Analysis of structural nets

Analysis of Structural Nets

Etude des réseaux de câbles

Untersuchung von Netzwerken

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Introduction

A network of high strength cables, if properly utilized, can offer an exceptionally efficient solution to many problems confronting the structural engineer now and in the future. A prime example is that of covering large areas such as urban sections or even cities for protection from atmospheric conditions. A more common situation is the need for an economical roof system for large structures such as stadiums, arenas and shopping centers. The structural net is a system which allows the ingenious designer to achieve aesthetically pleasing solutions to problems of large space enclosure. As a discrete system, the net is easily modified in design to control the behavior of its elements, e.g., by inserting a heavier cable for the thread which must sustain the larger forces. When used for relatively small roofs, more care is required lest wind flutter create difficulties.

The series of complete analyses of a structural net, required for a general design, may suggest the solution of a number of discrete field problems in the subject areas of statics, mechanics of materials and dynamics. This study is devoted to a closed form treatment of some of the basic second order field problems in the first area, statics. Specifically, the equilibrium displacement field will be determined for a general node loading on a number of different net configurations. This is the two dimensional analogue of the classical string polygon problem. In all cases, the node loads will have only components perpendicular to the net's reference plane and the configuration will be such that the in-plane equilibrium equations are satisfied identically. The principal function to be determined, with no restrictions as to magnitude, will be the out-of-plane displacement consistent with equilibrium conditions for a given final field of cable tensions. The solutions will serve a number of useful purposes, including use: 1. as a valid elasticity solution if deflections are small so that changes in cable tensions are negligible; 2. to determine load intensity for simultaneous yielding of all cables for small deflections; 3. as the key relations, from statics, for an iterative solution to the mechanics of materials problem of large deflections under large load changes; 4. to establish design shape for rigid lattices subject to the same load shape; and 5. as solutions to other mechanics problems with same or similar mathematical models.

String Polygon Equation

A discrete field problem of one dimension, useful as an intermediate step in dealing with the net problem, is that of finding the equilibrium position of a string polygon for parallel forces. An element of such a polygon is shown in Fig. 1.

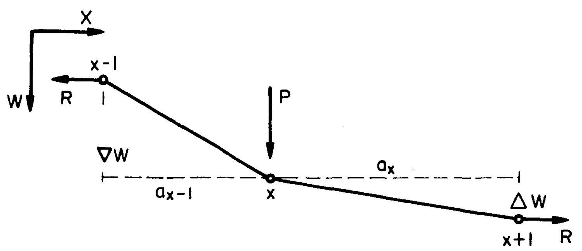

Fig. 1. Polygon element.

The governing equation for $W(x)$ results from summing forces normal to the reference line and utilizing the fact that the string has no shear component.

$$R\left[\frac{1}{a}\Delta W\right] - R E^{-1}\left[\frac{1}{a}\Delta W\right] = -P, \tag{1}$$

$$R\nabla\left[\frac{1}{a}\Delta W\right] = -P, \tag{2}$$

where R is the constant reference line component of string tension; $W(x)$ and $P(x)$ are the deflection and load, respectively, at node x perpendicular to reference line in the sense shown; and E, Δ, ∇ are, respectively, the displacement, forward difference, and backward difference operators; and $a(x)$ is the distance along the reference line between nodes x and $x+1$. For a constant, equally spaced loads, Eq. (2) becomes

$$\frac{R}{a}\Delta\nabla W(x) = -P(x), \tag{3}$$

where $\Delta\nabla$ denotes the product of the forward and backward differences or the second central difference operator, i.e., $\Delta\nabla = E - 2 + E^{-1}$.

Doubly Threaded nets

Consider the doubly threaded net shown in Fig. 2. The projections of the two sets of cables on the reference plane intersect at a constant angle, ω. These traces are selected as the coordinate reference lines. The plane components of cable tensions in the x and y cables are $R(y)$ and $S(x)$ respectively and the elemental lengths are $a(x)$ and $b(y)$. Then, for an out-of-plane node load function, $P(x,y)$, one can write the general governing partial difference equation by summing normal forces as in the case of the string polygon.

$$R(y)\,\nabla_x\left[\frac{1}{a(x)}\Delta_x W(x,y)\right] + S(x)\,\nabla_y\left[\frac{1}{b(y)}\Delta_y W(x,y)\right] = -P(x,y), \tag{4}$$

where Δ and ∇ are the forward and backward difference operators, e.g., $\Delta_x F(x,y) = F(x+1,y) - F(x,y)$ or $\nabla_y F(x,y) = F(x,y) - F(x,y-1)$. The indicated limitations, on variables with which the parameters may vary, are necessary for in-plane equilibrium. For the net with regular geometry, a and b constant, the equation becomes:

$$\left[\frac{R(y)}{a}\Delta\!\!\!\!\nabla_x + \frac{S(x)}{b}\Delta\!\!\!\!\nabla_y\right] W(x,y) = -P(x,y), \tag{5}$$

where the symbol, debla (a contraction of delta and nabla), denotes second central difference with respect to the variable shown as an index, e.g., $\Delta\!\!\!\!\nabla_x F(x,y) = F(x+1,y) - 2\,F(x,y) + F(x-1,y)$. The case of interest here is the net with uniform cable tensions, for which the model is:

$$[\Delta\!\!\!\!\nabla_x + \epsilon^2 \Delta\!\!\!\!\nabla_y]\, W(x,y) = -\frac{a}{R} P(x,y), \tag{6}$$

where $\epsilon^2 = \frac{S\,a}{R\,b}$. As the mathematical model is linear, one would hope that a solution could be found for the arbitrarily placed single load, i.e., a Green's function (see Appendix B), $K(x,y,\alpha,\nu)$, expressing the displacement at node x,y due to a unit load at node α,ν.

$$[\Delta\!\!\!\!\nabla_x + \epsilon^2 \Delta\!\!\!\!\nabla_y]\, K(x,y,\alpha,\nu) = -\frac{a}{R}\delta_x^\alpha \delta_y^\nu . \tag{7}$$

Solution for Quadrilateral Boundary Parallel to Cables

It is pertinent to the solution of the problem to be aware of a useful property of trigonometric functions as related to finite difference operators; namely, operation on a sine or cosine function (valid also for sinh and cosh) with a symmetric finite difference operator (one which can be written using only even differences), yields an expression containing that function as a factor if the independent variable is linear in the argument, e.g.,

$$(\Delta\!\!\!\!\nabla_x + 2)\sin\varphi x = 2\cos\varphi \sin\varphi x. \tag{8}$$

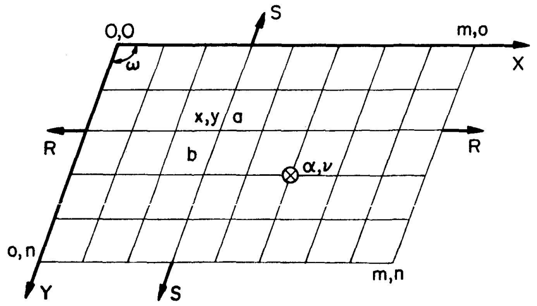

Fig. 2. The doubly threaded net with basic boundary.

This property leads one to consider use of a series of such functions as a closed from solution for (7); however, two theoretical points need to be taken into account before such a step is taken. First, does a unique solution exist? The answer is yes. In standard treatments of partial difference Eqs. [1, 3, 4], a study of the possibility of "walking through" solutions is used to demonstrate that, for a class of incomplete second order difference equations including (7), a unique solution exists when values of the dependent variable are given on a closed boundary. Second, can a trigonometric series be used to represent such a solution for a general loading? Again, the answer is yes, for a useful, if restricted, class of boundary conditions. From a study of finite half range Fourier Series (employing techniques closely paralleling those in the continuum [5]), it can be shown that one and two dimensional discrete fields, arbitrary at the interior nodes, can be represented as follows:

$$F(x) = \sum_{i=1}^{m-1} A_i \sin i\pi \frac{x}{m}, \qquad 1 \leqq x \leqq m-1 \tag{9}$$

$$F(x,y) = \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} A_{ij} \sin i\pi\frac{x}{m} \sin j\pi \frac{y}{n}, \qquad \begin{matrix} 1 \leqq x \leqq m-1 \\ 1 \leqq y \leqq n-1 \end{matrix} \tag{10}$$

where the Euler coefficients can be found from:

$$A_i = \frac{2}{m}\sum_{x=1}^{m-1} F(x) \sin x\pi \frac{i}{m} = \frac{2}{m} \Delta_x^{-1} F(x) \sin x\pi \frac{i}{m}\Bigg|_1^m, \tag{11}$$

$$\begin{aligned} A_{ij} &= \frac{4}{mn}\sum_{x=1}^{m-1}\sum_{y=1}^{n-1} F(x,y) \sin x\pi\frac{i}{m} \sin y\pi\frac{j}{n} \qquad \text{or} \\ &= \frac{4}{mn} \Delta_x^{-1}\Delta_y^{-1} F(x,y) \sin x\pi \frac{i}{m} \sin y\pi\frac{j}{n}\Bigg|_{1,1}^{m,n}. \end{aligned} \tag{12}$$

As a first case, consider the net shown in Fig. 2 subject to the boundary condition, zero displacement, i.e.,

$$K(0,y,\alpha,\nu = K(m,y,\alpha,\nu) = K(x,0,\alpha,\nu) = K(x,n,\alpha,\nu) = 0. \tag{13}$$

This problem is solved by writing the solution and the loading in the double series form (10), i.e., take

$$\delta_x^\alpha \delta_x^\nu = \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} B_{ij} \sin i\pi\frac{x}{m} \sin j\pi \frac{y}{n}, \tag{14}$$

then, by use of (12), the coefficients are found to be

$$B_{ij} = \frac{4}{m\,n} \sin i\,\pi \frac{\alpha}{m} \sin j\,\pi \frac{\nu}{n}, \tag{15}$$

$$K(x, y, \alpha, \nu) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} A_{ij} \sin i\,\pi \frac{x}{m} \sin j\,\pi \frac{y}{n}, \tag{16}$$

then, substituting (14) and (16) into (7) and matching completes the solution by yielding the coefficients in (16) as:

$$A_{ij} = \frac{a}{R\,m\,n} \frac{\sin i\,\pi \frac{\alpha}{m} \sin j\,\pi \frac{\nu}{n}}{\sin^2 \frac{i\,\pi}{2\,m} + \epsilon^2 \sin^2 \frac{j\,\pi}{2\,n}}. \tag{17}$$

Formulas (16) and (17) comprise a general solution to the stated problem. Solutions for specific loading conditions can be determined by either formal or numerical superposition of quantities determined by using it. For example, consider the case of the same net subject to a uniform node load of P_0.

$$W(x, y) = P_0 \sum_{\alpha=1}^{m-1} \sum_{\nu=1}^{n-1} K(x, y, \alpha, \nu) = P_0 \Delta_\alpha^{-1} \Delta_\nu^{-1} K(x, y, \alpha, \nu)\big|_{1,1}^{m,n}. \tag{18}$$

The result of performing the indicated operations formally is:

$$W(x, y) = \frac{a}{R\,m\,n} P_0 \sum_{i}^{m-1} \sum_{j}^{n-1} \frac{\cot \frac{i\,\pi}{2\,m} \cot \frac{j\,\pi}{2\,n}}{\sin^2 \frac{i\,\pi}{2\,m} + \epsilon^2 \sin^2 \frac{j\,\pi}{2\,n}} \sin i\,\pi \frac{x}{m} \sin j\,\pi \frac{y}{n}, \tag{19}$$

where the indices assume only odd values, i.e., $i, j = 1, 3, 5, \ldots$

While convergence of a finite series is not of formal interest, the subject is of practical concern due to its effect on the volume of computations. By comparing (15) and (17), it can be seen that, as contrasted with experience in the continuum, the convergence of the solution series is no more rapid than for the load series. The authors' have found that, for a highly irregular loading such as the single impulse load, one might need all terms of the series to obtain engineering accuracy. If the loading is "smooth", as in (19), there will be useful convergence (due to cotangent shrinking as the argument approaches $\pi/2$) so that terms with the higher indices can be discarded. Even so, many terms may be needed to get the desired accuracy. This is of little consequence if an automatic computer is available, but is inconvenient when dealing with fine nets by manual means. This difficulty is overcome (at some sacrifice in programming ease) by use of a single series solution form.

As a second case, of a doubly threaded net with basic boundary, consider the net with zero displacements along two parallel segments of the boundary and unspecified conditions elsewhere. A single series is indicated. Specifically, the partial statement of boundary conditions is:

$$K(0, y, \alpha, \nu) = K(m, y, \alpha, \nu) = 0, \tag{20}$$

$$\delta_x^\alpha = \frac{2}{m} \sum_{x=1}^{m-1} \sin i\pi\frac{\alpha}{m} \sin i\pi\frac{x}{m}, \tag{21}$$

$$K(x, y, \alpha, \nu) = \sum_{i=1}^{m-1} Y_i(y) \sin i\pi\frac{x}{m}. \tag{22}$$

Eq. (21) is a result of using formula (11). Substitution of (21) and (22) into (7) and matching coefficients yields the governing difference equation for the set of discrete functions $Y_i(y)$.

$$[\Delta_y - 2\beta_i] Y_i(y) = -C_{0i}\delta_y^\nu, \tag{23}$$

where $\quad \beta_i = 2\epsilon^{-2}\sin^2\frac{i\pi}{2m}, \ \beta_i > 0, \quad$ and $\quad C_{0i} = \frac{2b}{Sm}\sin i\pi\frac{\alpha}{m}.$

for $Y_i(y)$, as shown in Appendix B, is:

$$Y_i(y) = C_{0i}\left[C_{1i}\sinh\gamma_i y + C_{2i}\cosh\gamma_i y - \frac{\sinh\gamma_i(y-\nu)}{\sinh\gamma_i} \not{U}(y-\nu)\right], \tag{24}$$

where $\quad \cosh\gamma_i = \beta_i + 1 \ \left(\text{or } \sin\frac{i\pi}{2m} = \epsilon\sinh\tfrac{1}{2}\gamma_i \text{ for above } \beta_i\right).$

The solution (22), (24) is the form one gets via the separation of variables approach and more "natural" than the double series solution, (16), (17). In addition to having far fewer terms than the double series, it "converges" so that higher index terms can be discarded consistent with engineering accuracy. It is a convenient form for manual computation.

The sets of constants C_{1i} and C_{2i} can be determined from the conditions along $y=0$ and $y=n$. For example, if boundary conditions (13) are imposed on the solution (22), (24), the result is:

$$K(x, y, \alpha, \nu) = \sum_{i=1}^{m-1} \frac{C_{0i}}{\sinh\gamma_i}\left[\frac{\sinh\gamma_i(n-\nu)}{\sinh\gamma_i n}\sinh\gamma_i y - \sinh\gamma_i(y-\nu)\not{U}(y-\nu)\right]\sin i\pi\frac{x}{m}. \tag{25}$$

As in case one, the general solution, (25), was summed to get the single series solution for a uniform node loading. The result is given below and may be compared with (19).

$$W(x, y) = \frac{2}{2Rm}P_0\sum_{i}^{m-1}\frac{\cot\frac{i\pi}{2m}}{\sin^2\frac{i\pi}{2m}}\left[1 - \frac{\cosh\gamma_i\left(y-\frac{n}{2}\right)}{\cosh\gamma_i\frac{n}{2}}\right]\sin i\pi\frac{x}{m} \quad (i \text{ odd}). \tag{26}$$

Formulas (25) and (26) are numerically illustrated at the end of the theoretical development.

Solution for Quadrilateral Boundary with Nodes

A useful doubly threaded net, with a configuration which apparently differs from the one shown in Fig. 2, is that shown in Fig. 3. Actually, only the slopes of the boundary lines are different. In this case, the boundary lines are in the direction of the diagonals of the elemental parallelograms rather than the sides as in the first case. For simple boundary equations, a transformation of coordinates is indicated. The first illustrative transformation in Appendix A permits the new boundary to be described by constant values of the coordinates. For a unit load at $r=\alpha$, $s=\nu$, and $\epsilon^2=1$, use of (65a, 65b) transforms the governing equation for the Green's function, (7), into the following form referred to the new coordinates:

$$[\Delta_r \Delta_s + 2(\Delta_r + \Delta_s)]\, K(r, s, \alpha, \nu) = -\frac{a}{R}\,\delta_r^\alpha\,\delta_s^\nu. \tag{27}$$

Eq. (27) is second order in r and s, but not all mathematical nodes have corresponding physical nodes. Real nodes exist only at the coordinates, $r+s$ even. The number of interior physical nodes is $\frac{1}{2}[(m-1)(n-1)+1]$. The solution at the virtual nodes, $r+s$ odd, may be disregarded.

As in case one, a solution is sought for the zero displacement boundary condition. This condition and the fact that the governing equation, (27), is composed of even differences suggest a double series solution form:

$$K(r, s, \alpha, \nu) = \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} A_{ij}\sin i\pi\frac{r}{m}\sin j\pi\frac{s}{n}. \tag{28}$$

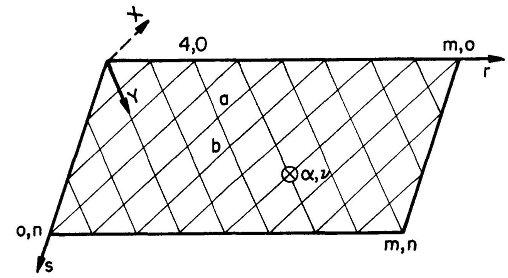

Fig. 3. The doubly threaded net with nodes on boundary.

As in the first case, the loading term is expressed in the same form and both series are substituted into (27) to determine the coefficients as:

$$A_{ij} = \frac{a}{R\,m\,n}\,\frac{\sin i\pi\frac{\alpha}{m}\sin j\pi\frac{\nu}{n}}{1-\cos\frac{i\pi}{m}\cos\frac{j\pi}{n}}. \tag{29}$$

Solution for Triangular Boundary

In this section, the two basic types of triangular boundaries for doubly threaded nets are considered. The first type, Fig. 4a, has two basic and one node type boundary segments. For the zero displacement boundary condition, the solution is easily written by use of the solution for zero displacement on the basic quadrilateral boundary. The principle of images is employed to create zero displacement along the node boundary segment ($y=x$ or $s=0$) by

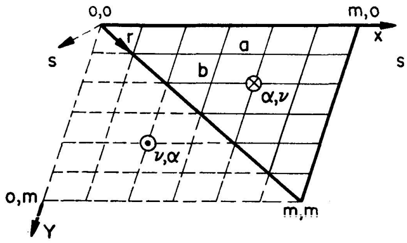

Fig. 4a.

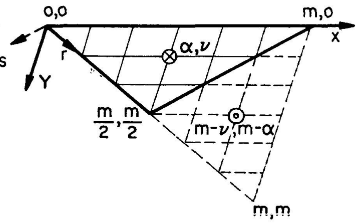

Fig. 4b.

The doubly threaded net with triangular boundaries.

making it a line of anti-symmetry in the $m x m$ basic boundary case. Solution (16), (17) or (25), Fig. 2, can be anti-symmetrically superimposed to give a Green's function for the case shown in Fig. 4a, i.e.,

$$K_{4a}(x,y,\alpha,\nu) = K_2(x,y,\alpha,\nu) - K_2(x,y,\nu,\alpha). \tag{28}$$

The second type, Fig. 4b, has two node and one basic type boundary segments. For the zero displacement boundary condition, one can write the general solution by anti-symmetrically superimposing the solution for the node boundary case, shown in Fig. 3, so as to make the displacements zero along the thread serving as a basic type boundary segment. This will not be shown in detail here. Instead, the solution for zero displacement on the boundary, shown in Fig. 4b, will be written in terms of the solution, for zero displacement on the basic boundary shown in Fig. 2, $K_2(x,y,\alpha,\nu)$. A single series solution, (25), is available for this purpose which does not require 1. $\epsilon = 1$, or 2. added terms resulting from virtual nodes (as did the alternative solution (28), (29)). The desired general solution, denoted $K_{4b}(x,y,\alpha,\nu)$, is obtained by repeated use of anti-symmetric superposition: once to obtain zero displacement along $y = x$ or $s = 0$; and once to obtain zero displacement along $y = m - x$ or $r = m$.

$$\begin{aligned} &K_{4b}(x,y,\alpha,\nu) = \\ &K_2(x,y,\alpha,\nu) - K_2(x,y,\nu,\alpha) + K_2(x,y,m-\alpha,m-\nu) - K_2(x,y,m-\nu,m-\alpha). \end{aligned} \tag{29}$$

An interesting single term solution is available for a special case of the net shown in Fig. 4b. The zero boundary condition is expressed as:

$$W(x,0) = W(x=y) = W(x+y=m) = 0. \tag{30}$$

This leads one to suspect that, for a uniform loading, i.e., $P(x,y) = P_0$ in (6), one might find a solution of the form,

$$W(x,y) = C_0\, y\,(x-y)\,(m-x-y), \tag{31}$$

which satisfies the boundary condition. Substitution into (6) shows that (31) satisfies that equation if

$$1)\ \epsilon = \frac{1}{\sqrt{3}} \quad \text{and} \quad 2)\ C_0 = \frac{3a}{2Rm} P_0 .$$

Triply Threaded Nets

The object of this portion of the paper is to determine the equilibrium displacement field for the triply threaded net, Fig. 5. As for the doubly threaded net, there are geometrical constraints and conditions on the variation of cable

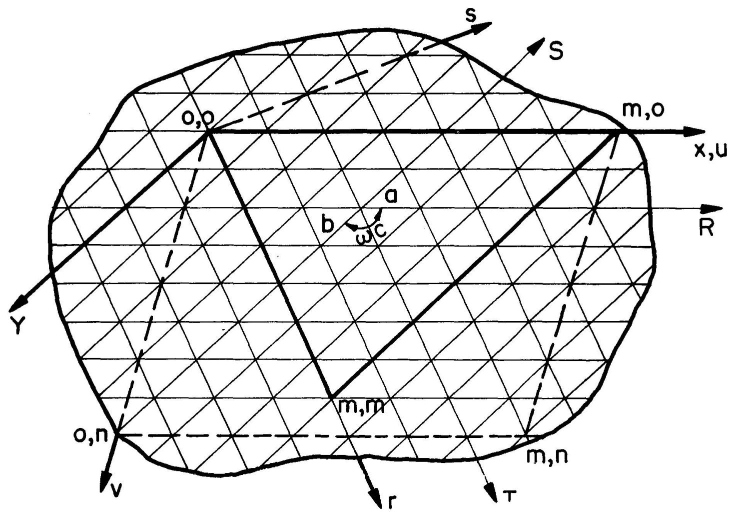

Fig. 5. The triply threaded net with various boundaries.

forces necessary for the identical satisfaction of the in-plane equilibrium equations:

$$\begin{aligned}
&\omega \quad \neq \omega(x,y); \quad a(x) \neq a(y); \quad b(y) \neq b(x); \quad c(r') \neq c(s'); \\
&R(y) \neq R(x); \quad S(x) \neq S(y); \quad T(s') \neq T(r'); \quad c^2 \quad = a^2+b^2+2\,a\,b\cos\omega,
\end{aligned} \tag{32}$$

where $r' = \frac{1}{2} r = \frac{1}{2}(x+y)$ and $s' = \frac{1}{2} s = \frac{1}{2}(x-y)$ and r, s are as shown in Appendix A and in the preceding section.

The variable coefficient form of the governing difference equation is similar to (2) and (4) and can be written by inspection.

$$R\,\nabla_x\left[\frac{1}{a}\Delta_x W\right] + S\,\nabla_y\left[\frac{1}{b}\Delta_y W\right] + T\,\nabla_{r'}\left[\frac{1}{c}\Delta_{r'} W\right] = -P(x,y), \tag{33}$$

where, it should be remembered, $E_{r'} = E_x E_y$.

Eq. (33) is still an incomplete second order difference equation and, as in the case of the doubly threaded net, three functions, or a specified closed boundary, are sufficient (and necessary) for a unique solution to exist. For the net with regular geometry, i.e., a, b and c constant, the equation is:

$$\left[\frac{R}{a}\Delta\!\!\!\!\nabla_x + \frac{S}{b}\Delta\!\!\!\!\nabla_y + \frac{T}{c}\Delta\!\!\!\!\nabla_{r'}\right] W(x,y) = -P(x,y). \tag{34}$$

Note that, even with R, S and T constant, the operator, in terms of the independent variables x and y, can not be written with even differences. Also, the separation of variables technique is not applicable.

For this net, there are three directions in which a straight boundary segment may run parallel to the cables and, therefore, qualify as a basic boundary segment. These three directions are also node boundary directions. A basic triangular boundary is shown in Fig. 5. For zero displacement on this boundary and a constant node loading, P_0, one is again tempted to seek a single term algebraic solution in a form which satisfies the boundary condition, i.e.,

$$W(x, y) = C_0 y (m-x)(x-y). \tag{35}$$

Substitution into (34) shows that for 1. $T/c = S/b = R/a$ and 2. $C_0 = \frac{a}{2Rm} P_0$, (35) is a solution for the uniformly loaded triply threaded net with zero displacement on the basic triangular boundary.

Solution for Quadrilateral Boundary with Nodes and Threads

Consider, now, the triply threaded net with the quadrilateral boundary shown in Fig. 5. The second coordinate transformation illustrated in Appendix A enables one to express this boundary by, $(0, v)$, (m, v), $(u, 0)$, (u, n). Of greater significance is the fact that this change of coordinates transforms the governing equation, (34), to a form with only even differences in the operator for the special case, $T/c = S/b$. Here again the transformation gives a mathematical model with nearly twice as many node points, corresponding to integer values of u and v, as exist in the physical model. The indicated transformation and substitution of an impulse load function yield:

$$[\Delta_u(\Delta_u + 4) + \epsilon^2(\Delta_u \Delta_v + 2\Delta_u + 2\Delta_v)]\, K(u, v, \alpha, \nu) = -\frac{a}{R}\delta_u^\alpha \delta_v^\nu, \tag{36}$$

where $\epsilon^2 = \frac{Sa}{Rb}$, an equation of fourth order in u and second order in v. The higher order is a result of the virtual nodes which, here, are such that an additional boundary condition must be specified along $(0, v)$ and (m, v), namely, the displacement of the threads cut by those boundary segments or, actually, conditions at the two real nodes it connects, For example, consider the suitability of a double sine series, i.e.,

$$K(u, v, \alpha, \nu) = \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} A_{ij} \sin i\pi\frac{u}{m} \sin j\pi\frac{v}{n} \tag{37}$$

for the conditions 1. displacement zero at boundary nodes and 2. displacement zero at mid-points of cables "cut" by $(0, v)$, (m, v). As (37) gives zero displacement for all boundary nodes, real and virtual, it satisfies the first condition. Due to anti-symmetry with respect to the boundary segments, displacement is zero at mid-points of threads "cut" by boundary; therefore, the second condition is also satisfied by (37). Furthermore, it is apparent, as operator has even differences only, that it will satisfy (36). Repeating the procedure used in the first case, one finds:

$$A_{ij} = \frac{a}{R\,m\,n} \frac{\sin i\,\pi \frac{\alpha}{m} \sin j\,\pi \frac{\nu}{n}}{\sin^2 \frac{i\,\pi}{m} + \epsilon^2 \left(1 - \cos \frac{i\,\pi}{m} \cos \frac{j\,\pi}{n}\right)} \tag{38}$$

thus, completing the Green's function solution for the subject problem of this section.

Quadruply Threaded Nets

In this section, a brief study is made of the net with four cables intersecting at each node, Fig. 6. (The two-thread intersections are unloaded and not regarded as nodes.)

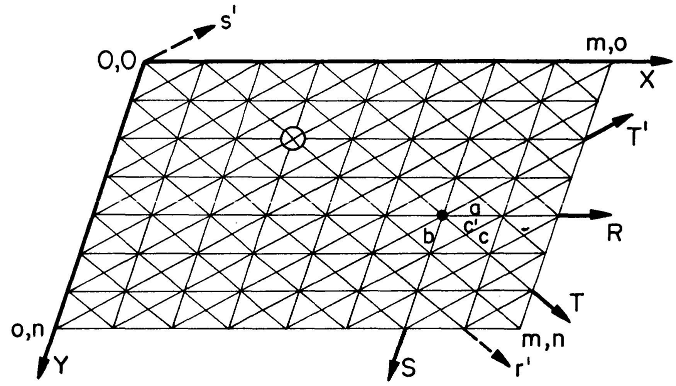

Fig. 6. The quadruply threaded net with basic boundary.

Triply threaded nets can be viewed as the result of adding a set of cables across one diagonal of the basic element of corresponding doubly threaded nets; then, for a quadruply threaded net, one adds two sets of cables, one for each diagonal of the basic element. The governing equation, expressing node equilibrium in terms of its displacement and that of neighboring nodes directly linked to it, can be written by inspection. For the case with regular geometry, the governing equation, for a displacement Green's function, is:

$$\left[\frac{R}{a}\Delta_x + \frac{S}{b}\Delta_y + \frac{T}{c}\Delta_{r'} + \frac{T'}{c'}\Delta_{s'}\right] K(x, y, \alpha, \nu) = -\delta_x^\alpha \delta_y^\nu, \tag{39}$$

where r' and s' are coordinate coefficients of basis vectors which are parallel, and equal in magnitude, to the diagonals of the basic parallelogram of the corresponding doubly threaded net. As in the preceding case, it should be remembered in (39) that $E_{r'} = E_x E_y$ and $E_{s'} = E_x E_y^{-1}$; then, it is seen that the equation is a complete second order difference equation, i. e., it contains all nine possible operators for an equation of order two. It can be shown that four independent functions must be given before a solution can be determined. For the net, this means that four boundary segments must be specified or that one can not find a solution for a triangular boundary which has only node displacements given. This is as would be expected on physical grounds.

Any triangular boundary would "cut" at least one set of threads; thus, boundary information, in addition to node displacements, would be required to uniquely determine a solution, i.e. the "fourth" function.

For a special condition within the constant coefficient case, i.e., $T/c = T'/c'$, (39) can be written in terms of even differences with respect to x and y and, thus, becomes tractable by the separation of variables technique. For this condition, the equation is:

$$\{\Delta\!\!\!/_y [\lambda^2 (\Delta\!\!\!/_x + 2) + \epsilon^2] + (1 + 2\lambda^2) \Delta\!\!\!/_x\} K(x, y, \alpha, \nu) = -\frac{a}{R} \delta_x^\alpha \delta_y^\nu, \tag{40}$$

where $\epsilon^2 = \frac{S a}{b R}$ and $\lambda^2 = \frac{T a}{R c}$. Consider the quadrilateral basic boundary shown in Fig. 6 with $K(0, y, \alpha, \nu) = K(m, y, \alpha, \nu) = 0$ as a partial statement of the boundary conditions. Then, as for the doubly threaded net, one can use a single series solution, i.e.,

$$K(x, y, \alpha, \nu) = \sum_{i=1}^{m-1} Y_i(y) \sin i\pi \frac{x}{m}. \tag{41}$$

By the procedure used for (21), (22) and (23), the governing difference equation for the coefficient functions is determined:

$$(\Delta\!\!\!/_y - 2\beta_i) Y_i(y) = -C_{0i} \sin i\pi \frac{\alpha}{m} \delta_y^\nu, \tag{42}$$

where $$\beta_i = \frac{2(1+\lambda^2)\sin^2 \frac{i\pi}{2m}}{2\lambda^2 \cos\frac{i\pi}{m} + \epsilon^2} \quad \text{and} \quad C_{0i} = \frac{2a}{Rm\left(2\lambda^2\cos\frac{i\pi}{m} + \epsilon^2\right)} \tag{43}$$

and $\beta_i > 0$ if $\frac{S c}{b T} > 2$. For general conditions at $y = 0, m$, the functions are:

$$Y_i = \frac{C_{0i} \sin i\pi \frac{\alpha}{m}}{\sinh \gamma_i} \left[C_{1i} \sinh \gamma_i y + C_{2i} \cosh \gamma_i y - \frac{\sinh \gamma_i (y - \nu)}{\sinh \gamma_i} U\!\!\!/ (y - \nu) \right], \tag{44}$$

where $\cosh \gamma_i = \beta_i + 1$.

For zero displacement at $y = 0, m$, the sets of constants are determined, resulting in the same solution found for the doubly threaded net, (25), except, (43) is used to evaluate the parameters here. For a uniform node loading, the analogous solution for the doubly threaded net is not applicable as special properties of the formula for β_i were used. The solution for a uniform node loading on the quadruply threaded net with the basic boundary of zero displacement, includes (26) as a special case and is:

$$W(x, y) = \frac{1}{4} P_0 \sum_i^{m-1} \frac{C_{0i} \cot \frac{i\pi}{m}}{\sinh^2 \frac{1}{2}\gamma_i} \left[1 - \frac{\cosh \gamma_i \left(y - \frac{n}{2} \right)}{\cosh \frac{1}{2}\gamma_i} \right] \sin i\pi \frac{x}{m} \quad (i \text{ odd}). \tag{45}$$

Illustrative Example

In order to illustrate use of the formulas and to facilitate an understanding of their application for preliminary design computations, some results are shown for the diamond shaped net shelter shown in Figs. 7, 8 and 9. The displacement fields where computed for a uniform node loading and for an impulse

Doubly threaded structural net with center pole.

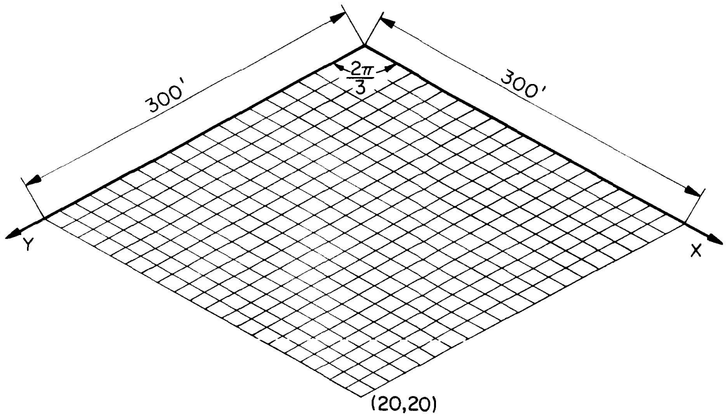

Fig. 7. Plan.

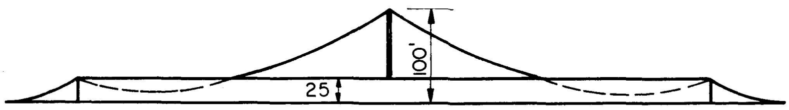

Fig. 8. Elevation.

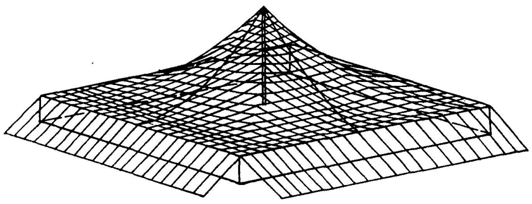

Fig. 9. Sketch.

load at the center. The results were combined so as to match the given displacement due to a pole at the center, $W_t(10, 10) = -75.0$. (Both fields contain the cable tension as a linear factor; therefore, it is not necessary to duplicate their evaluation in order to study effect changes in that parameter.) The double series, (16, 17) and (19) where used primarily, with "spot" checks performed by use of the single series, (25) and (26). The results were identical.

Data were: $R/a = S/b = \frac{150}{15} = 10\,kpf$; $m = n = 20$; $P_0 = 11.25$ (due to $0.05\,ksf$ loading on horizontal projection).

The combined deflection field is given by:

$$W_t(x, y) = W_u(x, y) + P_c K(x, y, 10, 10), \tag{46}$$

where W_t and W_u are, respectively, the total or combined displacement field and the field due to load of P_0 on each node, and P_c is the net load due to the center pole. The latter was determined from:

$$W_t(10,10) = W_u(10,10) + P_c K(10,10,10,10) \text{ or}$$
$$-75.0 = 33.087 + P_c(0.06357) \text{ or } P_c = -1700.3\,k, \text{ i.e.,}$$

the pole must sustain 41.87 per cent of the total load. Due to symmetry about both center axes and the diagonals, the displacement field in (46) can be completely shown by listing the values for one-eighth of the nodes. The coordinates of the distinct points and their displacements, as computed by (16, 17), (19) and (46), are given in table 1.

Table 1. Coordinates and Displacements for Net in Fig. 9.

(1, 1)	1.478	(1, 2)	2.393	(1, 3)	2.970	(1, 4)	3.316	(1, 5)	3.499
(1, 6)	3.566	(1, 7)	3.556	(1, 8)	3.520	(1, 9)	3.480	(1, 10)	3.463
(2, 2)	4.000	(2, 3)	5.044	(2, 4)	5.672	(2, 5)	5.988	(2, 6)	6.080
(2, 7)	6.029	(2, 8)	5.915	(2, 9)	5.810	(2, 10)	5.769	(3, 3)	6.410
(3, 4)	7.214	(3, 5)	7.576	(3, 6)	7.611	(3, 7)	7.436	(3, 8)	7.176
(3, 9)	6.953	(3, 10)	6.866	(4, 4)	8.072	(4, 5)	8.367	(4, 6)	8.228
(4, 7)	7.804	(4, 8)	7.274	(4, 9)	6.835	(4, 10)	6.663	(5, 5)	8.467
(5, 6)	8.046	(5, 7)	7.153	(5, 8)	6.155	(5, 9)	5.325	(5, 10)	4.992
(6, 6)	7.046	(6, 7)	5.524	(6, 8)	3.743	(6, 9)	2.195	(6, 10)	1.528
(7, 7)	3.029	(7, 8)	− 0.028	(7, 9)	− 2.942	(7, 10)	− 4.394	(8, 8)	− 7.599
(8, 9)	− 10.667	(8, 10)	− 14.344	(9, 9)	− 22.280	(9, 10)	− 32.775	(10, 10)	− 75.00

The manner in which the static formulas are used to deal with the associated mechanics of materials problem is illustrated by considering the following problem: What should be the field of initial tensions, i. e., when net is plane, for this static solution with its assumption of a uniform final tension field of $150\,K$ (horizontal components) to be valid? For example, consider cable $(5, y)$. Its final length is given by:

$$L_f(5,y) = \sum_{y=1}^{n-1} \{b^2 + [\Delta_y W_t(5,y)]^2\}^{1/2} \tag{47}$$

and

$$L_f(5,y) = n\,b\left[1 + \frac{S - S_0(5,y)}{K_s(5,y)}\right], \tag{48}$$

where K_s is the modulus of "stretch", product of area and elastic modulus, for the cable under study. For a given K_s, (47) and (48) can easily be solved for the initial tension $S_0(5, y)$. If the deflections are relatively small, one can use two terms of the binominal expansion for the expression giving length of deformed cable element in (47). This leads to the following expression:

$$S - S_0(5,y) = \frac{K_s(5,y)}{2\,n\,b^2} \Delta_y^{-1} [\Delta_y W_t(5,y)]^2 \Big|_0^n . \tag{49}$$

Acknowledgements

The authors' gratefully acknowledge two sources of support which made possible the research leading to this paper. It is based on work done as part of a National Science Foundation project at the University of Kansas in 1960. Continuation of the work and preparation of the paper, including automatic computation, was under the sponsorship of the University of Delaware.

Appendix A

Transformation of Coordinates in Discrete Field Mechanics

As in continuum mechanics, one frequently must transform the governing difference equation for a discrete field problem to a form which serves as the mathematical model referred to a new coordinate system. For example, the equation may be intractable as derived in terms of the first coordinate system, or the boundary equations may not be sufficiently convenient. The chain rule, as used to express the derivative with respect to a new variable in terms of derivatives with respect to the original independent variables, is the basic relation required to perform a transformation of variables in the continuum. Analogously, one requires an expression for the displacement operation with respect to the new variables in terms of the same operation with respect to the original variables for transformations in the discrete case.

Consider the regular plane two-dimensional lattice shown in Fig. 10. The position vector of a general node, in terms of the original independent variables x and y, referred to the smallest pair of basis vectors, is:

$$\bar{R}(x,y) = x\,\bar{i} + y\,\bar{j}, \tag{50}$$

where $\bar{i}$ and $\bar{j}$, as shown, are usually neither orthogonal nor of unit lengths. Assume that it is desired to transform a governing difference equation associated with the lattice,

$$F(E_x, E_y)\,W(x,y) = P(x,y) \quad \text{to} \quad F'(E_r, E_s)\,W(r,s) = P'(r,s), \tag{51}$$

where r and s are new independent variables which locate the general node through

$$\bar{R}(r,s) = r\,\bar{\zeta} + s\,\bar{\eta}, \tag{52}$$

where the new basis vectors $\bar{\zeta}$ and $\bar{\eta}$ are given by

$$\begin{Bmatrix} \bar{\zeta} \\ \bar{\eta} \end{Bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} \bar{i} \\ \bar{j} \end{Bmatrix}. \tag{53a, 53b}$$

The coefficients a, b, c and d are often integers so as to cover the discrete field, but may be other rational numbers, e.g., $\frac{1}{2}$ or $-\frac{1}{2}$.

The displacement operators, with respect to the new variables, in terms of operations, with respect to x and y, and the original variables in terms of r and s are:

$$E_r = E_x^a E_y^b\,; \qquad E_s = E_x^c E_y^d, \tag{54a, 54b}$$

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{Bmatrix} r \\ s \end{Bmatrix}. \tag{55a, 55b}$$

In the process of performing a complete transformation, one usually requires the corresponding inverse operations.

$$\begin{Bmatrix} \bar{i} \\ \bar{j} \end{Bmatrix} = \begin{bmatrix} d' & b' \\ c' & a' \end{bmatrix} \begin{Bmatrix} \bar{\zeta} \\ \bar{\eta} \end{Bmatrix}. \tag{56a, 56b}$$

The coefficients d', b', c' and a' of the inverse transformation are given in terms of the coefficients of the direct transformation by the classical matrix inversion formula.

$$d' = \frac{d}{D}; \quad b' = -\frac{b}{D}; \quad c' = -\frac{c}{D}; \quad a' = \frac{a}{D}; \quad D = ad - cb \neq 0, \tag{57}$$

$$E_x = E_r^{d'} E_s^{b'}\,; \qquad E_y = E_r^{c'} E_s^{a'}, \tag{58a, 58b}$$

$$\begin{Bmatrix} r \\ s \end{Bmatrix} = \begin{bmatrix} d' & c' \\ b' & a' \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}. \tag{59a, 59b}$$

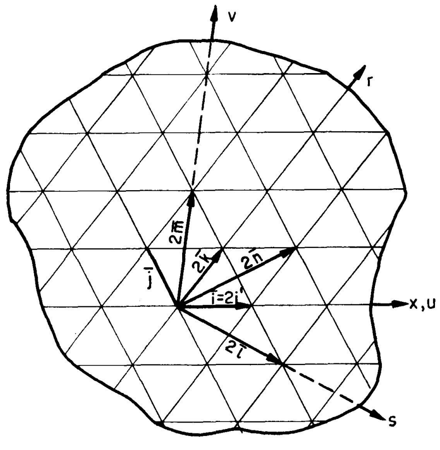

Fig. 10. Lattice coordinates.

For example, consider the transformation to coordinates referred to basis vectors $\bar{k}$ and $\bar{l}$ which are parallel to the diagonals of the $\bar{i}$ and $\bar{j}$ parallelograms.

$$\bar{R}(r, s) = r\bar{k} + s\bar{l}, \tag{60}$$

$$\bar{k} = \tfrac{1}{2}(\bar{i} + \bar{j}); \qquad \bar{l} = \tfrac{1}{2}(\bar{i} - \bar{j}), \tag{61a, 61b}$$

$$x = \tfrac{1}{2}(r + s); \qquad y = \tfrac{1}{2}(r - s), \tag{62a, 62b}$$

$$\bar{i} = \bar{k} + \bar{l}; \qquad \bar{j} = \bar{k} - \bar{l}, \tag{63a, 63b}$$

$$E_r^2 = E_x E_y\,; \qquad E_s^2 = E_x E_y^{-1}, \tag{64a, 64b}$$

$$E_x = E_r E_s\,; \qquad E_y = E_r E_s^{-1}, \tag{65a, 65b}$$

For a second example, consider the transformation to coordinates referred to basis vectors $\bar{i}'$ and $\bar{m}$ which are parallel to $\bar{i}$ and a diagonal of the $\bar{j}$ and $2\bar{k}$ parallelogram.

$$\bar{R}(u,v) = u\bar{i}' + v\bar{m}, \tag{66}$$

$$\bar{i}' = \tfrac{1}{2}\bar{i} + 0\bar{j}; \qquad \bar{m} = \tfrac{1}{2}\bar{i} + \bar{j}, \tag{67a, 67b}$$

$$E_u = E_x^{1/2}; \qquad E_v = E_x^{1/2} E_y, \tag{68a, 68b}$$

$$x = \tfrac{1}{2}(u+v); \qquad y = v, \tag{69a, 69b}$$

$$\bar{i} = 2\bar{i}'; \qquad \bar{j} = -\bar{i}' + \bar{m}, \tag{70a, 70b}$$

$$E_x = 2E_u; \qquad E_y = E_u^{-1} E_v. \tag{71a, 71b}$$

Appendix B

Solutions for Difference Equations with Symmetric Operators

A recurring problem in the mechanics of regular structural lattices is that of solving a constant coefficient ordinary difference equation with a symmetric operator, i. e., one which can be written in terms of even difference. The homogeneous solution can be determined routinely, so that only the second order case need be reviewed here. Consider the difference equation (72).

$$(\Delta\!\!\!\nabla - 2\beta)\, W(x) = 0, \tag{72}$$

where the second central difference operator $\Delta\!\!\!\nabla = E - 2 + E^{-1}$. The solution may be written:

$$W(x) = C_1 \cosh \gamma x + C_2 \sinh \gamma x, \tag{73}$$

where $\cosh\gamma = |(\beta+1)|$. If β is negative, γ is not real so the solution (73) is modified for computation, according to complex variable theory, as follows: 2. if $\beta < -2$, multiply solution by $(-1)^x$; 3. if $-2 < \beta < 0$, replace the hyperbolic functions in the solution and the definition of γ by trigonometric functions; 4. if $\beta = 0$ or $\beta = -2$, case is degenerate with repeated roots and the respective solutions are $W(x) = C_1 + C_2 x$ and $W(x) = (-1)^x (C_1 + C_2 x)$.

Although some applicable techniques and formulas are available [1, 2], sufficient formulas are not available for solving the subject class of difference equations with general inhomogeneous or loading terms, i. e.,

$$(\Delta\!\!\!\nabla - 2\beta_1)(\Delta\!\!\!\nabla - 2\beta_2)\ldots(\Delta\!\!\!\nabla - 2\beta_n)\, W(x) = P(x). \tag{74}$$

The necessary and sufficient solution, for generality as regards loading, is to solve the linear equation for an arbitrarily placed unit load, i.e., $P(x) = \delta_x^{\alpha}$, where the Kronecker delta loading function is defined in the usual manner,

$$\delta_x^{\alpha} = \begin{Bmatrix} 0 \text{ for } x \neq \alpha \\ 1 \text{ for } x = \alpha \end{Bmatrix}. \tag{75}$$

The solution for this loading, a Green's or Kernel function, is frequently denoted $K(x, \alpha)$. The "constants" of summation in the homogeneous part of such a solution are found as "functions" of α when the boundary conditions are imposed. $K(x, \alpha)$ is used to determine $W(x)$ for a given load function, P, as follows:

$$W(x) = \sum_{\alpha=0}^{m} K(x, \alpha)\, P(\alpha), \tag{76}$$

where m is the terminal node in the field.

The particular part of $K(x, \alpha)$, $K_p(x, \alpha)$, will be derived in some detail for the second order case. The fourth order and general cases can then be written by induction.

$$K_p(x, \alpha) = \frac{\delta_x^{\alpha}}{\nabla - 2\beta} = \left[\frac{b_1}{E - r} + \frac{b_2}{E - r^{-1}}\right] \delta_x^{\alpha-1}, \tag{77}$$

where r and r^{-1} are the roots of the characteristic equation,

$$r = \beta + 1 + \sqrt{\beta(\beta + 2)} = e^{\gamma}, \tag{78}$$

and the coefficients, b_1 and b_2, are determined from theory of partial fractions [1] as: $b_1^{-1} = -b_2^{-1} = r - r^{-1}$. The solution can now be determined by use of a standard formula from theory of finite calculus:

$$\frac{1}{E - r} \delta_x^{\alpha-1} = r^{x-1} \Delta^{-1} [r^{-x} \delta_x^{\alpha-1}] = r^{x-\alpha}\, \mathcal{U}(x - \alpha), \tag{79}$$

where the finite step function is defined as:

$$\mathcal{U}(x - \alpha) = \begin{Bmatrix} 0 \text{ for } x < \alpha \\ 1 \text{ for } x = \alpha \\ 1 \text{ for } x > \alpha \end{Bmatrix}. \tag{80}$$

The substitution into (77) yields:

$$K_p(x, \alpha) = \frac{1}{r - r^{-1}} [r^{(x-\alpha)} - r^{(\alpha-x)}]\, \mathcal{U}(x - \alpha) \tag{81}$$

or

$$K_p(x, \alpha) = \frac{\sinh \gamma (x - \alpha)}{\sinh \gamma}\, \mathcal{U}(x - \alpha), \tag{82}$$

where, as above, $\cosh \gamma = |(\beta + 1)|$. The modifications of (82) for computation when β is negative or assumes special values, leading to degenerate cases, are as follows: 2. if $\beta < -2$, multiply solution by $-(-1)^{x-a}$; 3. if $-2 < \beta < 0$, replace the hyperbolic functions in the solution and definition of γ by the corresponding trigonometric functions. 4. if $\beta = 0$, $K_p(x, \alpha) = (x - \alpha)\, \mathcal{U}(x - \alpha)$ and if $\beta = -2$, same except multiplied by $-(-1)^{x-a}$.

Consider the fourth order case, $n = 2$. Then:

$$K_p(x, \alpha) = \frac{\sinh \gamma_1 (x - \alpha)}{2(\beta_1 - \beta_2) \sinh \gamma_1}\, \mathcal{U}(x - \alpha) + \frac{\sinh \gamma_2 (x - \alpha)}{2(\beta_2 - \beta_1) \sinh \gamma_2}\, \mathcal{U}(x - \alpha), \tag{83}$$

where $\cosh\gamma_1 = |(\beta_1+1)|$ and $\cosh\gamma_2 = |(\beta_2+1)|$. For real and distinct values of β_1 and β_2, modification of (83) for computations is as given for the second order case. Another possibility, for which the form (83) requires modification for numerical use, is that β_1 and β_2 are complex conjugates. Thus, r, as given by formula (78), will be complex. One requires the real numbers σ and φ which are respectively the real and imaginary parts of $\log_e r$ or, in other words, e^σ and φ, respectively, are the amplitude and argument of the complex quantity, r. For this case, the working form for (83) becomes:

$$K_p(x,\alpha) = \frac{1}{\cosh 2\sigma - \cos 2\varphi}\left[\frac{\cosh\sigma(x-\alpha)\sin\varphi(x-\alpha)}{\tan\varphi} - \frac{\sinh\sigma(x-\alpha)\cos\varphi(x-\alpha)}{\tanh\sigma}\right]\not{U}(x-\alpha). \tag{84}$$

A second possibility, requiring special modification for the fourth order case, is that $\beta_1=\beta_2$. By either a limiting process or the theory of partial fractions, the working form should be:

$$K_p(x,\alpha) = \frac{1}{2\sinh^2\gamma}\left[(x-\alpha-1)\cosh\gamma(x-\alpha) - \frac{\sinh\gamma(x-\alpha)}{\tanh\gamma}\right]\not{U}(x-\alpha). \tag{85}$$

The general case can now be written as follows:

$$K_p(x,\alpha) = \sum_{i=1}^{n} A_i \frac{\sinh\gamma_i(x-\alpha)}{\sinh\gamma_i}\not{U}(x-\alpha), \tag{86}$$

where

$$\cosh\gamma_i = |(\beta_i+1)|, \tag{87}$$

$$\frac{1}{A_i} = 2^{n-1}\prod_{j=1}^{n}{}'(\beta_i-\beta_j), \tag{88}$$

$$\prod_{j=1}^{n}{}'(\beta_i-\beta_j) = \frac{d}{d\beta}\prod_{j=1}^{n}(\beta-\beta_j)|_{\beta=\beta_i} = (\beta_i-\beta_1)(\beta_i-\beta_2)\ldots(\beta_i-\beta_{i-1})(\beta_i-\beta_{i+1})\ldots(\beta_i-\beta_n). \tag{89}$$

Modifications for special values of the β's are as indicated in the cases for $n=1$ and $n=2$.

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Summary

The field approach, for analyzing two-dimensional structural lattices, was emphasized. The governing second order partial difference equations were derived for the three major categories of net configurations, that is, those with two, three and four sets of cables. Various types of boundaries were studied by transforming coordinates for the discrete field. In all three of the net categories, closed form solutions were obtained to the governing equations for a single arbitrarily located node load. These net-displacement Green's functions were written as double and/or single finite Fourier's series. Solutions were also derived for the uniform loading, as a single algebraic term in some cases. Formulas for the doubly threaded structural net were illustrated numerically.

Résumé

Les auteurs étudient les réseaux bidimensionnels à l'aide de la théorie des champs. Ils établissent les équations aux différences partielles du second ordre pour les trois principaux types de réseaux, c'est-à-dire les réseaux à 2, 3 et 4 familles de câbles. Ils étudient diverses formes du contour en transformant les coordonnées pour un champ limité. Pour les trois types de réseaux, les auteurs ont obtenu une solution exacte pour une charge isolée, agissant en un nœud quelconque. Les fonctions de Green exprimant le déplacement des nœuds sont exprimées sous forme de séries de Fourier finies, simples et/ou doubles. On a également établi des solutions pour une charge uniforme, dans certains cas sous forme de relations algébriques simples. A titre d'illustration, on applique dans un exemple numérique les formules relatives aux réseaux à 2 familles de câbles.

Zusammenfassung

Die Beanspruchungen von zweidimensionalen Netzwerken wurden untersucht. Partielle Differenzengleichungen zweiter Ordnung wurden für die drei Hauptarten von Netzanordnungen mit zwei, drei und vier Gruppen verschiedener Kabelrichtungen hergeleitet. Anschließend wurden verschiedene Grundrißformen mittels Koordinatentransformation eingehender untersucht. Für eine Einzellast an einem beliebig gelegenen Knoten wurden geschlossene Lösungen in Form von Greens Funktionen für alle drei Hauptarten von Netzanordnungen gefunden. Diese für Knotenverschiebungen geltenden Greens-Funktionen wurden als doppelte oder einfache endliche Fouriersche Reihen dargestellt. Lösungen für gleichmäßig verteilte Belastungen wurden ebenfalls gezeigt. In einzelnen Fällen wurde dabei ein einzelner, nur aus Faktoren bestehender Ausdruck erhalten. Formeln für Netzwerke mit zwei Kabelrichtungen wurden für die praktische Anwendung zahlenmäßig ausgewertet.