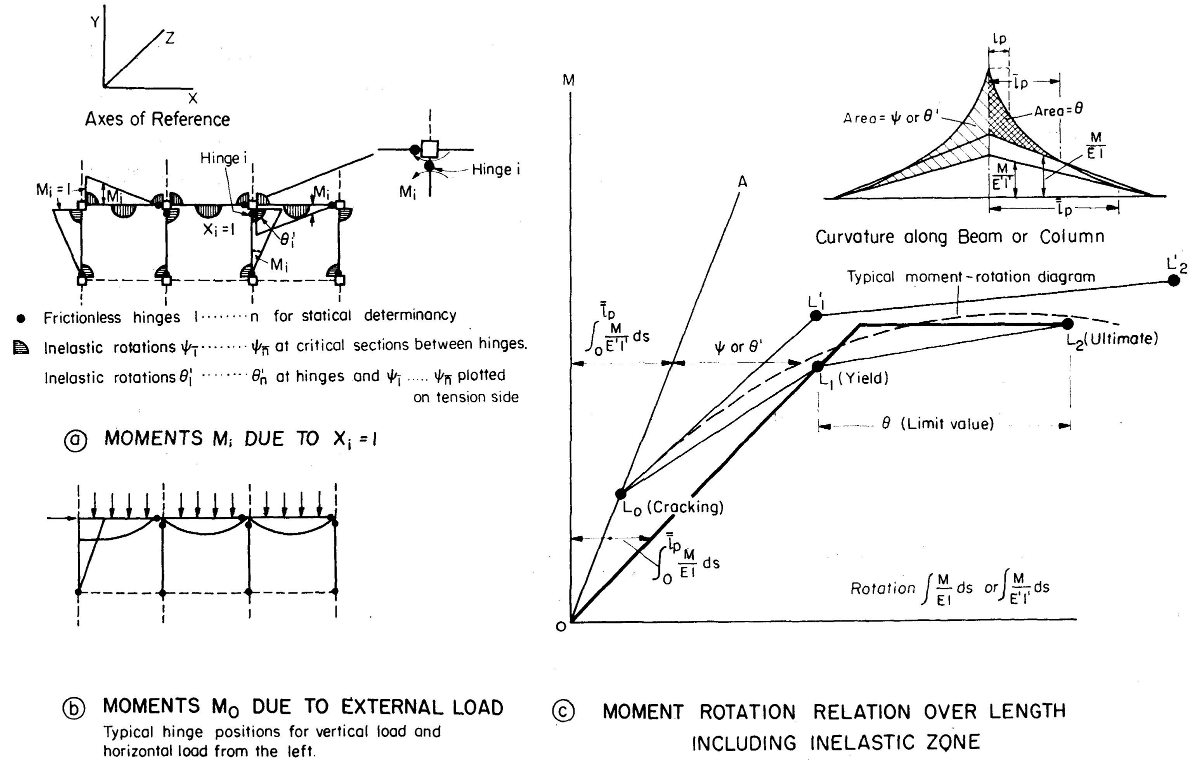

Fig. 1. Typical Basic Characteristics of a Frame.

length of the member to include all inelastic deformation adjacent to a critical section. In the tri-linear assumption, for a rigorous calculation characteristic upper and lower limits statistically based on test results should be assumed, distributed throughout the frame in the worst possible way in relation to live load.

For any frame, compatibility equations are established by inserting sufficient releases and unknowns X to make the frame statically determinate, but stable (ref. Fig. 1a). In the tri-linear assumption, integrated inelastic deformations say for bending θ' occur at, and adjacent to, hinge sections and ψ at, and adjacent to, critical sections between release sections.

In the greatly simplified bi-linear assumption, the frame, in effect, is assumed to remain elastic with a cracked EI value up to failure, except for inelastic deformations θ at hinge sections, which, for compatibility of restraint and deformation, must have the correct sign and not exceed the permissible value for the section equal to $L_1 L_2$. The frame may be designed by trial and adjustment, so that L_1 values are not exceeded between hinges, and restraints at hinges have appropriate plastic values. The ultimate state is assumed to be reached when all X values are plastic, so that the system is statically determinate. Further adjustment of X values to give θ values approximately $=0$ provides a solution for the restraints and hence stresses at working load.

Basic Compatibility Equations [2]

The basic compatibility equations may be derived by the Principle of Virtual Work. The virtual work done by internal restraints as deformations take place due to various causes is considered, due to external load and the various unknowns acting at release sections, as well as pre-stress moments and forces, temperature, shrinkage and creep movements.

Space Frame [2]

At a release k in direction X, since continuity is sustained, the total virtual work done by $X_k^x = 0$. Restraint-deformation diagrams, such as $0\,L_0, L_1, L_2$ are assumed at critical sections, giving the typical virtual work compatibility equation.

$$\delta_k^{x0} + X_k^x \delta_{kk}^{xx} + \sum X_i^x \delta_{ki}^{xx} + \sum w_k^x \bar{s}_l^x + \int w_k^x (s_{sc}^x + s_T^x)\, d s + (\bar{a}_k^x + a_{ksc}^x + a_{kd}^x) = 0, \quad (1)$$

where

$$\delta^{xx} = \int w_k^x w_i^x e^x\, d s.$$

For each release section, there is an equation, such as (1) above. To determine the restraint distribution at any stage of load, values of X must be found to satisfy the equations and so that all inelastic deformations are compatible with the resultant restraint values at the various critical sections. For repeated or reversed loading, a step by step calculation would be necessary and a diagram giving inelastic deformations (say for bending, ref. Fig. 1) ψ and θ' at each step. It would only be possible to obtain a solution in very simple cases. In evaluating the integrals of Eq. (1), care must be taken to ensure that restraints and deformations are of the same category in regard to direction. This may be done with the aid of suitable diagrams [2].

Plane Frame

(Compatibility of bending deformation and restraint only considered)

In a plane frame, when all deformations except those due to bending are ignored, the general compatibility Eq. (1) reduces to:

$$\int \frac{M_i M_0}{E' I'} d s + \sum X_k \int \frac{M_i M_k}{E' I'} d s + M_i \theta_i' + \sum M_i \psi_{\bar{n}} = 0. \tag{2}$$

A tri-linear moment-rotation line, such as $0\,L_0, L_1, L_2$ is assumed. Applying Eq. (2) to the simple case of a three-span, symmetrically loaded, continuous beam having mid-span l and end-spans $k\,l$, if X_1 is the support bending moment due to inelastic rotations only, $\psi_{\bar{1}}$ and $\psi_{\bar{3}}$ the mid-span values of ψ, and θ' the support section value of θ'.

$$X_1 = \frac{6\,E'\,I'\,(\psi_{\bar{1}} + \psi_{\bar{3}} - 2\,\theta')}{l\,(2\,k + 3)}. \tag{3}$$

An equation obtained by MACCHI [1 and 3].

This is a typical, useful, simple equation for studying the influence of inelastic deformations on the distribution of bending moments in a three-span continuous beam bridge and applies at any stage of loading uniformly distributed throughout. Such an investigation is often well justified. Alternative distributions of live load in conjunction with Limits $0\,L_0\,L_1\,L_2$ and $0\,L_0\,L_1'\,L_2'$ should be considered.

Eq. (2) may be used for pre-stressed plane frames, if terms are included, when expanding Eq. (1) for deformations due to pre-stress, both for bending and axial forces. The pre-stress forces may be regarded as external loads acting on the frame made statically determinate by sufficient hinges. Thus for a hinge k:

$$\int \frac{M_k M_0}{E' I'} d s + X_k \int \frac{M_k M_k}{E' I'} d s + \sum X_i \int \frac{M_k M_i}{E' I'} d s + \sum M_k \psi$$
$$+ \int \frac{M_k M_p}{E' I'} d s + \int \frac{N_k N_0}{E' A} d s + X_k \int \frac{N_k N_k}{E' A} d s + \sum X_i \int \frac{N_k N_i}{E A} d s$$
$$+ \sum N_k \bar{s}_l + \int \frac{N_k N_p}{E' A} d s = -\theta_k'. \tag{4}$$

Eq. (4) is greatly simplified if axial deformations are ignored and sway is assumed to be negligible, and the cracked value of flexural rigidity EI, based on actual curvature, is used. The Eqs. (6), (7) and (8) below may then be applied to obtain compatibility of rotation and moment at the plastic hinges adjacent to each joint, positioned as in Fig. 2.

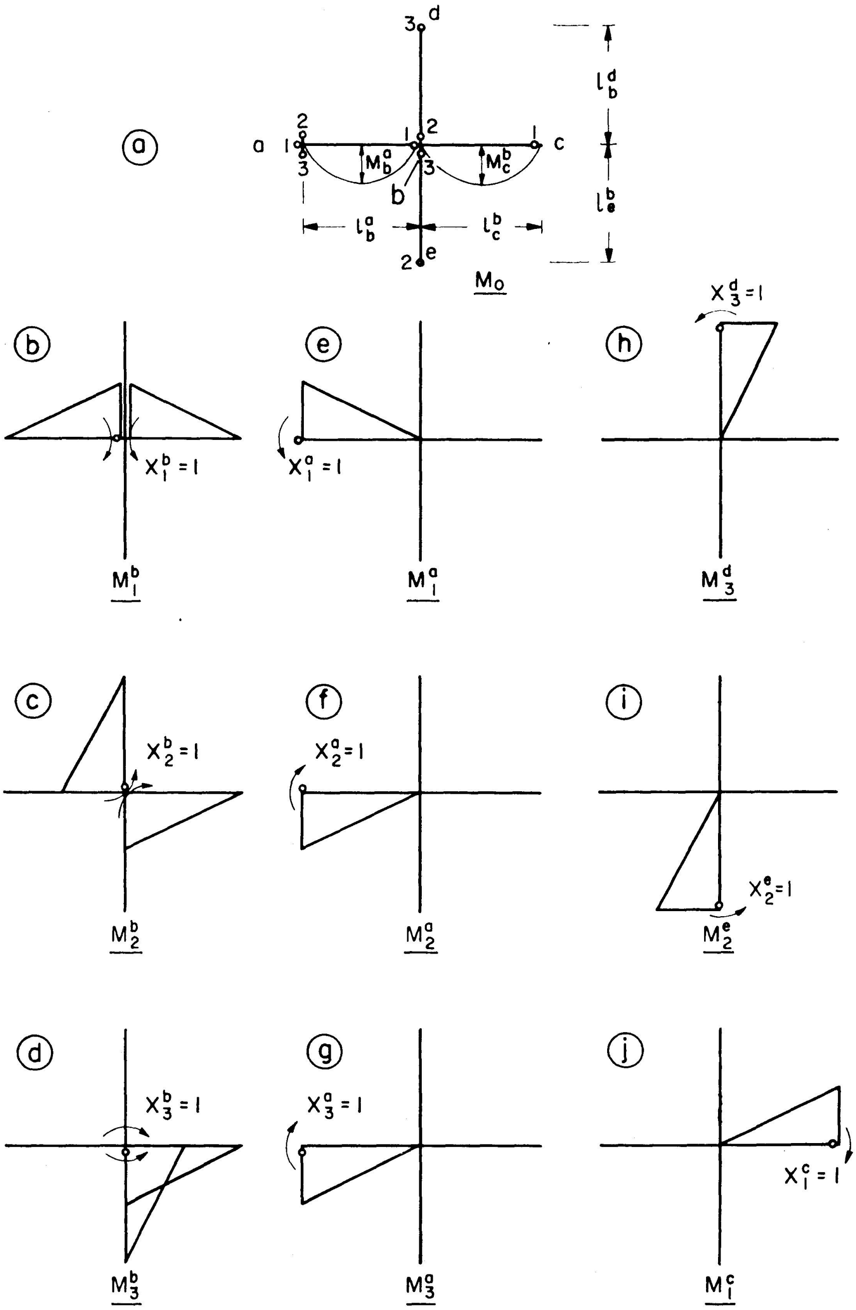

Fig. 2. Typical Joint and Hinges in a Building Frame.

Plane Frame — Simplified Limit Method

When the simplified limit line $0\,L_1, L_2$ is assumed, Eq. (4) becomes:

$$\int \frac{M_i M_0}{E I} d s + \sum X_k \int \frac{M_i M_k}{E I} d s + \theta_i = 0. \tag{5}$$

Using Eq. (5), a frame can be designed by trial and adjustment, by assuming values of X_k at all release sections, designing intermediate sections so that the bending moment value at L_1 is not exceeded and checking that θ_i has the correct sign and is not excessive in value.

In many cases of building frames, it may be an advantage to assume, first, release hinge sections and X values, and to separate the X values and loads into two parts, i.e. 1. vertical load and corresponding X values, 2. sway load and corresponding X values. It might then be apparent that X values and θ values, due to sway load and sway caused by lack of symmetry, were negligible. Adjustment to obtain compatibility in the equations for the three hinges adjacent to each joint may then readily be made, considering each joint in turn separately, and adjusting θ values by varying only the X values at the hinges adjacent to that joint, thus giving minimum interference to compatibility at other joints. Since frame members are assumed to be elastic between hinge sections, loadings and resulting bending moments may be separated into two or more cases, assuming the same set of hinges, and the resultant moments and rotations at hinge sections for the various cases checked and adjusted for compatibility. Thus, if sway moments and rotations are not negligible, additional X and θ values for sway could be included, and a check made to ensure that a permissible value of θ at each hinge section for total loads and X values was not exceeded.

Neglecting sway and assuming a bi-linear moment-rotation diagram, gives the following compatibility equations from Eq. (5) for the three hinge sections at a joint (ref. Fig. 2). Hinges 1, 2, 3 form at each joint *a*, *b*, *c*, *d*, *e*.

X_1^b = value of the bending restraint at joint b hinge 1.
M_0 = the bending moment caused by external load (see Fig. 2a).
θ_1^b = inelastic rotation at hinge b_1.

$$\frac{l_b^a}{6\,E\,I_b^a}(-2\,M_b^a + 2\,X_1^b + X_1^a - X_2^a - X_3^a) + \frac{l_c^b}{6\,E\,I_c^b}(-2\,M_c^b + 2\,X_1^b - 2\,X_2^b - 2\,X_3^b + X_1^c) = -\theta_1^b, \tag{6}$$

$$\frac{l_c^b}{3\,E\,I_c^b}\left(M_c^b - X_1^b + X_2^b + X_3^b - \frac{X_1^c}{2}\right) + \frac{l_b^d}{3\,E\,I_b^d}\left(X_2^b - \frac{X_3^d}{2}\right) = -\theta_2^b. \tag{7}$$

$$\frac{l_c^b}{3\,E\,I_c^b}\left(M_c^b - X_1^b + X_2^b + X_3^b - \frac{X_1^c}{2}\right) + \frac{l_e^b}{3\,E\,I_e^b}\left(X_3^b - \frac{X_3^e}{2}\right) = -\theta_3^b, \tag{8}$$

Adjustments to θ_1^b would be made by adjusting the X^b terms only. θ^a and θ^c terms would then be least affected and would probably remain within permissible limits, no further adjustments being required at joints *a* and *c*. The above equations may be used for fairly close ultimate solutions of pre-stressed frames, using cracked EI values based on curvature which allow for the influence of the pre-strain in the cables on the effective strain and hence on the position of the neutral axis. A similar method has been proposed by GUYON [3]. Permissible θ values, particularly when closed binding is used, at critical sections alow considerable variations of X values in the equations, so that adjustment to achieve compatibility at each hinge section need not be difficult.

By suitable detailing, the inelastic rotations in the beams may be split and hinges positioned, each side of the columns thus sometimes reducing the θ value for the column hinges and increasing the total permissible value of θ for the beam. It is important to do this in cases in which loading on columns is close to being symmetrical (ref. Fig. 3).

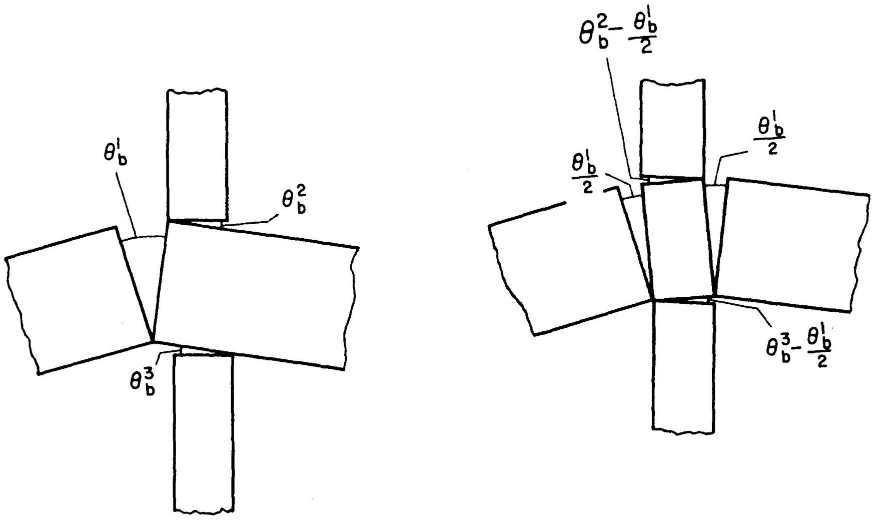

Fig. 3. Rotations at a Joint.

Practical Design

Computation Curves and Simple Design Formulae

Electronic computor solutions using tri-linear limits as in Fig. 1 are being attempted to check simplified limit theory. Some manually prepared computation curves which readily give rotation values at typical plastic hinges in typical frames, have been published [5, 6, 7]. Such curves may be used to check hinge rotations or as a guide in preliminary design towards adopting sections of suitable stiffness to avoid excessive rotations. Another promising aid to practical design is the use of rules for common structural forms, as given in [1] (reply to discussion). This rule, for continuous beams of four or more spans, defines the limits of variation of span, ratio of live to dead load, slenderness ratio and diameter of main reinforcement for which the support bending moments may be assumed equal to adjacent mid-span bending moments without risking unsatisfactory serviceability. The rule is developed from formulae for permissible values of θ [1] and the C.E.B. formula for crack width [8]. The criteria for design are: attainment of ultimate resistance of sections without excessive hinge rotations, for the worst distributions of live load, using a load factor of 2 for dead plus live load, and limited crack widths at working load to ensure satisfactory serviceability in a moderately corrosive atmosphere. The rule is very easy to apply and demonstrates that savings can be made in the reinforcement at supports, the area required being 15% to 20% less than when elastic theory is applied to the various possible distributions of live load. The elastic calculations give higher bending moments at the supports for the worst distributions of live load and little reduction at mid-span.

The simple design rules of British, Danish and Russian regulations [1] permit the transfer of bending moments between support and mid-span sections up to specified limits. Such rules are easy to apply, but a check should be made in regard to rotation values at ultimate load and crack widths or steel stress at working load for transfers over 15%, particularly if the live loads are high in relation to dead load and likely to be applied. The limiting depth of neutral axis generally specified provides some safeguard.

The various design rules for wind stress calculations in building frames, based on points of contraflexure at mid-height of the columns, it is often not realised depend for their validity in many cases on the redistribution of bending moments caused by inelastic deformation as ultimate load is approached. When further information on the moment-rotation characteristics of columns is available, it will be interesting to see to what extent this frequently made assumption is valid even by ultimate criteria.

Criteria for Optimum Design

Since the relative stiffness of members can be varied and the ultimate strength of critical sections by adjusting the reinforcement area, the distribution of bending moments in a frame can be controlled to give an optimum design for specific criteria at ultimate or working load. This problem has been studied and useful proposals made by Cohn and Tichy [1, discussion] and Burnett [9]. Attempts have also been made [14] to produce expressions for optimum design suitable for computor programmes, based on a given ratio of cost of steel to concrete. In a building frame, there are many factors to be considered. It is generally an advantage to transfer bending moments from supports to mid-span. Steel can be saved, congestion of steel in the top of the beam is avoided, risk of excessive cracking and deflection at mid-span is reduced. The transfer, however, must be limited, so that excessive rotations at ultimate load and excessively wide cracks or steel stress at working load are avoided. The requirements of shear resistance and architectural considerations such as maximum head-room have to be considered. Saving in the cost of formwork by maximum re-use is often much more important than optimum relative stiffness values for uniform Load Factor Values. Optimum design calculations are interesting and may give guidance, but designers need simple design rules as an aid in evolving sound economic designs which satisfy many different requirements, such as those which have been mentioned.

Moment Rotation Characteristics

One of the tasks of the C.E.B. Commission for Hyperstatism has been the co-ordination of tests to determine moment-rotation characteristics for typical frame members. This work is not yet complete, but it would appear that the bi-linear and tri-linear curves of Fig. 1 may provisionally be defined, as follows [1]:

Curve	Point	Concrete	Steel
$0 L_0\ L_1\ L_2$	L_0	Ultimate tension	
	L_1	0.002 strain (over-reinforced)	strain at 0.001 offset strain (under-reinforced)
	L_2	0.0035 strain	mean of ultimate and stress L_1 (failure by steel)
$0 L_0\ L_1'\ L_2'$	L_1'	M for L_1 plus 10%	
	L_2'	M for L_2 plus 20%, θ for L_2 plus 50%	

Bending moment calculations are based on the usual assumptions [1, 10]:

1. Concrete stress is parabolic — maximum stress cylinder strength $(m-k\,0)$.
2. Strain distribution across sections is linear.

Permissible values of θ (simplified limit method) are calculated from the following formulae which may require slight amendment when more research results are available [1]:

$$\theta_p = \frac{e_{cu}-e_{ce}}{n_u d}\, l_p \text{ (tension occurs)},$$

$$\theta_p = \frac{e_{cu}-e_{ce}}{d}\, l_p \text{ (no tension)},$$

$$e_{ce} = 0.002 \text{ (or less in under reinforced cases)},$$

$e_{cu} = 0.0035$ unbound concrete,

$e_{cu} = 0.012$ well-bound concrete ([5], for greater detail),

$l_p = k_1 k_2 k_3 \left(\frac{z}{d}\right)^{\frac{1}{4}} d,$

$k_1 = 0.7$ mild steel 0.9 cold worked steel,

$k_2 = \left(1 + 0.5 \frac{P}{P_u}\right)$, where P_u = ultimate axial load no bending present,
P = ultimate axial load bending present,

$k_3 = 0.6$, when $c_u = 6{,}000$ p.s.i.,
$= 0.9$, when $c_u = 2{,}000$ p.s.i. — other values by proportion.

Security

Stress or strain values at critical sections under working load give a good indication of the serviceability of a structure and its potential durability, but its security or lack of risk of failure is more logically and reliably expressed by a Load Factor of Safety defined as the ultimate strength divided by the working load. The principal reason for this is, that stresses at critical sections often do not increase in proportion to load often more rapidly, due to such inelastic influences as cracking particularly in pre-stressed members, buckling, yield of reinforcement and plastic deformation of concrete. The latter two influences may be used to advantage by allowing plastic hinges to form before failure occurs, so that a better distribution of bending moment may be obtained which facilitates detailing and produces saving of reinforcement.

The ultimate strength of a structure can only be calculated within limits for a given load distribution, but the possible variations of stress due to working load can only be predicted, as a rule, within very broad limits. The determination of a Load Factor value, providing an adequate safeguard against collapse of a structure, therefore requires statistical considerations. This does not mean that design calculations can be made entirely on a statistical basis. It is reasonable to relate strengths assumed in calculations to a statistical appreciation of test results and to adopt near-minimum values likely to be realised only on, say, one in 100 occasions in practice. Similarly, maximum working loads may be assumed which are not likely to be exceeded on more than one in N occasions where N is a member appropriate for the structure. On such a basis of design, adopting a Load Factor of only one and assuming $N = 100$ and that nothing abnormal happens, between one in 100 and one in 4,000 structures would fail, depending on the statistical distribution of the variables.

If however a load factor of 2 is used, and calculations allow for such things as buckling, relative settlement, and weakening by fatigue, the probability of failure is reduced to 1 in a very high number which cannot be calculated, but experience shows that a reasonable margin of security is provided. The standard of quality required by such a margin is sufficient to ensure that excessively weak material can easily be detected and rejected by inspectors, generally an excessive load would be obviously dangerous and therefore avoided. This situation alone helps to provide security, but cannot be treated statistically. Failure of a correctly designed structure having a load factor of 2 could therefore only be caused by some abnormal condition, such as omission or wrong placing of reinforcement or by an unpredicted settlement, or other calamity which could not necessarily be avoided, even by the use of a higher value of the Load Factor.

If a general value of 2 is adopted for the Load Factor of structures whose collapse would be calamitous, it is reasonable to reduce the value for structures made under excellent quality control and whose failure would not be at all serious. It is reasonable for Load Factor values to range from 1.25 for the latter type of structure, up to 2 for the former. The appropriate value in a given case can partly be considered in terms of probability of failure as influenced by quality control, load control, efficiency of maintenance, etc., and partly in terms of the seriousness of failure. Simple procedures for dealing with this problem are given in [1, 5, 10]. It is neither possible nor desirable to base designs on an acceptable failure rate of, say, 1 in X, where X is a very high number. Failure of Civil Engineering structures must be avoided altogether as far as is humanly possible, but a statistical consideration of parts of the problem, such as in determining permissible design strengths, is possible and is a help, and encourages reliable quality control and a rational treatment of the assessment of the loading to be assumed in the calculations. The statistical concept also assists in evaluating the degree of approximation required in design calculations. In a statically indeterminate frame, not only does the variation of ultimate strength of critical sections vary the ultimate strength of the whole structure, but the distribution of inelastic deformations influencing the distribution of bending moments axial and shear forces in conjunction with the distribution of live load has a very great influence, but, provided parts of the frame are not brittle, adjustments of stress distribution due to this cause are likely to be to advantage. When the moment-rotation characteristics $0\,L_1, L_2$ (Fig. 1) are used as a basis of Limit design, it can be shown that the errors of calculation are on the safe side, except in the most extreme cases; even then, they are not serious [3, 11].

Frame Instability [1, 5]

There appears to be no direct method of dealing with this problem. The Limit Method of design, based on the line $0\,L_1, L_2$ (Fig. 1) may be used to avoid frame instability at the statically determinate stage of plastic hinge formation, i.e. at the assumed ultimate stage. A building frame is designed so that plastic hinges develop, as shown in Figs. 1 or 4. The column hinges then isolate each storey, so that at each column hinge, the axial force and shear are known, and the bending moment is limited by the hinge to the L_2 value. One corner in each storey, for example the outside corner, is designed to remain elastic when the bending moment value has reached L_2 at all hinge sections. This ensures the lateral stability of each storey. Since the forces acting on each storey are known, if the sway and buckling deflections of the columns are assumed, the bending moments acting — including those due to

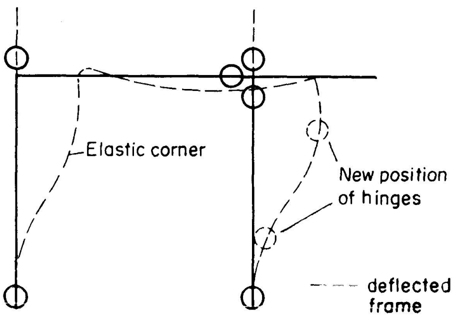

Fig. 4. Deformation of Unstable Frame.

vertical load in conjunction with sway, buckling deflection and deflection due to hinge rotation — are then known; hence, the sway, buckling deflection and hinge rotation deflection can be calculated. If the calculated values prove to be less than the assumed values, the columns of that storey are stable when all hinge sections have developed bending moment values at L_2, since all members between hinges are designed to have bending moments equal to, or less than, values at L_1 at the ultimate stage. The resultant bending moments in the columns may have maximum values at the quarter points or close to them, due to buckling (ref. Fig. 4). These hinge positions then must be assumed and bending moment diagrams plotted accordingly in the usual process of determining the terms of the compatibility equations. For compatibility in the usual way, calculated rotations of hinges must have the correct sign and not exceed the permissible value of θ for the section.

It has been suggested that failure could occur due to instability before all plastic hinges for the theoretical, statically determinate but stable stage has been reached. This would not appear to be possible since vertical loads are assumed to act at safe limiting values of eccentricity and the L_2 values, for which the column hinges are designed are limiting values of bending moments which are transmitted at each column hinge level down the frame. A step by step consideration of the formation of plastic hinges under increasing load of constant distribution would show that hinges must eventually form at sections which satisfy the compatibility equations, provided the frame members have the moment rotation characteristics assumed.

Alternative Loadings and Reversible Hinges

If several different distributions of live load may act on a frame, or due to unusual design details and combinations of vertical and sway load, it is possible for plastic deformation to occur at low load at a particular section in the reverse direction to that ultimately followed, then, for precise calculation, it would be necessary to carry out a step by step calculation, based on tri-linear restraint-deformation diagrams for each distribution of live load. At each step, the inelastic deformation assumed at any critical section, and included in the compatibility equations, would need to be compatible with the resultant restraint at that section. Either a tri-linear or bi-linear restraint-deformation diagram would only be displaced a small amount by residual plastic deformations caused by reversal in the early stages of direction of restraint, which would only have a small influence on ultimate strength.

When a "probability of failure" concept of the design problem is adopted, a satisfactory criterion of safe design for various possible loadings would appear to be, after investigating each significant distribution of live load, to design all sections of the frame at least as strong as is required by the most critical distribution of load for that section. The actual frame theoretically could be weakened for a particular live load distribution by strengthening some sections as required for other distributions of live load and so altering relative stiffness values. It is very unlikely to happen and the designer must avoid the use of brittle sections and unless he is prepared to keep repeating analyses for every case of load, judge whether the ultimate strength could seriously be reduced in this way. It must be remembered that, while the positions of hinges and ultimate inelastic rotations may be considerably altered in value, the corresponding variation of ultimate restraint value is small, since the restraint-deformation curve only has a small variation of restraint value beyond L_1 for relatively large deformations.

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Summary

The paper reviews contemporary approaches to Ultimate Load Design of frames and discusses differences of opinion of designers in regard to the relative importance of accuracy in providing satisfactory serviceability or an adequate margin of safety against collapse. General compatibility equations for space frames which apply at all stages of load and include terms for inelastic deformations are given, together with simplified forms suitable for practical design. The problem of optimising designs based on ultimate criteria is discussed, and a method of designing when frame instability is involved. The assessment of Safety Factor values, partly based on considerations of probability of failure and statistical treatment of test results, is briefly discussed.

Résumé

L'auteur discute le calcul à la rupture des ossatures en béton; il traite également les problèmes relatifs à la sécurité, à la ruine et au comportement en service. Pour des ossatures tridimensionnelles, il propose des équations générales de compatibilité, s'appliquant à tous les niveaux de charge et tenant compte des déformations non élastiques; il les présente sous une forme simplifiée pour le calcul pratique. Il évoque le problème du dimensionnement optimum à partir des critères de rupture et donne une méthode de calcul lorsqu'il faut tenir compte d'une instabilité de l'ossature. Pour terminer, l'auteur discute brièvement la détermination des coefficients de sécurité, en partie basée sur les probabilités de rupture et sur l'exploitation statistique des données fournies par les essais.

Zusammenfassung

Die Bemessung von Rahmen nach dem Traglastverfahren wird besprochen. Weiter werden Fragen der Bruchsicherheit und des Verhaltens im Gebrauchszustand behandelt. Es werden allgemeine Verträglichkeitsgleichungen für räumliche Rahmen entwickelt, die für alle Laststufen anwendbar sind und Glieder für unelastische Formänderungen enthalten. Gleichzeitig werden vereinfachte Ausdrücke für die praktische Bemessung angegeben. Der Autor behandelt das Problem der Optimalisierung, die sich auf Traglastkriterien stützt, und gibt eine Bemessungsmethode für den Fall, daß die Stabilität des Rahmens in Betracht gezogen werden muß. Schließlich wird kurz auf die Festlegung von Sicherheitswerten eingegangen, die teilweise durch Betrachtungen über die Wahrscheinlichkeit des Versagens und eine statistische Behandlung von Versuchsergebnissen bestimmt werden.