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Indeterminate Analysis

Calcul des systèmes hyperstatiques

Allgemeine Methode zur Berechnung statisch unbestimmter Tragsysteme

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This paper proposes a general indeterminate analysis of structures that is applicable to first order theory structures with linearly elastic members. Special cases of this analysis are the force and the deformation methods but the analysis also admits of solving a structure by all possible combinations of force and deformation method expedients.

Members and joints. The analysis begins by completely subdividing the structure into definite "members" with known elastic behavior (flexibility). The members connect at "joints" to other members, and to supports. Points of application of loads and displacements are also treated as joints.

Structure action. Mode. Loads *and* displacements are applied to joints upon the structure. Their components are arrayed in a column matrix Q called the "action" upon the structure.

The specific choice of forces and displacements in Q will be called the "mode" of the action. When Q contains only forces or only displacements we speak of a "pure flexibility mode" or a "pure rigidity mode", otherwise of a "mixed mode".

Member action. The structure action Q produces individual *member* actions N^m, consisting of forces *and* displacements, in a definite mode. The associated member "response" n^m is a column matrix, consisting of the displacements *and negative* force components that correspond to the components of N^m, that is, to its forces and displacements.

Action and response are terms that are only helpful in describing this analysis. Indeed, in the theory of elastic structures no proper relations of cause and effect should be inferred in any verbal statement.

Structure response. Clebsch's theorem. Consider for a moment a structure under an action Q producing the member action N. In another state of stress an enforced member deformation n' is accompanied by the compatible structure displacement q'. At unloaded joints the forces of N balance so their virtual work cancel. At loaded joints the forces of N are equivalent to the loads Q. Therefore the same virtual work will be produced by all Q displaced along q', as by *all* N displaced along n':

$$Q^* q' = \sum N^{m*} n^{m'} = N^* n' \tag{1}$$

asterisk denoting transposition. This holds also when applied displacements are generalized into forces at the same time as the negative associated forces are generalized into associated displacements.

In a first order theory structure a rectangular matrix B with constant elements transforms the structure action Q into the member action N: $N = BQ$. To find this "transaction" B, *elastic properties* must in general be employed. If found (1) gives $Q^* q' = Q^* B^* n'$ which must hold for any Q^*, consequently,

$$\text{if} \quad N = BQ, \quad \text{then} \quad q' = B^* n'. \tag{2}$$

This was stated by A. Clebsch (in his book Elasticität fester Körper, Leipzig 1862, p. 414, 415), and, for scalar B, by R. Krohn (in Zeitschr. Arch. u. Ing. Ver. Hannover 1884, p. 269). G. Kron, (in Journ. Franklin Inst. 238, 1944), expressed the non-generalized matrix version of (2). The fundamental relation (2) will be here referred to as Clebsch's theorem.

Flexibility. For each member a "flexibility" matrix f^m transforms the action N^m into the member response n^m. We assume f^m to be *known*. Thus we can calculate each member response $n^m = f^m N^m$. All individual member actions and responses N^m and n^m are now arrayed into two block columns that will be called simply the member action N and the member response n. Simultaneously all the individual flexibility matrices f^m are arrayed in a block *diagonal* flexibility matrix f so as to validate the formula $n = fN$. Since all f^m are known, the "unconnected flexibility matrix" f is also known.

For linearly elastic members all elements of f are constants. Then by (2): $q' = B^* n' = B^* f N' = B^* f B Q' = e Q'$ where e is the strukture flexibility. Such members and member action can always be found that f becomes diagonal ($= f^*$). Then $e^* = (B^* f B)^* = B^* f^* B = B^* f B = e$. We conclude that all flexibilities are symmetric (which is a result of our conventions for measuring action and response).

Part-inversion. We interpose here a discussion of the *part-solution* of a system of linear equations:

$$\begin{aligned} p &= c P + d' R, \\ r &= d P + k R. \end{aligned} \tag{3}$$

In (7) the letters may also signify conform matrices with non-singular k. The system can be *partly solved for R only.* We premultiply the second equation by $-k^{-1}=K$ and substitute R in the first equation:

$$\begin{gathered} R = K\,d\,P - K\,r, \qquad p = c\,P + d'\,K\,d\,P - d'\,K\,r, \\ p = e\,P - d'\,K\,r, \quad \text{with} \quad e = c + d'\,K\,d. \end{gathered} \tag{4}$$

Consider (3) or

$$\begin{bmatrix} p \\ r \end{bmatrix} = \begin{bmatrix} c & d^* \\ d & k \end{bmatrix} \begin{bmatrix} P \\ R \end{bmatrix} = g \begin{bmatrix} P \\ R \end{bmatrix} \tag{5}$$

in the case that g is *symmetric,* thus c and k symmetric and $d'=d^*$. This makes K, $d^*\,K\,d$, and $e=c+d^*\,K\,d$ symmetric. There are only two possible ways of obtaining in (4) a *symmetric* matrix h. One of these is to reverse the sign of R in (4):

$$\begin{bmatrix} p \\ -R \end{bmatrix} = \begin{bmatrix} e & -d^*\,K \\ -K\,d & K \end{bmatrix} \begin{bmatrix} P \\ r \end{bmatrix} = h \begin{bmatrix} P \\ r \end{bmatrix}, \quad \begin{aligned} K &= -k^{-1} \\ e &= c + d^*\,K\,d \end{aligned}. \tag{6}$$

When g and h denote flexibility matrices it is most convenient always to insist upon maintaining their symmetry. In such instances we shall employ instead of (4) the form (6) of the part-inversion. The action in (5) is P, R, and the response p, r. In (6) the action is P, r and the response $p, -R$. Note that r in (5) is moved directly into the action of (6) while R in (5) is moved into the response of (6) *with a reversed sign.* This agrees wholly with our previous definition of action and response in the mixed modes.

Change of flexibility mode. Part-inversion will be first applied in transforming member flexibilities g given in one mode into flexibilities h of another mode. In the action and response columns of the first mode arrange all L^m, l^m, that shall remain unchanged before those M^m, m^m that shall be interchanged. This requires interchange of the rows and columns of the flexibility matrix f^m into g^m validating the formulas

$$g = \begin{bmatrix} c & d^* \\ d & k \end{bmatrix}, \qquad h = \begin{bmatrix} e & -d^*\,K \\ -K\,d & K \end{bmatrix}, \quad \begin{aligned} K &= -k^{-1} \\ e &= c + d^*\,K\,d \end{aligned}. \tag{7}$$

and, for the required part-inversion,

$$\begin{bmatrix} l^m \\ m^m \end{bmatrix} = g^m \begin{bmatrix} L^m \\ M^m \end{bmatrix} \quad \text{where} \quad g^m = \begin{bmatrix} c & d^* \\ d & k \end{bmatrix} \tag{8}$$

with K and e as before.

$$\begin{bmatrix} l^m \\ -M^m \end{bmatrix} = h^m \begin{bmatrix} L^m \\ m^m \end{bmatrix}, \qquad h^m = \begin{bmatrix} e & -d^*\,K \\ -K\,d & K \end{bmatrix} \tag{9}$$

In general f^m is first established in the *pure flexibility mode.* It can be transformed by the described part-inversion into any desired mode. However, as a result of a previous analysis of a smaller structure it may often be that f^m is known in a *mixed mode.* By the described part inversion this mode can still be transformed into the flexibility $h^m=f^m$ in any desired mode.

Determinacy and indeterminacy. When B can be found by only static and kinematic considerations, the structure (with the preassumed subdivision in members m of known flexibilities f^m) will be termed *determinate.*

A given structure can be changed by cutting and locking joints in the following sense. Each *cut* consists in a physical release that nullifies a pair of force components that is transmitted by the joint. The cut is in general accompanied by a displacement between the two faces of the joint made by the cut. Each *lock* consists in the installation of a rigid lock arm between the joint and the foundation. When the structure is loaded all movement of the locked joint in the direction of the joint force component is prevented; the lock arm transmits a lock force to the foundation.

When a given non-determinate structure is changed into a determinate structure by cutting c joints and locking l other joints, the given structure will here be called *c times statically indeterminate and l times kinematically indeterminate.* Obviously this definition is meaningful only with reference to a *preassumed subdivision of the structure into members with preassigned modes of action.*

The cut and locked determinate structure will be called the auxiliary (structure). It is going to be employed in the analysis of the given indeterminate structure.

Indeterminate structure equations. So far undetermined gap forces (pairs of) and lock movements are applied to the cuts and locks in the auxiliary. Their magnitudes are arrayed in a "redundant action" column matrix R. Together with the given applied action P they make up the total action Q upon the auxiliary:

$$\text{action } Q = \begin{bmatrix} P \\ R \end{bmatrix}, \text{ with response } q = \begin{bmatrix} p \\ r \end{bmatrix}. \tag{10}$$

The state of stress and deformation in the given "loaded" structure is simulated in the auxiliary by adjusting the redundant action components R so as to *close all gaps* and to *reduce all lock forces to zero.* This is simply expressed by $r=0$ where r is the "redundant response", that is the column of gap displacements and negative lock forces that is associated to the column R.

In a *determinate* auxiliary we can find by definition, using statics and kinematics, the force transformation A; that is, we can find the transaction matrix A that applied to the action Q results in the member action N; that is, we can by statics and kinematics find A in $N=A\,Q$. It is possible to write A as a sparse "topological matrix", containing only one- and zero-elements, premultiplied by coordinate transformation matrices, but this factorization is inessential in the present treatment.

The transaction matrix A can be partitioned into two groups of columns: one, C, that transforms the given structure action P and one, D, that transforms the redundant action R, both into member action N:

$$A = [C \,\vdots\, D], \qquad N = A\,Q = C\,P + D\,R. \tag{11}$$

The flexibility f of all members was assumed to be known in their prescribed modes of loading. By this the associated elastic member response is fN. Adding non-elastic member "response" n^t due to "action" by temperature, misfit, initial elongations, settlements, plasticity, etc. we find the complete member response

$$n = f\,N + n^t. \tag{12}$$

Since $N = A\,Q$, we find by CLEBSCH's theorem (2):

$$q = A^*\,n \tag{13}$$

for the auxiliary structure response. Substitution of (11) and (12) gives

$$\begin{aligned} q &= A^*\,f\,N + A^*\,n^t = A^*\,f\,A\,Q + q^t, \\ q - q^t &= g\,Q, \quad g = A^*\,f\,A, \quad q^t = A^*\,n^t. \end{aligned} \tag{14}$$

Solution of the indeterminate equations. For a second application of part-inversion let us solve Eq. (14). We repeat it, splitting the matrices q, Q, A by (10), (11):

$$\begin{bmatrix} p \\ r \end{bmatrix} - \begin{bmatrix} p^t \\ r^t \end{bmatrix} = g \begin{bmatrix} P \\ R \end{bmatrix}, \qquad \begin{bmatrix} p^t \\ r^t \end{bmatrix} = \begin{bmatrix} C^* \\ D^* \end{bmatrix} n^t, \tag{15}$$

$$g = \begin{bmatrix} C^* \\ D^* \end{bmatrix} f\,[C\;D] = \begin{bmatrix} C^*fC & C^*fD \\ D^*fC & D^*fD \end{bmatrix} = \begin{bmatrix} c & d^* \\ d & k \end{bmatrix}. \tag{16}$$

In (15) the given action P is known, but the redundant action R is unknown. However, R are to be adjusted so as to nullify the redundant response r (gap displacements and lock forces) which thus are known and should be moved into the second member of (15). For this purpose (15) is part-inverted by (5), (6):

$$\begin{bmatrix} p - p^t \\ -R \end{bmatrix} = h \begin{bmatrix} P \\ r - r^t \end{bmatrix} = h \begin{bmatrix} P \\ -r^t \end{bmatrix} = h \begin{bmatrix} P \\ -D^*\,n^t \end{bmatrix}, \tag{17}$$

$$h = \begin{bmatrix} e & -d^*K \\ -K\,d & K \end{bmatrix}, \qquad \begin{aligned} K &= -k^{-1} \\ e &= c + d^*K\,d \end{aligned} \tag{18}$$

The second row of (17)

$$R = K\,d\,P + K\,D^*\,n^t \tag{19}$$

is substituted into (11) or $N = C\,P + D\,R$ to give

$$N = (C + DKd)\,P + D\,K\,D^*\,n^t \quad \text{or}$$

$$N = N^i\,P + N^c\,n^t, \quad N^i = C + DKd, \quad N^c = D\,K\,D^* \tag{20}$$

and into (12) or $n = fN + n^t$ to give

$$n = n^i\,P + n^c\,n^t, \quad n^i = f\,N^i, \quad n^c = f\,N^c + I. \tag{21}$$

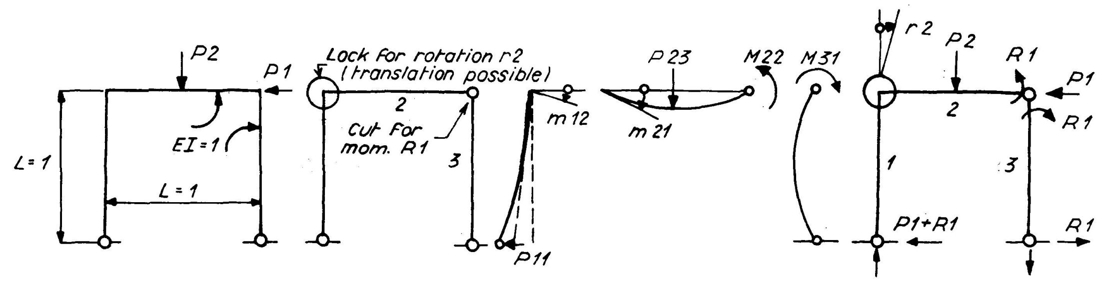
The first row of (17) gives

$$
\begin{gathered}
p-C^{*} n^{t}=e P+d^{*} K D^{*} n^{t}, \\
p=p^{i} P+p^{c} n^{t}, \quad p^{i}=e=c+d^{*} K d, \quad p^{c}=C^{*}+d^{*} K D^{*}=N^{i *} .
\end{gathered} \tag{22}
$$

The formulas (20), (21), (22) are written in influence coefficient form. For instance, the member action N in the indeterminate structure is given as the structure action P premultiplied by influence coefficients N^i, in exactly the same manner as employed in conventional influence line theories.

In some cases the complete analysis (20), (21), (22) of the structure is not required. In such cases the analysis can often be considerably simplified by omitting the rows that are not used in these equations.

Example. The principal characteristics of this method of analysis are illustrated in the following simple example.



Given structure — Fig. 1; Auxiliary — Fig. 2, Fig. 3; Members — Fig. 4, Fig. 5; Transaction — Fig. 6

The frame in Fig. 1 is given. It is locked and cut as in Fig. 2 to form an auxiliary, using the members Fig. 3, 4, 5. This is determinate because it is possible by statics and kinematics only to find as in (11) the transaction A:

$$
\begin{bmatrix} P_{11} \\ m_{12} \\ M_{22} \\ P_{23} \\ m_{21} \\ M_{31} \end{bmatrix}
=
\left[\begin{array}{cc:cc}
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{array}\right]
\left[\begin{array}{c} P_1 \\ P_2 \\ \hdashline R_1 \\ r_2 \end{array}\right],
\quad N=A Q=[C \,\vdots\, D]\left[\begin{array}{c} P \\ \hdashline R \end{array}\right]. \tag{31}
$$

The transaction A transforms the structure action, given load P_1, P_2 and redundants R_1, r_2 as shown in Fig. 6, into the member action N.

The pure flexibility matrix of the beam 2 is found by conventional simple integrations ($L=EI=1$):

$$
\begin{bmatrix} m_{22} \\ p_{23} \\ m_{21} \end{bmatrix}
=
\left[\begin{array}{cc:c}
1/3 & 1/16 & 1/6 \\
1/16 & 1/48 & 1/16 \\ \hdashline
1/6 & 1/16 & 1/3
\end{array}\right]
\begin{bmatrix} M_{22} \\ P_{23} \\ M_{21} \end{bmatrix},
\quad n=g N . \tag{32}
$$

By a part inversion (8), (9) this is changed into the flexibility h for the mode used in Fig. 4.

$$k = 1/3, \quad K = -3, \quad Kd = -[1/2 \quad 3/16],$$

$$d^* K d = -\begin{bmatrix} 1/12 & 1/32 \\ 1/32 & 3/256 \end{bmatrix}, \qquad e = \begin{bmatrix} 1/4 & 1/32 \\ 1/32 & 7/3\cdot 256 \end{bmatrix},$$

$$h = \begin{bmatrix} 1/4 & 1/32 & 1/2 \\ 1/32 & 7/3\cdot 256 & 3/16 \\ 1/2 & 3/16 & -3 \end{bmatrix}, \quad \begin{bmatrix} m_{22} \\ p_{23} \\ -M_{21} \end{bmatrix} = h \begin{bmatrix} M_{22} \\ P_{23} \\ m_{21} \end{bmatrix}. \tag{33}$$

For the members 1 and 3 i Fig. c we find

$$\begin{bmatrix} p_{11} \\ -M_{12} \end{bmatrix} = \begin{bmatrix} L^3/EI & L \\ L & 0 \end{bmatrix} \begin{bmatrix} P_{11} \\ m_{12} \end{bmatrix} = \begin{bmatrix} 1/3 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} \\ m_{12} \end{bmatrix}, \tag{34}$$

$$m_{31} = (1/3)\, M_{31}. \tag{35}$$

Thus the "unconnected" flexibility f in the auxiliary, with member action N ordered as in (31), is

$$n = fN \quad \text{or}$$

$$\begin{bmatrix} p_{11} \\ -M_{12} \\ m_{22} \\ p_{23} \\ -M_{21} \\ m_{31} \end{bmatrix} = \begin{bmatrix} 1/3 & 1 & & & & \\ 1 & 0 & & & & \\ & & 1/4 & 1/32 & 1/2 & \\ & & 1/32 & 7/3\cdot 256 & 3/16 & \\ & & 1/2 & 3/16 & -3 & \\ & & & & & 1/3 \end{bmatrix} \begin{bmatrix} P_{11} \\ m_{12} \\ M_{22} \\ P_{23} \\ m_{21} \\ M_{31} \end{bmatrix}.$$

Blank spaces denote zeros. We obtain for $g = A^* f A$ in (14)

$$fA = \begin{bmatrix} 1/3 & 0 & 1/3 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1/32 & 1/4 & 1/2 \\ 0 & 7/3\cdot 256 & 1/32 & 3/16 \\ 0 & 1/16 & 1/2 & -3 \\ 0 & 0 & 1/3 & 0 \end{bmatrix}, \qquad A^* f A = \left[\begin{array}{cc|cc} 1/3 & 0 & 1/3 & 1 \\ 0 & 7/3\cdot 256 & 1/32 & 3/16 \\ \hline 1/3 & 1/32 & 11/2 & 3/2 \\ 1 & 3/16 & 3/2 & -3 \end{array}\right]$$

with partitioning of g in given and redundant loads shown. Part-inversion of g by (16), (18) gives

$$K = -k^{-1} = \begin{bmatrix} -3 & -3/2 \\ -3/2 & 11/2 \end{bmatrix} /5 = \begin{bmatrix} 36 & 18 \\ 18 & -11 \end{bmatrix} /(-60),$$

$$Kd = \begin{bmatrix} 30 & 9/2 \\ -5 & -3/2 \end{bmatrix} /(-60), \qquad d^* K d = \begin{bmatrix} 5 & 0 \\ 0 & -9/64 \end{bmatrix} /(-60) = \begin{bmatrix} 1/12 & 0 \\ 0 & 3/5\cdot 256 \end{bmatrix},$$

$$e = c + d^* K d = \begin{bmatrix} 1/4 & 0 \\ 0 & 11/15\cdot 64 \end{bmatrix}.$$

This is the flexibility matrix of the given structure. We further find

$$DKd = \begin{bmatrix} -1/2 & -3/40 \\ 1/12 & 1/40 \\ -1/2 & -3/40 \\ 0 & 0 \\ 1/12 & 1/40 \\ -1/2 & -3/40 \end{bmatrix}, \quad N^i = C + DKd = p^{c*} = \begin{bmatrix} 1/2 & -3/40 \\ 1/12 & 1/40 \\ -1/2 & -3/40 \\ 0 & 1 \\ 1/12 & 1/40 \\ -1/2 & -3/40 \end{bmatrix}.$$

This N^i is the transaction matrix (20) of the given indeterminate structure, or its transposed temperature flexibility p^c, see (22). Several superfluous rows of N^i could have been suppressed in this example.

The temperature transaction (20) is

$$N^c = DKD^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 36 & 18 \\ 18 & -11 \end{bmatrix} (1/-60) \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$N^c = \begin{bmatrix} 36 & 18 & 36 & 0 & 18 & 36 \\ 18 & -11 & 18 & 0 & -11 & 18 \\ 36 & 18 & 36 & 0 & 18 & 36 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & -11 & 18 & 0 & -11 & 18 \\ 36 & 18 & 36 & 0 & 18 & 36 \end{bmatrix} /-60, \qquad \begin{bmatrix} P_{11} \\ m_{12} \\ M_{22} \\ P_{23} \\ m_{21} \\ M_{31} \end{bmatrix} = N^c \begin{bmatrix} p_{11} \\ -M_{12} \\ m_{22} \\ p_{23} \\ -M_{21} \\ m_{31} \end{bmatrix}. \tag{38}$$

A temperature elongation of the beam 2 by ΔL causes $m_{31} = \Delta L$,

$$n^t = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \, \Delta L]^*.$$

$$\begin{bmatrix} P_{11} \\ m_{12} \\ M_{22} \\ P_{23} \\ m_{21} \\ M_{31} \end{bmatrix} = - \begin{bmatrix} 36 \\ 18 \\ 36 \\ 0 \\ 18 \\ 36 \end{bmatrix} \Delta L/60.$$

Remarks. This example is by no means the easiest way of analyzing the frame selected but the example serves the purpose of illustrating simply the main points of the indeterminate analysis described in this paper. Very complicated structures composed of bars, beams, plate members, and even solid body members, can be treated quite analogously by the method. The calculations then should be performed by a *computer*. In the author's opinion an interpretive matrix routine is an indispensable complement in this. Then the hand preparations needed for even a large analysis may be less than needed for the hand computations spent in the preceding simple example. Discrimination is required in the establishment of cuts and locks to create a determinate auxiliary structure in such a manner that the total computational work becomes well conditioned and a minimum. The task of attaining this is not treated in this paper but it is well worth a treatment of its own. In the choice of members, cuts, and locks a wide variation is possible, from all members being pure flexibility members with all joints cut, to all members being pure rigidity members with all joints locked. Both the force and the deformation methods are included in the present analysis but all kinds of mixed modes are applicable as well, *under one and the same set of quite simple formulas.*

The hand preparations mentioned include the establishment of the determinate transaction matrix and the member flexibility matrices in proper modes. Both of these tasks can often be considerably expedited by matrix formulation and computer application, see for instance S. O. Asplund , Theory of Trusses, AIPC Mémoires, Vol. 19 (1959), p. 1. The determination and positioning of the design load etc. upon the influence lines also belongs to the preparations but the evaluation of the resultant member actions can usually be done by the computer.

Action not at proper joints. Action not at proper member joints can be handled by establishing further member joints at the points of action. The problem can also be dealt with in another way that is *exactly* applicable to distributed loads as well.

According to the latter method the member section forces and the member deformations, both at and between the existing joints, are first determined in the auxiliary by conventional elastic structure methods for the action between joints. Thereby each member joint is cut or locked as the case may be in the auxiliary. The section force and deformation diagrams are saved to be added at the end of the following analysis. The member response (joint reactions and displacements) caused by this action between joints, is treated as elements of n^t for the auxiliary structure. They are entered into (14), causing $q^t = A^* n^t$, that are additional responses $p^t = C^* n^t$ of this action between joints and additional gaps or lock forces $r^t = D^* n^t$ to be observed in the reduction of r to zero by proper adjustment of the redundants R.

For example a uniform load $= P_3$ upon the horizontal beam in Fig. 1 and 6 produces axial column forces of no effect, a left knee moment $-M_{21} = = P_3/8$ and a right knee rotation $m_{22} = P_3/48$ (see standard tables), thus $n^t = [0 \;\; 0 \;\; 1/48 \;\; 0 \;\; 1/8 \;\; 0]^* P_3$. The member force obtained by (20) is $N = N^c n^t = [-144 \;\; 48 \;\; -144 \;\; 0 \;\; 48 \;\; -144]^* / (-60 \cdot 48) P_3$.

Summary

Indeterminate analysis is generalized by treating displacements as forces and the *negative* associated forces as associated displacements. The generalization makes possible the application of redundant cuts and simultaneous redundant locks. The resultant analysis follows lines parallel to the so-called flexibility matrix method. Needed definitions of determinacy and indeterminacy are formulated. Simple formulas are deduced for the general method. A method of "part-inversion" is devised for their solution.

The force and the deformation methods for indeterminate analysis are seen to be two extreme special cases of the general method here demonstrated.

Résumé

L'auteur généralise le calcul des systèmes hyperstatiques en traitant de la même façon les déformations et les forces ainsi que les forces correspondantes *négatives* et les déformations correspondantes, ce qui permet de travailler simultanément avec des blocages et des coupes surabondantes. Le calcul qui en résulte présente certaines analogies avec la méthode dite «de la matrice de flexibilité». L'auteur introduit les définitions concernant les systèmes déterminés et indéterminés. Pour cette méthode générale, il établit des relations simples et il les résout à l'aide d'une méthode «d'inversion partielle».

La méthode des forces et celle des déformations sont deux cas particuliers et extrêmes de la méthode générale exposée.

Zusammenfassung

Der Autor gibt eine Verallgemeinerung der Theorie der statisch unbestimmten Systeme. Darin werden die Verschiebungen wie Kräfte behandelt und die entsprechenden negativen Kräfte als entsprechende Verschiebungen. Diese Verallgemeinerung erlaubt es, mit überzähligen Schnitten und gleichzeitig wirkenden überzähligen Verriegelungen zu arbeiten. Die daraus entwickelte Berechnungsmethode folgt Richtlinien, die der sogenannten «Flexibility Matrix Method» verwandt sind. Der Autor formuliert die notwendigen Definitionen betreffend Bestimmtheit und Unbestimmtheit. Diese allgemeine Methode führt auf einfache Gleichungssysteme, welche mit einer auf Teilinvertierung beruhenden Methode aufgelöst werden.

Es zeigt sich, daß Kräfte- und Deformationsmethode zwei extreme Spezialfälle der hier behandelten allgemeinen Methode darstellen.