Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 19 (1959)

Artikel: A criticism of the plastic-hinge concept used in the plastic theory of structures

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DOI: https://doi.org/10.5169/seals-16954

A Criticism of the Plastic-Hinge Concept Used in the Plastic Theory of Structures

Examen critique de la notion de «rotule plastique» utilisée dans la théorie de la plasticité appliquée aux constructions

Eine Kritik an der Anordnung der plastischen Gelenke beim Traglastverfahren

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It is quite obvious that the complicated phenomena occurring in unelastically strained elements of a structure cannot be composed in the frame of a mathematical theory without certain working hypotheses, assumptions and simplifications.

The basic assumptions introduced in plastic design are that the volume during plastic deformation remains constant, that the stress-strain relations are the same in tension and in compression and that they are of such a kind that Hooke's law is valid till the yield point is reached, that strains subsequently increase under constant stress (without work-hardening) and that the plane cross-sections remain plane and normal to the deformed longitudinal axis during the whole bending process.

These suppositions are correct when used in the mathematical theory of plasticity [1], which ought to relate (by definition, of course) to an ideal material. But the engineer can only do this if these assumptions do not change the character of the process considered, because he has to take account of the properties of the material he actually uses.

Though it is very difficult to see why a steel member, stressed beyond its yield point, should behave, in respect of its deformation, as if it were an ideal fluid and although the volume constancy assumption may lead on some occasions to errors of up to 30 per cent, in our engineering structures, due to the cross-sections normally used, it is permissible.

The ideal stress-strain diagram (fig. 1), first introduced by KIST, leads. on the contrary, to absurd conclusions such as that the ultimate resistance of a continuous beam depends only on static conditions of equilibrium, independently of elasticity conditions, and that in plastic hinges all angle changes occur without work-hardening, although it is quite easy to show that any necessary angle change in plasticity is impossible without it [2].

An idealized moment-angle change diagram (fig. 2) corresponding to the idealized stress-strain diagram, consists of two straight lines with a transition curve of some kind.

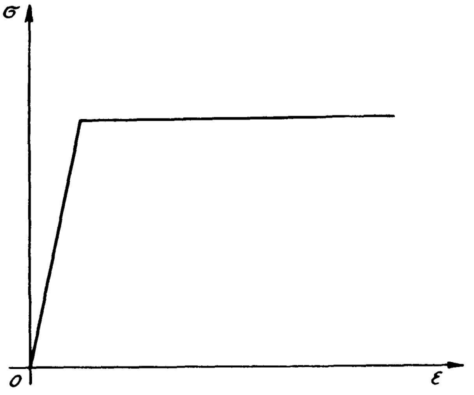

Fig. 1.

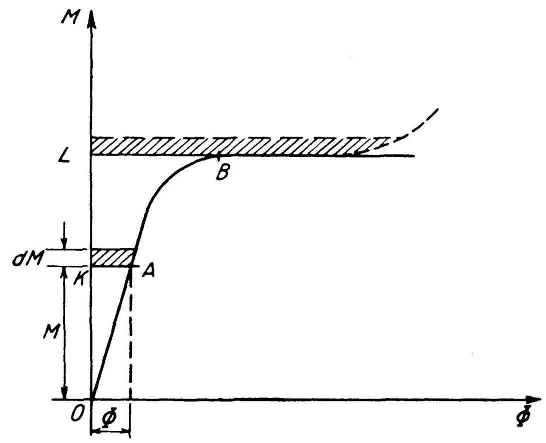

Fig. 2.

The shearing force V is equal to

$$V = \frac{d\,M}{d\,l},$$

where $d\,l$ is an element of the length of the beam axis. If ϕ is an angle change occurring on a unit length corresponding to the moment M, the angle change on the length $d\,l$ will be:

$$\phi\,d\,l = \frac{\phi\,d\,M}{V},$$

therefore the angle change between two points A and B is equal to

$$\phi = \int_A^B \phi\,d\,l = \int_A^B \frac{\phi}{V}\,d\,M.$$

If V is constant we have

$$\phi = \frac{\int_A^B \phi\,d\,M}{V} = \frac{\text{area}\,A\,B\,K\,L}{V}.$$

The area in the numerator is bounded by the axis of M, the curve $M-\phi$ and two abscissæ, corresponding to the points on the curve between which the angle change is evaluated. With the $M-\phi$ curve extending horizontally without limit, no contribution to the angle change is provided by the horizontal or plastic part of the curve, because advancing the point B along the curve to the right will not increase the area $ABKL$. Thus, contrary to what might be supposed, the plastic part of the $M-\phi$ curve proves incapable of contributing to the necessary angle change at the plastic hinge while other hinges are developing. The situation is different if the $M-\phi$ curve exhibits a strain hardening part, the area in the numerator begins to increase again, thus providing the necessary angle change at the hinge. Therefore, if the formation of plastic hinges is possible at all, strain hardening must occur and the expected equalization of different moments on critical sections cannot be fully achieved in any manner. The magnitude of the error that is made is dependent on the elasticity conditions and the loading programmes. If the difference to be equalized between the moments is great and other conditions are favourable, the error in the calculation based on plastic hinge design is also great and vice versa. Therefore, the agreement obtained between the collapse load, calculated by the plastic hinge method, and that obtained in a test does not in general prove the accuracy of this method or rather does not make it possible to decide upon the order of magnitude of the possible error in other cases. This variability of accuracy and therefore of safety is the main defect of the plastic hinge method. If the mode of failure of a structure is a local one, as it almost always is in the case of continuous beams, the discrepancy is also larger.

The tests carried out in Zurich by Professor F. Stüssi and C. F. Koll-

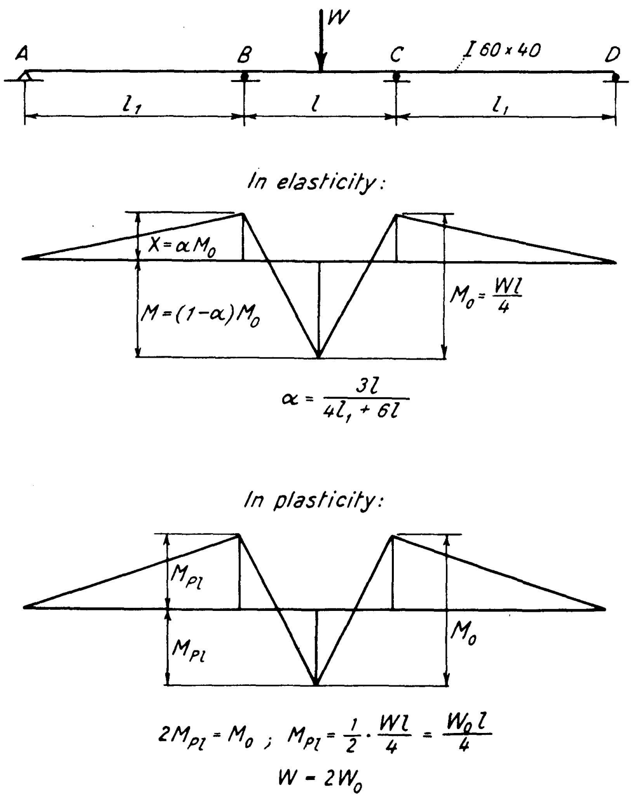

Fig. 3.

BRUNNER [3] were the first which proved that the collapse load of a continuous beam is dependent on the elasticity conditions. Tests recently repeated in a somewhat more elaborate form and extended to the case of dynamic loading by the same author [1] confirmed the previous results. The so-called "classical" continuous beam of Professor MAIER-LEIBNITZ with a steady middle span $l = 60$ cm was taken, but the side spans were varied as $l_1 = 180, 120, 60, 30$ and 15 cm respectively (fig. 3). In the elastic range, if $X = \alpha M_0$ denotes the moment at the internal supports, the field moment would be:

$$M = (1-\alpha) M_0 = (1-\alpha)\frac{Wl}{4}.$$

For a simply-supported control beam of the same span l and the same type of loading, the moment will be $\frac{W_0 l}{4}$, and the relation between these two loads in the elastic range are

$$(1-\alpha)\frac{Wl}{4} = \frac{W_0 l}{4}; \qquad W = \frac{1}{1-\alpha} W_0 \ldots \tag{1}$$

namely, the collapse load is dependent on the elasticity conditions of the beam, determined by the relation of the successive spans.

In the plastic range we should always have the collapse load of these continuous beams as twice the collapse load of the simply-supported beam. In the diagram (fig. 4) are given the results of the above mentioned tests for static and dynamic loading. In the last case of fatigue tests the loads are given which produced fracture under 10^4 or 10^5 load variations between 0 and W.

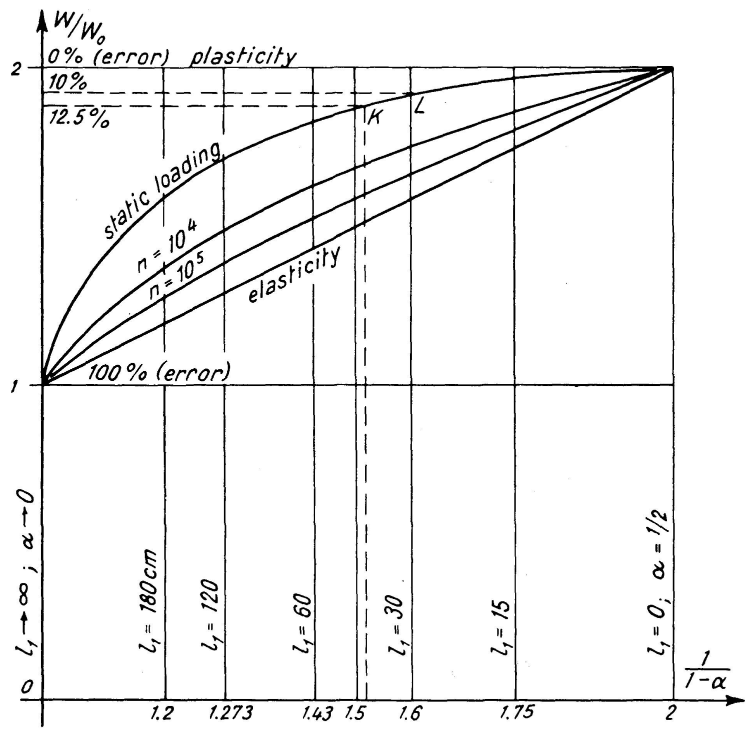

Fig. 4.

The actual collapse load was always between the two loads calculated according to the theories of plasticity and elasticity. In the case of dynamic loading with $n = 10^5$ load variations, the actual collapse load was very close to that given by the theory of elasticity.

The diagram in fig. 5 gives the actual behaviour of the continuous beam with $l_1 = 2l$; the field moment was always greater and remained so. There is only a tendency towards equalization of the respective moments and the strain hardening range can be seen easily, which agrees well with what has already been stated.

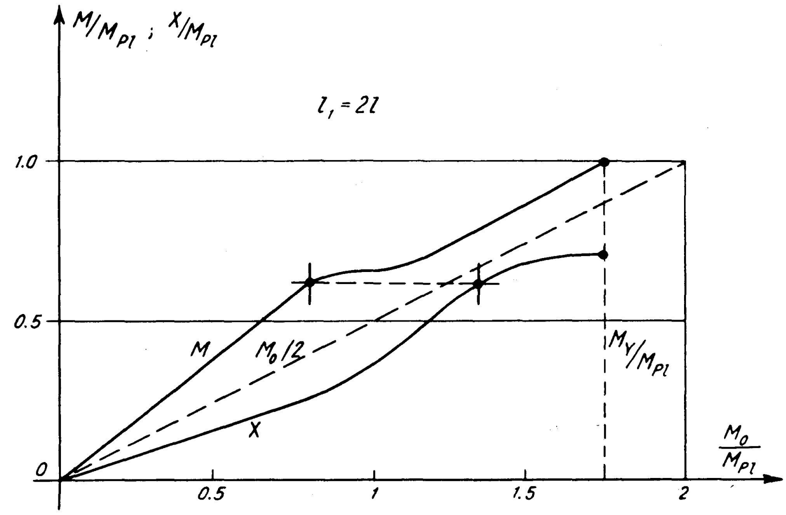

Fig. 5.

Nevertheless, it is often said that the tests carried out at Cambridge by Professor BAKER and his co-workers on continuous beams have proved a controversial statement, namely that because the calculated and actual collapse loads agreed well, the plastic hinge concept is the actual description of the behaviour of continuous beams. It will now be shown what the results do, in fact, prove.

The results of tests cannot be used directly, as was possible in the case already described, because the experiments were carried out for other purposes rather than to prove the equalization of moments. The diagram in fig. 4 must also be used in the absence of an actual diagram, supposing that the general behaviour is of the same character, which can be done because the equations are of the same kind, as will be shown.

Professor BAKER's tests [4] were carried out on two-span continuous beams with three types of loading and on simply supported control beams, which had not always the same span and type of loading as we should require. In fig. 6 the first type of loading is reproduced.

The actual moments in the elastic range are

$$X = \frac{3}{16} W l \quad \text{and} \quad M = \frac{5}{32} W l;$$

the difference being $1/32\,Wl$ or 16% and this is the upper limit of the possible discrepancy, if there was no equalization at all. It must be noticed that the moment on the internal support is now greater than the field moment and this is more favorable for the process of equalization than the previous case in which the field moment was greater.

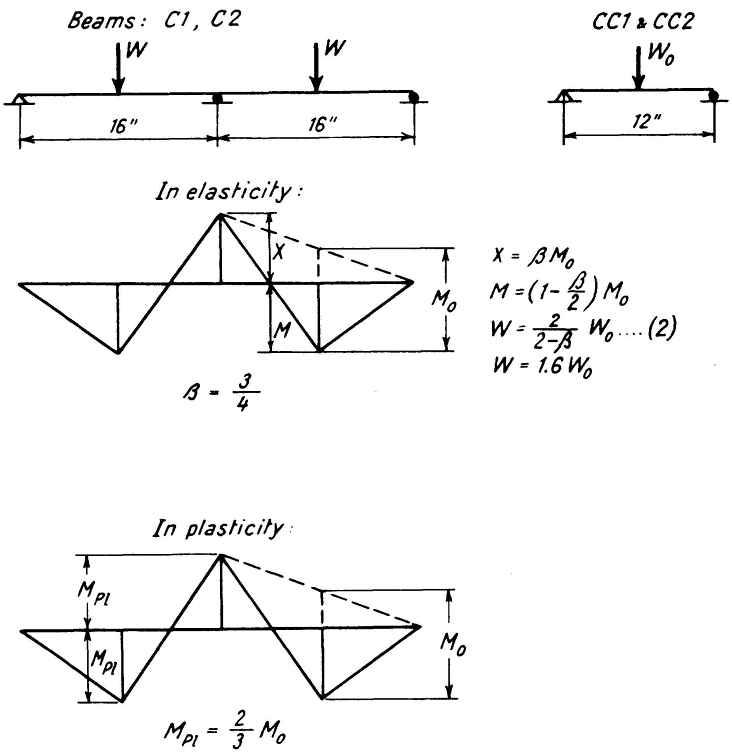

Fig. 6.

According to the diagram in fig. 4 (point L) and eq. (2), it can be seen that the expected difference must be small, approximately ten times smaller than the possible error or about $1{,}6\%$. And in fact the measured collapse loads were:

$$W_{c1} = 1{,}125\,t; \quad W_{c2} = 1{,}138\,t; \quad W_{cc1} = 1{,}000\,t \quad \text{and} \quad W_{cc2} = 1{,}025\,t.$$

The calculated collapse loads should be obtained by equating M_{Pl} for the continuous and simply-supported beam:

$$\frac{2}{3}\frac{Wl}{4} = \frac{W_0 l_0}{4}; \quad \text{or} \quad W = \frac{3}{2}\frac{l_0}{l} W_0 = 1{,}125\, W_0.$$

Then the calculated collapse loads are:

$$W = 1{,}125 \times 1{,}000 = 1{,}125 \text{ tons and}$$

$$W = 1{,}125 \times 1{,}025 = 1{,}138 \text{ tons}$$

exactly the same as the measured loads and according to what has already been shown, this was to be expected.

In the second type of loading, where the field moment was still less than the moment at the internal support with two concentrated loads in both spans, the difference of the moments in elasticity was about 33% and the equation similar to eq. (1) was $W = 1{,}52\ W_0$. Therefore, it can be said in advance that the discrepancy must be now greater than in the previous case and according to fig. 4 (point K) about $\frac{1}{8}$ of the upper limit, or about 4,2%. The differences found were actually: 8,7%, 6,2%, 3,5%, and 3,9% or an average of 5,6%, compared with the expected 4,2%.

In the third type, with four loads in each span, and the field moment still less than the moment on the support, the difference between elastic moments was about 41%, but the relation between collapse loads for continuous and simply-supported beams was $W = 1{,}63\ W_0$. This means, according to fig. 4, that the expected discrepancy should again be about 1/10 of the upper limit or 4,1%. The actual differences were 5,6%, 5,6%, 4,4% and 7,5% or an average of 5,8%, compared with the expected 4,1%.

It can be seen that the greatest error made in this prediction is only 1,7%. The other tests on fixed-ended beams cannot be analysed in this way because in the first part of the tests, with a concentrated load in the middle of the span, there is the natural equalization of moments, beginning in the elastic range, and no equalization is needed at all and in the second part of the tests, with asymmetric loading positions, we have no results on adequately loaded control beams.

It may be concluded that the results of both series of tests, Professor Stüssi's and Professor Baker's, are in a very close agreement, which is at first a sure sign of the high quality they possess and secondly that the simple plastic method of design is an approximation, as is the elasticity method, but that one must have an appropriate safety factor when using it on continuous beams, to be on the safe side. The accuracy of the simple plastic method is variable and dependent on the elasticity and loading conditions of the actual case and the contradictions inherent in the simple plastic method may give rise to uncertainly in the application of the simple plastic theory in general. It is true that a simply-supported beam in the plastic range behaves as a redundant structure and therefore it cannot be true that structures redundant from the outset become statically determined if they are sufficiently strained. The simplification of the natural problem, as it is made in the plastic hinge concept, must give uncertain results with the errors dependent upon many factors. When the loads are not specially chosen and when they are somewhat asymmetrical, the descrepancy can be very great [5], especially in the expected deflections of the structure.

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Summary

The assumptions usually made in the simple plastic theory may lead to incorrect results. Using two different series of tests, made by Professor STÜSSI and Professor BAKER, it is shown that the expected error in the results, computed on the base of the plastic hinge concept in continuous beams, is dependent on the elasticity conditions and the loading programmes of the beam under consideration. This includes a factor of uncertainty and the safety of the structure may be doubtful.

Résumé

Les hypothèses sur lesquelles se fonde généralement la théorie élémentaire de la plasticité peuvent conduire à des résultats erronés. Partant de deux series d'essais, réalisées l'une par le Professeur STÜSSI et l'autre par le Professeur BAKER, l'auteur montre que, dans les poutres continues calculées à l'aide de la théorie des rotules plastiques, l'importance de l'erreur dépend des conditions d'élasticité et du programme de mise en charge de la poutre considérée. Il en résulte quelque incertitude et l'on peut avoir des doutes sur la sécurité de la construction.

Zusammenfassung

Die Voraussetzungen, die normalerweise für das Traglastverfahren gemacht werden, können zu falschen Resultaten führen. Anhand zweier Versuchsreihen von Professor Stüssi und Professor Baker wird gezeigt, daß der Fehler an Resultaten, die auf Grund der Annahme von plastischen Gelenken in durchlaufenden Balken berechnet werden, von den Elastizitätsbedingungen und den Belastungsfällen des betrachteten Balkens abhängt. Daraus ergibt sich eine Unsicherheit, und die Tragfähigkeit eines plastisch berechneten Bauwerkes kann in Frage gestellt sein.