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# **Riveted Web Connections in Bending**

*Joints d'âme rivés travaillant en flexion*

*Genietete Stegverbindungen unter Bieungsbeanspruchung*

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## **Part I**

### *1.1. Introduction*

Many research workers [1]<sup>1)</sup> have investigated riveted joints subjected to axial forces and to shear combined with bending. The latter has usually been studied in beam-to-stanchion connections [2], and not in web joints of plate girders or beams. As the mechanism of deformation of such joints differs considerably, it is necessary to study web joints separately from beam-to-stanchion connections and the conclusions arrived at in the case of latter are not applicable to both cases.

The riveted joints of girders, so common in engineering practice, have rarely been the subject of research. Usually, the maximum loading of the whole joint has been investigated and not the distribution of forces in the rivets. Furthermore, as the flanges of the girder work co-operate with the web joint in forming the mechanism of deformation, it is obviously necessary to investigate the behaviour of the web joint as part of the girder and not in isolation nor to treat it as any other joint subjected to shear and bending. The author knows of some published papers [3] dealing with plate girder splices and joints, but these do not discuss the distribution of forces in rivets. The introduced assumption as to the centre of deformation of the whole rivet group which is now standard practice, seems to be taken for granted and beyond suspicion, though, in the author's opinion, what Professor C. BATHO

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<sup>1)</sup> References are given at the end of the paper.



said many years ago about the complexity of the whole problem is still true [4]. In a previous paper [5], the author started the study of the true behaviour of riveted joints in bending, and the tests reproduced in this paper are a further step in this direction.

The usual basis for computation of a riveted flexural joint, designed as a double-strap butt joint, is the assumption that the deformations of all the rivets are proportional to the distance between the rivet under consideration and the centre of rotation of the whole group of rivets, which point is taken as its centroid. If the height of the rivet group is large in comparison with the width (more than twice the width), then the vertical component of the rivet force taken by individual rivets is usually neglected, which implies the introduction of Bernoulli's hypothesis that plane sections remain plane: linear distribution of rivet forces is assumed along the height of the connection.

The deformation of a riveted joint is never elastic, and there is no proportionality between the average slip of the joint and the force in the rivet. The plastic deformation, occurring in the early stage of loading, so changes the stress distribution that a rigorous mathematical analysis is only of academic interest. The introduction of coefficients, obtained in particular tests, is not very helpful because the procedure is fundamentally wrong. The numerical values hold only for the conditions of the tested joint and those of the test itself, and cannot be generalized. Any theory dependent on such data, cannot claim to be a generally valid principle.

Hence a better way to tackle the problem is by considering limit design.

From the outset the calculation of riveted joints has been considered in this way, and this surely constitutes good practice. But if the assumed initial conditions of functioning of the joint are subsequently altered during the deformation of the connection until collapse, this alteration must be taken into account. Depending on the deformations occurring in the rivets, the web and the splices during bending, the above-mentioned basic assumption is at first correct: each group of rivets on either side of the butt joint undergoes a relative rotation about its own centre (or rather axis) of rotation, coinciding with the centroid of the rivet group, and the splices rotate about their neutral axis. But, when the two connected portions of the web come into contact with each other in the compressed parts of the end cross sections, which must occur sooner or later if the rivets and splices are strong enough to enable sufficient deformation to take place, the conditions of deformation are changed in this second phase of functioning of the rivets.

The effective cross section of the splices is actually enlarged by the amount of the compression area which is in contact, and instead of the previous three axes of rotation (two for each group of rivets and one for the splices), there is now only one. The position of this axis is at first at the top of the web, but deformation of the end cross sections of the web causes it to move gradually downwards.

The simple bending of splices, in the first phase, is now changed into eccentric tension. If the end supports of the connected beam are suitable, the simply-supported beam will thereafter behave as a three-pinned arch. The "hinge" in the middle, coinciding with the joint itself, functions as a hinge in which considerable friction occurs, and it thus changes to some extent the behaviour compared with a genuine three pin arch.

The forces in the rivets, at first dependent on their position in relation to their centroid, are also modified. The described displacement of the centre of rotation can be the only reasonable explanation for the higher values of the bending moment at collapse actually acting on the joint as compared with the values that the mechanism in the first phase would lead us to expect.

To prove the validity of this general idea and to yield basic information on what occurs in a riveted flexural web joint, tests had to be devised which would enable both stages to develop properly. Furthermore, to get a clear picture of all the changes in behaviour, and not just a mass of miscellaneous data simply added to what was already known, the number of possible variables taking part in the interaction between web, splices and rivets had to be limited to a reasonable minimum so as to permit an accurate structural analysis. Finally, instability had to be avoided throughout bending and until collapse took place.

In accordance with these ideas, a rolled I-section joist ( $9" \times 4"$ ) was taken and cut at mid-span provided with cover plates on the web only and subjected to a simple bending moment. A clearance of  $\frac{1}{8}"$  was provided between the two parts of the joint so as to permit the full development of the first phase, and also to conform to normal engineering practice.

The number of rivets and the cross section of the cover plates was so designed that, in one case, the end of the first phase would coincide with the practical maximum of the force taken by a rivet when it first starts yielding; and, in the second case, that this limit would be determined by the cover plates. Therefore the joints were of two different kinds: with 3 rivets of  $\frac{3}{4}"$  nominal diameter in only one vertical row, and with rivets of the same diameter in two vertical rows of 3; the cross section of the cover plates was the same in both cases, however, viz.,  $7" \times \frac{1}{4}"$ .

To enable the effect of friction upon the work of the riveted joint also to be studied, some tests were designed using fitted bolts of  $\frac{3}{4}"$  diameter. The bolts were tightened only by hand using a 7" spanner, and were machined in advance so as to obtain a good fit in the holes.

### *1.2. Experimental Equipment and Methods Used*

As already stated, it was necessary to obtain data about conditions in all three parts of the joint: in the web, the splices and the rivets. Therefore for the recording of strains electrical strain gauges (Philips Pr. 9210 of  $600 \Omega \pm 0.5\%$

with  $k = 2.12 \pm 1.5\%$ ) were used, glued in strain rosettes around the holes (Fig. 1) on the splices and on the web (in the case of bolted connections only). The position of the strain rosettes close to the holes and their relative position being exactly the same in all rivets, it was possible to obtain a relative (if not an absolute) value of the active force in the rivet under consideration and also to compute the directions of principal strains, i. e., of the rivet forces, and also to study their changes during the progress of the loading. Readings were taken by means of a Philips measuring bridge PR 9300, and for more than 10 strain gauges a Philips switch box OM 5545 was used.

Fig. 2 gives the positions of the dial gauges used. The relative rotation of the end cross sections of the beam at the joint was recorded by a horizontal 0.001" Batty dial gauge, measuring the change in the gap at the bottom of the lower flange (dial gauge No. 1, in Fig. 2). The deflections were measured by means of the dial gauges of the same type as the first one, in points 2 and 4, just beneath the applied loads and in the middle. The recorded deflections were used for the study of the general behaviour of the connected beam and of the mechanical properties of the joint itself, because these recorded data give a summarized result of the interaction of all parts involved.

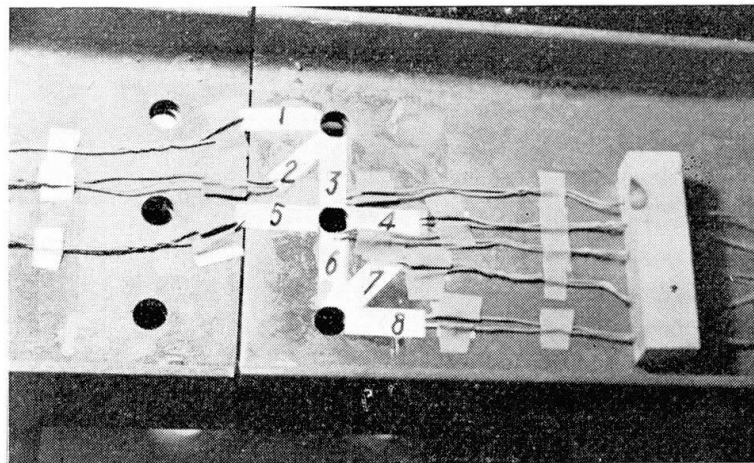


Fig. 1.

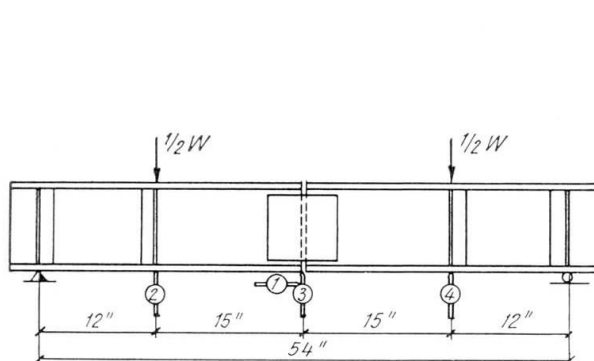


Fig. 2.

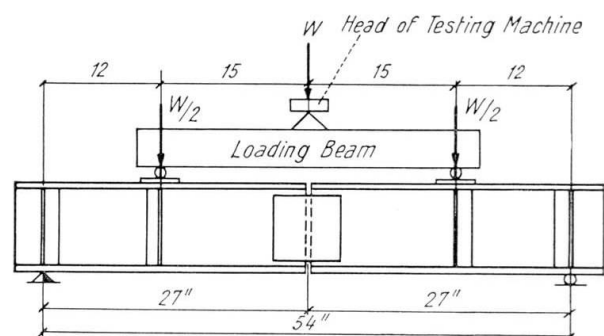


Fig. 3.

The loading arrangement is shown in Fig. 3. The necessary forces were produced with the aid of a 200-ton Amsler Compression Testing Machine, type 200 DB 76 (Fig. 3a), with the usual rate of load increase.

The load was increased, in the case of almost elastic deformations, at intervals of 2 minutes, the readings being taken in the meantime; and during plastic behaviour the readings were taken when their change after 10 minutes was less than 0.0005" and the loads were increased.

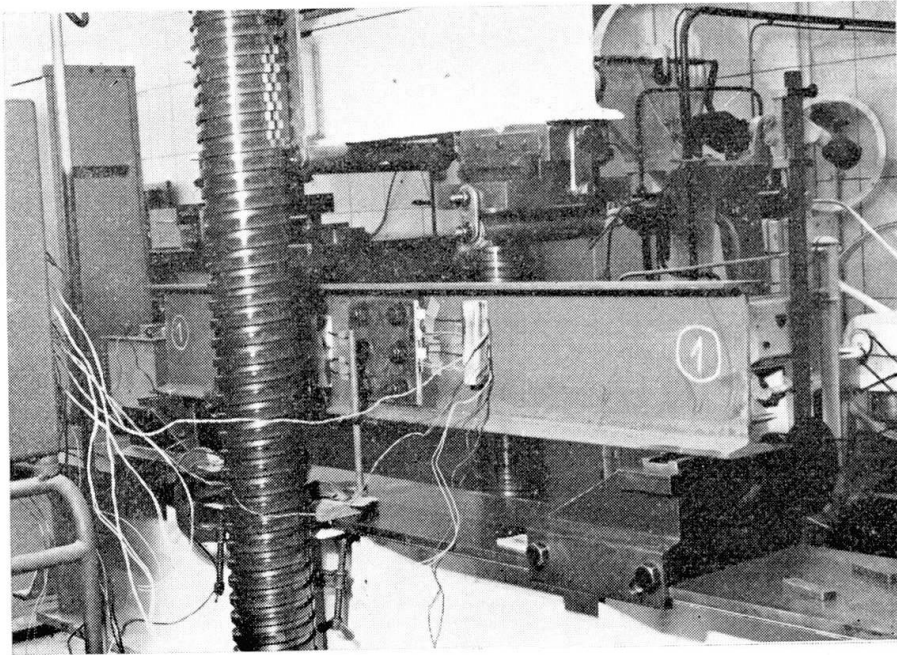


Fig. 3a.

### *1.3. Material and Design Computations*

The steel for the joist and splices was a low carbon structural steel. From the web and the splices 6 specimens were taken and tested in tension and bending. From the tensile tests data were obtained on the ultimate strength and the percentage elongation. The bending tests, on small control beams of 6" span, provided data on the upper and lower yield stress and on Young's modulus. As only one load — at mid-span — was applied to the control beam, the corrected formula proposed by RODERICK and PHILLIPPS [6] was used. The upper yield stress was computed according to the expression given by ROBERTSON and COOK [77].

The results are summarized and given as mean average values in Table 1.

The steel used for rivets and fitted bolts was carbon steel as normally used for the purpose and in view of the fact that the mechanical properties of this steel are of some value only if determined during the actual work of the riveted or bolted joint, it was not tested in the cold state.

Table 1.

Description 1	Yield Point stress in t/in. <sup>2</sup>		Ultimate tensile strength in t/in. <sup>2</sup> 4	Percentage Elongation on 4 in. 5	Modulus of elasticity t/in. <sup>2</sup> 6
	Upper $f_u$ 2	Lower $f_u$ 3			
web	25.0	18.9	31.3	17.65%	10,000
cover plates	25.8	18.8	29.0	20.96%	12,400

Table 2.

Description 1	Section Modulus in. <sup>3</sup>		Moment in pure bending and Testing machine load in. t and tons		
	Elastic $Z$ 2	Plastic $S$ 3	in first yielding 4	in full plasticity 5	According to allowable stresses 6
cover plates	3.08	6.125	79.5; 13.2	115.5; 19.2	50.0; 8.3
joist	17.7	—	456; 76	—	297; 49.3

Table 3.

Description  1	Yield Force			Ultimate Force					Allowable Force according to B.C. 449		
	without friction			with friction		without friction					
	Single shear tons 2	Double shear tons 3	Bear- ing tons 4	Bear- ing tons 5	Double shear tons 6	Single shear tons 7	Double shear tons 8	Bear- ing tons 9	Single shear tons 10	Double shear tons 11	Bear- ing tons 12
rivets	6.66	13.32	13.20	22.7	27.4	8.95	17.9	19.6	2.6	5.3	2.7
fitted bolts	5.0	10.0	10.2	14.7	20.0	7.65	15.3	15.6	2.6	5.3	2.7

The necessary properties of the joints themselves were obtained during the tests carried out, and were later successively reproduced.

According to the given dimensions and the quality of the material used, the necessary characteristic values of the cover plates and the joist itself were computed for the design and are presented in Table 2. To compute column 4, net cross-sectional values were used.

The design calculation of rivets, presented in Table 3, was done in accordance with normal practice for shear and bearing force of rivets. The stresses adopted for the purpose were based on the values for the web and cover plates already mentioned and on available experience. These values will subsequently be checked by the results obtained in tests.

To compute the yield force of rivets in shear and bearing the stress ratio was taken as  $\sigma:\tau:\rho = 1:0.8:3.1$  where  $\sigma$ ,  $\tau$ ,  $\rho$ , are stresses in tension, shear and bearing respectively. The figures 0.8 and 3.1 are based on published

results of the appropriate tests [8]. If we adopt for  $\sigma$  the lower yield stress of the web (Table 1), then the values for rivets, given in columns 2, 3 and 4 of the Table 3, can be obtained.

For bolts the stress ratio is taken as 1:0.6:2.4 (i. e., only approx. 80% of the above values), considering that the material of the bolts was not improved as it was in the case of rivets by the process of riveting.

Table 4.

Description 1		First yielding in. t and tons 2	Second yielding (collapse) in. t and tons 3
cover plates	Moment in pure ben.	79.5	115.5
	Testing machine load	13.2 tons	19.2 tons
rivets	Force in rivet	13.2 tons	22.7 tons

For ultimate strength the ratio for rivets [9] was 1:0.7:3.0 and that for bolts 1:0.6:2.4 and the figures in the columns 7, 8 and 9 of Table 3 were computed; for the ultimate tensile strength the value of the cover plate was taken.

Both the values of yielding and of ultimate force must be augmented by the value of friction arising from the tilting of the rivets [10], caused by the severe deformation preceding fracture. For the coefficient of friction the value 0.575 is adopted in accordance with the results of the large-scale tests performed in Germany [11], and for the tensile force that of full yielding in the shank of the rivet. Though the value of the coefficient may be regarded as high, it will be seen that the calculated values were in good agreement with the results of the tests.

Therefore this additional force is equal to:

$$F_f = \mu m P_v = 9.5 \text{ tons,}$$

where:  $\mu$  is the friction coefficient (0.575),  $m$  is the number of friction surfaces (2), and  $P_v$  is the tensile force in the rivet (equal to  $0.442 \cdot 18.8 = 8.3$  tons).

Then, the second limit of the yielding force in the rivet is obtained when  $F_f$  is added to the minimum value of the first yielding force of 13.2 tons. The same was done for the ultimate force, and thus the value for rivets in columns 5 and 6 in Table 3 was obtained. This additional force in bolts must be far less in tested cases, and only one half of the above value for rivets could be adopted.

In Table 4 are summarized the characteristic values for cover plates, already given in Tables 2 and 3, which determined the design limits between the first and second phase as well as giving the value at collapse produced by yielding.





## 2.2. Test Results

In Table 5 are presented the summarized results of all the tests carried out with characteristic values of loading and deflections.

The average moment at the end of the first phase, for single row and double-row bolted connections, 45.0 in.-tons and 103.8 in.-tons respectively. The corresponding values for rivets were 67.8 and 112.0 in.-tons. The corresponding mean values of the deflections at mid-span were:

for bolts 0.389 in. and 0.624 in.;  
for rivets 0.375 in. and 0.384 in.

The results of each particular test are presented in the form of graphs in Figs. 7—11 and 17. The respective curves are plotted with the testing machine loads as ordinates. For each test the relative rotations are given (in radians) of both end cross sections of the connected part of the beam, computed from the measured change in the clearance at mid-span. In the first phase this

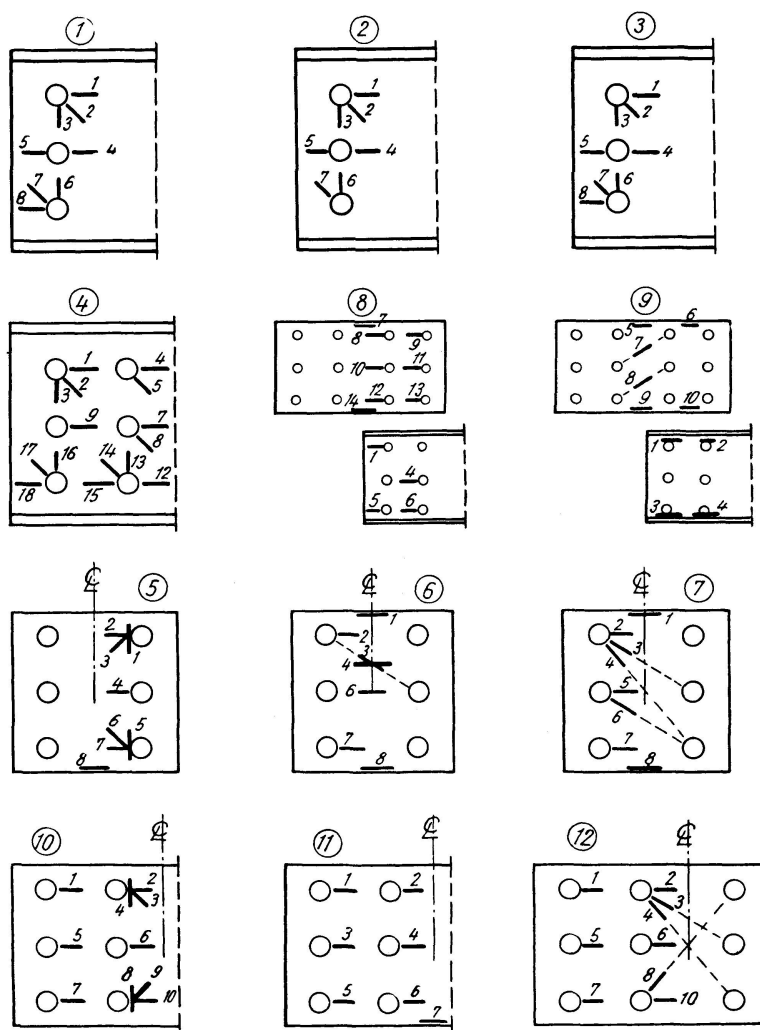


Fig. 6.

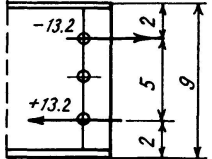
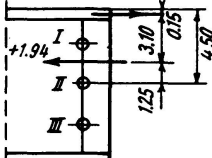
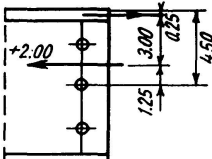
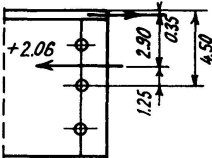
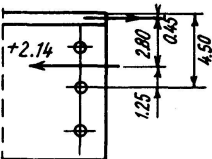
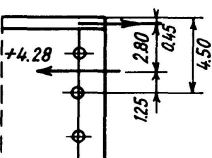
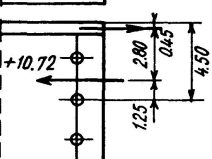
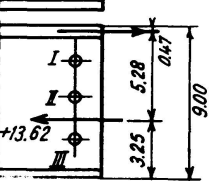


change was devided by half the depth of the beam, but after contact in the upper part of the end sections it was divided by half the reduced depth in accordance with the values given on the page 161, Table 6.

Table 5.

Description	No. of specimen	Clearance	Loading	Load in tons at				Deflections					
				First prop. limit	End of the phase 1	Second prop. limit	End of the phase 2	in the middle in.			Rotations radians		
								First prop. limit	End phase 1	End phase 2	First prop. limit	End phase 1	End phase 2
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Bolts	single row	1	W M	4.0 24.0				0.085			0.00523		
		2	W M	4.0 24.0	8.0 48.0			0.119	0.437		0.00634	0.0284	
		3	W M	4.5 27.0	7.5 45.0			0.123	0.339		0.01000	0.0262	
	double row	4	W M	6.0 36.0	18.0 108.0	32.0 192.0		0.125	0.4295		0.00533	0.0266	
		8	W M	10.0 60.0	17.0 102.0	30.0 180.0	40.0 240.0	0.150	0.437	1.175	0.02000	0.0635	0.12465
		9	W M	6.0 36.0	17.0 102.0	30.0 180.0	37.0 222.0	0.156	0.406	1.250	0.01554	0.0506	0.14905
	single row	5	W M	4.0 24.0	11.0 66.0	20.0 120.0	32.0 192.0	0.064	0.337	1.295	0.00511	0.0300	0.0676
		6	W M	4.0 24.0	11.0 66.0	20.0 120.0	34.0 204.0	0.064	0.304	0.994	0.00422	0.0246	0.0951
		7	W M	4.0 24.0	12.0 72.0	22.0 132.0	34.0 204.0	0.0745	0.4865	1.4275	0.00400	0.0351	0.0939
Rivets	double row	10	W M	10.0 60.0	18.0 108.0	32.0 192.0	40.0 240.0	0.123	0.280	0.990	0.00089	0.0245	0.0578
		11	W M	14.0 84.0	20.0 120.0	30.0 180.0	40.0 240.0	0.107	0.466	0.885	0.00554	0.0306	0.0527
		12	W M	10.0 60.0	18.0 108.0	30.0 180.0	42.0 252.0	0.126	0.408	0.969	0.00667	0.0278	0.0561

Table 6.

Phase	Load stage		Exter. Bending moment		During considered stage		Forces in rivets, tons			Remarks
	from	to	at beginning	incr. $\delta M$	Mechanism of Deformation	Distance $h$	At beginning of stage	Due to incr. $\delta M$	At End of stage	
1	2	3	4	5	6	7	8	9	10	11
1	0	11	0	66		5.00	0 0 0	-13.20 0 +13.20	-13.20 0 +13.20	Force only in outer rivets
	11	12	66	6		3.10	-13.20 0 +13.20	+0.97 +0.97 0	-12.23 +0.97 +13.20	Additional Tension received by rivets I and II
	12	13	72	6		3.00	-12.23 +0.97 +13.20	+1.00 +1.00 0	-11.23 +1.97 +13.20	
	13	14	78	6		2.90	-11.23 +1.97 +13.20	+1.03 +1.03 0	-10.20 +3.00 +13.20	
	14	15	84	6		2.80	-10.20 +3.00 +13.20	+1.07 +1.07 0	-9.13 +4.07 +13.20	
	15	17	90	12		2.80	-9.13 +4.07 +13.20	+2.14 +2.14 0	-6.99 +6.21 +13.20	
	17	22	102	30		2.80	-6.99 +6.21 +13.20	+5.36 +5.36 0	-1.63 +11.57 +13.20	
	22	34	132	72		5.28	-1.63 +11.57 +13.20	0 +6.81 +6.81	-1.63 +18.38 +20.01	Additional Tension received by rivets II and III
	34	—	204	—					36.76	

### 2.3. Theoretical Analysis of Tests

#### 2.31. Joint with Single Row of Rivets

If during the whole flexural test, i. e., from the beginning until the collapse, the mechanism of deformation remained unchanged, it would be impossible to explain how the joint could sustain such a large collapse load. For instance, if simple bending is assumed to apply throughout the test, and not only in the first phase, the cover plates could be strained at most by their full plastic moment  $M_f = 115.5$  in.-tons or by the force in the testing machine  $W_f = 19.2$  tons (Table 2), and the one row of rivets by a moment of full plasticity (Table 3)  $22.7 \times 5.0 = 113.5$  in.-tons or by  $W_f = 18.9$  tons. However, the effective measured collapse moment for such a joint was (Table 5) 192 and 204 in.-tons or 32 and 34 tons. Evidently, simple bending does not last throughout the whole flexural test, and as the joint is externally bent in simple bending, the change in the mechanism of deformation, capable of changing also the conditions of strain, is the only possible explanation for this collapse load, which is 66% larger than that obtained in simple bending.

The tests on specimens 5, 6, 7, i. e., with a single row of rivets will be used for the demonstration of the actual behaviour of the joint, because this type of joint has the simplest conditions.

There are two phases in the whole work of the joint. The first phase in the deformational response of the joint is from the beginning of the loading until contact occurs in the upper outer fibre of the end cross section of the joist.

This stage is represented in Fig. 8 by the part 0-B of the deflection curve.

The second phase is from contact until collapse of the joint (the part of the curve B-E in Fig. 8).

In both phases there is a portion in which linear proportionality exists, such as 0-A and C-D. Between two phases there is always a transition portion which is irregular in character (the part of the curve B-C, Fig. 8), depending on the finished condition of both end cross sections of joist and their parallelism.

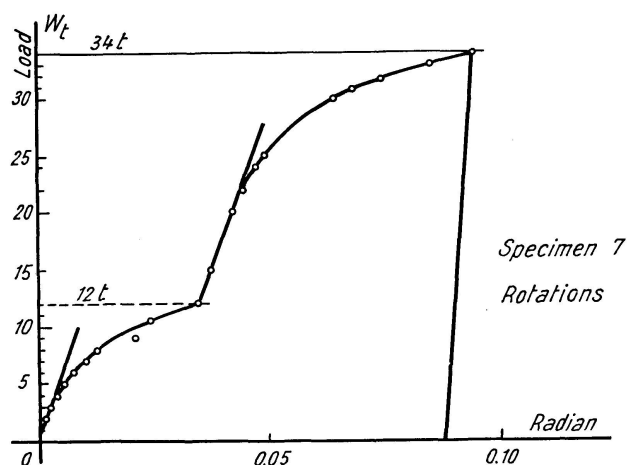


Fig. 7.

In the first stage each part of the joint rotates about its central rivet. The measured strains in horizontal directions in all three rivets, reproduced in Fig. 11 as lines 2, 5, 7 prove this statement, already known before. The strains in 2 and 7 are almost symmetrical and that in 5 is near to zero. The existence of strains at the neutral axis is due without doubt to the imperfection in the manner of loading and to the unequal slip in different rivets, when the initial friction was overcome.

The same is true of the cover plates. For example, the lines 1, 6 and 8 in Fig. 10 prove that the centre of plate rotation is in the middle of it.

After contact, the axes of rotation move and become united in only one, situated at the top of the end cross sections, where they are in contact. From that point onwards, the cover plates are subjected to an eccentric tension and in the upper part of the cross section the strains must diminish, and of course, the lines 1, 2 (Fig. 11) reverse their directions and are, at the end of loading,

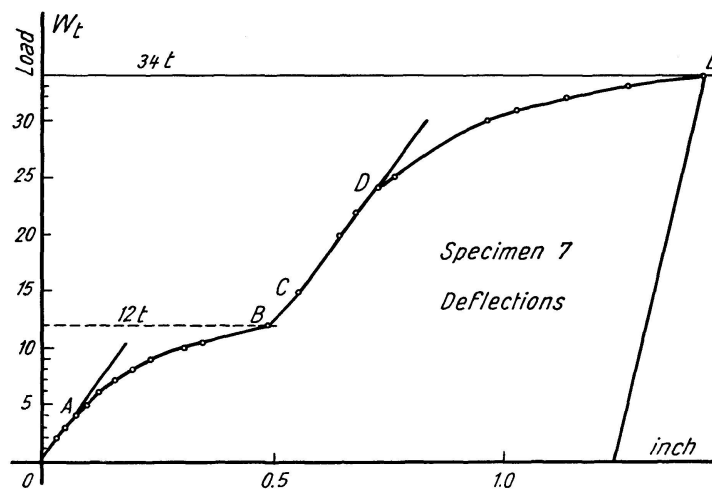


Fig. 8.

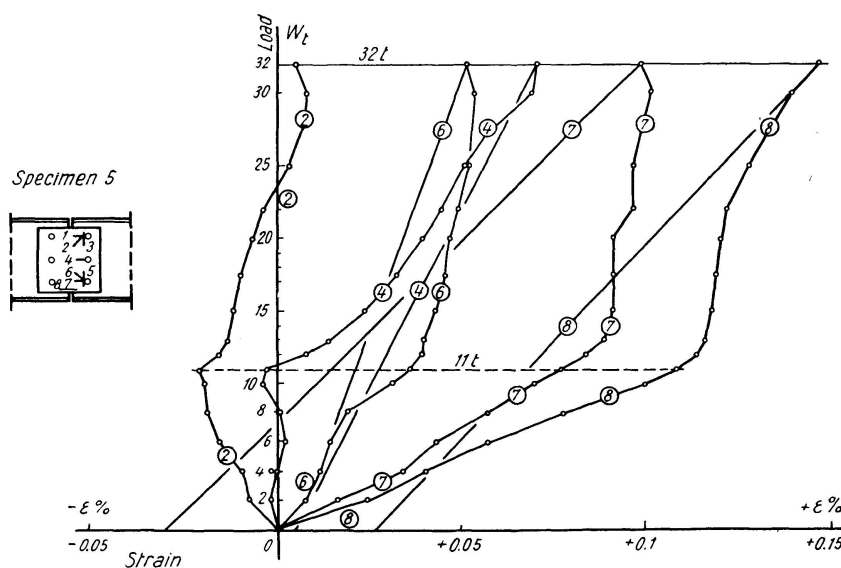


Fig. 9.

near zero. As a matter of fact, the neutral axis is no longer in the middle of the cross section or passing through the central rivets, therefore the strains, shown by lines 5 (Fig. 11) and 6 (Fig. 10), must steadily increase in magnitude. The same should happen with the horizontal strain in the lowest rivet, line 7, if the first plastic limit force of the rivet is not attained at the stage of contact. One can see that the line 7 in Fig. 11 shows temporarily an increase of strain after the contact, and from Fig. 10 we see that the increase had already ceased. For two joints having the same mechanical properties, the above behaviour is determined by the amount of clearance in the joint. If the gap is sufficient so that the first plastic flow of rivets is attained just at the instant of contact, then the increase of force in the lowest rivet is also temporarily stopped if the cover plates were so designed as to be sufficiently strong to be still in the elastic range at that instant.

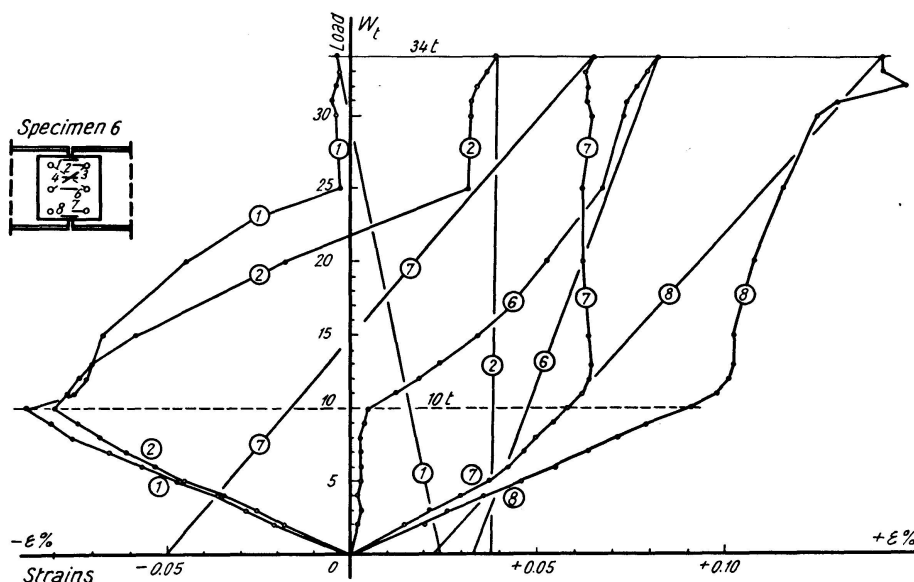


Fig. 10.

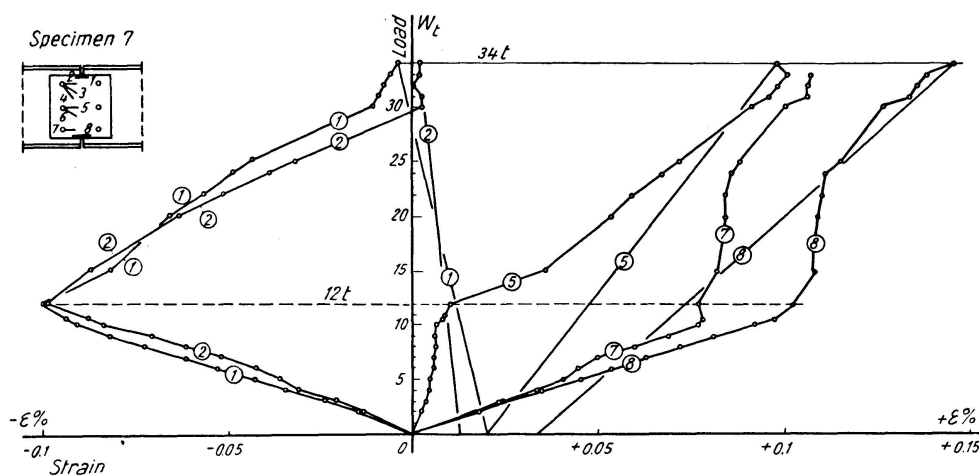


Fig. 11.

The gap in specimens 5, 6, 7 was 0.0985 in., 0.0850 in. and 0.126 in. respectively. The largest was in the case of specimen 7 and therefore the load producing the contact was also the largest (see Table 5); 12.0 tons or 72.0 in.-tons, but still less than 79.5 in.-tons, which moment is necessary to produce first yielding in the cover plates (Table 4).

From the shape of the load — rotation curve (given in Fig. 7) it is clear that the joint deformation was inelastic and also that the cover plates could not be the reason for this, as they were undergoing only elastic strain.

If we consider that at this stage, for  $W_T = 12$  tons or  $M_T = 72$  in.-tons, the first yielding limit of rivets is attained, the force in the rivet will be equal to  $72/5 = 14.4$  tons. The other two specimens 5 and 6 were in contact at a load of 11 tons, and the force in the outer rivet was  $66/5 = 13.2$  tons. The average value would be 13.6 tons. As the contact was actually not abrupt and began in specimen 7 at a load of 11.75 tons and in the other two specimens between 10 and 11 tons, it is advisable to take the limit as 13.2 tons. In Table 3 was given the computed limiting force for first yielding limit of the rivet (without friction), which is also equal to the same value of 13.2 tons. Therefore this limit is assumed to be 13.2 tons. When this force of  $\pm 13.2$  tons in the outer rivets is attained and contact has occurred, and if the moment is increased by  $\delta M$ , the lower rivet with the force  $\pm 13.2$  tons cannot take any increase, and plastic flow occurs. This produces a further rotation of the joint, but this time about the top of the end cross section, and therefore the middle rivet begins to take load as well as the top one which has a force equal to  $-13.2$  tons. They both take the tensile force of the internal couple, and the compressive force is taken by that part of the end cross sections in contact. This contact area is very irregular, according to the state of the end sections, and it takes some time before stabilization in the behaviour of this compressed zone is achieved. This is indicated by the transitional parts of the load-deformation curves. When a sufficient contact area is obtained, the behaviour reverts to linear as at the beginning. Further increase in loading produces further rotation of the joint, and the contact area is enlarged. This produces a slow downward movement of the centre of gravity of the contact area, and the decrease in strain in the upper part of the joint and the cover plate becomes steeper (lines 1 and 2 after 15 tons loading in Fig. 10). But, by this time, the force in the central rivet is sufficiently large to induce inelastic strains in it and its load — strain curve does not show the progressive increase of strain (line 4, Fig. 9). When the first plastic limit is attained also in the central rivet, another change in the organization of the inner couple takes place. At that moment, the deformation and the relative movements in the joint are so well developed that the additional friction must be induced, so that both lower rivets working in tension can take more of the loading. The friction itself, of course, is induced gradually and not suddenly as it happens in this analysis, but the result is almost the same and the analysis easier and simpler. The

additional tensile force of the inner couple is taken therefore by the lowest and by the middle rivet. This is well expressed in the loadstrain diagrams of line 7, e. g., in Fig. 9 at the load of 20 tons, in Fig. 10 at 25 tons and in Fig. 11 at 22 tons. The line of the strain in the central rivet does not show any such period of equal strain under increased loading (line 4 in Fig. 9 and line 5 in Fig. 11). The decrease in compressive strain in the top rivet can either stop at this stage (line 2 at load 25 tons in Fig. 10) or continue to drop further as in Fig. 9 and 11. In the first case the distribution of the tensile force of the internal couple is statically determinate and amenable to calculation, and in the attempted calculation it will be supposed that this is actually so. If the top rivet continues to participate in receiving the tension force the problem is practically insoluble. What kind of behaviour actually will take place obviously depends on the relative deformations of the compressed area and the cover plates, but cannot be predicted in advance.

If we consider that only the two lower rivets participate in taking the additional tensile force of the inner couple, the distance between the compressive and tensile forces is increased again and the increase is again slower. This deformation mechanism will act until collapse occurs.

The position of the compressive force is in general uncertain, and cannot be exactly determined but the shape of the designed specimens helped to locate it or at least to indicate the limits of its position. In the tests that were performed it was observed that at collapse the whole flange and about  $\frac{1}{3}$  of the web were in contact. Therefore, though the stress distribution is unknown, it could safely be assumed that the compressive force will never be outside the flange, because the ratio between the areas of flange and that part of the web was 1.828:0.762. The average thickness of the flange is 0.457 in., and the smallest value of the distance between the forces of the internal couple was taken as 2.80 in. so that the upper limit of the possible error in judgment would be less than 16.3 %, though an error of 100 % was assumed in the distance from the top to the position of the compressive force for this computation. In reality, the practical error would be about 5 %.

According to the stated changes of the mechanism in the deformation of the joint with the limiting values of the rivet forces mentioned, the computation of forces under various states of loading is given in Table 6.

As the result of the assumed mechanism of deformation and of the adopted procedure of distribution of the forces in the rivets, the rivet forces were found to be (beginning from the top): -1.63 tons, +18.72 tons and +20.35 tons. In the Table 4 the collapse rivet force was defined as 22.7 tons, which is a higher value than 20.35 tons. Apparently the rivets were near to collapse but still not actually collapsing. Therefore it could be said that the collapse of the joint was not produced by the collapse in the rivets, but in the cover plates.

This fact affords the possibility of checking the method used for the

distribution of forces among the rivets and of comparing the results obtained with those of tests with double joints.

Firstly, if a single row (according to the analysis given) was stronger than the cover plates used, it is even more certain that this must be true in the case of a double row joint with the same cover plates. And, indeed, with the double row joint, thanks to the difference in strains, it was possible to obtain the ultimate strength of the cover plates and to bring them to fracture. Each collapse of the specimens with double row joints was initiated by cracks in the cover plates and in no case was the fracture of any rivet observed.

Secondly, as the analysis shows that the first plastic yielding of rivets was responsible for bringing the end cross sections into contact (p. 165), if the clearances in specimens with one and two rows in the joints were approximately the same, than the moment necessary to produce the contact, must always be larger for specimens with double row joints. It is easy to see from the Table 7 that this is so.

Finally, if the cover plates were responsible for collapse and as they were the same in both cases (single and double row), then the total amount of the

Table 7.

Specimen	Single Row Rivets			Specimen	Double Row Rivets		
	Moment in. t		Difference of Moments		Moment in. t		Difference of Moments
	at contact	at collapse			at contact	at collapse	
1	2	3	4	5	6	7	8
5	66.0	192.0	126.0	10	108.0	240.0	132.0
6	66.0	204.0	138.0	11	120.0	240.0	120.0
7	72.0	204.0	128.0	12	108.0	252.0	144.0
average	68.0	200.0	132.0	average	112.0	244.0	132.0

Table 8.

Specimen 1		Collapse Load		Collapse Deflections	
		Testing Machine Load tons 2	Bending Moment in. t 3	Relative Rotations radians 4	Deflections in the middle of the beam in. 5
Rivets	10	40.0	240.0	0.0578	0.990
	11	40.0	240.0	0.0527	0.885
	12	42.0	252.0	0.0561	0.969
	average	40.7	244.0	0.05553	0.948
Bolts	4				
	8	40.0	240.0	0.12465	1.175
	9	37.0	222.0	0.14905	1.250
	average	38.5	231.0	0.13685	1.212





as represented in the diagram, according to the observations made at the final stage in testing. The centroid of the compressed area  $A_c = 2.99$  sq.in. is at point  $C$ . The tension area  $A_t = 2.875$  sq.in. is in full plasticity at collapse, but the compressed area is not necessarily so, and therefore the neutral axis is not at the same time, as usually happens, the axis of equal areas. The position adopted for  $C$  is based on uniform stress distribution, which must be regarded as only a rough approximation to the actual phenomenon. The full plastic moment of the cover plates will then be:

$$M_F = 2.875 \cdot 18.8 \cdot 4.405 = 237 \text{ in.-tons,}$$

compared with the obtained value of 204 in.-tons, a difference of 16.2 %. The actual difference was less than this, because the moment of 204 in.-tons obtained in the tests was not the real collapse moment. The whitewashed surfaces of the cover plates did not display many Lüder's lines. The load increase was stopped at this stage close to collapse, to enable the deflection readings to be taken after complete unloading, which would otherwise be impossible if the stage of total collapse load was previously attained.

Furthermore, it is possible that not only the bending moment of the value  $204 - 66 = 138$  in.-tons (i.e., during the second phase, Table 6) was acting, but a larger moment involving part of the moment from the first phase, if the plastic deformation in the end cross section was sufficient for this, the whole process being a highly statically indeterminate one.

### 2.32. Joint with Double Row of Rivets

When, instead of one row of rivets, there are two rows, the phenomenon of force distribution is still more complex. Considering the results of the experiments on specimens 10, 11, 12 it could be said, that in general, the participation of the second row of rivets, i.e., the row nearer to the end of the cover plate is less than that of the row nearer to the central cross section of the splice. In bolted specimens 4, 8, 9 this difference was not so serious as in

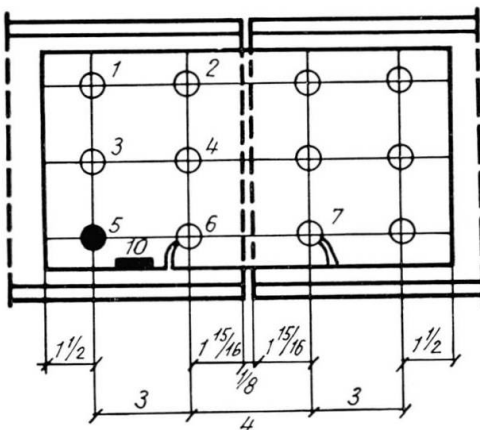


Fig. 13.

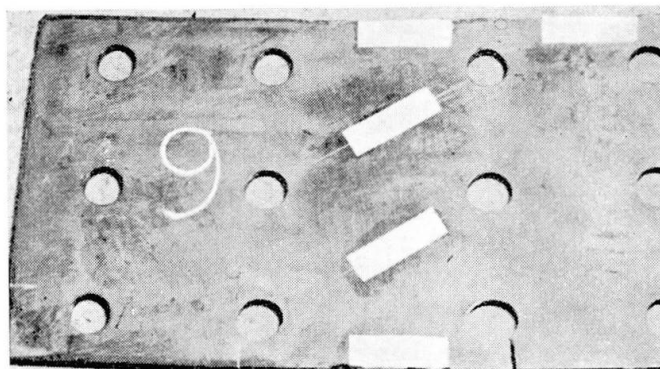


Fig. 14.

riveted joints, but still there was a phenomenon common to both types of joints: the rivet or bolt 5 (Fig. 13) in each test carried the least force.

The cracks in each case formed at 6 and 7, as indicated in Fig. 13 and shown on photograph Fig. 14.

This indicates that the plastic flow of the cover plates, which initially takes place between two inner rows induces larger forces in inner rivets or bolts. The development of Lüder's lines could be seen on the whitewashed surfaces of the cover plates of the specimen, for instance 11. In Fig. 15 are shown the first lines which developed in the compression part, when contact occurred, at a bending moment of 120.0 in.-tons. Then the yielding started in

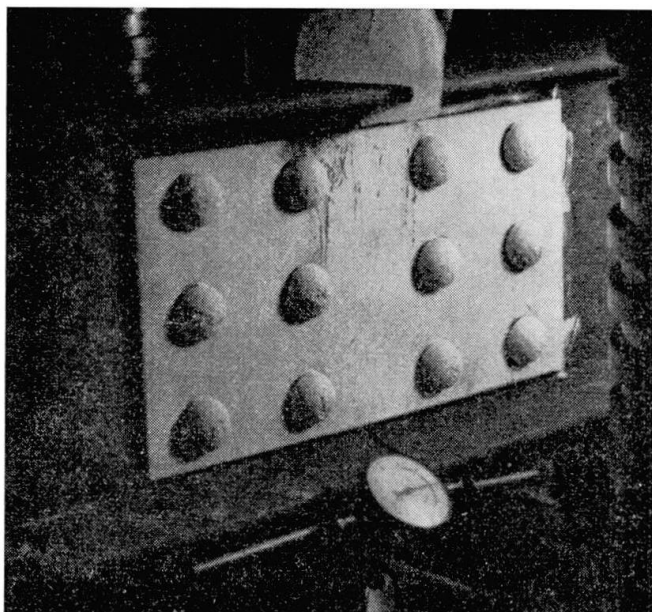


Fig. 15.

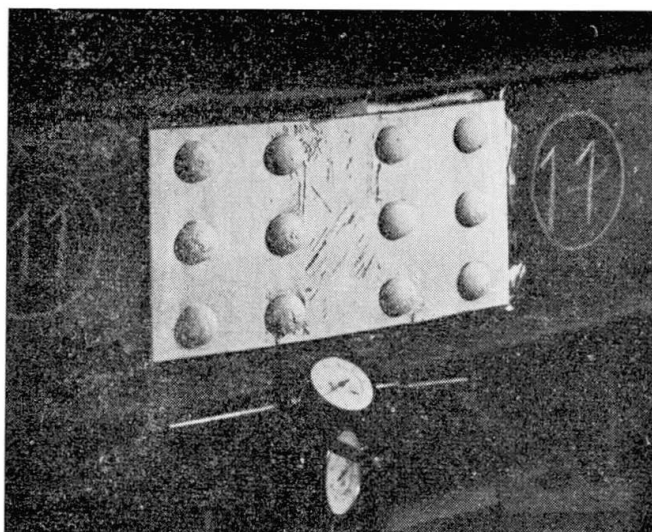


Fig. 16.

tension also, and the final position is shown in Fig. 16, with cracks as mentioned previously.

The deformation of the central part of the cover plate, of greater magnitude than the deformation of its outer parts (i. e., between two rows of rivets), make it possible for the rotation of rivet 5 about its instantaneous centre of rotation not to be proportional to the radius of rotation, i. e., the force is not proportional to the distance of rivet 5 from the centre. In the last stage the central load is so large that it could induce compression in the lower part of cover plate between 5 and 6.

The cracks first appeared at the free edge of the cover plate and passed through the rivet holes, under the bending moment of 240 in.-tons.

The distribution of forces in each rivet or bolt must be highly complex and the repetition of the method, used for a single row, cannot be successful, though the mechanism of deformation is the same. The problem is highly statically indeterminate and depends mainly on the mechanical properties of the central part of the cover plates and of each rivet.

In the above discussion the direction of the rivet forces was always taken as parallel to the longitudinal axis of the beam in both the first and second phases. In the first phase, for a single row joint, this is quite correct but nevertheless, in all the types of joints investigated, the calculated change in the direction of principal strains in the rivets was very small and the resultant nearly horizontal.

It thus seems justified to consider the rivet forces as always horizontal.

### Part III

#### *3.1. Prediction of Collapse Load for Riveted Joints*

##### *3.11. Single Row Joint*

The strength of the rivets and of the cover plates together determine the collapse load.

According to the mechanism of deformation of the single row joint of the same type as used in the tests, the first approximate value of plastic collapse moment for rivets could be obtained by making the assumption that the force in the top rivet is zero and then by equating the forces in the other rivets to the plastic collapse force (with friction, Table 3) and by assuming that the compressive force is located at  $1/6$  of the section depth from the top (Fig. 14). This would give, in our case, a limit according to rivets:

$${}_R M_F = 2 \cdot 22.7 \cdot 4.25 = 194 \text{ in.-tons.}$$

The other limit is obtained from the cover plates. The forces in the rivets in the final stage (Fig. 18) produce in the cover plates a tensile force +45.4

tons and a bending moment  $22.7 \cdot 2.5 = 56.8$  in.-tons. The cross-sectional dimensions of the plates must be such that the cover plates are in full plasticity under this loading.

In our case, according to the dimensions of the cover plates used in the tests, for the tensile force  $+45.4$  tons the full plastic moment could be computed (Fig. 19) in the usual way [12]:

$$\begin{aligned} e &= 45.4 / 18.8 = 2.42 \text{ in.}, \\ (S)_e &= 0.50 \cdot e^2 = 2.93 \text{ in}^3, \\ M_N &= 115.5 - 2.93 \cdot 18.8 = 60.5 \text{ in.-tons.} \end{aligned}$$

This result is therefore very close to  $56.8$  in.-tons and is in agreement with the already stated fact that in test specimens 5, 6, 7 the actual collapse was not obtained (p. 169).

In the tests, the actual average collapse moment for this type of joint was (Table 7)  $200.0$  in.-tons or only  $3\%$  more than that found above, as the limit for the rivets.

By making the assumption of zero force in the top rivet, one is on the safe side, because some force must remain in it and the result obtained proves this. Also the position of the compressive force has been assumed to be lower than its true position and this adds to the safety.

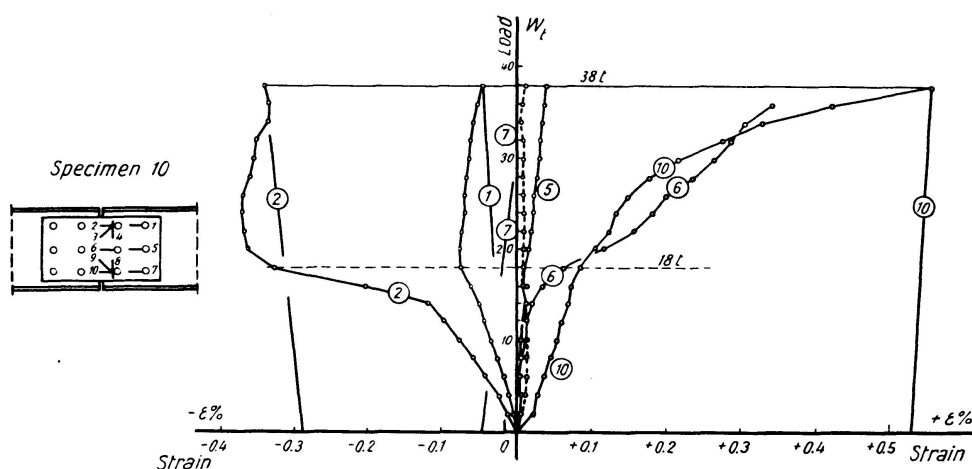


Fig. 17.

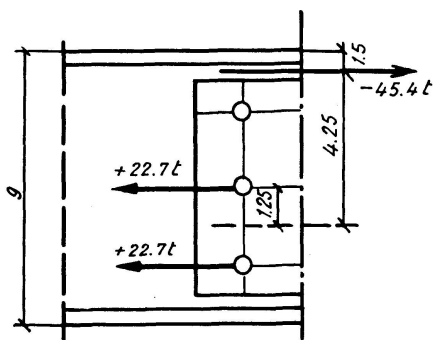


Fig. 18.

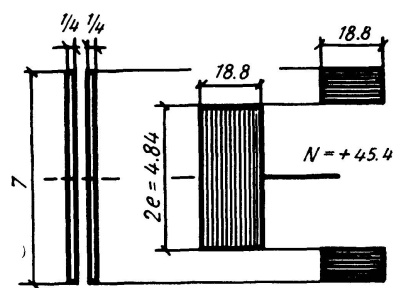


Fig. 19.

The procedure will be then to compute first  $R M_F$ , according to the assumed rivet group and then to check the dimensions of the cover plates, knowing the induced tensile force.

In the columns 10, 11, 12 of Table 3 the values of rivet forces are given according to B.C.449. The corresponding allowable moments will be for rivets in shear:

$$5.3 \cdot 5.00 = 26.5 \text{ in.-tons};$$

for rivets in bearing:

$$2.7 \cdot 5.00 = 13.5 \text{ in.-tons.}$$

If one now compares these moments with the average value of the collapse moment of 200 in.-tons (Table 7), obtained in tests, the following values of the safety factor will be obtained:

$$\begin{aligned} &\text{for rivets in shear} && 7.6 \text{ and} \\ &\text{for rivets in bearing} && 14.8. \end{aligned}$$

These are really high values and this means only that we have not taken advantage of the second phase.

Therefore, a more rational approach can be expected from the outlined procedure, with the values of the safety factor as normally used in structural analysis.

### 3.12. Double Row Joint

The procedure of predicting the collapse load can be the same as in the previous case of a single row, but modified in accordance with the results obtained for the distribution of forces. As is, for instance, shown in Fig. 17 in the case of specimen 10, the force in the top rivet of the inner row (No. 2 in Fig. 16) is not reduced to zero, as before. The efficiency of the external row, as obtained in the tests, was very low, compared with the inner row, i. e., only about 15 % of it. In general, the matter is not so bad as that because the flexural rigidity of the cover plates is of prime importance. If this is large

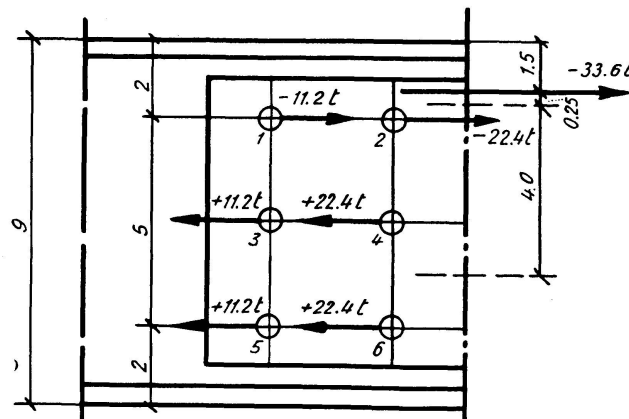


Fig. 20.

enough to involve the external row in sharing the bending moment, after the full engagement of the inner row, then the picture would be far better than in the tests performed.

Taking the force in the outer row to be 50 % of that in the inner row, the forces in the rivets Nos. 1, 3, 5 will be as indicated in Fig. 16. The position of the compressive force could be taken to be the same as in the previous case but less in magnitude (now only 33.6 tons). The resisting moment equivalent to the external moment will be:

$$2(11.2 + 22.4) \cdot 4.00 = 268.8 \text{ in.-tons.}$$

The bending moment induced in the cover plates by this rivet force is:

$$M = 33.6 \cdot 5.0 = 168.0 \text{ in.-tons.}$$

The tensile force is  $N = 33.6$  tons.

For the cover plates used in the tests (Fig. 15), the full plastic moment  $M_N$ , corresponding to the force  $N$  will now be only 85.4 in.-tons, i.e. less than 168.0 in.-tons. It is then obvious that such forces in the double row joint could not be developed because the cover plates were not strong enough.

The proof of this statement is again an indirect one in showing the impossibility of reaching the calculated moment. Only fresh tests with the cover plate dimensions increased in accordance with the above calculations would be sufficient for full proof of its accuracy.

### 3.2. *Significance of Test Results with Regard to Practical Joints*

It has already been explained why the type of specimens used in the tests were chosen (p. 153). In general practice, such a type of joint, with web splices only, is most unusual, and the question of the practical value could be raised.

The clearance in a joint is normally present, and the second phase could develop if cover plates on the flanges did not prevent it. For the size of gap between the connected joist length, as made in normal shop practice, the elastic deformation of flange splices would be insufficient, and it is reasonable to suppose that their deformation would have to take place before the beginning of phase two. This plastic movement can again happen in rivets or in splices. If the cross section of the jointed beam is, as is usually the case, symmetrical about the horizontal axis and if the top and bottom flange splices with their rivets are the same, then it can be expected that the movement will occur in both the tensile and the compressive flange. When the contact occurs, phase two will start, and sufficient yielding without breaking of the tensile part will allow full development of the deformation mechanism (as already described) to take place before collapse, if the instability or cracks are excluded.

Therefore, it could be said that both phases will take place in joints in practice also; only the timing of their occurrence is changed. The first phase



will last longer than in the specimens tested, and at the beginning of the second phase plastic movements in the flange splices will have to take place. The second phase will then arise and can fully develop if the fracture of parts in tension does not occur.

This also proves what common sense would indicate, namely, that the joint in which the gap is kept small is stronger than one in which a wider gap is present.

If the deflections of a jointed beam, in consequence of the plastic yielding in the flange cover plate, can be tolerated and the above-mentioned discontinuities are excluded, the total collapse load moment of the joint will be that obtained for the web connection at the end of phase two, plus the couple taken by the joint of the flanges.

Joist connections, designed in normal practice to withstand bending moment, do not make any use of the potential strength given by the second phase, which of course puts the design on the safe side. But this has a negative effect on economy. Such joints, designed primarily for static bending, should utilise the actual mode of their collapse, because this would be only the final logical conclusion from all the considerations in connection with rivets. The normal design of a riveted joint exposed to tension or compression is based on the conditions prior to collapse, but this is not applied to the case of joints subjected to static flexural load. It would be nearer the truth and more economical, in the case of such joints, to calculate the collapse load according to the final stage in the second phase as described and then to obtain the working load by deviding the collapse load by an appropriate safety factor.

From the deformation mechanism described it can be seen that the splice of the compression flange could be very weak and therefore economies could be effected in both rivets and splices. But the deflection of the jointed beam must always be checked, and more care is needed in keeping the gaps small.

### *3.3. Conclusions*

It has been shown, that the results of the tests, though obtained on specially designed specimens not common in usual practice, could be extended and generalized in some way to the normal type of joints subjected to bending. The full plastic moment of the splices under consideration was 115.5 in.-tons, and they actually collapsed at a moment of 200.0 in.-tons. Such an increase is partly made possible by specially selected spatial features of the specimens tested, but this still proves that the whole conception of the process, which is now the basis for design and calculation, must be inadequate. The bending of the cover plates is changed into eccentric tension, and that is why the difference between the computed value (based on bending) and that actually measured is so very great.

1. Therefore, the first conclusion is that the design method for joints



subjected to static bending, as used by the average designer, cannot provide a means of determining the collapse load, and the actual factor of safety is higher than expected.

2. The design of such joints should be based on failure load, as a logical extension of the procedures already adopted in general for the design of some riveted joints or members.

3. The direct contact of the compressed parts of the end cross section of the jointed beam changes the character of the deformation mechanism and increases the strength of the joint.

4. The most active rows of rivets are those nearest to the joint, and the flexural rigidity of splices in general determines the percentage of the participation of the other rows.

5. The distribution of forces during bending is highly statically indeterminate. With the progressive enlargement of the web area in contact the neutral line is shifted upwards, and gradually more rivets are strained in tension.

6. In a single row joint the collapse load can be predicted by assuming that all the rivets (except the top one) are strained in a direction parallel to their maximum forces, and that the compressive force, equal to the sum of all the rivet forces, is acting at a point located at one-sixth of the cross section depth from the top.

7. Further investigations are needed for obtaining information on the work of joints with more than one row of rivets in the web splices. The flexural rigidity of the splices used in these investigations should be varied over a wide range in order to obtain an exact picture of the distribution of rivet forces in such joints.

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### References

1. DE JONGE R. A. E., "Riveted Joints, a Critical Review of the Literature Covering their Developments". A Research Publication by Am. Soc. C. E., 1945, New York.
2. Second and Final Report of the Steel Structures Research Committee, published by H. M. Stationary Office, 1931 and 1934, London.

- MASSONNET, CH., "Essais d'assemblage à boulons ou rivets tirés". Publications I.A.B.S., Vol. XVII, 1957, Zurich, pp. 95—116.
- MARSHALL, W. T. and HOO, C. N.: "An Experimental Investigation of the Stresses in Angle Brackets". J. Instn. Civ. Engrs. Vol. 36, Dec. 1951, London, pp. 503—533.
- RATHBUN, J. CH., "Elastic Properties of Riveted Connections". Proc. Am. Soc. C.E. Vol. 61, January 1935, pp. 3—42.
3. GARRELTS, J. M. and MADSEN, J. E., "An Investigation of Plate Girder Web Splices". Trans. Am. Soc. C.E., Vol. 107, 1942, pp. 1303—1329.
- CASSENS, J., "Die Nietkräfte in den Verstärkungslaschen von Biegeträgern". Zeitung für Flugtechnik und Motorluftschiffahrt; Vol. 24, Feb. 1933, pp. 103—107.
- M'BROOM, H. L., "Deflection Characteristics of Bolted Joints". Royal Techn. College-Gl., V. 3, pt. 3, Jan. 1933, pp. 440—446, and V. 3, pt. 4, pp. 559—608, Jan. 1936.
- MARTIN, H. C., "Designing Web Flange Rivet Connections for Shear Stresses". Aero Digest, V. 41, Sept. 1942, pp. 162, 165, 166, 279, 280, and Oct. 1942, pp. 116—118.
4. BATHO, C., "Riveted and Bolted Connections". First Report of the Steel Structures Research Committee, published by H. M. Stationary Office, 1931, London, pp. 100—129.
5. KUZMANOVIĆ, B. O., "Behaviour of Riveted Connections of Plates and Rolled Beams under Statical Bending Moment". Publications of the Technical Faculty (in Serbian), V. 1, 1959, Sarajevo, pp. 49—52.
6. RODERICK, J. W. and PHILLIPPS, J. H., "Carrying Capacity of Simply Supported Mild Steel Beams". Colston Papers, Res. Engng. Struct. Suppl. 1949, pp. 9—48.
7. ROBERTSON, A. and COOK, G., "Transition from the Elastic to the Plastic State in Mild Steel". Proc. Roy. Soc., A. Vol. 88 (1913).
8. JONES, J., "Bearing-Ratio Effect on Strength of Riveted Joints". Trans. Am. Soc. C.E., Vol. 123, 1958, pp. 964—972.
- KAYSER, H., "Die Versuche über die Schär- und Lochleibungsfestigkeit der Nietverbindungen". Der Stahlbau, April 17, 1931 and October 13, 1933.
9. CHESSEON, E. and MUNSE, W. H., "Behaviour of Riveted Truss-Type Connections". Trans. Am. Soc. C.E., Vol. 123, 1958, pp. 1087—1126.
10. DAVIES, R. E. and WOODROFF, G. B., "Tension Tests on Large Riveted Joints". Trans. Am. Soc. C.E., Vol. 105, 1940, pp. 1193—1299.
11. STEINHARDT, O., "Vorgespannte Schrauben im Stahlbau". VDI-Z, Bd. 97 (1955), Nr. 21, pp. 701—708.
12. NEAL, B. G., "The Plastic Methods of Structural Analysis". Chapman and Hall Ltd., 1956, London, p. 353.

### Summary

This paper reports a series of tests designed to obtain basic information on the behaviour of riveted web connections, i. e. the mechanism of deformation and that of forces in rivets, when subjected to simple bending under a static loading.

There are two distinct phases in the deformational response of such connections, with the corresponding changes in the rotation point, both of web splices and rivets.

The collapse load of riveted connections is higher than computed according to normal practice, if cases of instability or other discontinuities are excluded.

According to the assumed mode of functioning of the connection, a tentative calculation is made of the actual forces in each rivet, and the results obtained are compared with the values adopted in normal practice.

### Résumé

L'auteur rend compte d'une série d'essais qui ont été effectués en vue d'obtenir des indications de base sur le comportement des joints d'âme réalisés par rivetage, c'est-à-dire sur le mécanisme de déformation et sur les efforts mis en jeu dans les rivets lorsque le joint est soumis à une flexion pure résultant d'une charge statique.

Le processus de déformation de ces assemblages comporte deux phases, avec modifications correspondantes du centre de rotation, aussi bien des couvre-joints que des rivets.

La charge de rupture des assemblages rivés est plus grande que celle donnée par le calcul habituel, lorsqu'il n'intervient ni instabilité ni discontinuités.

En tablant sur le comportement présumé du joint, l'auteur tente d'établir un mode de calcul des efforts effectifs dans chaque rivet, et il compare les résultats obtenus avec ceux de la technique courante.

### Zusammenfassung

In dieser Abhandlung wird über eine Reihe von Versuchen berichtet, die ausgeführt wurden zur Bestimmung von grundlegenden Angaben über das Verhalten von genieteten Stegverbindungen: d. h. über den Mechanismus der Deformationen und denjenigen der Kräfte in den Nieten bei reiner Biegebbeanspruchung unter einer statischen Last.

In der Formänderungsaufnahme von solchen Verbindungen gibt es zwei getrennte Phasen mit den entsprechenden Änderungen im Drehpunkt sowohl der Steglaschen wie auch der Nieten.

Die Bruchlast von genieteten Verbindungen ist größer als sie nach der üblichen Praxis berechnet wird, falls Instabilitätsfälle und andere Diskontinuitäten ausgeschlossen sind.

Entsprechend dem angenommenen Verhalten des Stoßes wird eine Berechnung der tatsächlichen Kräfte in jedem Niet versucht und dazu ein Vergleich mit der üblichen Praxis aufgestellt.