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Analysis of Suspension Bridges by the Minimum Energy Principle

Calcul des ponts suspendus d'après la méthode de l'énergie minimum

Die Berechnung von Hängebrücken nach der Methode der kleinsten Energie

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Synopsis

The purpose of this paper is to develop and solve the differential equations of displacements and cable stress in a suspension bridge due to live load or temperature change. The Principle of Minimum Energy is applied in the analysis, and by means of variational calculus two equations are obtained from which are derived expressions for the cable stresses and the deflection of the suspension bridge in the form of trigonometric series.

Introduction

The usual theory of suspension bridges necessitates the determination of two equations in order to calculate the stresses completely. This stems from the fact that the basic differential equation in terms of the vertical displacement of the cable and the stiffened truss includes a redundant quantity known as the additional cable stress due to live load or temperature change. One method of obtaining the second equation required for the solution is given by TIMOSHENKO[1]) and by JOHNSON, BRYAN, and TURNEAURE[2]). To do this, the increase in energy of the cable is equated to the work done by the load acting on the cable.

[1]) "The Stiffness of Suspension Bridges", S. TIMOSHENKO, Transactions Am. Soc. C. E. Vol. 94 (1930), p. 377.

[2]) "Modern Framed Structures", J. B. JOHNSON, C. W. BRYAN, and F. E. TURNEAURE. New York: John Wiley & Sons, 10th ed., 1929, part II, pp. 252ff.

However, two independent equations can be established by minimizing the total strain energy defined in terms of two displacements. Therefore it is possible to obtain directly two equations by means of variational methods if the total strain energy of a suspension bridge is expressed in terms of its vertical and horizontal displacements. In addition to the displacements, both equations include the redundant quantity mentioned above. This redundant can be evaluated from the known boundary conditions. Thus the problem of suspension bridges resolves into the determination of the differential equations and the solution of these equations with due regard to the boundary conditions.

Notation

The following notation is used throughout the paper.

q = Dead load per unit length of the truss, cable and hangers.
p = Live load per unit length on the truss.
P = Concentrated live load.
H = Horizontal component of cable stress due to dead load and mean temperature.
h = Additional horizontal component of cable stress due to live load or temperature change.
y = Ordinate of the cable under the action of dead load.
w = Vertical displacement of cable in excess of y (assumed to be equal to the vertical displacement of the truss) due to live load or temperature change.
v = Horizontal displacement of cable due to live load or temperature change.
EI = Flexural rigidity of the truss.
L = Span of the truss.
f = Sag of the cable.
ΔT = Change in temperature from mean temperature.
ω = Coefficient of thermal expansion.

The Total Strain Energy Expression and the Derivation of the Differential Equations

The usual assumption made in the analysis of suspension bridges is that the vertical displacement of the cable is equal to the deflection of the truss. This assumption is valid in view of the fact that the deflection of the truss and cable is large in comparison with the elongation of the hangers due to tension. It is also assumed that all the dead load, before the application of the live load, is carried by the cable, and under uniformly distributed load of constant magnitude the shape of the cable is a parabola given by the equation

$$H\frac{d^2 y}{d x^2} = -q \tag{1}$$

If the origin of the coordinate system is taken at the top of the left tower, the equation of the parabolic curve for the center span is

$$y = \frac{4 f_2}{L_2^2}(L_2 x - x_2) \tag{2}$$

and for the side spans eq. (2) holds except that y and f are measured from the diagonal drawn from the outer support to the top of the tower as shown in fig. 1.

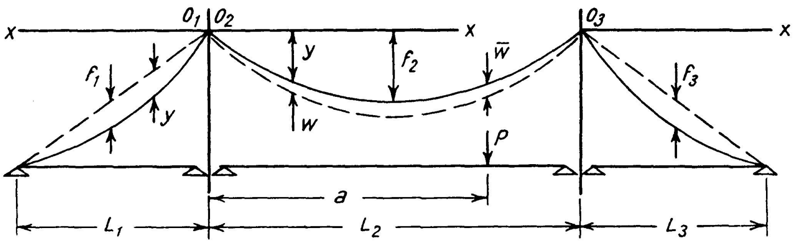

Fig. 1

In the derivation of the total strain energy expression both the vertical and horizontal displacements will be taken into consideration. As stated above, the vertical displacement w will be taken to be the same for the cable and the truss. Furthermore, for the time being, both displacements will be considered large. The strain energy in the cable will be investigated first.

Let there be taken a differential element of the cable ds long. After deformation of the cable due to the application of the live load or the temperature change, this length becomes ds_1. The unit strain in the cable is then given by

$$\epsilon = \frac{d s_1 - d s}{d s} \tag{3}$$

The change in length of ds corresponds to the change in x and y coordinates of the cable. That is, the original coordinates of a point on the cable (x, y) become $(x+v)$ and $(y+w)$ after deformation. As a result of this deformation the length ds expressed by

$$d s = (d x^2 + d y^2)^{1/2} = (1 + y_x^2)^{1/2} d x$$

(the subscript x will designate the derivative) becomes

$$d s_1 = [(d x + d v)^2 + (d y + d w)^2]^{1/2}$$

or

$$d s_1 = [1 + y_x^2 + 2 v_x + v_x^2 + 2 y_x w_x + w_x^2]^{1/2} d x \tag{4}$$

Thus it may be seen that the unit strain ϵ can be expressed in terms of the displacement functions of the cable. This unit strain can also be associated with the change in three quantities, namely, the increase in the horizontal cable pull, the change in the stress produced by H as a result of the change in slope of the cable, and finally the temperature change. It can therefore be said that the unit strain given by eq. (3) is the sum of the three unit strains produced by the change in the three quantities mentioned above. Denoting by dx_1, the horizontal component of ds_1, the unit strain due to h is given by

$$\epsilon_1 = \frac{1}{E} \frac{h}{A} \frac{ds_1}{dx_1}$$

and the increase in stress produced by H yields

$$\epsilon_2 = \frac{1}{E} \frac{H}{A} \left(\frac{ds_1}{dx_1} - \frac{ds}{dx} \right)$$

in which ds_1 is given by eq. (4), and

$$dx_1 = (1 + v_x)\, dx \tag{5}$$

and finally the unit strain produced by the temperature change is

$$\epsilon_3 = \omega \Delta T$$

Therefore the sum of the unit strain components becomes

$$\epsilon = \frac{1}{E} \left[\frac{h}{A} \frac{ds_1}{dx_1} + \frac{H}{A} \left(\frac{ds_1}{dx_1} - \frac{ds}{dx} \right) \right] + \omega \Delta T = \frac{ds_1 - ds}{ds}$$

This equation may be written as

$$\frac{h+H}{A} \frac{ds_1}{dx_1} = E \frac{ds_1 - ds}{ds} + \frac{H}{A} \frac{ds}{dx} - E \omega \Delta T \tag{6}$$

which is the unit stress in the cable at its displaced position. This stress is also composed of three terms. The first term, due to the extension of the cable, is given by

$$\sigma_1 = E \frac{ds_1 - ds}{ds}$$

the second term is the initial stress

$$\sigma_2 = \frac{H}{A} \frac{ds}{dx}$$

and the third is a constant stress due to temperature change

$$\sigma_3 = -E \omega \Delta T$$

These unit stresses are shown in fig. 2.

Thus the strain energy of deformation of a unit volume becomes, from the figure

$$\tfrac{1}{2} \sigma_1 \epsilon + \sigma_2 \epsilon + \sigma_3 \epsilon$$

Substituting the corresponding values for σ_1, σ_2, σ_3 and ϵ, this becomes

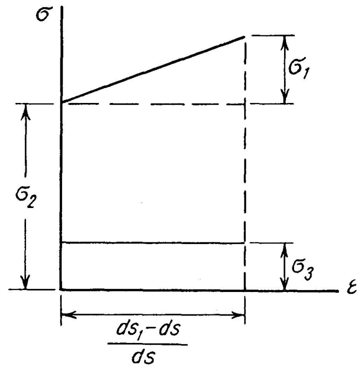

Fig. 2

$$\tfrac{1}{2} E \left(\frac{d s_1 - d s}{d s}\right)^2 + \frac{H}{A} \frac{d s}{d x} \frac{d s_1 - d s}{d s} - E \omega \Delta T \frac{d s_1 - d s}{d s}$$

and for a differential volume $A\,ds$ of the cable

$$d U_C = \tfrac{1}{2} E A \frac{(d s_1 - d s)^2}{d s} + H \frac{d s}{d x} (d s_1 - d s) - E A \omega \Delta T (d s_1 - d s)$$

or $$d U_C = \tfrac{1}{2} E A \left(\frac{d s_1}{d s} - 1\right)^2 d s + H \frac{d s}{d x} \left(\frac{d s_1}{d s} - 1\right) d s - E A \omega \Delta T \left(\frac{d s_1}{d s} - 1\right) d s$$

The strain energy due to bending of the truss is given by

$$U_T = \int \tfrac{1}{2} E I w^2_{xx} d x$$

and for the dead load

$$U_{D.L.} = \int q w d x$$

Summing up all the terms and remembering that in $d U_C$

$$d s = (1 + y_x^2)^{1/2} d x$$

there results

$$U = \int_0^L \left[\tfrac{1}{2} E A \left(\frac{d s_1}{d s} - 1\right)^2 (1 + y_x^2)^{1/2} + H \frac{d s}{d x} \left(\frac{d s_1}{d s} - 1\right) (1 + y_x^2)^{1/2} - \right.$$
$$\left. - E A \omega \Delta T \left(\frac{d s_1}{d s} - 1\right) (1 + y_x^2)^{1/2} + \tfrac{1}{2} E I w_{xx}^2 - q w\right] d x - P \overline{w} \qquad (7)$$

where $\overline{w}$ is the deflection under the concentrated live load P. To minimize the total strain energy one takes the variation of U with respect to w and v and sets it equal to zero. Since $d s_1$ includes w and v terms, the variation of $d s_1$ from eq. (4) is

$$\delta d s_1 = \frac{\delta v_x + v_x \delta v_x + y_x \delta w_x + w_x \delta w_x}{(1 + y_x^2 + 2 v_x + v_x^2 + 2 y_x w_x + w_x^2)^{1/2}} d x$$

or $$\delta d s_1 = \frac{\delta v_x + v_x \delta v_x + y_x \delta w_x + w_x \delta w_x}{d s_1} (d x)^2 \qquad (8)$$

and from eq. (5)

$$\delta\, d\, x_1 = \delta\, v_x\, d\, x \tag{9}$$

Taking the variation of U from eq. (7) there is found

$$\delta\, U = \int_0^L \left[E\, A \left(\frac{d\, s_1}{d\, s} - 1 \right) \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \delta\, d\, s_1 + H\, \frac{d\, s}{d\, x}\, \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \delta\, d\, s_1 - \right.$$

$$\left. - E\, A\, \omega\, \varDelta\, T\, \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \delta\, d\, s_1 + E\, I\, w_{xx}\, \delta\, w_{xx} - q\, \delta w \right] d\, x - P\, \delta\, \overline{w} = 0$$

Substituting for $\delta\, d\, s_1$ and $\delta\, d\, x_1$ from eqs. (8) and (9),

$$\delta_U = \int_0^L \left[E\, A \left(\frac{d\, s_1}{d\, s} - 1 \right) \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \frac{\delta\, v_x + v_x\, \delta\, v_x + y_x\, \delta\, w_x + w_x\, \delta\, w_x}{d\, s_1}\, (d\, x)^2 + \right.$$

$$+ H\, \frac{d\, s}{d\, x}\, \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \frac{\delta\, v_x + v_x\, \delta\, v_x + y_x\, \delta\, w_x + w_x\, \delta\, w_x}{d\, s_1}\, (d\, x)^2 -$$

$$- E\, A\, \omega\, \varDelta\, T\, \frac{(1+y_x{}^2)^{1/2}}{d\, s}\, \frac{\delta\, v_x + v_x\, \delta\, v_x + y_x\, \delta\, w_x + w_x\, \delta\, w_x}{d\, s_1}\, (d\, x)^2 +$$

$$\left. + E\, I\, w_{xx}\, \delta\, w_{xx} - q\, \delta\, w \right] d\, x - P\, \delta\, \overline{w} = 0$$

Writing

$$\frac{\delta\, v_x + v_x\, \delta\, v_x}{d\, s_1} = \frac{1+v_x}{d\, s_1}\, \delta\, v_x = \frac{d\, x_1}{d\, x\, d\, s_1}\, \delta\, v_x$$

remembering that $\quad (1+y_x{}^2)^{1/2}\, d\, x = d\, s,$

and factoring,

$$\delta_U = \int_0^L \left[EA \left(\frac{d s_1}{d\, s} - 1 \right) + H\, \frac{d\, s}{d x} - E\, A\, \omega\, \varDelta\, T \right] \frac{d\, x_1}{d\, s_1}\, \delta\, v_x\, d\, x$$

$$+ \int_0^L \left\{ \left[EA \left(\frac{d s_1}{d\, s} - 1 \right) + H\, \frac{d\, s}{d\, x} - E\, A\, \omega\, \varDelta\, T \right] \frac{(y_x + w_x)}{d\, s_1}\, d\, x\, \delta\, w_x + \right.$$

$$\left. + E\, I\, w_{xx}\, \delta\, w_{xx} - q\, \delta\, w \right\} d\, x - P\, \delta\, \overline{w} = 0 \tag{10}$$

From the above equation one can obtain two differential equations by considering terms in $\delta\, v$ and $\delta\, w$ independently. To do this, the terms are integrated by parts. Since the variation of the derivative of a function is equal to the derivative of the variation, i. e.,

$$\delta\, \frac{d\, v}{d\, x} = \frac{d}{d\, x}\, \delta\, v$$

the first integral can be written as

$$\int_0^L \phi \frac{d}{dx} \delta v \, dx$$

where

$$\phi = \left[EA\left(\frac{ds_1}{ds}-1\right)+H\frac{ds}{dx} - EA\omega\Delta T\right]\frac{dx_1}{ds_1} \tag{11}$$

Integrating by parts

$$[\phi\,\delta v]_0^L - \int_0^L \frac{d\phi}{dx}\delta v\,dx = 0$$

but since δv is arbitrary between $x=0$ and $x=L$

$$\frac{d}{dx}\phi = 0$$

from which there results

$$\phi = \text{constant.}$$

Eq. (6) may be rewritten

$$h+H = \left[EA\left(\frac{ds_1-ds}{ds}\right)+H\frac{ds}{dx}-EA\omega\Delta T\right]\frac{dx_1}{ds_1} \tag{6a}$$

Comparing now eq. (11) with the above, it is seen that

$$\phi = H+h = a \text{ constant.}$$

Substituting this constant in the second portion of eq. (10),

$$\int_0^L \left[\phi\frac{y_x+w_x}{1+v_x}\delta w_x + EI\,w_{xx}\,\delta w_{xx} - q\,\delta w\right]dx - P\,\delta\overline{w} = 0$$

The first and second terms of this last expression will now be integrated by parts in order to collect them under δw. From the first term is obtained

$$\int_0^L (H+h)\frac{y_x+w_x}{1+v_x}\left(\frac{d}{dx}\delta w\right)dx = \left[(H+h)\frac{y_x+w_x}{1+v_x}\delta w\right]_0^L -$$

$$-\int_0^L (H+h)\frac{d}{dx}\left(\frac{y_x+w_x}{1+v_x}\right)\delta w\,d\,x$$

and the second term after integrating twice yields

$$\int_0^L EI\,w_{xx}\left(\frac{d}{dx}\delta w_x\right)dx = [EI\,w_{xx}\,\delta w_x]_0^L - \int_0^L EI\left(\frac{d}{dx}w_{xx}\right)\left(\frac{d}{dx}\delta w\right)dx =$$

$$= [EI\,w_{xx}\,\delta w_x]_0^L - [EI\,w_{xxx}\,\delta w]_0^L + \int_0^L EI\left(\frac{d}{dx}w_{xxx}\right)\delta w\,dx$$

Thus the integral becomes

$$\int_0^L \left[E I w_{xxxx} - (H+h) \frac{d}{dx} \left(\frac{y_x + w_x}{1+v_x} \right) - q \right] \delta w \, dx - P \delta \overline{w} = 0 \qquad (12)$$

In order to find a solution for the problem eq. (12) must be satisfied. This equation contains three unknowns, namely, w, v and h. However eq. (11) or eq. (6) can be used to find the displacement function v which necessarily will contain the redundant h. But once the expression for v is obtained h can be evaluated from the consideration of the boundary conditions.

If it is assumed that v_x is small in comparison with unity, from eq. (12) there results

$$\int_0^L [E I w_{xxxx} - (H+h)(y_{xx} + w_{xx}) - q] \, \delta w \, dx - P \delta \overline{w} = 0$$

but $$H y_{xx} = -q$$

therefore this integral becomes

$$\int_0^L [E I w_{xxxx} - h y_{xx} - (H+h) w_{xx}] \, \delta w \, dx - P \delta \overline{w} = 0 \qquad (13)$$

This is the well-known equation used by previous investigators. A trigonometric series for w in the form

$$w = \sum_{n=1}^{n=\infty} A_n \sin \frac{n \pi x}{L}$$

can now be assumed, in which A_n are coefficients to be determined. Taking the concentrated load P at $x = a$, there is found

$$\delta w = \sum_{n=1}^{n=\infty} \sin \frac{n \pi x}{L} \, \delta A_n \quad \text{and} \quad \delta \overline{w} = \sum_{n=1}^{n=\infty} \sin \frac{n \pi a}{L} \, \delta A_n$$

Substituting these expressions, and $y_{xx} = -\frac{8 f}{L^2}$ from

$$y = \frac{4 f}{L^2} (L x - x^2)$$

there is obtained,

$$\int_0^L \sum \left[E I \left(\frac{n \pi}{L} \right)^4 A_n \sin \frac{n \pi x}{L} + \frac{8 f h}{L^2} + (H+h) \left(\frac{n \pi}{L} \right)^2 A_n \sin \frac{n \pi x}{L} \right]$$

$$\sum \sin \frac{n \pi x}{L} \, dx \, \delta A_n = \sum P \sin \frac{n \pi a}{L} \, \delta A_n$$

The integration between the limits yields

$$E I \left(\frac{n \pi}{L} \right)^4 \frac{L}{2} A_n + \frac{8 f}{L^2} h \frac{L}{n \pi} (1 - \cos n \pi) + (H+h) \left(\frac{n \pi}{L} \right)^2 \frac{L}{2} A_n = P \sin \frac{n \pi a}{L}$$

from which the coefficients A_n are found to be

$$A_n = \frac{P \sin \frac{n\pi a}{L} - \frac{8fh}{L^2}\frac{L}{n\pi}(1-\cos n\pi)}{\frac{L}{2}\left(\frac{n\pi}{L}\right)^2\left[EI\left(\frac{n\pi}{L}\right)^2 + H + h\right]} \tag{14}$$

However the preceding formula contains an unknown constant quantity h which must be determined. This will be found with the aid of eq. (11)

$$H+h = \left[EA\left(\frac{ds_1-ds}{ds}\right) + H\frac{ds}{dx} - EA\,\omega\,\varDelta T\right]\frac{dx_1}{ds_1}$$

in which dx_1 as previously found is

$$dx_1 = (1+v_x)\,dx$$

and
$$ds_1 = [(1+y_x^2)+(2v_x+v_x^2+2y_x w_x+w_x^2)]^{1/2}\,dx$$

which upon expanding by the binomial formula becomes

$$ds_1 = [(1+y_x^2)^{1/2}+\tfrac{1}{2}(1+y_x^2)^{-1/2}(2v_x+v_x^2+2y_x w_x+w_x^2)+\cdots]\,dx$$

neglecting terms after the second, this gives

$$ds_1 - ds = \frac{v_x+\frac{v_x^2}{2}+y_x w_x+\frac{w_x^2}{2}}{(1+y_x^2)^{1/2}}\,dx$$

whence

$$H+h = \left[EA\,\frac{v_x+\frac{v_x^2}{2}+y_x w_x+\frac{w_x^2}{2}}{1+y_x^2} + H(1+y_x^2)^{1/2} - EA\,\omega\,\varDelta T\right]$$

$$\left[\frac{1+v_x}{(1+y_x^2)^{1/2}+\frac{v_x+\frac{v_x^2}{2}+y_x w_x+\frac{w_x^2}{2}}{(1+y_x^2)^{1/2}}}\right]$$

If the assumption be made that the secant of the angle between the horizontal and the cable before deformation is equal to the secant of the angle after deformation

$$\frac{dx}{ds} \simeq \frac{dx_1}{ds_1}$$

the above formula then becomes

$$H+h = EA\,\frac{v_x+\frac{v_x^2}{2}+y_x w_x+\frac{w_x^2}{2}}{(1+y_x^2)^{3/2}} + H - \frac{EA\,\omega\,\varDelta T}{(1+y_x^2)^{1/2}}$$

Neglecting the relatively very small term $\frac{v_x^2}{2}$, this reduces to

$$v_x + y_x w_x + \frac{w_x^2}{2} = \frac{h}{AE}(1+y_x^2)^{3/2} + \omega\,\varDelta T\,(1+y_x^2)$$

Substituting

$$y_x = \frac{4f}{L} - \frac{8f}{L^2}x, \quad w_x = \sum\left(\frac{n\pi}{L}\right) A_n \cos\frac{n\pi x}{L}$$

this equation becomes

$$v_x + \sum\left(\frac{4f}{L} - \frac{8f}{L^2}x\right)\frac{n\pi}{L} A_n \cos\frac{n\pi x}{L} + \frac{1}{2}\left[\sum\left(\frac{n\pi}{L}\right) A_n \cos\frac{n\pi x}{L}\right]^2 =$$

$$= \frac{h}{AE}\left[1+\left(\frac{4f}{L} - \frac{8f}{L^2}x\right)^2\right]^{3/2} + \omega \Delta T\left[1+\left(\frac{4f}{L} - \frac{8fx}{L^2}\right)^2\right] \quad (15)$$

Denoting by Z the derivative of y

$$y_x = Z = \frac{4f}{L} - \frac{8f}{L^2}x$$

and integrating eq. (15), the following equation is found:

$$v + \sum\frac{4f}{L} A_n \sin\frac{n\pi x}{L} - \sum\frac{8f}{L^2} A_n \frac{L}{n\pi}\left[\cos\frac{n\pi x}{L} + \frac{n\pi}{L}x \sin\frac{n\pi x}{L}\right]$$

$$+ \frac{1}{4}\sum\left(\frac{n\pi}{L}\right)^2 A_n^2 x + \frac{1}{8}\sum\frac{n\pi}{L} A_n^2 \sin 2\frac{n\pi x}{L} +$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{\sin(m-n)\frac{\pi}{L}x}{\frac{2\pi}{L}(m-n)} + \frac{\sin(m+n)\frac{\pi}{L}x}{\frac{2\pi}{L}(m+n)}\right]^{3)}$$

$$= -\frac{h}{AE}\frac{L^2}{32f}\left\{Z(1+Z^2)^{3/2} + \frac{3}{2}Z(1+Z^2)^{1/2} + \frac{3}{2}\ln\left[Z+(1+Z^2)^{1/2}\right]\right\} -$$

$$-\frac{L^2}{8f}\left(Z+\frac{1}{3}Z^3\right)\omega \Delta T + c \quad (16)$$

in which c includes the constant

$$\frac{L}{2} + \frac{8f^2}{3L}$$

arriving from the integration of

$$\omega \Delta T\left[1+\left(\frac{4f}{L} - \frac{8f}{L^2}x\right)^2\right]$$

which is

$$-\frac{L^2}{8f}\left(Z+\frac{1}{3}Z^3\right)\omega \Delta T + \frac{L}{2} + \frac{8f^2}{3L}$$

Equations (14) and (16) are general and apply to all the spans of the bridge. Denoting the coefficients in the series representing w by a_n, b_n, c_n for the first, second and third spans of the bridge respectively,

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3) Where $m^2 \neq n^2$.

$$w_1 = \sum a_n \sin \frac{n \pi x}{L_1}, \quad w_2 = \sum b_n \sin \frac{n \pi x}{L_2}, \quad w_3 = \sum c_n \sin \frac{n \pi x}{L_3} \tag{17}$$

If, instead of a concentrated load P, uniformly distributed loads of intensities p_1, p_2 and p_3 occupy portions of all three spans denoted by αL_1, βL_2 and γL_3 (all measured from $x=0$), then substitution of the integrals

$$\int_0^{\alpha L_1} p_1 \sin \frac{n \pi x}{L_1} dx = p_1 \frac{L_1}{n \pi} (1 - \cos n \pi \alpha)$$

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in eq. (14) for

$$P \sin \frac{n \pi a}{L}$$

gives

$$a_n = \frac{p_1 (1 - \cos n \pi \alpha) - \frac{8 f_1}{L_1^2} h (1 - \cos n \pi)}{\frac{L_1}{2} \left(\frac{n \pi}{L_1}\right)^3 \left[E I_1 \left(\frac{n \pi}{L_1}\right)^2 + H + h\right]}$$

$$b_n = \frac{p_2 (1 - \cos n \pi \beta) - \frac{8 f_2}{L_2^2} h (1 - \cos n \pi)}{\frac{L_2}{2} \left(\frac{n \pi}{L_2}\right)^3 \left[E I_2 \left(\frac{n \pi}{L_2}\right)^2 + H + h\right]} \tag{18}$$

$$c_n = \frac{p_3 (1 - \cos n \pi \gamma) - \frac{8 f_3}{L_3^2} h (1 - \cos n \pi)}{\frac{L_3}{2} \left(\frac{n \pi}{L_3}\right)^3 \left[E I_3 \left(\frac{n \pi}{L_3}\right)^2 + H + h\right]}$$

Similarly eq. (16) can be written for all three spans by proper substitution of the coefficients a_n, b_n, c_n for A_n, and the values f_1, f_2, f_3, L_1, L_2, L_3 for f and L respectively.

There will now be determined an expression for h by considering the boundary conditions. Denoting by v_1, v_2, and v_3, the horizontal displacements of the three spans, one can write

$$v_1 = 0 \text{ at } x = L_1 \qquad \text{(a)}$$

$$-(v_1)_{x=0} = (v_2)_{x=0} \qquad \text{(b)}$$

$$(v_2)_{x=L_2} = (v_3)_{x=0} \qquad \text{(c)}$$

$$v_3 = 0 \text{ at } x = L_3 \qquad \text{(d)}$$

Since

$$Z = \frac{4 f}{L} \quad \text{at} \quad x = 0$$

and

$$Z = -\frac{4 f}{L} \quad \text{at} \quad x = L$$

Eq. (16) yields the following four equations based on the four expressions (a) to (d):

$$\sum \frac{8f_1}{L_1^2} a_n \frac{L_1}{n\pi} \cos n\pi - \frac{1}{4} \sum \left(\frac{n\pi}{L_1}\right)^2 a_n^2 L^1 +$$
$$+ \frac{h}{AE} \frac{L_1}{8} \left[\left(1+\frac{16 f_1^2}{L_1^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] -$$
$$- \frac{h}{AE} \frac{3L_1^2}{64_1} \ln \left[-\frac{4f_1}{L_1} + \left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] +$$
$$+ \frac{L_1}{2}\left(1+\frac{16 f_1^2}{3L_1^2}\right) \omega \Delta T + c_1 = 0 \qquad \text{(a')}$$

$$- \sum \frac{8f_1}{L_1^2} a_n \frac{L_1}{n\pi} + \frac{h}{AE} \frac{L_1}{8} \left[\left(1+\frac{16 f_1^2}{L_1^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] +$$
$$+ \frac{h}{AE} \frac{3L_1^2}{64 f_1} \ln \left[\frac{4f_1}{L_1} + \left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] + \frac{L_1}{2}\left(1+\frac{16 f_1^2}{3L_1^2}\right) \omega \Delta T - c_1 =$$
$$= \sum \frac{8f_2}{L_2^2} b_n \frac{L_2}{n\pi} - \frac{h}{AE} \frac{L_2}{8} \left[\left(1+\frac{16 f_2^2}{L_2^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] -$$
$$- \frac{h}{AE} \frac{3L_2^2}{64 f_2} \ln \left[\frac{4f_2}{L_2} + \left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] - \frac{L_2}{2}\left(1+\frac{16 f_2^2}{3L_2^2}\right) \omega \Delta T + c_2 \qquad \text{(b')}$$

$$\sum \frac{8f_2}{L_2^2} b_n \frac{L_2}{n\pi} \cos n\pi - \frac{1}{4} \sum \left(\frac{n\pi}{L_2}\right)^2 b_n^2 L_2 +$$
$$+ \frac{h}{AE} \frac{L_2}{8} \left[\left(1+\frac{16 f_2^2}{L_2^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] -$$
$$- \frac{h}{AE} \frac{3L_2^2}{64 f_2} \ln \left[-\frac{4f_2}{L_2} + \left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] + \frac{L_2}{2}\left(1+\frac{16 f_2^2}{3L_2^2}\right) \omega \Delta T + c_2 =$$
$$= \sum \frac{8f_3}{L_3^2} c_n \frac{L_3}{n\pi} - \frac{h}{AE} \frac{L_3}{8} \left[\left(1+\frac{16 f_3^2}{L_3^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_3^2}{L_3^2}\right)^{1/2}\right] -$$
$$- \frac{h}{AE} \frac{3L_3^2}{64 f_3} \ln \left[\frac{4f_3}{L_3} + \left(1+\frac{16 f_3^2}{L_3^2}\right)^{1/2}\right] - \frac{L_3}{2}\left(1+\frac{16 f_3^2}{3L_3^2}\right) \omega \Delta T + c_3 \qquad \text{(c')}$$

and

$$\sum \frac{8f_3}{L_3^2} c_n \frac{L_3}{n\pi} \cos n\pi - \frac{1}{4} \sum \left(\frac{n\pi}{L_3}\right)^2 c_n^2 L_3 +$$
$$+ \frac{h}{AE} \frac{L_3}{8} \left[\left(1+\frac{16 f_3^2}{L_3^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_3^2}{L_3^2}\right)^{1/2}\right] -$$
$$- \frac{h}{AE} \frac{3L_3^2}{64 f_3} \ln \left[-\frac{4f_3}{L_3} + \left(1+\frac{16 f_3^2}{L_3^2}\right)^{1/2}\right] +$$
$$+ \frac{L_3}{2}\left(1+\frac{16 f_3^2}{3L_3^2}\right) \omega \Delta T + c_3 = 0 \qquad \text{(d')}$$

If there be imposed the further restriction of a bridge with L_1 and f_1 equal respectively to L_3 and f_3, by eliminating c_2 between eqs. (b') and (c') there is found

$$\sum \frac{8f_2}{L_2^2} b_n \frac{L_2}{n\pi}(1-\cos n\pi)+\frac{1}{4}\sum\left(\frac{n\pi}{L_2}\right)^2 b_n^2 L_2-$$

$$-\frac{h}{AE}\frac{L_2}{4}\left[\left(1+\frac{16f_2^2}{L_2^2}\right)^{3/2}+\frac{3}{2}\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}\right]-$$

$$-\frac{h}{AE}\frac{3L_2^2}{64f_2}\ln\left[\frac{\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}+\frac{4f_2}{L_2}}{\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}-\frac{4f_2}{L_2}}\right]-L_2\left(1+\frac{16f_2^2}{3L_2^2}\right)\omega\Delta T=$$

$$=-\sum\frac{8f_1}{L_1^2}\frac{L_1}{n\pi}(a_n+c_n)+\frac{h}{AE}\frac{L_1}{4}\left[\left(1+\frac{16f_1^2}{L_1^2}\right)^{3/2}+\frac{3}{2}\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}\right]+$$

$$+\frac{h}{AE}\frac{3L_1^2}{32f_1}\ln\left[\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}+\frac{4f_1}{L_1}\right]+L_1\left(1+\frac{16f_1^2}{3L_1^2}\right)\omega\Delta T-(c_1+c_3)$$

By adding eqs. (a′) and (d′) there is obtained

$$-\frac{1}{4}\sum\left(\frac{n\pi}{L_1}\right)^2 a_n^2 L_1-\frac{1}{4}\sum\left(\frac{n\pi}{L_1}\right)^2 c_n^2 L_1+\sum\frac{8f_1}{L_1^2}a_n\frac{L_1}{n\pi}\cos n\pi+$$

$$+\sum\frac{8f_1}{L_1^2}c_n\frac{L_1}{n\pi}\cos n\pi+\frac{h}{AE}\frac{L_1}{4}\left[\left(1+\frac{16f_1^2}{L_1^2}\right)^{3/2}+\frac{3}{2}\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}\right]-$$

$$-\frac{h}{AE}\frac{3L_1^2}{32f_1}\ln\left[\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}-\frac{4f_1}{L_1}\right]+L_1\left(1+\frac{16f_1^2}{3L_1^2}\right)\omega\Delta T+(c_1+c_3)=0$$

Eliminating (c_1+c_3) between the last two equations

$$\frac{1}{4}\sum\left(\frac{n\pi}{L_1}\right)^2 a_n^2 L_1+\frac{1}{4}\sum\left(\frac{n\pi}{L_2}\right)^2 b_n^2 L_2+\frac{1}{4}\sum\left(\frac{n\pi}{L_1}\right)^2 c_n^2 L_1+$$

$$+\sum\frac{8f_1}{L_1^2}a_n\frac{L_1}{n\pi}(1-\cos n\pi)+\sum\frac{8f_2}{L_2^2}b_n\frac{L_2}{n\pi}(1+\cos n\pi)+$$

$$+\sum\frac{8f_1}{L_1^2}c_n\frac{L_1}{n\pi}(1-\cos n\pi)=$$

$$=\frac{h}{AE}\frac{L_1}{2}\left[\left(1+\frac{16f_1^2}{L_1^2}\right)^{3/2}+\frac{3}{2}\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}\right]+$$

$$+\frac{h}{AE}\frac{L_2}{4}\left[\left(1+\frac{16f_2^2}{L_2^2}\right)^{3/2}+\frac{3}{2}\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}\right]+$$

$$+\frac{h}{AE}\frac{3L_1^2}{32f_1}\ln\left[\frac{\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}+\frac{4f_1}{L_1}}{\left(1+\frac{16f_1^2}{L_1^2}\right)^{1/2}-\frac{4f_1}{L_1}}\right]+\frac{h}{AE}\frac{3L_2^2}{64f_2}\ln\left[\frac{\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}+\frac{4f_2}{L_2}}{\left(1+\frac{16f_2^2}{L_2^2}\right)^{1/2}-\frac{4f_2}{L_2}}\right]+$$

$$+2L_1\left(1+\frac{16f_1^2}{3L_1^2}\right)\omega\Delta T+L_2\left(1+\frac{16f_2^2}{3L_2^2}\right)\omega\Delta T \qquad (19)$$

Eq. (19) contains the coefficients a_n, b_n and c_n. When the values of these coefficients are substituted there is obtained an equation from which h can be evaluated.

Let there be considered now the case when only the middle span is loaded and the two side spans are of equal length. Furthermore if the flexural rigidities of the two side span stiffening trusses are equal, $a_n = c_n$, and eq. (19) simplifies to

$$\frac{1}{2}\sum\left(\frac{n\pi}{L_1}\right)^2 a_n^2 L_1 + \frac{1}{4}\sum\left(\frac{n\pi}{L_2}\right)^2 b_n^2 L_2 + \sum\frac{16 f_1^2}{L_1^2} a_n \frac{L_1}{n\pi}(1-\cos n\pi) +$$

$$+\sum\frac{8 f_2}{L_2^2} b_n \frac{L_2}{n\pi}(1-\cos n\pi) = \frac{h}{AE}\frac{L_1}{2}\left[\left(1+\frac{16 f_1^2}{L_1^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] +$$

$$+\frac{h}{AE}\frac{L_2}{4}\left[\left(1+\frac{16 f_2^2}{L_2^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] +$$

$$+\frac{h}{HE}\frac{3 L_1^2}{32 f_1}\ln\left[\frac{\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2} + \frac{4 f_1}{L_1}}{\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2} - \frac{4 f_1}{L_1}}\right] + \frac{h}{AE}\frac{3 L_2^2}{64 f_2}\ln\left[\frac{\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2} + \frac{4 f_2}{L_2}}{\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2} - \frac{4 f_2}{L_2}}\right] +$$

$$+2 L_1\left(1+\frac{16 f_1^2}{3 L_1^2}\right)\omega \Delta T + L_2\left(1+\frac{16 f_2^2}{3 L_2^2}\right)\omega \Delta T$$

Substituting for a_n and b_n from eq. (18)

$$\sum 2\left(\frac{8 f_1}{L_1^2}\right)^2 h^2 \frac{(1-\cos n\pi)^2}{L_1\left(\frac{n\pi}{L_1}\right)^4\left[E I_1\left(\frac{n\pi}{L_1}\right)^2 + Hh +\right]^2} +$$

$$+\sum\frac{\left[p_2(1-\cos n\pi\beta) - \frac{8 f_2^2}{L_2^2} h(1-\cos n\pi)\right]^2}{L_2\left(\frac{n\pi}{L_2}\right)^4\left[E I_2\left(\frac{n\pi}{L_2}\right)^2 + H + h\right]^2} -$$

$$-\sum 4\left(\frac{8 f_1}{L_1^2}\right)^2 h \frac{(1-\cos n\pi)^2}{L_1\left(\frac{n\pi}{L_1}\right)^4\left[E I_1\left(\frac{n\pi}{L_1}\right)^2 + H + h\right]} +$$

$$+\sum 2\left(\frac{8 f_2}{L_2^2}\right)\frac{\left[p_2(1-\cos n\pi\beta) - \frac{8 f_2}{L_2^2} h(1-\cos n\pi)\right]}{L_2\left(\frac{n\pi}{L_2}\right)^4\left[E I_2\left(\frac{n\pi}{L_2}\right)^2 + H + h\right]}(1-\cos n\pi) =$$

$$= \frac{h}{AE}\frac{L_1}{2}\left[\left(1+\frac{16 f_1^2}{L_1^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2}\right] +$$

$$+\frac{h}{AE}\frac{L_2}{2}\left[\left(1+\frac{16 f_2^2}{L_2^2}\right)^{3/2} + \frac{3}{2}\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2}\right] +$$

$$+\frac{h}{AE}\frac{3 L_1^2}{32 f_1}\ln\left[\frac{\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2} + \frac{4 f_1}{L_1}}{\left(1+\frac{16 f_1^2}{L_1^2}\right)^{1/2} - \frac{4 f_1}{L_1}}\right] + \frac{h}{AE}\frac{3 L_2^2}{64 f_2}\ln\left[\frac{\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2} + \frac{4 f_2}{L_2}}{\left(1+\frac{16 f_2^2}{L_2^2}\right)^{1/2} - \frac{4 f_2}{L_2}}\right] +$$

$$+2 L_1\left(1+\frac{16 f_1^2}{3 L_1^2}\right)\omega \Delta T + L_2\left(1+\frac{16 f_2^2}{3 L_2^2}\right)\omega \Delta T \qquad (20)$$

in which h is the only unknown quantity and can be evaluated by trial.

Numerical Example

The Manhattan Suspension Bridge will be used as an example. Eq. (20) will be applied to finding the additional cable stress due to temperature change and also to live load occupying part of the middle span. The following data will be used in the calculations:

$$
\begin{aligned}
L_1 &= 713.5 \text{ ft.}\\
L_2 &= 1446.7 \text{ ft.}\\
f_1 &= 37.2 \text{ ft.}\\
f_2 &= 145.3 \text{ ft.}\\
A &= 275 \text{ in.}^2\\
E\,I_1 &= 29\cdot 10^6\cdot 50860\cdot 144 \text{ lb.-in.}^2\\
E\,I_2 &= 29\cdot 10^6\cdot 43900\cdot 144 \text{ lb.-in.}^2\\
q_2 &= 5820 \text{ lb. per ft.}\\
p_2 &= 4000 \text{ lb. per ft.} = 334 \text{ lb. per in.}\\
\beta &= \tfrac{1}{4}\\
\omega &= 66\cdot 10^{-7} \text{ in. per in. per degree Fahrenheit}\\
\varDelta T &= +55^\circ \text{ F.}
\end{aligned}
$$

Substituting this data with units in pounds and inches, in eq. (20), it becomes:

$$
30.5\,h^2 \sum \frac{(1-\cos n\pi)^2}{n^4\,(28.7\cdot 10^6\,n^2 + H + h)^2} +
$$

$$
+ 53\,400\cdot 10^6 \sum \frac{\left[334\left(1-\cos\frac{n\pi}{4}\right) - 46.4\cdot 10^{-6}\,h\,(1-\cos n\pi)\right]^2}{n^4\,(6.02\cdot 10^6\,n^2 + H + h)^2} -
$$

$$
- 61.0\,h \sum \frac{(1-\cos n\pi)^2}{n^4\,(28.7\cdot 10^6\,n^2 + H + h)} +
$$

$$
+ 4.96\cdot 10^6 \sum \frac{\left[334\left(1-\cos\frac{n\pi}{4}\right) - 46.4\cdot 10^{-6}\,h\,(1-\cos n\pi)\right]}{n^4\,(6.02\cdot 10^6\,n^2 + H + h)}\,(1-\cos n\pi) =
$$

$$
= 4.549\cdot 10^{-6}\,h + 35.67\cdot 10^3\,\omega\,\varDelta T
$$

in which H is found from eqs. (1) and (2) to be

$$
H = \frac{q\,L_2^2}{8\,f_2} = \frac{5820\,(1446.7)^2}{8\cdot 145.3} = 10.48\cdot 10^6 \text{ lb.}
$$

Taking $n = 1, 2, 3, 4$ and 5, this equation becomes

$$122h^2\left[\frac{1}{(39.18\cdot 10^6+h)^2}+\frac{1}{81\,(268.78\cdot 10^6+h)^2}+\frac{1}{625\,(727.98\cdot 10^6+h)^2}\right]+$$

$$+53\,400\cdot 10^6\left[\frac{(97.9-92.8\cdot 10^{-6}h)^2}{(16.50\cdot 10^6+h)^2}+\frac{(334)^2}{16\,(34.56\cdot 10^6+h)^2}+\frac{(570-92.8\cdot 10^{-6}h)^2}{81\,(64.66\cdot 10^6+h)^2}+\right.$$

$$\left.+\frac{(668)^2}{256\,(106.80\cdot 10^6+h)^2}+\frac{(570-92.8\cdot 10^{-6})^2}{625\,(160.98\cdot 10^6+h)^2}\right]-$$

$$-\,244h\left[\frac{1}{39.18\cdot 10^6+h}+\frac{1}{81\,(268.78\cdot 10^6+h)}+\frac{1}{625\,(727.98\cdot 10^6+h)}\right]+$$

$$+\,9.92\cdot 10^6\left[\frac{97.9-92.8\cdot 10^{-6}h}{16.50\cdot 10^6+h}+\frac{570-92.8\cdot 10^{-6}h}{81\,(64.66\cdot 10^6+h)}+\frac{570-92.8\cdot 10^{-6}h}{625\,(160.98\cdot 10^6+h)}\right]=$$

$$=4.549\cdot 10^{-6}h+12.95$$

When there is no change in temperature the second term on the right side of the equation is zero, and for this case there is found by trial

$$h = 0.901\cdot 10^6 \text{ lb.}$$

If the effect of temperature change is included,

$$h = 0.703\cdot 10^6 \text{ lb.}$$

If there is no live load acting on the bridge, the p_2 terms vanish; a change of temperature $+55°$ F. will reduce the stress in the cable in the amount of

$$h = -0.191\cdot 10^6 \text{ lb.}$$

From eq. (20) it may be observed that the law of superposition does not apply in finding the value of h separately from live load and temperature change.

The following table presents a comparison of the calculated values of h as found above and those found by Messrs. JOHNSON, BRYAN, TURNEAURE[4]) and TIMOSHENKO[5]).

Table I. Comparative results for h, with or without temperature change

	Live Load	Temp. +55° F.	L. L. +Temp.
Author's results	901000 lb.	−191000 lb.	703000 lb.
JOHNSON, BRYAN, TURNEAURE			663400 lb.
TIMOSHENKO	897000 lb.	−250000 lb.	647000 lb.

[4]) "Modern Framed Structures", J. B. JOHNSON, C. W. BRYAN, and F. E. TURNEAURE. New York: John Wiley & Sons, 10th ed., 1929, part II, pp. 271ff.

[5]) "The Stiffness of Suspension Bridges", S. TIMOSHENKO, Transactions Am. Soc. C. E. Vol. 94 (1930), p. 391.

Moments, Shears, and Deflections

Once h is evaluated the coefficients a_n, b_n and c_n are obtained by substituting the value of h in eq. (18). The deflection of the mid-span truss is obtained from

$$w = \sum b_n \sin \frac{n \pi x}{L_2}$$

The shear and moment in the truss are given by

$$V = E I w_{xxx}$$

and

$$M = E I w_{xx}$$

If one substitutes the coefficient b_n in

$$w_{xx} = - \sum \left(\frac{n \pi}{L_2}\right)^2 b_n \sin \frac{n \pi x}{L_2}$$

the expression for moment becomes

$$M = - \frac{2 E I_2}{\pi} \sum \frac{p_2 (1 - \cos n \pi \beta) - \frac{8 f_2}{L_2^2} h (1 - \cos n \pi)}{n \left(E I_2 \frac{\pi^2}{L_2^2} n^2 + H + h\right)} \sin \frac{n \pi x}{L_2}$$

It may be observed that the series representing M does not converge as rapidly as the basic series from which h has been determined. For this reason more terms are necessary to find a good approximation for M.

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Summary

The complete solution of the suspension bridge necessitates the determination of two differential equations. In this paper it is shown that these equations can be obtained directly by the Principle of Minimum Energy. However, in order to solve these equations, the assumption is made that the slope of the cable does not change during deformation. On the basis of this assumption the solution is given in terms of a trigonometric series. The equations derived can be applied to a bridge with different lengths of side and main spans and for live loads covering any region of the bridge.

Résumé

Le calcul complet des ponts suspendus exige l'établissement de deux équations différentielles. L'auteur montre que ces équations peuvent être obtenues directement à l'aide de la méthode de l'énergie minimum. Toutefois, pour les résoudre, il admet que l'inclinaison du câble porteur reste la même au cours des déformations. En tablant sur cette hypothèse, il obtient la solution sous la forme d'une série trigonométrique. Les équations obtenues peuvent être appliquées au calcul de ponts de différentes portées, tant en ce qui concerne les travées principales que les travées d'accès et pour toutes les charges utiles qui peuvent être appliquées en un point quelconque de ces ponts.

Zusammenfassung

Die vollständige Durchrechnung von Hängebrücken macht die Aufstellung von zwei Differentialgleichungen notwendig. Es wird gezeigt, wie diese Gleichungen mit Hilfe der Methode der kleinsten Energie direkt gefunden werden können. Für die Auflösung dieser Gleichungen wird vorausgesetzt, daß die Neigungswinkel der Tragkabel bei den Formänderungen unverändert bleiben. Auf Grund dieser Voraussetzung erhält man die Lösung in Form einer trigonometrischen Reihe. Die entwickelten Gleichungen dienen zur Berechnung von Brücken verschiedener Spannweiten und für Nutzlasten, die an jedem beliebigen Brückenpunkt angreifen können.