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Autor: Distefano, Jose Nestor

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Creep Deflections in Concrete and Reinforced Concrete Columns

Déformations dues au fluage de colonnes en béton et béton armé

Kriechdeformationen in Beton und Stahlbetonstützen

JOSE NESTOR DISTEFANO
Professor, Argentine

I. Introduction

Previous literature [1], [2], [3], [4] on creep deflections in columns was based on different models of linear viscoelastic material and all the papers seem to prove that indefinitely large deflections can be reached after an indefinitely long time, and failure occurs when the ultimate bending moment is reached.

Other papers [5], [6] show that for every P (axial force) between zero and the Euler's critic load, deflections converge towards a finite value when time tends to infinity.

This differences in conclusions can be explained because of the different models used for representing linear viscoelastic materials. In the present paper, the integral expression of Volterra's theory of hereditary phenomena [7] is used, which permits a great generality, because the differential expressions used to represent the viscoelastic bodies e. g., Maxwell, standard, etc. are but particular cases of the Volterra's integral expression. Moreover, by means of this representation it is possible to take into account, as closely as we want, reduction of creep due to the age of concrete.

In the present paper, deflections due to creep in symetrically reinforced concrete columns are studied. It is shown that creep deflections tend to a finite limit as $t \to \infty$, if, and only if, the axial load is not equal to or less than a certain value $P^* < P_k$, where P_k is the Euler's critic load. When $P^* \leqq P \leqq P_k$, deflections will reach infinitely large values for $t \to \infty$. The upper limit P^* of loads (below which only finite deflections are possible) depend, on the asymptotic value of the specific creep strain — measured in the aged concrete —, on the inertia moment of the steel reinforcement and on the elastic modulus of both materials.

II. Creep in Concrete

It is generally accepted that creep in concrete is proportional at every time to the applied stress, and that it verifies the superposition principle in the Boltzmann's sense, provided 30% to 50% of the rupture stress is not exceeded [8], [9], [10], [11], [12].

In this form, it is possible to represent the creep strain of a concrete element by means of the general integral expression of Volterra's theory of linear hereditary phenomena. Let $\sigma(t)$ be the applied stress, variable with time t. Then it is possible to write the creep strain

$$\bar{\epsilon}(t) = \int_{\tau_0}^{t} \sigma(\tau) f(t,\tau)\, d\tau, \tag{1}$$

where τ_0 is the instant in which the stress is applied, and $f(t,\tau)$ is the creep coefficient defined by

$$f(t,\tau) = -\frac{\partial}{\partial \tau} \bar{\epsilon}_0(t,\tau), \tag{2}$$

where $\bar{\epsilon}_0(t,\tau)$ is the specific creep produced when a stress of 1 kg/cm^2 is applied at the instant τ. The specific creep for different ages τ of concrete can be found experimentally and the creep strain is well defined by mean of Eq. (1).

It is assumed that the instantaneous strain, produced when stress is applied, will obey Hooke's law, so that

$$\epsilon(t) = \frac{\sigma(t)}{E_b}, \tag{3}$$

where E_b is the elastic modulus of concrete.

III. Integro-Differential Equations of Bending

If we consider that bending takes place only in a principal inertia plane and that sections, plain before bending, remain plain, the total strain can be expressed by

$$\epsilon(t) + \bar{\epsilon}(t) = \lambda(t) + \mu(t) z. \tag{4}$$

Eliminating $\epsilon(t)$ between (3) and (4), and taking into account that in the zone occupied by the steel must be $\bar{\epsilon}(t) = 0$, stresses σ_b and σ_e in concrete and steel can be calculated by

$$\sigma_b(t) = E_b[\lambda(t) + \mu(t)z - \bar{\epsilon}(t)], \tag{5}$$

$$\sigma_e(t) = E_e[\lambda(t) + \mu(t)z]. \tag{6}$$

The conditions of static equilibrium when an axial load P and a bending moment M act in the cross section, are expressed by

$$P = \int_{A_b} \sigma_b \, d A_b + \int_{A_e} \sigma_e \, d A_e, \tag{7}$$

$$M = \int_{A_b} \sigma_b z \, d A_b + \int_{A_e} \sigma_e z \, d A_e, \tag{8}$$

where A_b and A_e are the concrete and steel areas, respectively. Substituting (5) and (6) in (7) and (8), these last take the form

$$P = \lambda E_b \bar{A} + \mu (E_b S_b + E_e S_e) - E_b \int_{\tau_0}^{t} f(t,\tau) \, d\tau \int_{A_b} \sigma_b(\tau) \, d A_b, \tag{9}$$

$$M = \mu E_b \bar{I} + \lambda (E_b S_b + E_e S_e) - E_b \int_{\tau_0}^{t} f(t,\tau) \, d\tau \int_{A_b} \sigma_b(\tau) z \, d A_b, \tag{10}$$

where $\bar{\epsilon}(t)$ was replaced by its equivalent (1), and where

$$\bar{A} = A_b + n A_e; \qquad \bar{I} = I_b + n I_e,$$

$$S_b = \int_{A_b} z \, d A_b; \qquad S_e = \int_{A_e} z \, d A_e.$$

From equilibrium Eqs. (7) and (8) and taking into account (6), it is then possible to calculate the following integrals

$$\int_{A_b} \sigma_b \, d A_b = P - \lambda E_e A_e - \mu E_e S_e,$$

$$\int_{A_b} \sigma_b z \, d A_b = M - \lambda E_e S_e - \mu E_e I_e.$$

These integrals, substituted in (9) and (10), give the following equations

$$\left.\begin{aligned} \lambda + \mu R + E_e \frac{A_e}{\bar{A}} \int_{\tau_0}^{t} \lambda f \, d\tau + E_e \frac{S_e}{\bar{A}} \int_{\tau_0}^{t} \mu f \, d\tau &= \frac{1}{E_b \bar{A}} [P + E_b \int_{\tau_0}^{t} P f \, d\tau], \\ \lambda R + \mu + E_e \frac{S_e}{\bar{I}} \int_{\tau_0}^{t} \lambda f \, d\tau + E_e \frac{I_e}{\bar{I}} \int_{\tau_0}^{t} \mu f \, d\tau &= \frac{1}{E_b \bar{I}} [M + E_b \int_{\tau_0}^{t} M f \, d\tau], \end{aligned}\right\} \tag{11}$$

where $\quad R = \frac{1}{\bar{A}} (S_b + n S_e).$

These equations are simplified if we choose the axis so that

$$S_b + n S_e = \bar{A} R = 0.$$

Moreover, in the present paper we will consider only the case in which the steel is symetrically placed with respect to the bending axis. With this disposition, it will be $S_e = 0$, and the system (11) will be reduced to the following independent integral equations:

$$\lambda + E_e \frac{A_e}{\bar{A}} \int_{\tau_0}^{t} \lambda f(t,\tau)\, d\tau = \frac{1}{E_b \bar{A}} [P + E_b \int_{\tau_0}^{t} P f(t,\tau)\, d\tau], \tag{12}$$

$$\mu + E_e \frac{I_e}{\bar{I}} \int_{\tau_0}^{t} \mu f(t,\tau)\, d\tau = \frac{1}{E_b \bar{I}} [M + E_b \int_{\tau_0}^{t} M f(t,\tau)\, d\tau]. \tag{13}$$

We will suppose now that the column have a little initial eccentricity $y_0(x)$, so that the bending moment M can be expressed by

$$M = P[y(x,t) + y_0(x)]. \tag{14}$$

Substituting (14) in (12) and (13), and the curvature μ by its approximate value $\mu = -\frac{\partial^2 y}{\partial x^2}$ and considering that the axial P is constant with time, Eqs. (12) and (13) are modified in

$$\lambda + E_e \frac{A_e}{\bar{A}} \int_{\tau_0}^{t} \lambda f(t,\tau)\, d\tau = \frac{P}{E_b \bar{A}} [1 + E_b \bar{\epsilon}_0(t,\tau_0)], \tag{15}$$

$$\frac{P}{E_b \bar{I}} y + \frac{\partial^2 y}{\partial x^2} + \frac{1}{\bar{I}} \int_{\tau_0}^{t} \left(P y + E_e I_e \frac{\partial^2 y}{\partial x^2}\right) f(t,\tau)\, d\tau = -\frac{P}{E_b \bar{I}} y_0(x) [1 + E_b \bar{\epsilon}_0(t,\tau_0). \tag{16}$$

First, we will solve the integral-differential equation for deflections (16), and then we will solve Eq. (15).

IV. Solution of Equation (16)

To solve Eq. (16) we will suppose that it is possible to develop the functions $y(x,t)$ and $y_0(x)$ in series of functions $\varphi_i(x)$

$$y_0(x) = \sum_1^\infty a_i \varphi_i(x), \tag{17}$$

$$y(x,t) = \sum_1^\infty b_i(t) \varphi_i(x), \tag{18}$$

where $\varphi_i(x)$ are the eigen-functions of the differential equation

$$[\varphi_i'' + k_i \varphi_i]'' = 0, \tag{19}$$

whose first derivates φ_i' are orthogonal functions, *when conditions of free, or hinged, or built-in end are satisfied* [13]. By means of this property of orthogonality coefficients a_i are immediately calculated by

$$a_i = \frac{\int_0^l y_0'(x) \varphi_i'(x)\, dx}{\int_0^l [\varphi_i'(x)]^2\, dx}.$$

With respect to coefficient $b_i(t)$, eliminating functions $y(x,t)$ and $\varphi_i(x)$ between Eqs. (16), (18) and (19), the following condition equations are obtained

$$b_i(t)+\beta_i\int_{\tau_0}^{t} b_i(\tau)\, f(t,\tau)\, d\tau = \frac{P}{k_i E_b \bar{I} - P} a_i [1+E_b \bar{\epsilon}_0(t,\tau_0)], \tag{20}$$

where
$$\beta_i = E_b \frac{k_i E_e I_e - P}{k_i E_b \bar{I} - P}.$$

To solve the above integral equation, it is convenient to reduce a little the generality of function $f(t,\tau)$ or what is the same, of the function $\bar{\epsilon}_0(t,\tau)$. It is noted that in materials like synthetic plastics or aged concrete, creep depends only on the difference of parameters $t-\tau$ [14]; in this case, Eq. (20) can be rapidly solved by means of the Laplace's transformation, and the asymptotic properties of solutions can be easily investigated [15]. But in young concrete, creep depends not only on the difference $t-\tau$, but also on the age τ. Aroutiounian [16] proposes to represent the specific creep with the function

$$\bar{\epsilon}_0(t,\tau) = \psi(\tau)\, F(t-\tau) \tag{21}$$

where $\psi(\tau)$ is always positive and decreases monotonely towards the finite limit $\psi(\infty)=\gamma_0$, and it represents the damping of creep due to age. The function $F(t-\tau)$ is also positive and monotonely grows towards an upper limit equal to unity, for great values of parameters $t-\tau$. Taking

$$\begin{aligned} \psi(\tau) &= \gamma_0 + \frac{C}{\tau}, \\ F(t-\tau) &= 1-e^{-\delta(t-\tau)} \end{aligned} \tag{22}$$

and choosing convenient values of the constants γ_0, C, δ, it is possible to follow as closely as we want, the creep of the concrete, with its dependence on age. The creep coefficient $f(t,\tau)$ can be calculated by means of Eq. (2) and its value is in this case

$$f(t,\tau) = -\frac{\partial}{\partial \tau}[\psi(\tau)(1-e^{-\delta(t-\tau)})]. \tag{23}$$

Substituting (23) in Eq. (20), and rewriting, we have

$$b_i(t) - \beta_i \int_{\tau_0}^{t} b_i(\tau)\, \psi'(\tau)\, d\tau + \beta_i \int_{\tau_0}^{t} b_i(\tau)(\psi' + \delta\psi)\, e^{-\delta(t-\tau)}\, d\tau = h_i(t,\tau_0). \tag{24}$$

where $h_i(t,\tau_0)$ is the second member of Eq. (20).

Differentiations above equation with respect to t

$$b_i'(t) + \beta_i \delta \psi(t)\, b_i(t) - \delta \beta_i \int_{\tau_0}^{t} b_i(\tau)(\psi'+\delta\psi)\, e^{-\delta(t-\tau)}\, d\tau = h_i'(t,\tau_0). \tag{25}$$

Eliminating the integral

$$\int_{\tau_0}^{t} b_i(\tau)(\psi'+\delta\psi)\, e^{-\delta(t-\tau)}\, d\tau$$

between Eqs. (24) and (25), and differentiating again with respect to t, the following differential equation is obtained

$$b_i''(t) + \delta [1 + \beta_i \psi(t)] b_i'(t) = 0 \tag{26}$$

with the following boundary conditions

$$\begin{aligned} b_i(\tau_0) &= h_i(\tau_0, \tau_0) = \frac{P a_i}{k_i E_b \bar{I} - P}, \\ b_i'(\tau_0) &= E_b \frac{P a_i}{k_i E_b \bar{I} - P} \psi(\tau_0) \delta \left(1 - \frac{k_i E_e I_e - P}{k_i E_b \bar{I} - P}\right). \end{aligned} \tag{27}$$

The general solution of Eq. (26) is obtained by means of two successive integrations

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0) \int_{\tau_0}^{t} e^{-J_i(\xi)} d\xi, \tag{28}$$

where
$$J_i(t) = \delta \int_{\tau_0}^{t} [1 + \beta_i \psi(\tau)] d\tau.$$

Substituting for $\psi(\tau)$ from (22), the integral becomes transformed into

$$J_i(t) = \delta (1 + \beta_i \gamma_0)(t - \tau_0) + \ln \left(\frac{t}{\tau_0}\right)^{\beta_i C \delta},$$

which substituted in (28) gives

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0) e^{\delta \tau_0 (1 + \beta_i \gamma_0)} \tau_0^{\beta_i C \delta} \int_{\tau_0}^{t} e^{-\delta(1+\beta_i \gamma_0)\tau} \tau^{-\beta_i C \delta} d\tau. \tag{29}$$

Introducing the incomplete gamma function $\Phi(\alpha, t)$ defined by

$$\Phi(\alpha, t) = \int_0^t e^{-\tau} \tau^{\alpha - 1} d\tau.$$

Eq. (29) becomes

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0) e^{\nu_i \tau_0} \tau_0^{1-\alpha_i} \nu_i^{-\alpha_i} [\Phi(\alpha_i, \nu_i t) - \Phi(\alpha_i, \nu_i \tau_0)], \tag{30}$$

where
$$\begin{aligned} \alpha_i &= 1 - \beta_i C \delta, \\ \nu_i &= \delta (1 + \beta_i \gamma_0). \end{aligned}$$

V. Convergence of Deflections

The integral that figures in Eq. (29) is convergent for $t \to \infty$, if, and only if,

a) $1 + \beta_i \gamma_0 > 0$,
b) $1 - \beta_i C \delta > 0$,

remembering the value of β_i given by Eq. (20), the first condition is verified if

$$0 \leqq P < \frac{E_b \bar{I} k_m}{1 + E_b \gamma_0} (1 + E_b \gamma_0 \rho),$$

where k_m is the smaller eigenvalue of differential Eq. (19) and where $\rho = \frac{n I_e}{\bar{I}}$ But it is easy to show that $E_b \bar{I} k_m$ is just Euler's critic load of the reinforced concrete column calculated in perfectly elastic range. Consequently, the above unequality can be written

$$0 \leqq P < \frac{P_k}{1 + E_b \gamma_0} (1 + E_b \gamma_0 \rho). \tag{31}$$

With respect to condition (b) it can be expressed as follows:

$$E_b C \delta \frac{\rho P_k - P}{P_k - P} < 1. \tag{32}$$

Because ρ can be varied between zero and one, when ρ is placed in the interval

$$0 \leqq \rho \leqq \frac{P}{P_k}$$

the condition (32) is always verified, for every value of $E_b C \delta$ coefficient. But, when ρ is in the interval (columns very much reinforced)

$$\frac{P}{P_k} < \rho \leqq 1$$

condition (32) is verified if, and only if,

$$E_b C \delta < 1. \tag{33}$$

We are justified in assuming that this inequality is satisfied because a large number of experiments [8] show that generally the factor $E_b C \delta$ does not exceed 0.5.

Once the conditions of convergence of functions $b_i(t)$ are established, we will examine the deflections. These are given by (18).

From conditions of convergence of $b_i(t)$ functions, we can conclude that

a) *When the axial load P is placed in the interval (31), deflections will tend to the following finite limit, for $t \to \infty$*

$$y(x, \infty) \to \sum_1^\infty b_i(\tau_0) \varphi_i(x) + \sum_1^\infty b_i'(\tau_0) e^{\nu_i \tau_0} \tau_0^{1-\alpha_i} \nu_i^{-\alpha_i} [\Phi(\alpha_i, \infty) - \Phi(\alpha_i, \nu_i \tau_0)]. \tag{34}$$

b) *When the axial Load P is placed in the range*

$$\frac{P_k}{1 + E_b \gamma_0} (1 + E_b \gamma_0 \rho) \leqq P < P_k \tag{35}$$

the functions $b_i(t)$, and consequently the deflections, will tend to infinity for $t \to \infty$.

Naturally, when $P \geqq P_k$, large deflections will be produced instantaeously.

VI. Stresses in Concrete and Steel

Having calculated functions $b_i(t)$ by means of (30), curvature μ is immediately calculated by means of

$$\mu(x,t) = -\sum_1^\infty b_i(t)\,\varphi_i''(x). \tag{36}$$

It is possible to calculate the function $\lambda(t)$ by means of the integral Eq. (15). With a similar treatment given to integral Eq. (20), Eq. (15) reduces to the following differential equation

$$\lambda'' + \delta[1+\theta\psi(t)]\lambda' = 0; \qquad \theta = E_e\frac{A_e}{\bar{A}} \tag{37}$$

associated with the following boundary conditions

$$\lambda(\tau_0) = \frac{P}{E_b\bar{A}},$$

$$\lambda'(\tau_0) = n\frac{P}{\bar{A}}\psi(\tau_0)\,\delta\left(1+\frac{A_e}{\bar{A}}\right).$$

General solution of (37) is then obtained

$$\lambda(t) = \frac{P}{E_b\bar{A}}\left[1 + E_e\psi(\tau_0)\,\delta\left(1+\frac{A_e}{\bar{A}}\right)e^{\delta\tau_0(1+\theta\gamma_0)}\tau_0^{\theta C\delta}\int_{\tau_0}^{t} e^{-\delta(1+\theta\gamma_0)\tau}\,\tau^{-\theta C\delta}\,d\tau\right]. \tag{38}$$

The conditions of convergence for $t\to\infty$ of the integral are

$$1+\theta\gamma_0 > 0,$$

$$1+\theta C\delta > 0,$$

which are always verified because $\theta = E_e\frac{A_e}{\bar{A}} \geqq 0$. Knowing λ and μ by means of (36) and (38), the elastic stresses in the steel can be immediately calculated from

$$\sigma_e = E_e(\lambda+\mu z). \tag{39}$$

The stresses σ_b in concrete will be calculated in the following manner: we will calculate the bending moment M_b and the axial force P_b absorbed by the concrete only

$$M_b = P(y+y_0) - \int_{A_e}\sigma_e z\,dA_e,$$

$$P_b = P - \int_{A_e}\sigma_e\,dA_e$$

and then we can apply the well know formula of Strength of Materials.

$$\sigma_b = \frac{P_b}{A_b} + \frac{M_b}{I_b}z.$$

VII. Generalization of the Problem in the Case of the Beam-Column

Supposing a system of lateral loads are applied to the column, which produce a bending moment $\mathfrak{M}(x)$ in the absence of the axial load P, then the total bending moment will be

$$M = P\,y(x,t) + P\,y_0(x) + \mathfrak{M}(x). \tag{40}$$

Taking
$$\kappa(x) = y_0(x) + \frac{1}{P}\mathfrak{M}(x).$$

The previous developments remain unchanged if instead of coefficients a_i we take coefficients a_i^* given by the formula

$$a_i^* = \frac{\int\limits_0^l \kappa'(x)\,\varphi_i'(x)\,dx}{\int\limits_0^l [\varphi_i'(x)]^2\,dx}.$$

Then the condition of convergence of deflections will not change.

VIII. Discussion

In section V we have shown that deflections will tend to a finite limit with $t \to \infty$, if, and only if, the axial load belongs to the interval (31). While, when P is in the interval (35), the deflections will tend towards infinity for $t \to \infty$. These conclusions are based on the linear creep theory, which is valid only when the stresses in the concrete are not greater than 30% to 50% of the rupture stress. Experiments show that when the stresses are greater than this limit, creep is not linear, increasing more rapidly with increasing stresses [17]. For this reason, axial loads P placed in the interval (35), are always creep failure loads.

It is important to note that the amplitude of the interval of the safe loads (31), can be modified by varying the proportion of steel in the column. When the columns have no steel, that is $\rho = 0$, the interval (31) is reduced to

$$0 \leqq P < \frac{P_k}{1 + E_b\,\gamma_0}$$

while, when the column is reduced (by increasing the proportion of steel) to a purely steel one, then $\rho = 1$, and the interval (31) is enlarged to

$$0 \leqq P < P_k$$

this naturally corresponds to a purely elastic column.

We can see now in a particular case the influence of a steel reinforcement of 3.33% in a rectangular concrete column. Suppose that the reinforcement is formed by 4 steel bars placed in the corner of the rectangle, with this disposition, and considering $n=10$, ρ has the value $^1/_2$. We suppose now that factor $E_b \gamma_0 = 2$. The range of the safe load (31) of a column without steel will be

$$0 \leqq P < \frac{1}{3} P_k .$$

While for the same column reinforced with 3.33% of steel, the range will be

$$0 \leqq P < \frac{2}{3} P_k .$$

That is to say, the amplitude of the range was duplicated.

Finally, we wish to mention the creep-buckling experiments carried out by RENÉ PERZO [18], which seem to confirm the previous criterion. Plain concrete plates of 250/4/50 cm, built-in at the ends, were axially loaded at the age of 7 days. These tests showed that the time of rupture by creep, increases quickly when load is decreased, and that loads below $0.30\, P_k$ are not capable of producing rupture.

On the other hand, creep tests made at the same time, on concrete aged 21 days, showed that factor $\frac{1}{1+E_b \gamma_0}$ has the value 0.33.

Notation

P	=	Axial load
P_k	=	Euler's critical load
M	=	Bending moment
ϵ	=	Elastic strain
$\bar{\epsilon}$	=	Creep strain
$\bar{\epsilon}_0$	=	Specific creep produced by $\sigma = 1\, k\, \mathrm{cm}^{-2}$
σ	=	Stress
t	=	Time
τ	=	Age of concrete
E_e	=	Elastic modulus of steel
E_b	=	Elastic modulus of concrete
n	=	$\frac{E_e}{E_b}$
λ	=	Shortening of the axis of the column
μ	=	Increment of curvature of the axis of the column, referred to the unloaded incurved position
z	=	Ordinate of a generic point of the cross section with respect to the neutral axis
A_e	=	Steel area
A_b	=	Concrete area
$\bar{A}$	=	$A_b + n\, A_e$

$I_e \quad = \int\limits_{A_e} z^2 \, d A_e$

$I_b \quad = \int\limits_{A_b} z^2 \, d A_b$

$\bar{I} \quad = I_b + n \, I_e$

$y_0(x) \quad =$ Initial deflection of the column axis

$y(x,t) \quad =$ Deflections of the column axes with respect to the unloaded incurved positions

$\rho \quad = \dfrac{n \, I_e}{\bar{I}}$

$\Phi(\alpha, t) = \int\limits_0^t e^{-\tau} \tau^{\alpha-1} \, d\tau =$ Incomplete Gamma function

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Summary

In the present paper, deflections due to creep in reinforced concrete columns with symetrical steel reinforcement are studied. It is shown that creep deflections tend to a finite limit for $t \to \infty$, if, and only if, axial load is not equal or smaller than a certain value $P^* < P_k$, where P_k is the Euler's critic load. When $P^* \leqq P \leqq P_k$, deflections will reach infinitely large values for $t \to \infty$. The upper limit P^* of loads (below which only finite deflections are possible) depends on the asymptotic value of the specific creep, measured in the aged concrete, on the inertia moment of the steel reinforcement and on the elastic modulus of both materials.

Résumé

L'auteur étudie les déformations dues au fluage de colonnes en béton armé à armature symétrique. Pour $t \to \infty$, les déformations dues au fluage ne tendent vers une valeur finie que si la charge axiale n'atteint pas une certaine valeur $P^* < P_k$, P_k désignant la charge critique d'Euler. Lorsque l'on a $P^* \leqq P \leqq P_k$, les déformations atteindront des valeurs infinies pour $t \to \infty$. La limite supérieure P^* (au-dessous de laquelle seules des déformations finies sont possibles) dépend de la valeur limite du fluage spécifique, mesurée sur le béton vieilli, du moment d'inertie de l'armature et du module d'élasticité des deux matériaux.

Zusammenfassung

Im vorliegenden Beitrag werden die Kriechverformungen von symmetrisch bewehrten Stahlbetonstützen untersucht. Für $t \to \infty$ zeigt sich, daß die Kriechdeformation einem endlichen Grenzwert zustrebt, solange und nur solange als die zentrische Belastung kleiner als ein bestimmter Wert $P^* < P_k$ ist, wo P_k die Eulersche Knicklast bedeutet. Liegt P zwischen P^* und P_k ($P^* \leqq P \leqq P_k$), so wächst für $t \to \infty$ die Kriechdeformation ins Unendliche. Die obere Lastgrenze P^* (unterhalb welcher nur endliche Deformationen eintreten können) wird bestimmt durch den Endwert des spez. Kriechmaßes (an gealtertem Beton gemessen), durch das Trägheitsmoment der Bewehrung und den Elastizitätsmodul beider Baustoffe.