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Optimization of Framing Arrangements for Large Metal Roof Systems

Détermination de la disposition la plus économique des éléments porteurs métalliques pour couvertures de grandes dimensions

Bestimmung der optimalen Trägerverteilung für weitgespannte Dachkonstruktionen aus Stahl

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Introduction

The principles of structural design which establish the optimum proportions of a structure so as to minimize weight, have been well formulated by F. R. SHANLEY (Reference 1). His approach may validly be described as design in that the methodology is one of synthesis as contrasted to analysis.

The underlying philosophy is implicit in the statement: Given load W and distance L over which it must be supported, determine the best structure. The traditional approach has been: Given a structural system, determine its performance. In general, this has entailed estimating stresses. Optimum design has brought about a major step forward in the field of aircraft structure where weight and cost have a one-to-one correspondence. A similar methodology is needed for civil engineering structures. However, in this case Optimal principles entail minimization of costs which are not related directly to weight of material.

The design of any structure may be shown to depend on a parameter referred to as the Structural Index. This is a ratio of load to a distance squared. It is as useful for the case of minimum cost as it is for that of minimum weight.

The objective of this paper is to apply an optimum design technique for beams and trusses. It includes an optimum design approach for a flat roof system supported on an orthogonal grid of either beams or trusses. The entire roof is supported at four corners of a rectangle (Fig. 1). This rectangular module may be repeated continuously in either direction.

The introduction of the digital computer further extends the possible application of methods of design synthesis. It is this newest tool which is particularly suited to solving the general expressions which are developed below. It should be noted, that the use of the Structural Index makes it possible to eliminate considerations of scale. The parameters which determine the design may be expressed in non-dimensional form. In this respect the title of this paper may be misleading in referring to large roof systems. On the other hand the advantage of the technique becomes more apparent when it is applied to the design of a large system.

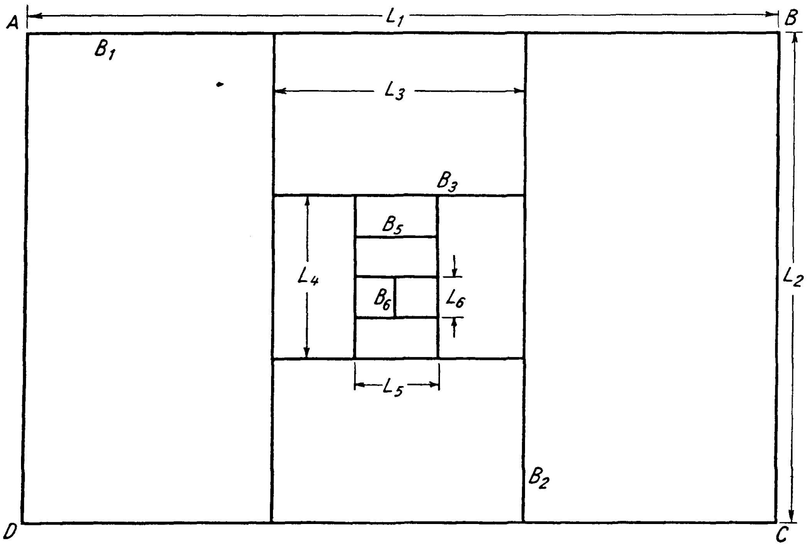

Fig. 1. General Roof Framing Plan. Pattern Repeats Over Entire Roof Only One of Each Subdivision Shown for Clarity.

Cost

The cost of a roof system will be the sum of the costs of material, fabrication and erection.

$$C = f(C_v, V) + f(C_L, L) + f(C_n, n). \tag{1}$$

In other words the cost is a sum of a function of the volume and unit cost of material, a function of the lengths of the members and the unit cost of fabrication, and a function of the number of connections and the unit cost per connection.

The total cost of a system of beams or trusses may be written as a cost ratio by dividing by a reference cost, which for convenience will be taken as the cost of material per unit volume.

Thus
$$\frac{C}{C_1} = \sum V + \sum \alpha_0 L + \sum \beta_0 n, \tag{2}$$

where α_0 and β_0 are relative costs of fabrication per unit length and of erection per connection respectively.

It may reasonably be assumed that these relative costs will have two components — a fixed unit cost conditioned by the nature of the beam or connection and a variable one which depends on size. For any class of beams it may be reasonable to assume linearity. Hence

$$\begin{aligned}\alpha_0 &= \alpha A + B,\\ \beta_0 &= \beta V + D.\end{aligned} \tag{3}$$

Thus
$$\frac{C}{C_1} = \sum (1+\alpha+\beta n)\, V + \sum B L + \sum D n. \tag{4}$$

In order to non-dimensionalize the cost, a cost parameter which will be called the "Cost Index" is defined as

$$\Gamma = \frac{C}{C_1 L_1^3}. \tag{5}$$

The use of the "Cost Index" will become apparent later.

The various unit costs will vary depending on the class of structure to be designed. They should be essentially constant within reasonable ranges for any one class. For example, a beamed system of rolled wide flange sections may be expected to have considerable different unit cost factors from those of a truss system. This variation also will extend to the unit cost of material. On the other hand such variations may be reflected entirely within the values chosen for α, β, B and D. Since the solution is one which inherently is readily accomplished by means of digital type computer, it should not be difficult to extend it to examine the effects of varying these cost factors. By the same reasoning, the use of linear relations for the fabrication and erection cost parameters is justifiable even where it is recognized that the range over which linearity may be assumed might prove to be small. Furthermore, the functional cost relations which have been chosen somewhat arbitrarily, may well prove upon further research to be better represented by some other parameters than those indicated. Such changes, however, would be in the nature of refinements and should not seriously affect the results obtained herein.

Optimum Beam or Truss

Except for very short beams the primary design condition is moment rather than shear. The maximum moment which may occur anywhere along the length will determine the basic section property.

$$M = K_b w L^3, \tag{6}$$

where K_b is a constant which depends on the spanwise distribution of load and on the end constraints. w is the Structural Index, defined as the total load W (regardless of its distribution) divided by the span length squared

$$w = W/L^2.$$

The resisting moment may be expressed in terms of the tension-compression cross-sectional areas

$$M = \{A_t + f(A_w)\}\, h\, \sigma_t = \{A_c + f_1(A_w)\}\, h\, \sigma_c. \tag{7}$$

$f(A_w)$ is a term to account for the effectiveness of web area in resisting moment. Solving for the total area and introducing $\eta = \sigma_t/\sigma_c$.

$$A = \frac{M}{h\,\sigma_t}(1+\eta) + A_w - f(A_w) - f_1(A_w) = \frac{M}{h\,\sigma_t}(1+\eta) + K_w A_w. \tag{8}$$

K_w is a web bending effectiveness factor which ranges between 0.67 for an idealized beam to 1.00 for a truss in which no bending is resisted by the web system.

The average cross sectional area of the web A_w is all the material of the web divided by the span length. In turn it leads to an effective web thickness $t_e = A_w/h$. This effective t_e may or may not directly resist shear, some part of it may be stiffening material. The flanges may be reduced in regions of reduced moment. To allow for this a flange reduction factor K_m is introduced. Thus the total volume of the beam is

$$V = K_m \frac{M\,L}{h\,\sigma_t}(1+\eta) + K_w t_e h\, L. \tag{9}$$

By rearranging Eq. (9) and substituting Eq. (6), the non-dimensional volume becomes

$$\frac{V}{L^3} = K_m K_b \left(\frac{w}{\sigma_t}\right)\left(\frac{1+\eta}{h/L}\right) + K_w \left(\frac{t_e}{h}\right)\left(\frac{h}{L}\right)^2. \tag{10}$$

The effective web thickness depends on the section configuration which has been selected for the design. In general, t_e/h tends to remain fairly uniform for particular classes of structures. For economical rolled wide flange sections it is approximately 1/45; for built up beams with unstiffened webs, which are designed in accordance with the AISC specification, tends to be 1/70; for plate girders it may be 1/200. In general the thickness is dictated by selection of one of relatively few standard gauges. The truss presents a somewhat different relation. Consider a panel of a Pratt truss, Fig. 2, with 45^0 diagonals. The volume of material in the web system will depend on the magnitude of the shear. This may be expressed as $K_s w L^2$. If shear is constant in each panel

$$\left(\frac{V}{L^3}\right)_{web} = (2+\eta) K_s' \left(\frac{w}{\sigma_t}\right)\left(\frac{h}{L}\right). \tag{11}$$

A riveted or bolted system will require a reduction in average tensile stress to allow for holes; shear will not be constant along the span; the midspan section may be governed by practical design rules rather than stress; the material and fabrication costs for a web of a truss will be more than for a beam. All of these design variables may be lumped into the single web coefficient K_s. Thus Eq. (10) for the truss becomes

$$\frac{V}{L^3} = K_m K_b \left(\frac{w}{\sigma_t}\right) \frac{(1+\eta)}{h/L} + K_s \left(\frac{w}{\sigma_t}\right) \left(\frac{h}{L}\right). \tag{10a}$$

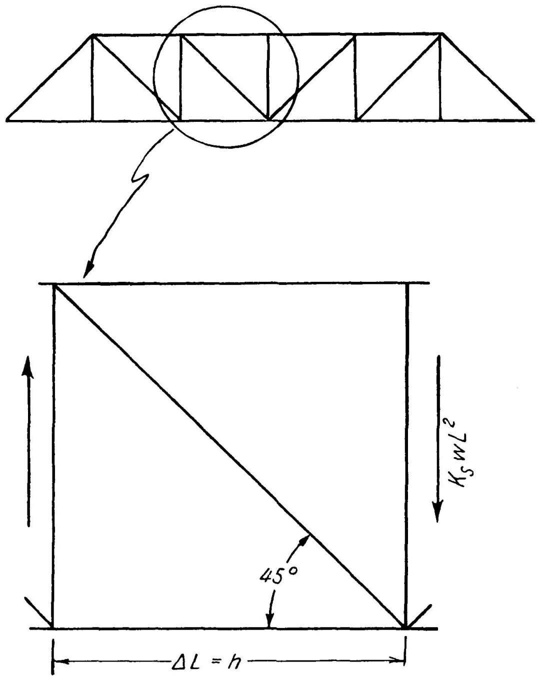

Fig. 2.

Using the AISC specification for allowable shear as one possible criterion

$$\tau = 64 \times 10^6 \left(\frac{t}{h}\right)^2 = K_s' \frac{w L^2}{t h},$$

from which
$$\left(\frac{t}{h}\right)^3 = \frac{K_s'}{3200} \frac{w}{\sigma_t} \left(\frac{L}{h}\right)^2.$$

For this equation $\sigma_t = 20{,}000$ psi.

Again using k_s to allow for web stiffening material, extra cost of fabrication, etc.

$$\left(\frac{t_e}{h}\right)^3 = \frac{k_s}{3200} \left(\frac{L}{h}\right)^2 \frac{w}{\sigma_t}.$$

Substituting in Eq. (10)

$$\frac{V}{L^3} = K_m K_b \frac{(1+\eta)}{h/L} \frac{w}{\sigma_t} + K_w \left(\frac{K_s w}{3200\, \sigma_t}\right)^{1/3} \left(\frac{h}{L}\right)^{4/3}. \tag{10b}$$

As an example this relationship is represented graphically in Fig. 3 for Eq. (10) only; $K_m = 1$, $K_b = 1/8$, $\eta = 1$, $K_w = 0.67$.

The two terms of each equation are plotted separately in order to indicate the intersections which determine the value of h/L for minimum effective volume. It is of interest to note that as loads increase the optimum depth/span ratio also increases.

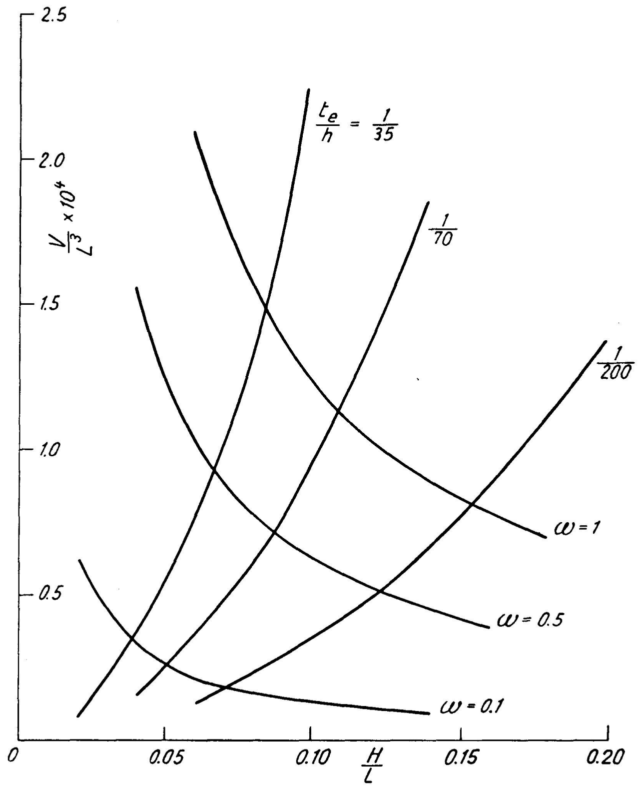

Fig. 3.

In order to establish the minimum effective volume analytically, differentiate Eqs. (10) with respect to (h/L) and set equal to zero. Thus

$$\left(\frac{h}{L}\right)_{opt.} = \left\{\frac{K_m K_b (1+\eta)}{2 K_w}\right\}^{1/3} \left\{\frac{h}{t_e} \frac{w}{\sigma_t}\right\}^{1/3} \tag{12}$$

for beams where t_e/h = constant,

$$\left(\frac{h}{L}\right)_{opt.} = \left\{\frac{K_m K_b (1+\eta)}{K_s}\right\}^{1/2} \tag{12a}$$

for the truss,

$$\left(\frac{h}{L}\right)_{opt.} = 2.84 \left\{\frac{K_m K_b (1+\eta)}{K_w}\right\}^{3/7} \frac{1}{K_s^{1/7}} \left\{\frac{w}{\sigma_t}\right\}^{2/7} \tag{12b}$$

for beams where shear stress governs.

Using these values for h/L above and substituting back into Eq. (10)

$$\left(\frac{V}{L^3}\right)_{opt.} = 1.83\{K_m K_b (1+\eta)\}^{2/3}\left\{K_w \frac{t_e}{h}\right\}^{1/3}\left\{\frac{w}{\sigma_t}\right\}^{2/3} \tag{13}$$

for beam with t_e/h = constant,

$$\left(\frac{V}{L^3}\right)_{opt.} = 2\{K_m K_b K_s (1+\eta)\}^{1/2}\frac{w}{\sigma_t} \tag{13a}$$

for a truss,

$$\left(\frac{V}{L^3}\right)_{opt.} = 0.45\, K_w^{3/7} K_s^{1/7}\{K_m K_b (1+\eta)\}^{4/7}\left(\frac{w}{\sigma_t}\right)^{5/7} \tag{13b}$$

for beams where shear stress governs.

To account for all three cases and others which may be developed on the basis of a different criteria for web design, let Eq. (13) be generalized.

$$\left(\frac{V}{L^3}\right)_{opt.} = \lambda\, w^{\gamma}. \tag{14}$$

The minimum relative volume is expressed in terms of the structural index and a dimensionless parameter which accounts for the distribution of loads, distribution of material, in the flanges and web, and, if desired, extra cost of web material. For the latter inclusion, V/L^3 will be a parameter representing a relative rather than true volume of material. If so desired it is readily corrected to a weight of material parameter by multiplying by the density.

Optimum Beam Grid

Consider a horizontal panel supported at four corners of a rectangle $ABCD$. This grid is assumed to be made up from a series of constant section beams $B_1 B_2 B_3$ spanning in alternate orthogonal directions, Fig. 1. B_2 is assumed to span between the B_1's, B_3 between the B_2's, and so on. The condition of support and constraint may be any desired. They may be simple and determinate; they may be constrained to satisfy an elasticity and continuity assumption; they may be constrained to satisfy a criteria of plastic design.

A cost parameter is desired such that the cost may be determined for any arrangement of beam sizes or for any overall scale.

Let the ratios of lengths of the beams be $L_1/L_2 = r_1$, $L_2/L_3 = r_2$, $L_m/L_m + 1 = r_m \ldots$.

The length of any beam L_i may be expressed in terms of the ratios and L_1 thus

$$L_i = \frac{L_1}{\prod_{j=1}^{j=i-1} r_j}. \tag{15}$$

It will be assumed that each beam is an optimum section. The minimum is an optimum section. The minimum volume of material will then be

$$\left(\frac{V}{L_1^3}\right)_i = \frac{\lambda_i w_i^{\gamma_i}}{\prod_{j=1}^{j=i-1} r_j^3}. \tag{16}$$

Assuming a single panel and including all boundary beams. The number of beams N will be

$$\begin{aligned} n(B_1) &= 2, \\ n(B_2) &= (r_1 r_2 - 1) + 2, \\ n(B_3) &= (r_2 r_3 - 1)(r_1 r_2), \\ n(B_i) &= (r_i r_{i-1} - 1)(r_{i-1} r_{i-2}) \ldots (r_2 r_1), \\ n(\text{Total}) &= (r_1 r_2 + 3) + \sum_{i=3}^{i=n} \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \prod_{j=1}^{j=i-1} r_j^2, \end{aligned} \tag{17}$$

the total length

$$\sum L = L_1 \left\{ r_2 + \frac{1}{r_1} + 2 \right\} + L_1 \sum_{i=3}^{i=n} \frac{r_{i-1} r_i - 1}{r_1 r_{i-1}} \prod_{j=1}^{j=i-1} r_j. \tag{18}$$

The total volume of material will be

$$\frac{V}{L_1^3} = 2\lambda_1 w_1^{\gamma_1} + \frac{r_1 r_2 + 1}{r_1^3} \lambda_2 w_2^{\gamma_2} + \sum_{i=3}^{i=n} \lambda_i w_i^{\gamma_i} \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \cdot \frac{1}{\prod_{j=1}^{j=i-1} r_j}. \tag{19}$$

Choose a relative value $\lambda_0 w_0^{\gamma_0}$ which may be factored and leaving a relative structural parameter a_i as the significant variable in the summation.

By introducing the dimensionless cost parameter Γ from Eq. (5) and substituting into (4), the total cost may now be written as

$$\begin{aligned} \Gamma = (1 + \alpha + \beta n) \lambda_0 w_0^{\gamma_0} &\left\{ 2a_1 + \frac{r_1 r_2 + 1}{r_1^3} a_2 + \sum_{i=3}^{n} a_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \cdot \frac{1}{\prod_{j=1}^{i-1} r_j} \right\} \\ &+ \frac{1}{L_1^2} \left\{ 2B_1 + \frac{r_1 r_2 + 1}{r_1} B_2 + \sum_{i=3}^{n} B_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \prod_{j=1}^{i-1} r_j \right\} \\ &+ \frac{1}{L_1^3} \left\{ 2D_1 + (r_1 r_2 + 1) D_2 + \sum_{i=3}^{n} D_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \prod_{j=1}^{j=i-1} r_j^2 \right\}. \end{aligned} \tag{20}$$

The problem will now be specialized by assuming a load distribution such as might be specified for a roof system. This may involve a uniform load plus a single concentrated load which may act any where on the roof. Thus for any beam

$$w_i = w_0 \frac{L_{i+1}}{L_i} + \frac{K_p P}{L_i^2}. \tag{21}$$

In this case w_0 is the uniformly distributed load (i.e. lbs. per square foot). w_0 becomes the structural index for the square panel. In terms of r's

$$w_i = \frac{w_0}{r_i} + \frac{K_p P}{L_1^2} \prod_{j=1}^{j=i-1} r_j^2 . \tag{22}$$

Introducing this into Eq. (20), the a_i as coefficients, may be replaced by

$$a_i = b_i \left\{ \frac{1}{r_i} + \frac{K_p P}{w_0 L_1^2} \prod_{j=1}^{j=i-1} r_j^2 \right\}^{\gamma_i} w_0^{\gamma_i - \gamma_0}, \tag{23}$$

where b_i are the ratios of the coefficient λ_i/λ_0. Because a different K_b will apply for P than for w_0 the coefficient K_p representing the ratio of moment coefficients, $\frac{K_b(P)}{K_b(w_0)}$, is introduced. Eq. (20) becomes

$$\begin{aligned} \Gamma = (1+\alpha+\beta n)\lambda_0 w_0^{\gamma_0} \Bigg[& 2 b_1 \left\{ \frac{1}{r_1} + \frac{K_p P}{w_0 L_1^2} \right\}^{\gamma_1} w_0^{\gamma_1 - \gamma_0} + \frac{r_1 r_2 + 1}{r_1^3} b_2 \left\{ \frac{1}{r_2} + \frac{K_p P}{w_0 L_1^2} r_1^2 \right\}^{\gamma_2} w_0^{\gamma_2 - \gamma_0} \\ & + \sum_{i=3}^{n} b_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \frac{1}{\prod_{j=1}^{i-1} r_j} \left\{ \frac{1}{r_i} + \frac{K_p P}{w_0 L_1^2} \prod_{j=1}^{j=i-1} r_j^2 \right\}^{\gamma_i} w_0^{\gamma_j - \gamma_0} \Bigg] \\ & + \frac{1}{L_1^2} \left[2 B_1 + \frac{r_1 r_2 + 1}{r_1} B_2 + \sum_{i=3}^{n} B_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \prod_{j=1}^{j=i-1} r_j \right] \\ & + \frac{1}{L_1^3} \left[2 D_1 + (r_1 r_2 + 1) D_2 + \sum_{i=3}^{n} D_i \frac{r_i r_{i-1} - 1}{r_1 r_{i-1}} \prod_{j=1}^{j=i-1} r_j^2 \right]. \end{aligned} \tag{20 a}$$

This equation may be solved for a large number of sample configurations by means of a small-size digital computer. It is possible to program it so that the machine will select a minimum Γ or a series of combinations of r's which will produce a Γ in the range of the minimum.

Conclusion

The purpose of this paper is to demonstrate a technique of design-synthesis as opposed to design-analysis. At one time the amount of computational work required to examine so many variables over rather wide ranges would have been considered prohibitive. The digital computer now makes this feasible. Aside from this time saving aspect, new and basic methodology is formulated. No longer is it necessary for the designer to make an assumption of the final structural configuration, proceed to analyze it to establish its adequate performance; adjust or modify it as indicated; reanalyze it until he has converged on an acceptable and economical solution, which incidentally he can never know for certain represents the optimum. With the new method, the designer arrives directly at a cost as a function of configuration. He can now determine a minimum cost for any given loading and set of boundary dimensions. This approach to a method of design-synthesis as opposed to design-analysis does not automatically eliminate a need for all design-analysis methods. The roof system described herein is a class of roof system which may be described as a one-way-span beam system. This should, of course, be compared to other classes of systems. The important thing is that the optimum for each class may be established without resorting to sample analysis of one or two conceived arrangements. As further development of such methodology proceeds, and as similar programs are developed, the complexity of the structural system which can be optimized by this and similar techniques can be expanded.

Nomenclature

A	cross sectional area of beam
A_w	cross sectional area of beam web
a	relative beam factor
B	unit fabrication cost for beam
b	factor
C	total cost
C_1	cost per unit volume
c	factor relating moment coefficients K_b
D	unit fabrication and erection cost of connections
h	beam depth
K_b	bending moment coefficient
K_m	flange reduction coefficient
K_p	load distribution coefficient
K_s	web coefficient
K_w	web bending effectiveness factor
L	length of beam
M	Moment
P	concentrated load
t	thickness
t_e	effective web thickness
V	volume of material in the beam
W	total distributed load
w	structural index
α	cost coefficient
β	cost coefficient: also an exponent
Γ	dimensionless cost parameter
γ	beam coefficient
μ	ratio of tension to compression allowable stress
λ	beam coefficient
σ	stress tension or compression
τ	shear stress

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Summary

A system of beams or trusses is arranged in a horizontal orthogonal pattern in such a way that roof loads are delivered eventually to four corner columns. An optimum design approach for establishing the proportions of individual beams or trusses is developed to minimize cost in terms of a defined cost parameter. It is assumed that beams or trusses of the grid are individually optimized. A parameter is derived for the cost of a complete grid of beams and trusses. A methodology for examining all possible geometric configurations and combination of beams and trusses is developed in order to determine the minimum cost system. The complete system is optimized with respect to the geometry of the pattern, and in terms of beam versus truss elements, in order to obtain the minimum cost. The cost parameters which include material, fabrication and erection are developed in a non-dimensional form in order to eliminate scale effects. The problem is solved by means of a digital computer.

The complete development provides a design-synthesis technique for long span roof systems. More significantly it presents a point of view which, in the opinion of the author, may lead to a more efficient design method for all structures.

Résumé

L'auteur considère un système maillé orthogonal, situé dans un plan horizontal et formé de poutres (profilés, poutres à âme pleine) ou de treillis; leur disposition est telle que les charges de la couverture soient reprises finalement par quatre colonnes d'angle. L'auteur développe une méthode permettant de fixer les dimensions de chacun des éléments porteurs, poutres ou treillis, de manière à en réduire au minimum le coût en fonction d'un paramètre des frais déterminé. Il admet que chaque élément du système remplit pour soi les exigences d'une économie maximale. Il établit un paramètre qui donne le coût du réseau complet de poutres et de treillis. Afin de trouver le système le moins dispendieux, l'auteur développe une méthodologie permettant d'examiner toutes les dispositions géométriques et toutes les combinaisons possibles de poutres et de treillis. Pour rechercher le système entier donnant la dépense la plus réduite, on tient compte des dimensions géométriques du réseau et on compare les poutres et les treillis. Les paramètres des frais comprennent le coût des matériaux, de la fabrication et du montage; ils sont développés sous

forme de grandeurs sans dimensions, afin d'éliminer les effets d'échelle. Le problème est résolu à l'aide d'une calculatrice digitale électronique.

Le développement complet permet d'obtenir une méthode pour le calcul et l'étude des couvertures à grande portée, méthode basée sur une synthèse. D'une manière plus significative, cette méthode présente une façon d'envisager les problèmes qui, d'aprés l'auteur, peut conduire à un procédé d'étude et de calcul plus efficace pour toutes les sortes d'ouvrages.

Zusammenfassung

Ein aus Vollwand- oder Fachwerkträgern bestehendes System ist in horizontal orthogonaler Weise derart angeordnet, daß die Dachlasten schließlich auf vier Eckpfeiler abgegeben werden. Der Verfasser entwickelt eine Methode, um im Entwurf die Abmessungen der einzelnen Balken oder Fachwerke so gut als möglich zu bestimmen, so daß die Kosten mit Rücksicht auf einen gegebenen Kostenparameter auf ein Minimum reduziert werden. Dabei wird angenommen, daß für jeden Balken oder jedes Fachwerk des Rostes die optimalen Abmessungen individuell bestimmt werden. Für die Kosten des ganzen aus Vollwand- oder Fachwerkträgern bestehenden Rostes wird ein Parameter abgeleitet. Zur Bestimmung des Systems mit den geringsten Kosten wird eine Methodik zur Untersuchung aller möglichen geometrischen Konfigurationen und Kombinationen von Balken- oder Fachwerkträgern entwickelt. Um die geringsten Kosten zu erhalten, wird nach dem Optimum des ganzen Systems gesucht, wobei die Geometrie des Rostes und die Ausbildung der Elemente als Vollwand- oder Fachwerkträger berücksichtigt werden. Die Kostenparameter, welche Material, Fabrikation und Montage berücksichtigen, werden in einer dimensionslosen Form entwickelt, um die Maßstababhängigkeit auszuschalten. Das Problem wird mit Hilfe eines digitalen Rechengerätes gelöst.

Die ganze Herleitung verschafft uns eine auf einer Synthese beruhenden Entwurfsmethode für weitgespannte Dachsysteme. Bedeutend wichtiger ist, daß diese Methode einen Standpunkt darlegt, der, nach der Meinung des Autors, zu wirksameren Entwurfsmethoden für alle Konstruktionen führen dürfte.