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Probable Fatigue Life of Under-Reinforced Prestressed Concrete Beams

Résistance probable à la fatigue de poutres en béton précontraint faiblement armées

Wahrscheinliche Dauerfestigkeit von unterarmierten Balken in vorgespanntem Beton

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Introduction

The possible modes of fatigue failure in a prestressed concrete flexural member are comparable in some ways to the corresponding modes of failure under static loading, and a similar method of classification is useful. Thus, failures involving fatigue in the component materials may be referred to conveniently as *under-reinforced, over-reinforced,* or *balanced,* depending upon whether the primary failure takes place in the steel, in the concrete, or in the two materials more or less simultaneously. However, it should be noted that a beam which is under-reinforced with respect to fatigue failure is not necessarily under-reinforced from the point of view of static ultimate strength.

In certain circumstances it may be possible also for the concrete-steel bond to break down progressively along the length of the beam as a result of fatigue loading. Bond-fatigue failure, like bond-failure under static loading, will occur only in regions where relatively steep moment gradients exist; it is therefore unlikely in flexural members in which the moment-to-shear ratio is large, and is more conveniently treated in association with a study of shear failure. Shear-fatigue and bond-fatigue failures are not considered in this investigation.

A recent review of research on concrete fatigue [1] indicates that in beams of normal design the fatigue failure is far more likely to occur in the steel than in the concrete. Indeed, in the literature, only one case of an over-

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reinforced failure is reported: LE CAMUS [2] was able to force a concrete fatigue failure by using a special reinforced concrete test beam of champignon design with a reduced concrete compression region.

In the work summarized in this paper[1]), a study was made of the fatigue life of under-reinforced prestressed concrete flexural members subjected to repeated loadings of either constant or varied magnitude. The work consisted of three main phases. First, an experimental study was made of the fatigue properties of high strength steel strand, a type of prestressing steel used extensively in the United States in the manufacture of pretensioned prestressed concrete structures. Second, a theoretical analysis was made of the stresses and deformations in concrete members subjected to repeated flexural loadings. The results of the study of strand fatigue properties, used in conjunction with the theoretical analysis, provide a method for estimating probable fatigue life of under-reinforced beams. Finally, a series of beams was tested to determine the accuracy of the method developed for estimating beam fatigue life.

Fatigue Properties of Steel Prestressing Strand

The fatigue properties of $^{7}/_{16}$ inch diameter, seven wire, high strength prestressing strand were studied in an investigation involving static tests, constant cycle fatigue tests, and cumulative damage tests on almost 150 specimens.

A special frame with a 22 kip capacity Amsler jack coupled to an Amsler pulsator was used to transmit load to the specimen at a rate of 500 cycles per minute. The specimens were held in a manner designed to minimize stress concentrations in the gripping regions and hence prevent premature fatigue failure. The gripping arrangement transmitted the force to the test specimen partly through a cement grout bond anchorage and partly through a mechanical anchorage at the end of the strand piece.

Fatigue Life Under Constant Cycle Loading

The constant cycle tests were conducted with minimum stress levels of 40 and 60 percent of the static ultimate strength; maximum stress levels were chosen to give fatigue lives varying between 50 thousand and 5 million cycles. Apart from several tests which yielded fatigue lives outside of this main region of interest, at least six replications of each test were made.

The results of the constant cycle tests are given in Fig. 1. The data are replotted in Fig. 2 on coordinates of $\log N$ and $R = (S_{max} - S_L)$, where S_L is the fatigue limit corresponding to the minimum stress level S_{min} and S_{max} is

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[1]) A more complete description of the investigation is contained in Reference 3.

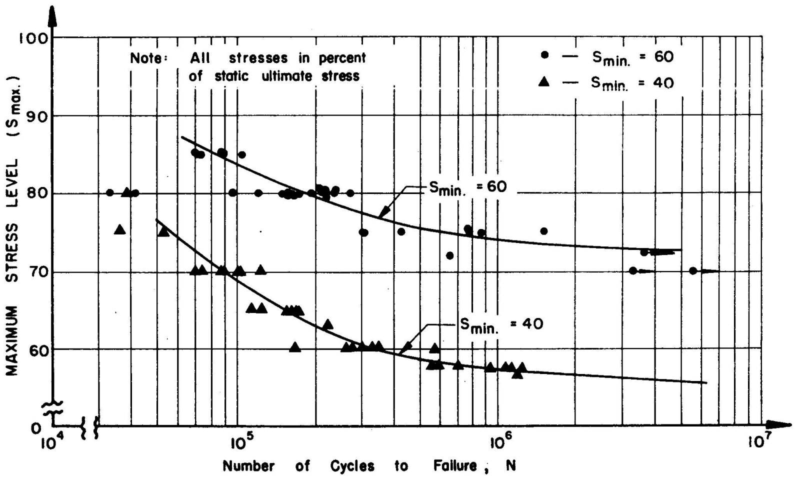


Fig. 1. Maximum Stress Level Versus Fatigue Life, Constant Cycle Tests

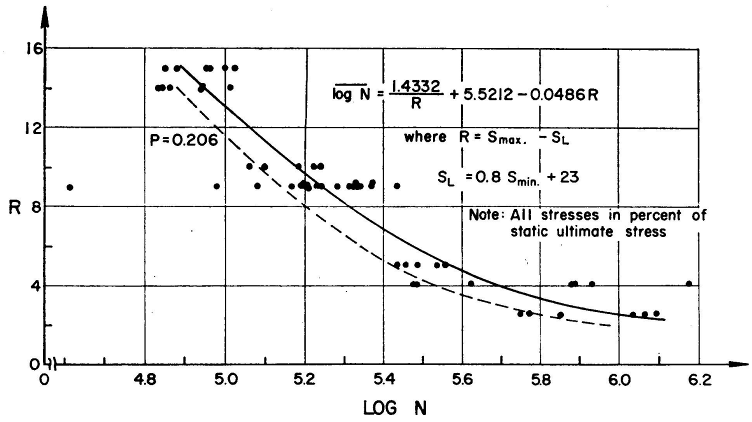


Fig. 2. R Versus Log N.

the maximum stress level. The method of least squares was used to obtain the following equations for mean fatigue life.

$$\overline{\log N} = \frac{1.4332}{R} + 5.5212 - 0.0486\,R, \tag{1}$$

$$R = S_{max} - S_L, \tag{2}$$

$$S_L = 0.8\,S_{min} + 23. \tag{3}$$

In the above equations, all stress levels are expressed as percentages of the static ultimate strength.

The standard deviation, D, of the set of data for each particular maximum stress level is plotted against R in Fig. 3. Eq. (4) was obtained by the method of least squares.

$$D = 0.2196 - 0.0103\,R. \tag{4}$$

Recent studies of the fatigue properties of materials [4, 5] clearly indicate that the variability inherent in fatigue test data requires that fatigue phenomena be treated in probabilistic terms. Variability may be associated with fatigue failure by treating the values of fatigue life observed in test replications as a sample taken from an infinite population of values which is distributed in some manner about a central value and represented by some distribution function. Thus, for any repeated loading which might be applied to the specimen, we consider the probability of failure, P, to vary between zero and unity. With each value of P we associate a number of cycles, N, such that the probability is P that failure will occur at a number of cycles equal to or less than N.

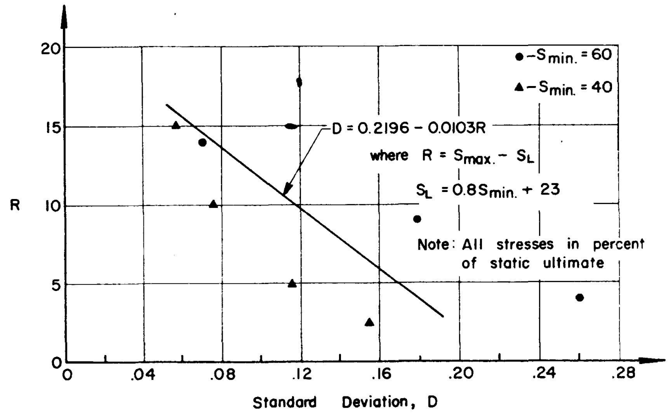


Fig. 3. R Versus Standard Deviation.

The logarithmic-normal distribution, which has the probability density function

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}} \tag{5}$$

and cumulative distribution function

$$P = F(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{(X-\mu)^2}{2\sigma^2}} dX, \tag{6}$$

where $$X = \log N$$

and μ and σ are the mean and standard deviation of the population of N values, has in previous investigations [5, 6] been associated with the phenomenon of fatigue failure, and is adopted in this study. The distribution of the grouped constant cycle test data about the mean is shown in Fig. 4. The different sets of data have been grouped together in this figure by the change of variable

$$Z = \frac{\log N - \overline{\log N}}{D},$$

where $\overline{\log N}$ and D are the mean and standard deviation of the set of data being grouped. A χ^2 goodness-of-fit test [7] conducted on the data indicated the "fit" to be well within the 0.05 significance level.

The S-N-P relation is thus given by the equations

$$P = F(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{(X-\mu)^2}{2\sigma^2}} dX, \tag{6}$$

where
$$X = \log N$$

$$\mu = \overline{\log N} = \frac{1.4332}{R} + 5.5212 - 0.0486\,R, \tag{1}$$

where
$$R = S_{max} - (0.8\,S_{min} + 23)$$

and
$$\sigma = D = 0.2196 - 0.0103\,R. \tag{4}$$

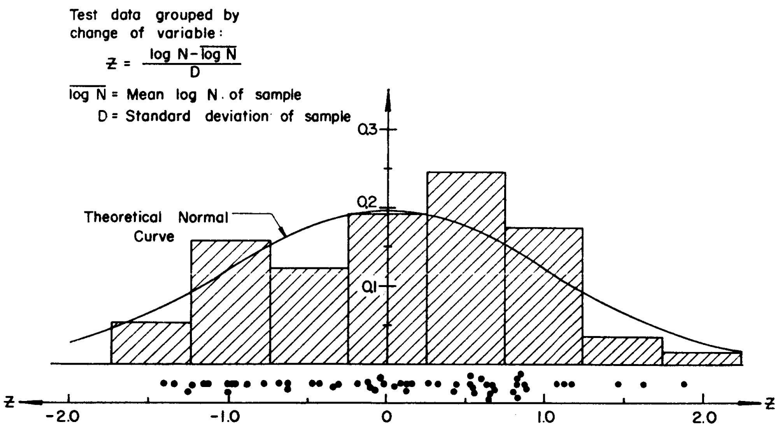


Fig. 4. Frequency Distribution of Grouped Constant Cycle Test Data

Values of P corresponding to values of X in Eq. (6), and vice versa, can of course be obtained most easily from standard tables [7].

It should be noted that the above equations were derived from tests conducted on $^7/_{16}$ inch diameter high strength steel prestressing strand for the following range of variables:

$$40 \leqq S_{min} \leqq 60$$
$$0 < R \leqq 15$$

Fatigue Life Under Varied Repeated Loadings

In cumulative damage tests conducted on approximately fifty strand specimens, the fatigue loading fluctuated between a constant minimum level and either two or three different maximum levels. Load blocks containing one and two overloads are shown in Fig. 5; the overloads are represented by S_{01} and S_{02}, the predominant loading by S_{pred}, and the minimum by S_{min}. The main variables in the tests were the magnitudes and relative frequencies of occurrence of load levels.

The test results showed good agreement with the predictions of the Palmgren-Miner cumulative damage theory [8] which states that failure will occur when

$$\sum_{i=1}^{q} \frac{n_i}{\overline{N}_i} = 1\,, \tag{7}$$

where n_i is the number of cycles of S_i loading, and $\overline{N}_i$ is the mean fatigue life corresponding to S_i. The mean of the $\sum \frac{n_i}{\overline{N}_i}$ values for all tests was 0.97, with extreme values of 0.48 and 1.65 and a standard deviation of 0.224.

Eq. (7), which is used for the prediction of mean fatigue life, may be written in the form

$$\sum_{i=1}^{q} \frac{a_i N(0.5)}{N_i(0.5)} = 1\,, \tag{8}$$

where a_i is the relative frequency of load S_i, $N_i(0.5)$ is the mean fatigue life corresponding to S_i, and $N(0.5)$ is the mean fatigue life under cumulative damage loading. Eq. (8) may be generalized to the form

$$\sum_{i=1}^{q} \frac{a_i N(P)}{N_i(P)} = 1$$

or

$$N(P) = \frac{1}{\sum_{i=1}^{q} \frac{a_i}{N_i(P)}}\,, \tag{9}$$

for application at any probability level P.

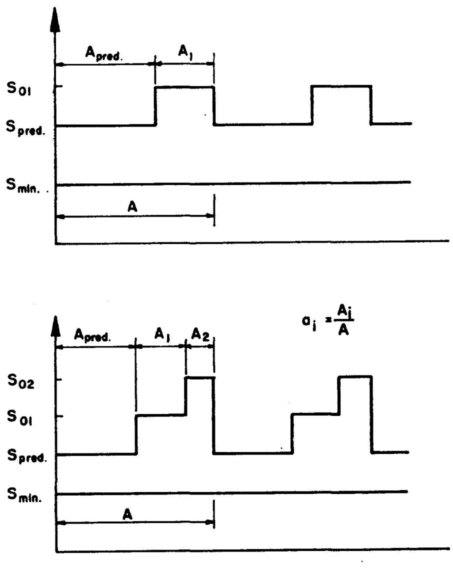


Fig. 5. Load Blocks for Cumulative Damage Tests on Strands

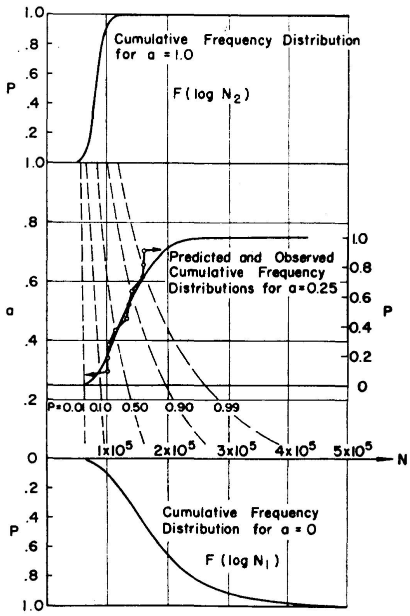


Fig. 6. Predicted and observed Frequency Distributions for Cumulative Damage Test.

Eq. (9), together with the S-N-P relation represented by Eqs. (1), (4), and (6), may be used to predict probable fatigue life of strand specimens. In Fig. 6a comparison is made between the frequency distribution predicted by Eq. (9) and the results of ten replications of one cumulative damage test.

Beam Behavior Under Repeated Flexural Loadings

To use the strand fatigue data for the prediction of the fatigue life of a prestressed member, it is necessary to know or to be able to predict the response of the beam to load. In particular, it must be possible in any given load cycle to determine the relation between steel stress and applied moment so that the loading history of the beam can be transformed into the corresponding stress history for the steel.

An analysis of the stresses in a rectangular section with the steel reinforcement placed at one horizontal layer is briefly described in this paper[2]).

Loading is considered in two stages; zero moment to M_{on}, and M_{on} to static ultimate moment, where M_{on} is the moment at which cracks begin to open in the n-th load cycle. In the first loading stage, increments of strain in the steel and concrete are relatively small and linear stress-strain relations are assumed for both materials. Previously formed flexural cracks are closed in this initial loading stage by the internal prestressing force in the steel, F_n, and so the cracked regions are assumed to behave elastically provided the stresses remain compressive, i. e., provided M_{on} is not exceeded.

Conditions in the second stage of loading are considerably more complicated. The analysis of beam behavior is based on a consideration of the following:

a) Stress-strain relations for concrete and steel.
b) An assumed pattern of deformation in the beam in the region of flexural cracking.
c) Equilibrium of internal forces.

Initial Loading Stage, $M \leqq M_{on}$

With the assumption of linear stress-strain relations for both concrete and steel, the equations for the top and bottom concrete fiber stresses and the steel stress are:

$$f^t_{cn} = -F_n \left[\frac{1}{A_c} - \frac{e\,h}{I_c 2}\right] - \frac{M}{I}\left[\frac{h}{2} + e - \bar{x}\right], \tag{10}$$

2) A more complete treatment is contained in Reference 3, together with analyses for members with I-shaped sections and for members with the steel placed at several different levels.

$$f_{cn}^{b} = -F_n\left[\frac{1}{A_c}+\frac{e\,h}{I_c\,2}\right]+\frac{M}{I}\left[\frac{h}{2}-e+\bar{x}\right], \tag{11}$$

$$f_{sn} = +\frac{F_n}{A_s}+m\frac{M}{I}\bar{x}. \tag{12}$$

In the above equations A_c and A_s are the areas of concrete and steel, respectively, while I_c and I are the moments of inertia of the concrete section and the concrete-steel transformed section with a modular ratio of m. Other terms are defined in the figure accompanying Table 1.

In general, the prestressing force in the steel prior to the application of the n-th load cycle, F_n, will vary slightly during the lifetime of the member. The value of F_n will be reduced by shrinkage of the concrete; creep of the concrete may tend either to increase or decrease it, depending upon whether the loading on the beam is for most of the time greater than or less than M_{on}; imperfect closing of the cracks may tend to increase the value slightly; creep in the steel strand will cause a reduction in F_n. Appropriate values must be chosen in each particular instance on the basis of an analysis for creep and shrinkage losses and an estimation of the other possible effects.

For the first cycle of loading, i.e. when $n=1$, the value of F_1 will either be known by measurement or estimated in the design calculations. Cracking will take place in this initial load cycle when f_{c1}^{b} becomes equal to f_t', the modulus of rupture of concrete. Thus

$$-F_1\left[\frac{1}{A_c}+\frac{e\,h}{I_c\,2}\right]+\frac{M_{o1}}{I}\left[\frac{h}{2}-e+\bar{x}\right] = f_t'$$

and the value of M_{on} in the first load cycle is

$$M_{o1} = I\frac{f_t'+F_1\left[\frac{1}{A_c}+\frac{e\,h}{I_c\,2}\right]}{\frac{h}{2}-e+\bar{x}}. \tag{13}$$

In subsequent load cycles, i.e. when $n>1$, cracks will begin to open when the value of f_{cn}^{b} is zero; thus,

$$M_{on} = I\frac{F_n\left[\frac{1}{A_c}+\frac{e\,h}{I_c\,2}\right]}{\frac{h}{2}-e+\bar{x}} \tag{14}$$

Second Loading Stage, $M > M_{on}$

The loading portion of the concrete stress-strain relation is represented in non-dimensional terms in this investigation by the cubic parabola

$$F = \alpha\,E+(3-2\,\alpha)\,E^2+(\alpha-2)\,E^3 \tag{15}$$

in the range

$$0 \leqq E \leqq 1\,;$$

where

$$F = \frac{f_c}{k_3 f_c'}, \tag{16}$$

$$E = \frac{\epsilon_c}{\epsilon_u}, \tag{17}$$

$$\alpha = E_{cn} \frac{\epsilon_u}{k_3 f_c'} \leqq 3. \tag{18}$$

The term $k_3 f_c'$ is the static strength of the concrete in the beam, ϵ_c is the concrete strain, ϵ_u is the concrete strain corresponding to the maximum stress value of $k_3 f_c'$, and E_{cn} is the concrete modulus of elasticity as determined by the initial tangent of the stress-strain curve at the n-th load cycle.

The unloading portion of the curve is represented by a second order parabola of the form

$$F = 1 - \frac{\beta}{\gamma^2}(E-1)^2 \tag{19}$$

for the region

$$1 \leqq E \leqq 1 + \gamma.$$

The complete stress-strain curve is shown in Fig. 7.

The area under the curve represented by Eq. (15) in the region $0 \leqq E \leqq E_1$, $E_1 \leqq 1$, is

$$A = \frac{\alpha}{2} E_1^2 + \frac{3 - 2\alpha}{3} E_1^3 + \frac{\alpha - 2}{4} E_1^4. \tag{20}$$

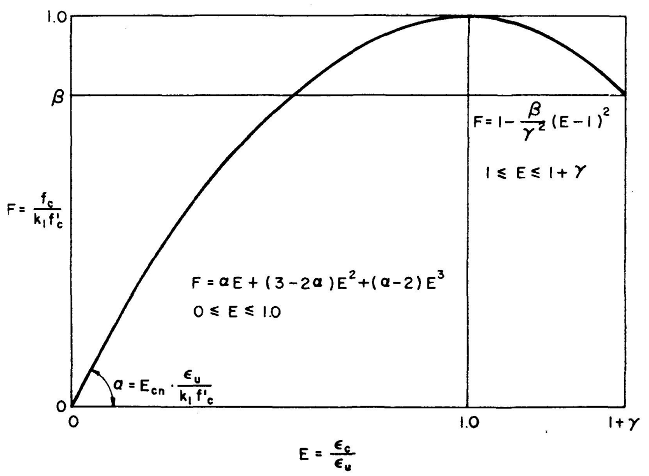


Fig. 7. Complete Stress-Strain Relation for Concrete.

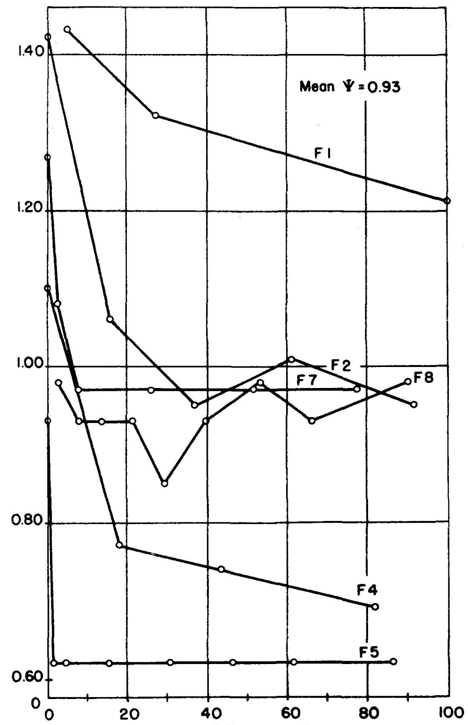


Fig. 8. Experimental Values of Bond Parameter, Ψ.

With the interval between E_1 and the center of gravity of this area defined as $k_2 E_1$, one obtains

$$k_2 = 1 - \frac{\frac{\alpha}{2} + \frac{3-2\alpha}{4} E_1 + \frac{\alpha-2}{5} E_1^2}{\frac{\alpha}{2} + \frac{3-2\alpha}{3} E_1 + \frac{\alpha-2}{4} E_1^2}. \tag{21}$$

In the idealized case of perfect bonding between steel and concrete, Bernoulli's linear strain hypothesis leads to the following compatibility equation

$$\epsilon_{s1} = \epsilon_{sF} + \epsilon_{cF} + \frac{1-k}{k} \epsilon_{c1}, \tag{22}$$

in which k is the ratio of depth of compression block to the effective depth d, ϵ_{s1} and ϵ_{c1} are the strains in the steel reinforcement and the concrete top fiber respectively, and ϵ_{sF} and ϵ_{cF} are the strains in the steel and in the concrete at the steel level due to the prestressing force F_n. Eq. (22) quantitatively describes a condition of uniform bending in which the tensile deformations consist of infinitesimal hair cracks at infinitesimal spacings; in actual fact, however, cracks of finite width form at finite spacings and the deformations in the beam correspond, approximately, to a slight kink at each cracked section. Since infinite strains cannot exist in the steel reinforcement, full or partial breakdown of bond must exist over some finite length on either side of each crack. To take account of this situation a bond parameter, ψ, is introduced into Eq. (22) as follows

$$\epsilon_{s1} = \epsilon_{sF} + \epsilon_{cF} + \frac{1-k}{k} \epsilon_{c1} \psi. \tag{23}$$

A theoretical expression for the dimensionless term ψ has been derived from several reasonable but approximate assumptions [3]; however, in most practical situations it is necessary to evaluate this term empirically from deformations measured during actual beam tests. In general, the value of ψ for a particular beam is not constant, but will vary with the number of cycles of fatigue loading. In Fig. 8 values of ψ are shown which were obtained during fatigue tests conducted on six rectangular beams made from high strength concrete and 7-wire prestressing strand. Values of ψ were usually in excess of unity during the first load cycles, but during an initial sequence of repeated loadings in which the beam settled down to a fairly regular and consistent response to load, the ψ values for the beams reduced in value considerably and then became reasonably constant. The mean value of ψ for the six beams, disregarding values in the initial loading sequences, is approximately 0.93. However, considering the scatter shown in Fig. 8, it is reasonable to adopt the simpler value

$$\psi = 1.0$$

for beams containing strand reinforcement. It is emphasized that values considerably different from unity may be expected for beams with other types of reinforcement.

Assuming a linear distribution of strain above the crack, one obtains for the total compressive force,

$$C = \frac{b\,d\,k_3 f_c' k}{E_1} \int_0^{E_1} F\,d\,E$$

and substituting from Eq. (20)

$$C = b\,d\,k_3 f_c' k \left[\frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3\right].$$

Horizontal forces may now be equated to yield

$$\frac{f_{s1} A_s}{b\,d\,k_3 f_c'} = k \left[\frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3\right] \tag{24}$$

and equating internal and external moments,

$$M_1 = f_{s1} A_s d\,(1 - k_2 k), \tag{25}$$

where k_2 is given in terms of E_1 in Eq. (21).

Eqs. (21), (23), (24), and (25), together with the steel stress-strain relation, may be used to evaluate, for a moment M_1, the unknowns f_{s1}, ϵ_{s1}, k, k_2, and E_1. It will be noted that the value of F_n for the n-th load cycle must be known or estimated since it is used to determine ϵ_{sF} and ϵ_{cF}. In general, the calculations for the stress-moment relation will be simplified if values of f_{s1} and ϵ_{s1} are first chosen and substituted in Eqs. (23) and (24) to obtain k, E_1, and hence k_2 from Eq. (21). The corresponding values of M_1 may then be obtained by substitution of values in Eq. (25). The process may be repeated to obtain different points on the $M_1 - f_{s1}$ curve.

When a large number of stress-moment calculations are to be made, it may be convenient to plot Eqs. (23) and (24) in the form of an intercept chart on coordinates of E_1 and k, for appropriate values of α. Eq. (23) may be divided throughout by ϵ_u and rearranged to the form

$$\frac{1}{\psi \epsilon_u}[\epsilon_{s1} - \epsilon_{sF} - \epsilon_{cF}] = E_1 \frac{1-k}{k}. \tag{23a}$$

Eq. (23a) and (24) both contain the terms E_1 and k in the right hand sides, while their left hand sides are functions of either f_{s1} or ϵ_{s1}. Each equation may be used to plot a family of curves on the $E_1 - k$ coordinates. The resulting intercept chart provides values directly for k and E_1 without a trial and error procedure. To avoid a trial and error method for obtaining k_2, it is convenient also to plot k_2 against E_1 using Eq. (21).

Probable Fatigue Life of Prestressed Concrete Beams

In order to predict the fatigue life of a given beam, it is necessary first to determine, from the known or assumed load history, the corresponding stress history for the reinforcing steel. The transformation from load history to stress history is made by using stress-moment curves which are computed using the equations derived above together with the steel stress-strain relation. If the response of the beam to load remains constant throughout the major portion of its fatigue life, only one stress-moment relation has to be obtained. If, however, the response of the beam varies as a result of the fatigue loading, the load history must be broken into a number of intervals, the size of the interval depending upon the rate of change of beam response, and a stress-moment relation must be computed for each interval. It is assumed here that the response of the beam to load remains constant over the majority of its fatigue life.

When the beam is subjected only to repeated load cycles of constant magnitude, the stress history will consist of repeated stress cycles of constant magnitude. After the magnitude of the stress cycle has been determined from the stress-moment relation, Eqs. 1, 2, 3, and 4 may be used to determine the mean fatigue life, $\bar{N}$, and the standard deviation of fatigue life, D, for a single strand element subjected to this stress cycle. These two values may be used in Eq. 6 to determine the number of cycles, N, corresponding to *any* probability level P.

If there are u similar strands present in the beam section at the same level, then the probability of beam failure at or before N cycles is

$$Q = 1-(1-P)^u. \tag{26}$$

In the case of a beam subjected to cumulative damage loading, the load history may be expressed as a curve relating load magnitude and relative frequency of occurrence (load-frequency distribution), a load-frequency histogram, or a block of load cycles similar to those shown in Fig. 5. In each case the load history can be represented, either exactly or approximately, by a block of load cycles, and the stress-moment relation can be used to make the transformation into a corresponding block of stress cycles as in Fig. 5. Eq. (9) will then indicate, for a strand element subjected to this repeated load block, the probability of failure, P, corresponding to any number of load cycles, N. Eq. (26) may be used to determine from P the probability of fatigue failure, Q, of the beam at N cycles.

Beam Tests

A small series of static and fatigue tests was conducted on eight prestressed concrete beams of rectangular section to provide test data to check the accuracy of the methods developed in this investigation for the prediction of beam fatigue life. Two beams were tested to failure statically, three were tested in fatigue under constant cycle loading, and three were tested in fatigue with varied repeated loadings.

The beams were twelve feet long, with a rectangular cross section approximately six inches wide and twelve inches deep. The longitudinal reinforcement consisted of three $^{7}/_{16}$ inch diameter high strength steel prestressing strands placed at a depth of eight inches below the top surface of the beam. Details of the beams are contained in Table 1.

Table 1. Prestressed Concrete Test Beam Properties

Beam	b (in.)	d (in.)	h (in.)	f'_c* (ksi)	p	ϵ_{ce}	$\Delta\,\epsilon_c$	ϵ_{se}	F_1 (kips)
F 1	6.12	8.09	12.12	7.04	0.00661	22	68	580	51.0
2	6.06	8.09	12.12	7.21	0.00669	23	71	576	50.6
4	6.19	8.00	12.06	6.98	0.00660	24	71	575	50.6
4	6.00	8.00	12.06	6.98	0.00681	24	70	576	50.6
F 5	6.09	8.12	12.12	6.42	0.00660	17	62	436	37.8
6	6.19	8.00	12.06	6.88	0.00660	16	63	431	37.5
7	6.31	8.00	12.06	6.22	0.00648	16	70	414	35.6
8	6.25	8.00	12.06	6.83	0.00653	16	65	419	36.0

* Average f'_c in test section

Notes: All strains in in/in $\times 10^{-5}$ $\quad p=\frac{A_s}{b\,d}$

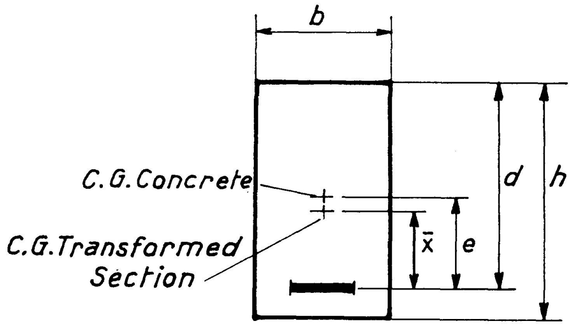
All moments in in-kips

ϵ_{se} = steel strain corresponding to F_1

ϵ_{ce} = elastic concrete strain at steel level corresponding to F_1

$\Delta\,\epsilon_c$ = inelastic concrete strains in concrete at steel level prior to fatigue loading

F_1 = steel prestressing force prior to first load cycle



The beams were tested on a ten foot span, with two loads symmetrically placed four feet apart. Static tests were conducted at regular intervals during the fatigue loading to allow deformation measurements to be made on the sides of the beams. These deformation readings were used to obtain the values of ψ shown in Fig. 8.

A comparison is made in Table 2 between observed fatigue life and mean fatigue life predicted by the method described above for the six beams tested in fatigue. Since three strands were contained in the cross section, values of $u=3$, and $Q=0.5$ are substituted in Eq. (26) to obtain the value $P=0.206$. The line of 0.206 probability plotted in Fig. 2 was used in the computations for mean fatigue life.

Table 2. Comparison of Predicted and Observed Beam Fatigue Lives

BEAM	Rate of Loading (cpm)	Applied Moments *)				Observed Moments		Wire Failures, Million Cycles					
									Observed				
		M_{min}	M_{pred}	M_{01}	M_{02}	M_{o1} (Cracking)	M_{ult} (Static)	$\bar{N}_p$	N_1	N_2	N_3	N_4	N_5
F 1	250	162	436	—	—	309.6	—	0.179	0.225	0.233	0.258	0.258	0.258
2	250	162	436	—	—	308.0	—	0.191	0.164	0.200	0.215	0.226	0.226
3		(Static Test)				316.6	536.4		(Crushing of Concrete Top Fibers)				
4	250	162	436	—	—	300.5	—	0.170	0.139	0.146	0.164	—	—
F 5	500	137	255	329	366	263.3	—	2.300	1.947	2.516	2.817	2.817	2.820
6		(Static Test)				257.5	545.4		(Crushing of Concrete Top Fibers)				
7	250	137	254	328	374	248.4	—	1.120	1.167	1.437	1.467	1.552	1.580
8	250	137	256	327	375	241.2	—	1.310	1.136	1.557	1.586	1.587	—

*) Including dynamic effect.

Notes: All moments in in-kips.

$\bar{N}_p$ = predicted mean fatigue life corresponding to N_1.

$N_1, N_2, \ldots$ are the number of load cycles at which first, second, ... wire failures occurred.

In making the computations, a ψ value of 1.0, and a k_3 value of 0.85 were used. Creep relaxation losses in the steel were not measured during the beam tests, and a value of four percent was adopted on the basis of figures quoted by Kommendant [7] for prestressing wires. Although Table 2 shows a slight trend for the method to over-estimate fatigue life, agreement is generally quite satisfactory, especially considering the variability of the phenomenon being studied.

Discussion

It will be noted that although the effect of the presence of a number of strands in the section was taken into account in Eq. (26), no consideration was given to any "size effect" to take account of variations in the length of the beams. Such an effect would be difficult to analyze quantitatively since the steel stress varies with moment along the length of the beam, with additional stress concentrations existing at each cracked section. In the present analysis, it has been assumed that failure will always occur in the region of maximum steel stress which will exist at the section of maximum moment and hence at the widest crack.

An examination of the results of the strand fatigue tests shows extreme sensitivity of fatigue life to small changes in the maximum and minimum stress levels. This sensitivity becomes very pronounced in the range of large

N and small S values, where the mean curve is approaching its asymtotic value. Beam fatigue life shows like sensitivity to small variations in the loadings, particularly in the maximum load level. Small errors in the computed steel stresses can thus cause large errors in computed beam fatigue life.

In the stress computations a number of factors are involved, such as k_3, ψ, f'_c, and prestress losses, which cannot be evaluated precisely in most practical situations, and it is important to observe that variations in these quantities have a very significant effect on computed fatigue life. In this situation, it would seem desirable to treat not only the fatigue properties of the materials as random variables but also the response of the beam to load. Thus, quantities such as f'_c, k_3, α, F_n, and ψ would be considered not as single valued parameters but as statistics with associated frequency distributions. Such a procedure, however, is clearly not feasible until very extensive experimental work is conducted to determine the frequency distribution for each random variable.

The reasonable agreement obtained between predicted mean fatigue life and observed fatigue life for the test beams does however indicate the appropriateness of the methods developed in this investigation. By adopting suitably conservative values for parameters which are not known exactly, the equations may be used to check the safety against fatigue failure of partially prestressed members which are cracked under load.

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References

1. Nordby, G. M., Fatigue of Concrete — A Review of Research. Journal, American Concrete Institute, Vol. 30, No. 2, August 1958.
2. Le Camus, B., Recherches sur le comportement du béton et du béton armé soumis à des efforts répétés. Compte Rendu des Recherches Effectuées en 1945/46, Laboratoires du Bâtiment et des Travaux Publics, Paris, 1946.
3. Warner, R. F.; Hulsbos, C. L., Probable Fatigue Life of Prestressed Concrete Flexural Members. Fritz Laboratory Report No. 223.24a, Lehigh University, Bethlehem, Penna.
4. Freudenthal, A. M., Planning and Interpretation of Fatigue Tests. Symposium on Statistical Aspects of Fatigue, Special Technical Publication No. 121, ASTM, 1952.
5. Epremian, E.; Mehl, R. F., Investigation of Statistical Nature of Fatigue Properties. National Advisory Committee for Aeronautics, Technical Note 2719, June 1952.

6. GROVER, H. J.; GORDON, S. A.; JACKSON, L. R., Fatigue of Metals and Structures. U.S. Government Printing Office, Revised Edition, June 1960.
7. CROW, E. L.; DAVIS, F. A.; MAXFIELD, M. W., Statistics Manual. Dover Publications, New York, 1960.
8. MINER, M. A., Cumulative Damage in Fatigue. Journal of Applied Mechanics, Vol. 12, P.A-159, September 1945.
9. KOMMENDANT, A. E., Prestressed Concrete Structures. McGraw-Hill Book Co., Inc. 1952.

Summary

This paper summarizes the results of an investigation into the fatigue life of under-reinforced prestressed concrete flexural members subjected to repeated loadings of either constant or varied magnitude. The work consisted of an experimental study of the fatigue properties of high strength steel prestressing strand, a theoretical analysis of the stresses and deformations in prestressed concrete flexural members under fatigue loading, and a small series of beam fatigue tests. The results of the theoretical analysis may be used together with the experimentally determined strand fatigue properties to estimate the fatigue life of beams failing by fatigue of the steel reinforcement.

Résumé

Les auteurs résument les résultats d'une étude sur la résistance, à la fatigue de poutres en béton précontraint faiblement armées, travaillant à la flexion et soumises à des efforts répétés de grandeur constante ou variable. Cette étude a comporté des recherches expérimentales concernant le comportement à la fatigue des fils en acier à haute résistance, une analyse théorique sur les déformations et les contraintes dans des poutres en béton précontraint sollicitées à la fatique, ainsi qu'une petite série d'essais à la fatigue sur des poutres. Les résultats des études théoriques peuvent être utilisés, conjointement avec les caractéristiques de résistance à la fatigue des fils trouvées expérimentalement, pour estimer le nombre de cycles supportés par des poutres dont la ruine est due à une rupture par fatigue des armatures.

Zusammenfassung

Die Resultate einer Untersuchung über die Dauerfestigkeit unterarmierter Balken aus vorgespanntem Beton unter konstanter und variabler Wechsellast sind zusammengefaßt. Experimentelle Untersuchungen der Dauerfestigkeitseigenschaften hochwertiger Stahldrähte und vorgespannter Biegebalken sowie theoretische Untersuchungen über Formänderungen und Spannungen im Vorspannbeton sind Inhalt der Arbeit. Die Resultate der theoretischen Studie können zusammen mit den experimentell ermittelten Dauerfestigkeitseigenschaften dazu verwendet werden, die Wahrscheinlichkeit der Ermüdung von Vorspannbiegebalken durch Stahlbruch zu beurteilen.