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Some Aspects of the Plastic Design of Aluminium Structures

Quelques problèmes relatifs aux alliages légers. Calcul dans le domaine plastique

Einige Probleme der Leichtmetall-Berechnung im plastischen Bereich

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Introduction

In the last three decades there has been a continuous upheaval in the development of structural design, the most conspicuous manifestation of which has been in the evolution and in the practical adaptability of the Theory of Plasticity — forming a rational refinement of the age old Theory of Elasticity. *The behaviour of materials under stress so great that plastic action is set up is of interest in many spheres.* Thus, in the bending, punching, and stretching of cold material, stresses beyond the yield point of the material develop; in the phenomena of accidental overload as in car couplings, draft rigging, and side frames of railroad rolling stock, plasticity becomes the governing factor in design. In problems concerned with the stability of slender beams and columns, stress concentration and energy absorption, the theory of elasticity is no longer valid and the plastic theory comes into play. Among problems of the latter category are structures built for air raid protection, and the elements of a torpedo protection system in which the ship's hull functions under explosive loading [1] [1]). Early theoretical and experimental work undertaken to reveal the inelastic behaviour of stressed components of a structure indicated that the maximum utilisable load on that component could be considerably increased by allowing only a small amount of plastic deformation to take place. Experiments on mild steel beams indicated that the yield point of steel was raised in the presence of non-uniform stress distribution [2, 3, 4]. Similar conclusions were also reached by MORKOVIN [5], MARIN [6], and BRUSH [7], who based their findings on aluminium, magnesium and annealed

1) For references see end of the paper.

high carbon steel which exhibited strain-hardening characteristics throughout the inelastic range.

Need for the Adoption of Plastic Theory

The conventional form of engineering design based on the principles of the Theory of Elasticity, assume the material to obey Hooke's Law, that its stress is assumed to be proportional to strain. All stresses are supposed to lie in this range of linearity and not to exceed the elastic limit. In practice, however, stresses in the different components of a structure are never uniform nor adhere throughout the Hooke's Law. Thus the assumption of uniform distribution of load among rivets in a tension group is evidently incorrect since the end rivets carry proportionately greater stress. Similarly stress-concentration resulting from abrupt changes in cross section or size, as from flange to web in a I-beam; in rivet holes; cutout of various shapes; and welded connections of fixed-ended beams, are completely ignored in the theory of elasticity or at times assumed that the ductility of the member takes care of it. The latter is no doubt correct, as the ductility of the material has the effect of smoothening out all irregularities in the case of static loading. However, this does not take place in the elastic range and the unevenness of stresses are only regularised when appreciable plastic deformations take place. Quite logically, therefore, the plastic theory must come into play. The new theory also offers a means whereby a constant factor of safety can be selected which is much closer to the real safe margin than is provided by the conventional methods based on the elastic theory. Added to these is the special advantage of using the theory of plasticity for light metals, which display stress-strain curves radically different from those of mild steel. The peculiar shape of these curves constitutes an advantage in that relatively higher loads can be carried especially by redundant structures built in aluminium alloys (as compared to non-redundant systems) than can be carried by redundant steel structures (as compared to non-redundant systems) [8].

In the current methods of structural design, each member is so proportioned that the most unfavourable combination of external loads when combined with a suitable factor of safety will just produce yield in that member. It is quite evident that a redundant structure is by no means on the verge of collapse when yielding occurs in one of its members. If the external loads on such a structure are steadily increased then the excess load on the member which is assumed to have yielded, is automatically taken up by other members in which yielding has not commenced. It is therefore correct to design in a manner such that the most economical structure is one which is so proportioned that it would collapse only when subjected to the maximum specified loads multiplied by a correct factor of safety. It is for this reason that in the plastic range, the evaluation of safety factor can be carried out rationally unlike its evaluation and assumption in the elastic range. The earliest statements of the concept of plastic design were made by N. C. Kist [9] and M. Gruning [10]; applications of the method to the design of transmission towers by Goodrich in 1910 are described by Van den Broek [11], and a general review of the work in this field prior to 1931 was given by Friedrich Bleich [12]. At present flourishing schools of research in the development of the theory of plasticity with special applications to structural problems have been established by Baker and his colleagues at Cambridge University [13], by Prager and his associates at Brown University, USA [14], and by Ilyushin, Kachanov, Beliaev and others in Russia [15]. In India, the Board of Scientific and Industrial Research has recently sanctioned schemes for the development of this subject. Prof. S. C. Goyal of the M.B.M. Engineering College, Jodhpur, will work on, "Use of Plastic Theory in the Design of Statically Determinate and Indeterminate Steel Structures", while Dr. Jai Krishna, of Roorkee University will cover, "Plastic Theory as Applied to Steel Structures". Unfortunately none of these take within their wake the analysis of the new theory specifically for aluminium alloys. The fact that these light alloys do not exhibit the flat yield characteristics of mild steel, render them quite useful in carrying higher loads in redundant structures, principally due to continuity of structure and ductility which they possess to an appreciable degree. The continuity of structure transfers the stresses proportionately while the ductility of the light metal contributes towards evening out all secondary effects which are not usually taken care of in the design.

Bending

An understanding of the plastic theory necessitates a knowledge of the stress-strain relation of the metal used. In figs. 1 and 2, are shown typical forms of stress-strain curves for mild steel and aluminium. In the case of steel, which exhibits a sharp yield point, a clear difference between the elastic and plastic strains is noticeable, the latter being nearly as much as ten times the former. On the other hand in aluminium alloys, the transition from elastic

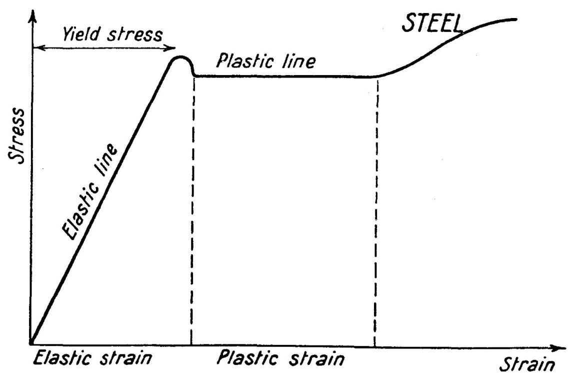


Fig. 1

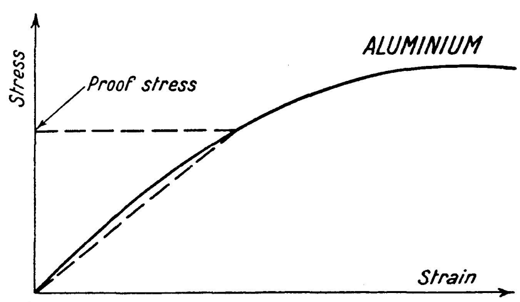


Fig. 2

to plastic state takes place very smoothly, there being no well defined yield point. The nature of the difference between the two metals can be further visualised from the moment-curvature curves, shown in figs. 3 and 4, for steel and aluminium respectively.

In a steel beam, the maximum bending moment goes on increasing till the point A, after which there is no increase and the curve flattens out. In the case of an aluminium beam, no definite maximum bending moment can be said to have been reached, since the curve goes on increasing without any definite indication of the carrying capacity. In other words, a steel beam can become fully plastic by carrying a maximum moment without any risk of fracture, whereas an aluminium beam does not have a well defined plastic state, although it carries proportionately higher moments. Loads can thus be increased till the very maximum when there is an iminent danger of fracture.

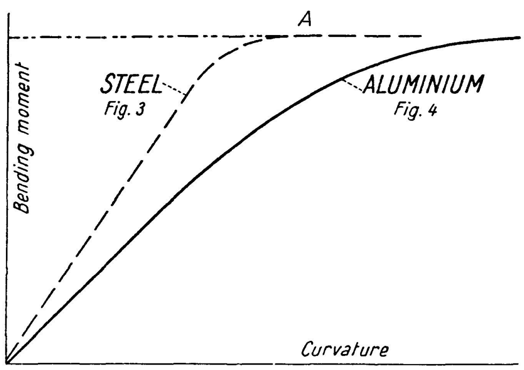


Fig. 3 and 4

An investigation of the plastic nature of bending of aluminium beams has been made by Dwight [17]. In the plastic design of beams, the Shape Factor and the Load Factor play an important part. The Shape Factor, Q [2]), is the ratio of the plastic modulus to section modulus, and is an intrinsic property of the section, being entirely governed by the shape of the section. The minimum value of Q, is unity and can only exist in an imaginary section in which all material is assumed to be lumped at the extreme fibres, such as in an I beam having no web, but only flanges of finite area and infinitesimal thickness. In the case of ordinary rolled steel joists, the value of Q is taken as an average, viz. 1.15, since it is presumed to lie between 1.14 and 1.17; while for a rectangle it is 1.5; and for a solid cylinder 1.7.

The well knit stress-strain curve of steel enables an easy assessment of the maximum plastic moment the member is capable of carrying. In aluminium, this presents difficulties, mainly because the stress-strain curves in tension and in compression differ, and it is not easy to replace them by simple forms as in steel. Working to approximations, but within the limits of practical feasibility, Dwight [17] has arrived at interesting conclusions. According to

[2]) Notation of symbols given at the end of the paper.

him the moment-curvature relation for a I, or a Channel section beam, would be of the form,

$$M = Z \cdot E \cdot F s \quad \text{when} \quad F s < \frac{F p}{E - E p},$$

$$M = Z \left[Q \cdot F p + E p F s - \frac{F p^3 (Q-1)}{F s^2 (E - E p)^2} \right]$$

$$\text{when} \quad F s > F p / E - E p.$$

In the case of solid stub members where the mass is closely distributed towards the neutral axis such as in solid rounds and rectangles, and where strength is not limited by buckling criterion, the normal flexure formula is inadmissible for stresses above the yield strength, since this yield produces a redistribution of stress. In such cases, tests have indicated [18] that the ultimate bending moment is given by the expression:

$$M = Q \cdot F t \cdot J / n.$$

The problems arising in the pure bending of beams in the plastic range with loads parallel to the principal plane of bending are far less complicated than those arising in cases when loads are not parallel to the principal plane. Here the complexity increases manifold. It can be conceived that pure bending as is commonly understood, does not exist in practice, except in very special and rare cases. GOODIER's [19] general solution suggests a possible line of attack. If the applied torsion and axial thrust are eliminated from Goodier's equation as suggested by ABRAMSON [20], it becomes evident that a torsional couple must exist and continue to act at any cross section, except at the centre of the length of the beam, unless all the deflection is parallel to the principal plane. This couple further causes twist, producing a rotation of the neutral axis.

The non linear relation between stress and strain forms a major hurdle in plastic design problems, which to some extent is further aggrevated by a modification in the standardised conceptions of shear flow. In this connection the work of ABRAMSON [20], on the plastic bending of 24 S-T aluminium alloy beams of angle cross section is worth noting.

In the case of torsion of members in the plastic range, the law of pure torsion fails and the relation between torque and twist becomes completely non-linear. The introduction of "proof-torque", aids in an analysis of this problem, as indicated by tests carried out at Bristol University [21, 78]. It was observed that the proof torque of aluminium angles instead of varying linearly with J/t, as per the simple theory, varies with J/d, where d is the diameter of the largest circle that can be inscribed in the junction of the section, whose wall thickness is t. On the basis of this theory, the proof torque can be expressed as $T p = 1.5\, F y \cdot J / d$ for sections with $R > 1.5$; and $T p = (1.2 + 0.2\, R) \cdot F y \cdot J / d$ for sections with $R < 1.5$.

Inelastic Instability

A general indication of the importance of elastic instability in designing aluminium structures, as well as a general appraisal of the plastic theory and its applications to light metals were discussed by the author elsewhere [22, 23]. An extended review of the present status of instability beyond the elastic range is given here.

Normally structural design is concerned with the evaluation of stresses based upon the primordial assumption that stable equilibrium exists between internal and external forces. In other words within certain limits, there is maintained equilibrium such that any slight change in the loading condition does not result in a disproportionate deviation of stresses or elastic distortion of the system. Thus the degree of safety of the structure lies in its adherence to that certain stress, which is recognised for design purposes as the allowable stress. The buckling problem presents an entirely new aspect viz. the evaluation of the potential *unstable* equilibrium between the external loading and the internal response of the structure. To this is added the fact that the buckling phenomena is controlled by the stress-strain relation of the metal in its entire range that is both in elastic and plastic zones resulting in complications in arriving at finite practical values.

Thus in arriving at column strength, it is not a question in merely avoiding a certain stress in the structure, by an adequate margin, but in preventing the occurrence of that peculiar condition of unstable equilibrium. This condition is characterised by disproportionately large increases, to which deformations and stresses are subject at slight increases in load. In the early investigations, it was not recognised clearly enough, that the elastic limit was exceeded before the occurrence of buckling.

The first to investigate torsional buckling of open thin-walled sections of the type now adopted in aluminium alloys was H. Wagner [24]. Unfortunately the results of his analysis were not correct since he based his theory on the assumption that the centre of rotation during buckling coincides with the shear centre — an assumption now disproved. Ostenfeld [25] investigated buckling by torsion and flexure through highly complicated analysis which received scant note. Thin-walled polygonal sections in bending, twisting and buckling received considerable impetus after the work F. and H. Bleich [26] and was followed by the refined theory of Kappus [27] and the independent observations of Lundquist and Fligg [28]. Later Goodier [29] opened out the way by reducing the buckling and flexure equations to the simpliest forms.

The theoretical treatment of the general buckling problem has been given in the most exhaustive manner by Timoshenko [30] and forms the stepping stone for practically all problems in the field of elastic instability.

Taking the cubic equation of Timoshenko as his starting point Sutter [31] has lucidly brought out the mode of application of this equation to aluminium alloys under different conditions of loading and failure in the elastic and plastic range.

According to SUTTER, the curves plotted for slenderness ratio against critical buckling stress give a complete indication of the instability range of sections. A master graph showing a set of seven curves covers the entire range of buckling phenomena in aluminium, and because of its general unfamiliarity is reproduced here from Sutter's paper (fig. 5).

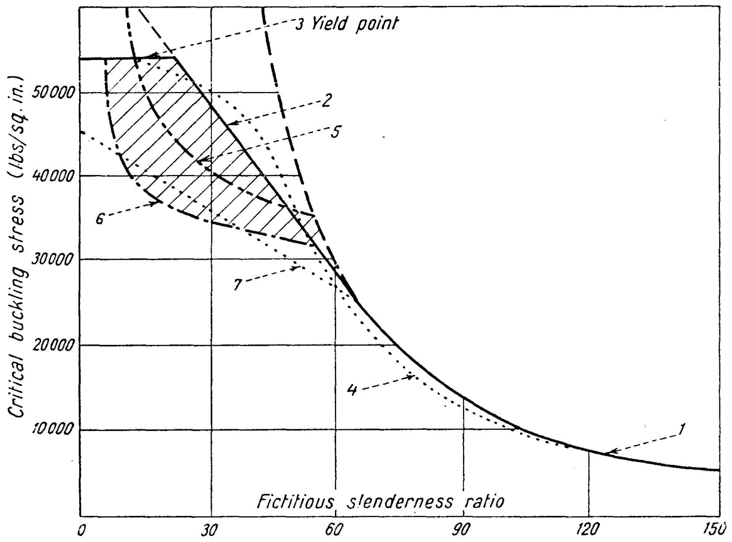


Fig. 5

At high values of slenderness ratio (l/r) the Euler hyperbola holds good as in *Curve 1*, while for low values of (l/r) the yield point represents the critical stress — *Curve 3*. In between these two lies the transition zone governed by the Tetmajer line — *Curve 2*. These three curves are at times replaced by a common curve from one unique formula as the Perry-Robertson equation — *Curve 4*.

All these cases cover buckling due to bending but not due to twisting. Sutter therefore has suggested the introduction of a fictitious slenderness ratio to take into account twisting. This modification in (l/r) takes care of the fact that at low slenderness ratios the failure is no longer elastic as is generally assumed in the Euler formula. This fictitious slenderness ratio (lk/r) is given by the expression:

$$\frac{l\,k}{r} = \sqrt{\frac{\pi^2 \cdot A \cdot k^2}{0.376\,J + \frac{\pi^2\,W}{l\,k^2}}}.$$

Since the term lk occurs on both sides of the equation, a further simplification is necessary to reduce this into a practically usable form. REINHOLD [32] gives this expression as

$$\frac{l\,k}{r} = \sqrt{\frac{10 \cdot A \cdot k^2}{0.38\,J + \frac{10\,W}{l^2}}}$$

where, k = radius of gyration of cross section, with reference to its shear centre.

Converting the value of A from square millimeters into square inches, and simplifying it further, we have

$$\frac{l\,k}{r} = \frac{k\sqrt{A}}{5\sqrt{J + \frac{26\cdot 6\,W}{l^2}}}.$$

The values of J and W, depend upon the cross section of the members and has been evaluated and explained by REINHOLD [32]. Thus the fictitious lk/r covers the individual cases of buckling failure by bending alone and by twisting alone, but not when they are in combination. *Curve 5* shows the critical buckling stress for this case and for a particular section calculated according to Timoshenko's equations, i. e. for elastic failure, while *Curve 6* shows the buckling failure for the same section taking into consideration the fact that at low values of l/r, failure is not elastic. It has been found that for a number of unequal angles, *Curves 5* and *6* are considerably lower than indicated here, mainly because of the tendency on the part of the designers to maintain a sufficiently high margin of safety. In other words unsymmetric sections cannot be designed exactly on the same lines as symmetrical sections, if maximum economy is to be achieved. Thus the shaded area enclosed by *Curves 2* and *6* in fig. 5, represents pure waste of material and penalises the really efficient sections where failure due to bending and failure due to twisting are independent. It is therefore best to use *Curves 1, 2, 3,* for buckling by bending alone and twisting alone and calculate, for a combination of bending and twisting, critical stress based on Timoshenko's formula modified for inelastic behaviour. This calculation can be carried out by the adoption of failing loads in bending with respect to the two principal axes of the section viz:

$$Pxx = \frac{\pi^2 E\, Ixx}{l^2}$$

$$Pyy = \frac{\pi^2 E\, Iyy}{l^2}$$

and the failing load by twisting as:

$$Pt = \frac{(G\,J + E\cdot W)}{k^2}\cdot\frac{\pi^2}{l^2}$$

where: Ixx and Iyy are the moments of inertia along xx and yy axes respectively, and

l = equivalent length of the compression member 'L' and assume the following values:

$l = L$ for columns with simple supports;

$l = L/2$ for fixed ends;

$l = 0.7\,L$ for one end fixed and one end simply supported;

$l = 2\,L$ for one end fixed and other free;

$l = 0.75\,L$ to $0.85\,L$ for main members of a framework;
$l = 0.6\,L$ to $0.7\,L$ for secondary compression members which buckle in the plane of the support.

A detailed analysis on these lines especially for formed aluminium sheets and thin-walled sections has been given by REINHOLD [32] and MARSH [33] and is worth studying.

A recent example of the use of fictitious slenderness ratio, is provided in the design of the high voltage aluminium transmission line towers constructed on the Kolda Pass section of the 300 Kv Kemano-Kitimat system of the Aluminium Company of Canada Limited in British Columbia. These light metal towers are braced H frame structures, comprising five thin-walled tubular aluminium legs of 38 inches overall diameter. The critical local buckling stress for the leg sections, manufactured from various thicknesses of sheet, was based on an expression involving fictitious slenderness ratio term whose value was taken as under:

$$\frac{l\,k}{r} = 5{\cdot}7\sqrt{d/2\,t}.$$

d, being the diameter of leg in inches and t being the thickness of sheet in inches [79].

Plate Stability

The history of the theory of stability of plates under edge compression dates back to 1891 when BRYAN [34] presented his classic paper to the London Mathematical Society, and was actually the first to apply the energy criterion of stability to the solution of a buckling problem. After nearly 15 years TIMOSHENKO [35] carried out investigations and presented extensively his findings, followed by REISSNER [36] and BLEICH [37]. Attempts to formulate a rational theory of stability of plates beyond the elastic limit were made by Roš and EICHINGER [38], BIJLAARD [39] and ILYUSHIN [40]. Later moment distribution method of analysis was applied by LUNDQUIST, STOWELL and SCHUETTE [41] for analysing the buckling behaviour, while large scale tests were conducted by KOLLBRUNNER [42] in Zurich. Extensive tests on inelastic behaviour are also reported by HEIMERL [43].

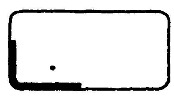
As compared to the theory of column stability, the problems of plate stability are complicated by the fact that the critical buckling load may be different from the ultimate load which the plate can carry. While for all practical purposes the buckling load is the largest load any column can carry, plates may be able to sustain in the buckled state ultimate loads noticeably exceeding the buckling load. In fact this difference between the buckling and ultimate loads which lie outside the elastic range, assumes considerable importance for very thin plates and for materials which have a low elastic modulus, like aluminium alloys. The tension field beam is a case in point [23].

In fact, in spite of these developments the practical applications of the theory of plasticity to plate buckling problems in aluminium has been rather controversial.

In the case of simply supported plates there exists discrepancies between the predictions of buckling based on deformation or finite theories of plasticity, and those based on flow or incremental theories of plasticity. Experimental evidence corraborates with deformation rather than flow-type theories. In the case of long simply supported flanges, the discrepancy between these theories is much more pronounced and the experimental investigations indicate good agreement with deformation-type theories and contradict altogether the flow type buckling theories. At high buckling stresses, that is for plates having a large thickness — width ratio the discrepancy between the theories becomes very great. However, it would not be correct to go far in this direction since the thin plate equations on which all the plastic buckling theories have been based would cease to be applicable. In view of this it would be interesting to study similar problems in the field of aeronautical engineering, wherein many theories have been involved and experimental verification made to arrive at finite conclusions. The primary structure of a modern aircraft contains many wide and thin plate elements loaded in compression, and their design offers scope for application in the general structural engineering field as well. A very large number of tests have been carried out by Needham [44] on formed 24 S-T 3 plain and 24 S-T 3 and 75 S-T 6 clad aluminium alloy equal and unequal angles and channels to determine the ultimate strength of these members in compression. These have led to the formulation of an equation for predicting directly the crippling stress of equal and unequal angles, channels, Z-sections and rectangular tubes as under:

$$Fc = \frac{c\sqrt{Fcy \cdot E}}{\left(\frac{a+b}{2t}\right)^{3/4}}.$$

c is a constant that depends on the degree of edge support along the edges of the contiguous angle elements. Specifically it is as under:

0.366 for angles with no free edge: 

0.342 for angles with one edge free: 

0.316 for angles with two edges free: 

For average conditions with $c = 0.342$, the value of Fc is given by:

$$Fc = \frac{0.574\, t^{3/4} \sqrt{E \cdot Fcy}}{(a+b)^{3/4}}.$$

If the member is to be limited to a certain crippling stress then the thickness of the section in inches can be worked out as under:

$$t = \frac{2 \cdot 1\,(a+b) \cdot Fc^{1.33}}{(E \cdot Fcy)^{2/3}}.$$

Compression in the Plastic Range

The well-known Euler formula gives the critical load for a column in the elastic range of stress, this load which is unique being the load at which bending starts. In the inelastic range, this is not so simple since the problem arises in the character of the stress-strain relations in the plastic range. In this range for uniaxial states of stress increments in stress are related to increments in strain by the tangent modulus of the light metal, but decreases in stress are related to strain by the original elastic modulus. Thus the flexural stiffness of a column in the bent state, upon which its strength in the plastic range depends is a function not only of load but also the amount of strain-reversal taking place. DUBERG and WILDER [45] carried out investigations at the Langley Aeronautical Laboratoy and arrived at significant results. According to them if the behaviour of a perfectly straight column is regarded as the limiting behaviour of a bent column, as its initial imperfection disappears then the tangent modulus load represents the critical load of the column at which bending starts. Further in computing the maximum load a column can support, a proper treatment of the phenomena of strain-reversal has to be included.

In the plastic range, the maximum load that can be carried by a column, is larger than the tangent modulus load and is greater the more gradual the change in the slope of the stress-strain curve in the vicinity of the yield stress. In terms of the latter, stainless steel columns can carry maximum loads significantly above the tangent modulus load, whereas in high strength aluminium alloy columns this is not the case. This becomes evident from the value of the parameter in the Ramberg Osgood non-dimensional stress-strain curve. Low values of this parameter in the vicinity of ten are associated with aluminium alloys, whereas about half of their value is applicable to stainless steels. In magnesium and low carbon steels, the value becomes 30 or even greater.

Plasticity of Frameworks

The difference between the limiting conditions for plastic and elastic design of frames is mainly determined by the nonuniformity in stress distribution or of elastic resistance. In the case of linear statically indeterminate frames it is the degree of redundancy and the resistance distribution characteristics such as areas and moments of inertia of the section that determine the difference. The total difference is made up of the actual difference between the limiting conditions associated with individual critical sections and the difference between the limiting states for the whole structure arising out of its redundancy. In a redundant structure although its resistance to external loading is well balanced with reference to certain loading conditions, it cannot by itself provide a carrying capacity exceeding that associated with its elastic design. However in view of the variable nature of loading existing in practice, no balanced distribution of stress resistance is possible with reference to any specific loading, and it is for this reason that an excess carrying capacity partly through elastic and partly through plastic design can be expected from the structure. An ideally plastic multifold redundant structure under increasing load gets transformed into a structure of gradually decreasing degree of redundancy, till just before it fails it behaves as a statically determinate structure under the action of a load at which the stable elastic-plastic equilibrium is completely changed into an unstable state of free plastic flow. In aluminium structures, exhibiting the work hardening characteristics the gradual reduction in the degree of redundancy does not take place. The limiting condition is therefore not determined by a condition of instantaneous instability but has to be considered by an additional criterion in terms of maximum stress leading to fracture or of an excessive deformation. In view of the fact that in aluminium alloys the redundants do not reach limiting values, but in the plastic zone increase steadily with increasing load at a rate which is determined from a work-hardening coefficient as explained by FREUDENTHAL [46] and elaborated by SWAINGER [47], it becomes difficult to determine the relation between the redundants of the system and the load, under increasing load intensity. The application of the plastic theory of design to statically indeterminate trusses is limited, because of the fact that a member, the carrying capacity of which is reached in compression cannot be considered in the same manner as one that fails in tension, as the force-deformation relation between the limiting load is different in both cases.

It can be said that in metals like aluminium which do not exhibit a sharp yield point and for which the stress strain curve of ideal plasticity does not represent a satisfactory approximation, the ultimate carrying capacity instead of being defined as the limit of stability has to be specified arbitrarily by introducing a separate criterion. The usual assumptions of moment equalisation by the formation of "plastic hinges" are not justified here, as the structure remains fully redundant even if the plastic deformations in all redundant components extends over the entire depth of the sections. In view of this an exact analysis becomes very tedious even in the case of simple structures, as well as with ordinary loading conditions.

Plasticity and Structural Resistance

A study of the impact of plasticity on structural resistance of engineering materials in general and steel in particular has been made by several investigators such as WAGNER [48], PFLUGER [49], RUHL [50], ITERSON [51], REINER [52], KACHANOV [15], KELDYSCH [54], DE PANDO [55], PETERS [56], NADAI [73] and HILL [74], in addition to the others already mentioned before. The phase of plasticity and elastoplasticity in aluminium structural design has however not been exhaustively dealt with, and whatever has trickled through has mainly been from the field of aeronautical engineering. Localised treatment on the subject with greater emphasis on elastic rather than on plastic behaviour, can be found in the works of TEMPLE [58], SCHAPITZ [59], JENSEN [60], LABARAQUE [61], DWIGHT [62], JAOUL [63], FRANKLAND [64], JOHNSON, FROST and HENDERSON [86], STÜSSI [53], and a few others [57, 65 to 68, 76, 77]. A critical survey of the principles of limit design and structural analysis has been recently given by DRUCKER [16] and though not directly concerned with light metals, contains much useful information and an excellent bibliography which can be judiciously utilised in aluminium design.

It is a well established engineering fact that the maximum strength of a structure to resist superimposed loads is determinable by one or more of the following four modes of behaviour:

a) Bending, buckling, shear and compression, which may either be elastic, elastoplastic or plastic. Buckling may either be localized or general.
b) Endurance against repeated or reversed loading.
c) Brittle fracture against static or dynamic loads.
d) Plastic failure due to excessive loading or large deflections.

With a low modulus of elasticity, alumining structures deflect considerably more than the conventional steel structures and as such the last mode of behaviour assumes prominence in light alloy design. A general conception of the plastic strength of structural members, and in particular of eccentric loaded aluminium columns is given in a symposium containing papers reporting the work sponsored by the Column Research Council of the Engineering Foundation [69].

The concept of plasticity has brought to light a new criterion of strength of materials. When a member is subjected to a combination of stresses as mostly arise in actual practice, these stresses may be reduced to two kinds viz. tangential and normal stresses. The ratio of these 2 stresses remains practically constant irrespective of the magnitude of loading. The corresponding factors opposing these stresses are resistance to slip or shearing, and resistance to tearing apart or cohesive strength. In the case of ductile materials like aluminium, a tension test reveals that the material first deforms elastically, then elasto-plastically and then under full plasticity. Tensile strength under plastic state cannot therefore be taken as an elementary measure of strength but represents a complex combination of strength, from which prediction of failure becomes problematic. Thus if the normal stress overcomes the cohesive resistance first the member fails by brittle fracture; if on the other hand the shearing strength is overcome first, the failure is ductile and is generally accompanied by appreciable deformation. The transition from elastic to plastic behaviour is fairly well known. However, surface changes of a different nature are also known to exist and represent deviations from the laws of elasticity. Thus if a material like aluminium is subjected to triaxial stresses and such a stress investigation were analysed and plotted 3-dimensionally, a certain surface would be found which represents the transition from elastic to plastic behaviour; another surface on which elastic behaviour is followed directly by cohesive fracture and probably a third surface on which elastic behaviour is directly followed by shear fracture. Out of the above three deviations of elastic behaviour, the first is well established, while very little is known regarding the laws governing the transition of the two latter types [70].

The determination of slip and cohesive strength for analysing the phenomenological behaviour of tensile specimens, has not received its due measure of importance, and there is hardly any institution where investigation on these lines has been carried out so far. The failure of the "Comet" type aircraft in terms of fuselage shell brittleness, on the basis of this new criterion has been investigated and discussed by Zaustin [71].

According to him, the propensity for brittleness or plasticity can be easily determined once the slip and cohesive strength of a particular material are evaluated by Neuber's theory of stress concentration [75] and torsion test of tubes. If resistance to slip is Rs and cohesive resistance is Rc, then if $Rs/Rc > 1$ the material is brittle under given conditions, while if this ratio is less than one, the material is ductile. Further if the ratio of tangential stress St to normal stress Sn i. e. St/Sn is > 1, the combined stress tends to produce ductile fracture, and if $St/Sn < 1$ the tendency is to produce brittle fracture. This relation can be visualised further by plotting the ratios of Rs/Rc and St/Sn.

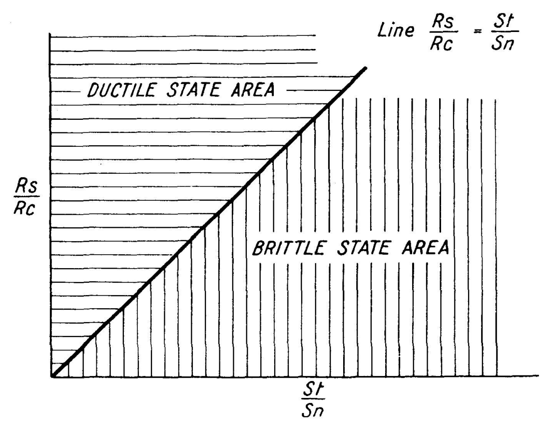


Fig. 6

The line $Rs/Rc = St/Sn$ passing through the point of origin divides the quadrant in 2 sections, the upper area of which corresponds to the ductile state and the lower area, the brittle state (fig. 6). It must, however, be made clear that this straight line characterising the state of stress is no longer a mere line in the plastic state, except under the most simple cases. Generally the transition from elastic to plastic state involves some redistribution of stress, and deformations cause deviations in the line. The conception of ductility and brittleness as inherent characteristics of individual materials requires to be modified in the light of these new formulations, since these can now be looked upon merely as different states of matter. A material can thus be ductile or brittle not only according to its temperature but also according to its speed of loading and its imposed stress pattern.

In the case of plastic design of statically indeterminate structures subjected to fluctuating loads, new factors come into play. These factors include residual stresses and as ZAUSTIN [71] terms it the phenomena of "incremental collapse", which in contrast to fatigue, does not require any appreciable number of fluctuations of the load to produce failure. In very ductile metals like aluminium, copper and nickel, it has to be pointed out that a tensile stress criterion for fracture is inapplicable and consequently true fracture stresses cannot be determined by tensile tests.

The phenomena of the theory of plasticity of light metals when developed to perfection will necessarily have to explain satisfactorily such criteria as:

1. The law of critical shear stress. This is based on the observation that in general the manner of plastic deformation of single crystals is by the slipping of sheets of crystallographic planes over one another;
2. The discrepancy between the observed low resistance to deformation as compared with the high theoretical resistance of crystal structure — attributable to "dislocations" and irregularities of atomic dimensions within the crystals [72].
3. The phenomena of work-hardening which is physically visible in the form of increased resistance to deformation with increasing deformations.

No theory of the plastic deformation of a metal crystal can be considered satisfactory unless it can account for the observed behaviour under conditions of alternating stress. Unfortunately the amount of detailed experimental data suitable for comparison with a proposed theory is, however, extremely limited. Thus it is doubtful whether a single crystal shows the BAUSCHINGER effect [83]. MASING [84] assumed that it would not and the same assumption has been generally made about the individual grains of a polycrystal and incorporated into the theories of plasticity given in the work of HILL [74]. With a view to obtain experimental data, THOMSON, COOGAN and RIDER [85] carried out a number of investigations on aluminium crystals at room temperature subject to slow cycles of alternating stress, which is worthy of study.

Fatigue

The problems of structural resistance of metals in the inelastic range are tied up with the phenomena of fatigue. Since the last century, considerable effort has been devoted to the study and exploration of fatigue, and although much new information has and is being released, no satisfactory fundamental theory of fatigue relating the phenomena with the established Natural laws has so far been put forward. The most striking instance of this fact can be found in the two serious disasters of the Comet I aircraft G-ALYP near Elba on January 10, 1954 and Comet G-ALYY near Naples on April 8, 1954. A review of the report of the Court of Inquiry into these accidents has been given by Prof. Duncan, who was one of the Assessors at the Court. His report [80] is a notable presentation of the advanced design aspects of structural work with special reference to modern aircraft, and is of exceptional importance to all engineers confronted with the establishment and evaluation of the laws of fatigue failure of structural metals. These accidents revealed that in spite of the advances recorded in aircraft structural design, the implication of fatigue phenomena was not clearly understood and an exhaustive set of investigations had to be opened out to reexamine the older theories to evaluate the modes of failure.

According to Duncan, one of the unexplained phenomena of fatigue in metals is Size Effect, observed through the fact that the fatigue strengths of geometrically similar specimens of the same metal are not proportional to their cross sectional areas — the safe stress being reduced with increase in linear dimensions. The member may have a number of inherent linear dimensions which are significant such as the average diameter of the crystal grains, the size of the crystal lattice itself, or some linear dimension associated with the intercrystalline state of the metal.

The phenomena of stress concentration, or as it is rightly referred to as "stress intensification" by Duncan, plays a significant part in plasticity of metals. It is a common knowledge that if a strip of homogeneous isotropic elastic metal with a uniform uniaxial tensile stress is punctured with a circular hole at right angles to the plate surface, the stress in the plane of the hole alters. If the ratio of width of the strip to the hole diameter is very large and if the elastic limit of the metal is not exceeded as it often happens in steel, then the greatest tensile stress on a plane section through the axis of the hole and perpendicular to the axis of the strip occurs at a surface of the hole and is equal in magnitude to three times the uniform uniaxial tensile stress.

On the other hand aluminium and some other metals which do not have a well defined yield point at any temperature do not have a fatigue limit above that defined by the extremely low slip resistance of a single crystal. Only very fine grained alloys in which the overall stress producing slip is higher than the maximum stress required to disrupt the intercrystalline regions can exhibit this limit without possessing a well defined yield point. The existence of this limit is closely connected with inherent stability of the metal, and when changes occur with time, either varying or independent of creep, recrystallisation or corrosion, no fatigue limit can exist.

In the case of aluminium alloys and other metals where there is no definite elastic limit, and plastic deformation occurs, it has been observed that this additional stretching, instead of increasing the stress actually relieves the stress where it is highest, but increases it in the adjacent regions.

As a result of this redistribution, the stress in the entire member becomes more uniform and stress intensification is reduced. Dependent upon the material, nature of loading and intensity, it is generally found that a slight amount of plastic deformation is capable of relieving considerable stresses which goes on reducing as the overall load in the member increases. The result of plastic deformation in a member having holes and cutouts or any other forms of stress raisers, influences its behaviour under repeated loading and may increase its fatigue life appreciably. In fact although not conclusively proved, the continued existence of many structures under highly fluctuating loads is a confirmation of the presence of plastic deformation around regions of high stress. The efficient designing of structures subjected to such loading conditions depends greatly on the inherent fatigue limit. In steel for example which has a finite fatigue limit, the design is based for an indefinitely extended life, which can be achieved when the maximum stress does not exceed the fatigue limit appropriate to that particular type of loading. On the other hand in aluminium alloys of the type which do not possess a fatigue limit, the aim is to design the member for a specified safe life, which for complex structures is extremely difficult. These problems require more intensive study and investigation if they are to yield useful possibilities of development in design [46, 81, 82, 88, 89, 90 to 94].

Among the other unsolved problems of design are the effects of creep at various temperatures and the determination of Poisson's ratio in the plastic range. Extant knowledge indicates that experimental values of Poisson's ratio for aluminium and some aluminium alloys increased rapidly with deformation upto one percent. During further increase of deformation the ratio increased more slowly and reached a value of 0.5 in aluminium. These experimental values differed slightly from the theoretical ones based on Hooke's Law and on the assumption of incompressibility of metals, but even the largest difference came to only five percent [87].

Conclusion

The theory and applications of plastic design of structures have not as yet attained a sufficiently mature status for practical engineering usage. It is therefore essential to understand the limitations of the procedure before standardising its use. Thus among others there is an overall increase in office design costs by the use of plastic theory due to involved calculations, although a more rational design results. There is a lack of understanding of the manner in which dead loads, vertical live loads, and unknown and arbitrary load allowances such as those due to wind, quake, and blasts, can be combined together suitably in an overall design. Problems such as design of tapered members passing through plastic zone over a considerable length; and analysis of tiered structures, in which large column sections are subjected to high axial stresses still remain unsolved.

In surveying the present status of the development of plasticity applicable to light alloys, the author has tried to evaluate the future potentialities and current limitations of this concept of structural analysis and laid bare the modus operandi of its applications, within the limits of extant knowledge. In the years to come the theories of plasticity will steadily replace the older theory of elasticity and form the fountainhead of the theory of structures, encompassing within their wake, increased strength in members, more definite and rational concepts of safety of structures, and a profound economy in the use of engineering materials. As an established structural medium, aluminium and its alloys therefore, will have to be of necessity, designed and analysed on the principles of these new theories if they are to keep in line with the advancing forefront.

The present contribution to some aspects of plasticity as a measure of structural resistance of aluminium, incorporating a fairly exhaustive bibliography, is presented here in the hope that it will lead the intrepid investigators to enter the yet unchartered realms, and establish finite laws of plasticity, intrinsically meant for a rational design of structures in light metals.

Notation Used

A	cross sectional area in sq. mm.
E	modulus of elasticity.
Ep	modulus of plasticity.
Fc	crippling stress.
Fp	mean 0.1% proof stress.
Fs	extreme fibre strain.
Ft	ultimate tensile strength.
Fy	yield stress in shear.
Fcy	compressive yield stress.
G	shear modulus.
I	moment of inertia of section about neutral axis.
J	torsion constant of the section.
L	actual length of member.
M	ultimate bending moment.

Pt	critical load for failure against twisting.
Pxx	Euler buckling load with bending along xx axis.
Pyy	Euler buckling load with bending along yy axis.
Q	shape factor.
Rc	cohesive resistance.
Rs	resistance to slip.
Sn	normal stress.
St	tangential stress.
Tp	proof torque.
W	warping constant.
Z	modulus of section.
a	width of channel or angle flange in inches.
b	width of flat plate or half-width of channel web in inches.
c	constant depending on degree of edge support.
k	radius of gyration of cross section with reference to shear centre.
l	equivalent length of compression member.
l/r	slenderness ratio.
lk/r	fictitious slenderness ratio.
n	distance of neutral axis from extreme fibre.
r	radius of curve.
t	thickness of section in inches.

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Summary

A rational evaluation of the theories of plasticity as applicable to the design of aluminium structures has been made in this paper. A brief introduction is followed by a digress on the need for the adoption of this new concept in terms of correct applications. The main survey of the present status of plasticity in the field of aluminium design covers topics on bending and torsion, inelastic instability and plate stability, compression, and frameworks. The last section on the role of plasticity as a measure of structural strength brings out some of the very recent ideas on the new criterion of strength in terms of slip and cohesive resistance. Reference is also made to combined stresses, the concept of fatigue, and the evaluation of Poisson's ratio in the plastic range, which have not been investigated and studied in an exhaustive manner in the domain of aluminium. The paper, together with its fairly exhaustive bibliography, forms a critical survey of the potentialities and limitations of the present state of plasticity as applicable to aluminium structural design, and is here presented as a means of inculcating a keener and increased interest in the subject than what has been taken up to now.

Résumé

L'auteur étudie les possibilités d'application de la théorie de la plasticité aux ouvrages en alliage léger. Après une brève introduction et en s'appuyant sur des applications judicieuses, il montre la nécessité d'envisager cette notion nouvelle. Un exposé sur l'état actuel de la théorie de la plasticité dans le domaine des alliages légers porte sur la flexion et la torsion, sur l'instabilité et sur la stabilité des dalles dans le domaine plastique, sur la compression, ainsi que sur les ouvrages en treillis.

Dans la dernière partie, dans laquelle la plasticité est considérée comme constituant une mesure de la capacité de charge, l'auteur expose quelques-uns des points de vue les plus récents sur le nouveau critérium de capacité de charge, en fonction du glissement et de la résistance de cohésion. Il étudie ensuite les contraintes combinées, la notion de la fatigue et la détermination du coefficient de Poisson dans le domaine plastique, détermination qui n'a pas fait jusqu'à maintenant l'objet d'études poussées à fond dans le cas de l'aluminium.

Outre une abondante bibliographie, le rapport contient un aperçu d'ensemble sur l'état actuel de la théorie de la plasticité, en ce qui concerne la construction des ouvrages en alliage léger. Son but est d'attirer plus étroitement l'attention sur les problèmes qui sont traités.

Zusammenfassung

Die Anwendungsmöglichkeiten der Plastizitätstheorie auf Leichtmetallkonstruktionen werden erörtert. Nach einer kurzen Einleitung wird anhand von richtigen Anwendungen die Notwendigkeit der Einführung des neuen

Begriffs gezeigt. Der Überblick über den heutigen Stand der Plastizitätstheorie im Gebiete des Leichtmetalls befasst sich mit Biegung und Torsion, Unstabilität und Plattenstabilität im plastischen Bereich, Druck und mit Fachwerken.

Der letzte Teil über die Plastizität als Maßstab der Tragfähigkeit behandelt einige der neuesten Ansichten über das neue Tragfähigkeitskriterium in Abhängigkeit von Gleiten und kohäsivem Widerstand. Im weitern werden untersucht kombinierte Beanspruchungen, der Begriff der Ermüdung und die Bestimmung der Poissonschen Zahl im plastischen Bereich, welche bis heute für den Fall des Aluminiums nicht erschöpfend erforscht worden ist.

Die Abhandlung enthält zusammen mit der angefügten reichhaltigen Bibliographie einen Überblick über den heutigen Stand der Plastizitätstheorie für die Konstruktion von Aluminium-Bauwerken. Ihr Zweck ist es, die Aufmerksamkeit vermehrt auf die behandelten Probleme zu lenken.