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A Method for Measuring Damping and Frequencies of High Modes of Vibration of Beams

Procédé pour la mesure de l'amortissement et de la fréquence des harmoniques dans les poutres

Verfahren zur Messung von Dämpfung und Frequenzen der Oberschwingungen von Trägern

Techn. lic. Bo Adamson, Stockholm

1. Introduction and Summary

A beam, subjected to a concentrated impact loading, will have a vibratory motion containing modes of vibration, which can be of very high orders. In computing deflections and stresses in such a beam it must be considered how the various modes of vibration are damped, in order to get a close agreement with experiments.

When lateral vibrations of beams are computed a distinction is made, according to classical theories, between external damping, which is caused by air resistance and internal friction, which has the character of a material property. The external damping is generally supposed to be proportional to the velocity of the vibrating system. If the amplitude is assumed to be constant the logarithmic decrement in the case of vibrations of beams will then be inversely proportional to the frequency of the modes of vibration. Detailed investigations of the external damping have recently been made by Voelz [11][1]). These investigations show that the logarithmic decrement of the external damping is proportional to $1/\sqrt{f}$, where f is the frequency of vibration. The damping caused by internal friction is most often supposed to be of viscous character, i. e. dependent on the strain rate of the vibrating element [2].

[1]) Numbers within [ ] refer to the Bibliography at the end of the paper.

If the amplitude is constant the logarithmic decrement in the case of a beam will then be directly proportional to the frequency.

Experimental investigations of the internal damping [3], [4] have given rise to doubts about the correctness of the supposition of the viscous damping. These investigations show that the logarithmic decrement of the internal friction is dependent on the frequency with a power less than 1, and that the logarithmic decrement is also dependent on the amplitude of vibration. The first discovery has caused MINDLIN [5] to construct the "constant Q theory", where the logarithmic decrement is supposed to be constant and consequently independent of the frequency.

In this paper an experimental arrangement is presented for separating the modes of vibration of a beam, so that the damping of each mode of vibration can be measured separately. The separation is arranged by using two different methods *at the same time,* namely

1. suitable load distribution and
2. suitable placing of two strain gauges connected in series.

The first method is in principle to make the load distribution similar to the normal function of the mode of vibration that is to be separated. Only this mode of vibration is generated, whereas the other modes are cancelled, i. e. their amplitudes become equal to zero. In the case of impact loading, however, it is a problem to start the different parts of the load at the same time. This problem has been solved by using small detonating charges, which can easily be brought to detonate at the same time.

To find out where the two strain gauges connected in series should be placed, the mathematical condition for the cancellation of various modes of vibration has been derived, and a diagram (fig. 2) has been designed which refers to beams with hinged ends. With the aid of the diagram it is easy to determine appropriate gauge placings.

By comparing the experimental values with the above-mentioned theories it was found that the test values agreed more closely with the "constant Q theory" than with the theory of viscous damping. The maximum strains have been about $0{,}5\,^0/_{00}$ at all the frequencies.

Besides the damping, the frequencies of the separated modes of vibration have also been determined. When the test values are compared with different theories we find a close agreement with those frequency values that are calculated, taking rotatory inertia and shearing forces into account.

2. Separation of a Mode of Vibration by an Appropriate Load Arrangement

When a vibrating system with several degrees of freedom is treated mathematically and the differential equation is linear, the load distribution is often expanded in a Fourier-series in such a way that each degree of freedom can be treated separately and then the parts of the solution can be superposed. In the case of an elastic beam each part of the load should be similar to a normal function of the beam.

If the load distribution on the beam is similar to the n-th normal function, the beam will vibrate only according to the n-th mode. This fact can be made use of for the separation of a certain mode of vibration of a beam. The load is then arranged in such a way that it agrees with the normal function of the desired mode of vibration and so this mode is the only one existing. If you want to separate for instance the 5-th mode of vibration of a beam with hinged ends the load distribution should be arranged according to fig. 1.

The load is applied in a transient way and it is of the greatest importance that all the distributed parts of the load start at the same time and that their time dependence is the same along the beam. To attain this the following procedure is used: The beam is loaded with detonating charges which are applied about one centimeter from the beam, so that the latter should not get too large plastic deformations. The charges must not be too large. In the tests which were made on steel-beams of $50 \times 50 \times 1060$ mm (fig. 3b), the charges consist of boxes, which are 2,5 cm of length and have a square cross section area of 2 cm^2. The boxes are filled with 4 grams of powder TNT with a unit weight of 0,8 gm/cm^3. Between the charges and the beam there is left a free space of about 1 cm.

As is already mentioned, it is of great importance that all the charges detonate at the same time. This requirement is satisfied if an electrical detonating cap initiates equally long pieces of detonating fuse, each of which initiates one charge. In order to get a reliable initiation of the charges the farther ends of the detonating fuses are provided with detonating caps, which are put right through the charges, see fig. 3b.

With this procedure the detonation moments of the charges will differ from each other by only a few microseconds. Another investigation [1] has shown that the maximum difference between the detonation moments of two charges that have been initiated in the above-mentioned way, can be kept less than 1,5 μs. The investigation comprised more than 20 tests.

In fig. 3b the test arrangement for measuring the 5th mode of vibration is shown.

3. Separation of a Mode of Vibration by Connecting Two Strain Gauges in Series

If two strain gauges are cemented to certain places of a beam and then connected in series, one or several modes of vibration of the beam can be completely cancelled, others partly cancelled, whereas one or in some cases several modes of vibration are near their maximum values. This fact can be used for the experimental separation of modes of vibration.

In the case of a beam with hinged ends, i.e. horizontal and rotatory motions are permitted at the supports whereas vertical motion is prevented, the symmetrical modes of vibration have the following normal functions (the origo of the coordinate system is taken in the middle of the beam)

$$\cos \frac{n \pi x}{l}$$

with symbols according to fig. 1.

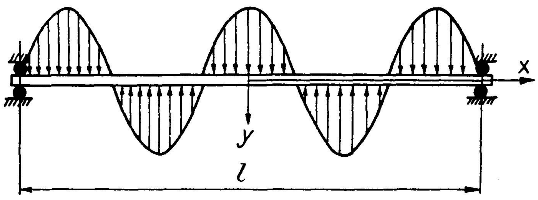


Fig. 1. Load distribution when the 5th mode of vibration of a beam with hinged ends is to be separated.

If two identical gauges, connected in series, are placed at a distance of x_1 and x_2 from the middle of the beam, the mathematical condition for the complete cancellation of the n-th mode of vibration will be

$$\cos \frac{n \pi x_1}{l} + \cos \frac{n \pi x_2}{l} = 0 \tag{1}$$

This equation has the solution

$$\pm \frac{x_1}{l} \pm \frac{x_2}{l} = \frac{2p+1}{n} \quad \text{where} \quad p = 0, 1, 2, 3, 4, 5 \ldots \tag{2}$$

The solution is shown graphically in fig. 2 with $\frac{x_1}{l}$ and $\frac{x_2}{l}$ as coordinate axes within the area

$$0 \leqslant \frac{x_1}{l} \leqslant 0{,}5$$

$$0 \leqslant \frac{x_2}{l} \leqslant 0{,}5$$

i.e. the gauge is placed somewhere between the middle of the beam and one of its ends.

The lines of the diagram in fig. 2 indicate where the different uneven modes of vibration $n = 1 - 17$ are completely extinguished. The lines are orthogonal, and those which refer to the same mode of vibration form squares. In the centre of such a square that particular mode of vibration has its maximum value, i.e.

$$\cos \frac{n \pi x_1}{l} + \cos \frac{n \pi x_2}{l} = max = |2|$$

With the aid of the diagram appropriate gauge places for separating the uneven modes of vibration $n = 1 - 11$ have been determined, and the amplitudes of the various modes of vibration have been calculated in every case as a percentage of the maximum value. The results are presented in table 1. On account of the many possibilities of varying the gauge placings it is possible that in certain cases more appropriate placings can be found than those given in the table. It is often a matter of estimate to determine which of the adjoining modes of vibration should be made equal to zero.

Table 1. Gauge places and amplitudes of the different modes of vibration when the n-th mode is separated

Separation of the mode $n=$	Gauge places		Amplitude of the n-th mode in percent of its maximum value								
	$\frac{x_1}{l}$	$\frac{x_2}{l}$	$n=1$	$n=3$	$n=5$	$n=7$	$n=9$	$n=11$	$n=13$	$n=15$	$n=17$
1	0,266	0,066	83	0	0	51	0	—	—	—	—
3	0,370	0,228	58	74	0	0	22	—	—	—	—
5	0,380	0,047	—	0	84	0	0	39	—	—	—
7	0,413	0,142	—	25	18	97	0	0	24	—	—
9	0,342	0,112	—	25	22	23	99	0	0	19	—
11	0,369	0,169	—	—	0	63	23	95	0	0	13

The percentage values given in table 1 are valid if the mode of vibration is separated by using only the principle of two gauges connected in series. The separation can be made more complete by arranging the load distribution in an appropriate way *at the same time*. This combined procedure has been used in the tests which are described in the next chapter.

4. Experiments

Both of the described methods for separating modes of vibration of a beam have been used in combination to determine the damping and the frequencies of the uneven modes of vibration $n = 1 - 11$ of a steel beam $50 \times 50 \times 1060$ mm with hinged ends. The separation is made by using simultaneously appropriate load distribution and appropriate placings of two gauges connected in series. The span of the beam is $l = 1000$ mm.

The beam is fastened at its ends by solid supports (fig. 3a), designed to tolerate large detonation loads and to imitate the end conditions of a beam with hinged ends. The beam is steadily fastened between two cylindrical surfaces with a radius of 50 mm. The rotatory centres of the supports are 75 mm below the centre line of the beam and permit rotatory as well as horizontal motion. Vertical motion, however, is eliminated.

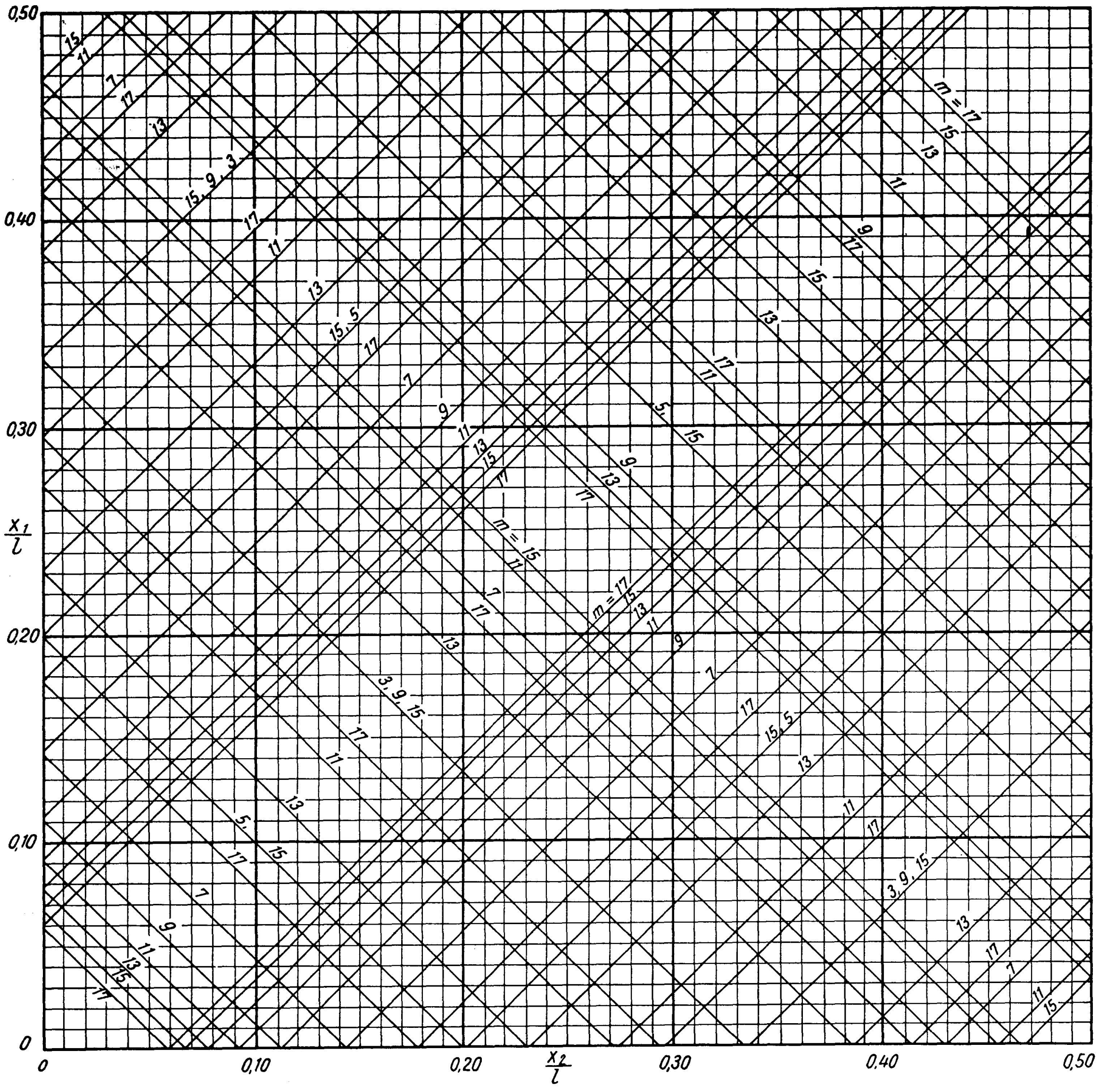


Fig. 2.

Comparisons between computed frequencies of a beam with hinged ends and measured frequencies of the modes of vibrations $n = 1 - 11$ show good agreement — see table 3. Unfortunately, it has not been possible to find out if the vibrations of the beam are damped by the supports.

The test arrangements are in other respects as described above. They are shown in fig. 3b.

The strain gauge used in the tests was of the type GH (made in Sweden) with the resistance wires embedded in a plastic layer. The gauges are cemented to the vertical side of the beam so as not to be spalled from the beam when the compressional shock wave is reflected at the bottom surface of the beam. The gauges are connected in series and the voltage change across the gauges is recorded by a cathode-ray oscilloscope *Tektronix,* type 512.

Table 2. Results of measurements of damping and frequency

Mode $n=$	Damping			Frequency		
	δ	δ_{mean}	Confidence limits [2])	f	f_{mean}	Confidence limits [2])
1	0,065 0,071 0,075 0,071	0,0705	± 0,0065	116,1 116,1 115,9 116,0	116,0	± 0,16
3	0,070 0,077 0,081 0,078	0,0765	± 0,0075	970 1007 968 971	979	± 30
5	0,250 0,245 0,250 0,270 0,285 0,256 0,280 0,242 0,232	0,257	± 0,0139	2740 2670 2720 2760 2660 2610 2640 2620 2690	2679	± 41
7	0,285 0,270 0,241 0,205 0,321 0,204 0,294 0,310 0,244 0,269 0,244	0,262	± 0,0263	5010 4980 4700 5040 5100 5200 4990 4950 4940 4850 4820	4962	± 88
9	0,157 0,171 0,155 0,174 0,137	0,159	± 0,0183	7500 7550 7360 7650 7360	7484	± 156
11	0,149 0,128 0,121 0,125 0,163 0,125	0,135	± 0,0177	10800 10310 10310 10910 10180 10900	10568	± 352

[2]) 95% probability.

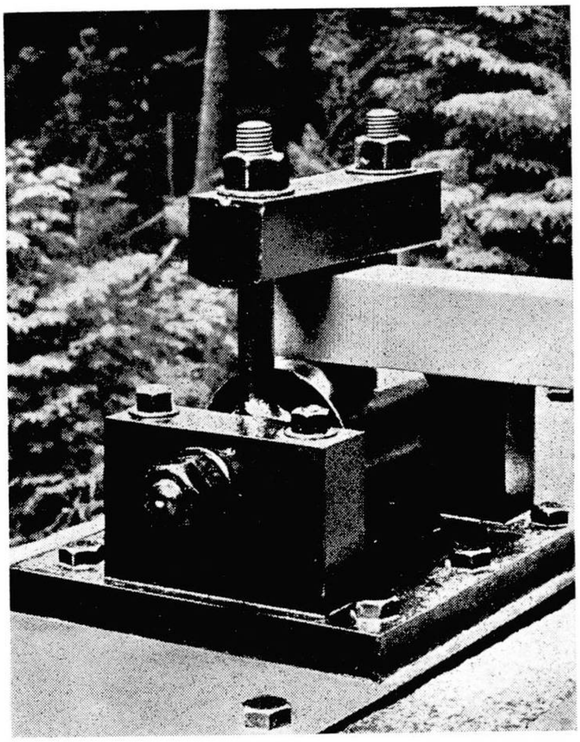


Fig. 3a. Photo of the support.

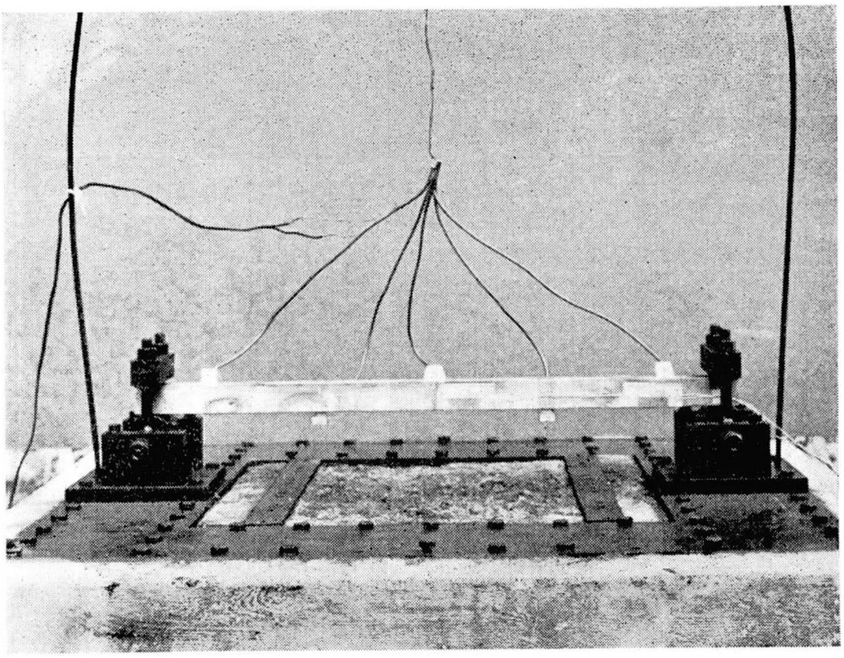
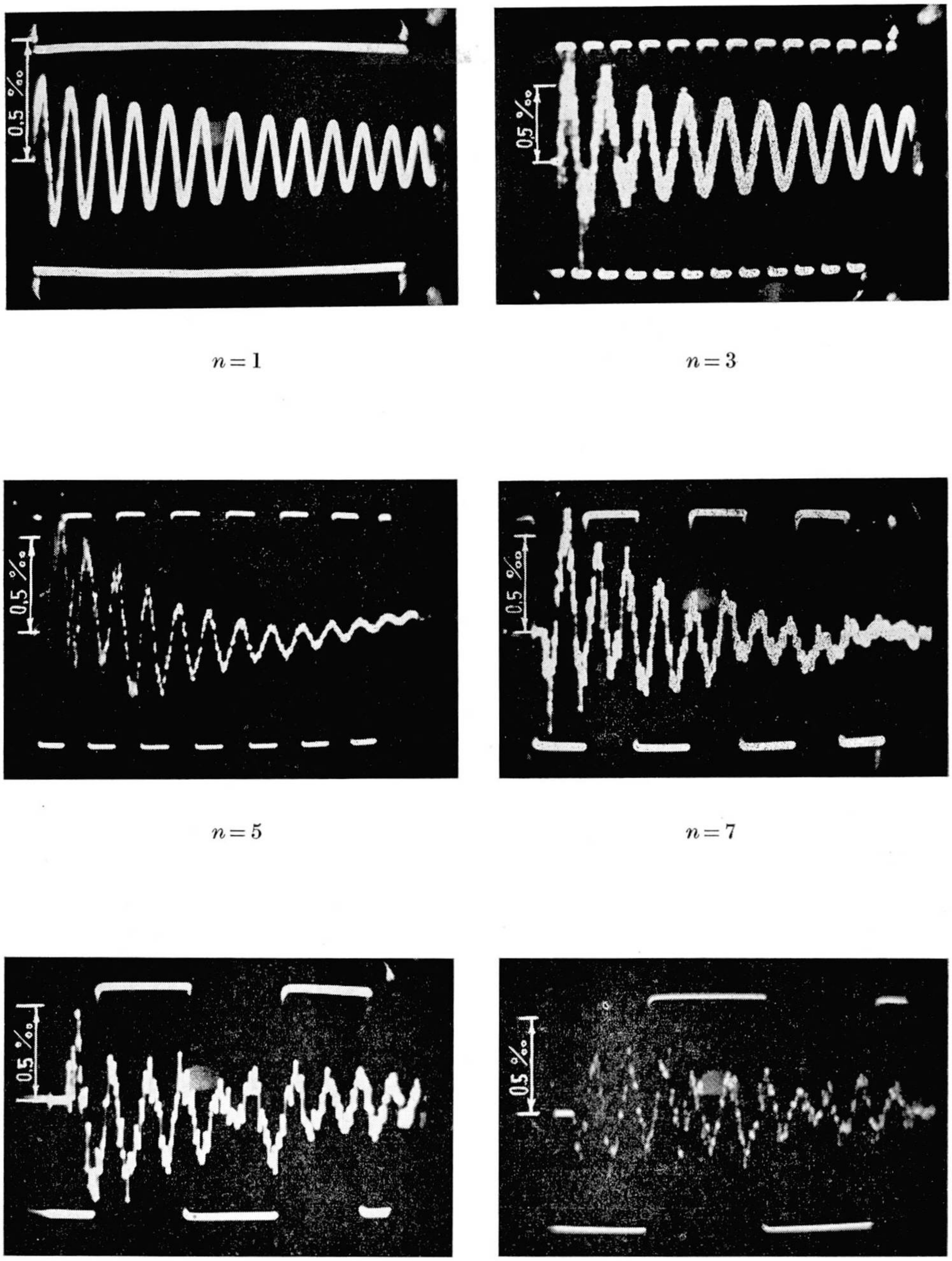


Fig. 3b. Photo of test arrangement for separation of the mode $n = 5$.



$n=1$ $n=3$

$n=5$ $n=7$

$n=9$ $n=11$

Fig. 4. Examples of oscillograms of modes $n=1$—11.

In fig. 4, oscillograms of the modes of vibration $n = 1 - 11$ are shown. The amplitudes decrease roughly exponentially, and thus it should be possible to compute a logarithmic decrement for each mode of vibration. This decrement, however, will not be quite constant over the whole time of vibration, and therefore it has been computed as a mean value over the first 7 vibrations. It is of course the damping during the first periods of the vibratory motion that is of interest in most practical cases.

The tests, described here, have only aimed at making clear in a special case the dependence of the logarithmic decrement upon the frequencies of some modes of vibration. The maximum strains have been about $0{,}5\,^0/_{00}$ in all the tests (see fig. 4). If the strains differ considerably from this value, the decrements will probably change, too.

The logarithmic decrements δ, which have been obtained experimentally for the modes of vibration $n = 1 - 11$ are shown in table 2, which also contains frequencies for the respective modes of vibration.

In fig. 5 the average values of δ have been drawn as a function of the frequencies. The confidence limits of the means corresponding to 95% probability have also been calculated.

5. Comparison Between Experiments and Theories

A. Damping

When damping problems are treated mathematically, the external damping — which is greatly due to air resistance — is often supposed to be proportional to the velocity of the beam, whereas the internal damping is supposed to be "viscous", i.e. proportional to the strain rate of the beam [2]. With these assumptions (and neglecting the influence of rotatory inertia and shear forces) the differential equation for a vibrating beam will be [10]

$$K_2 \frac{\partial^5 y}{\partial x^4 \partial t} + EI \frac{\partial^4 y}{\partial x^4} + A \rho \frac{\partial^2 y}{\partial t^2} + K_1 \frac{\partial y}{\partial t} = 0 \quad (3)$$

where y = transverse displacement (see fig. 1)
x = coordinate parallel to the axis of the beam
t = time
E = Young's modulus
I = moment of inertia
A = cross section area
ρ = density
K_1 = constant referring to external damping
K_2 = constant referring to internal friction.

If the equation (3) is solved, it will be found that the vibrations of a hinged beam are damped exponentially, and the logarithmic decrement will be equal to the sum of the decrements of the external and internal damping

$$\delta = \delta_{external} + \delta_{internal} \tag{4}$$

The logarithmic decrement of the external damping will be inversely proportional to the frequency (curve 1 in fig. 6)

$$\delta_{external} = \frac{k_1}{f} \tag{5}$$

whereas the decrement of the internal viscous damping will be directly proportional to the frequency (curve 2 in fig. 6).

$$\delta_{internal} = k_2 \cdot f \tag{6}$$

Voelz [11] has made a thorough investigation of the external damping (the friction against the surrounding medium). He then found that the logarithmic decrement is

$$\delta_{external} = \frac{k_1'}{\sqrt{f}}$$

This is a decreasing function, too, and hence the logarithmic decrement depends on the frequency in principally the same way as has been shown by curve 1 in fig. 6. It is merely the rate of decrease of the curve that is changed.

Experimental investigations of internal damping [3] and [4] have shown that the internal damping is not directly proportional to the frequency. Rather it approximates a constant value [3] or according to [4] a curve with a maximum (curve 3 in fig. 6). Mindlin [5] has stated that in practical cases the logarithmic decrement of the internal damping $\delta_{internal}$ should be regarded as independent of the frequency in such a way as is shown by curve 4 in fig. 6. This theory is called the "constant Q theory".

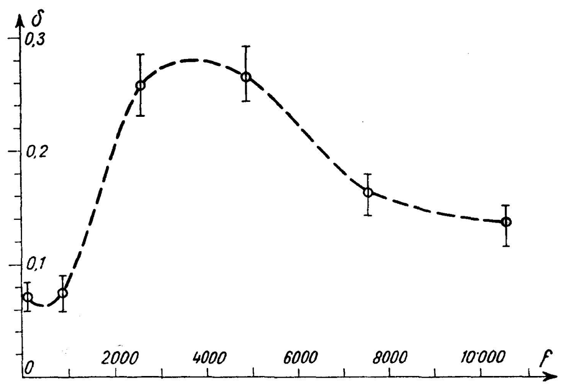


Fig. 5. Experimental mean values of the logarithmic decrements versus the frequency f for the modes $n = 1$—11. The confidence limits corresponding to 95% probability are shown.

To the experimental values in fig. 5 a curve can easily be adapted, that is a sum of the curves 1 and 3 in fig. 6. It should be noticed that the influence of the damping of the supports is then neglected.

A curve composed of external damping, internal "viscous" friction and a damping from the supports can hardly be adapted to the test values without making special assumptions about the frequency dependence on the last mentioned damping.

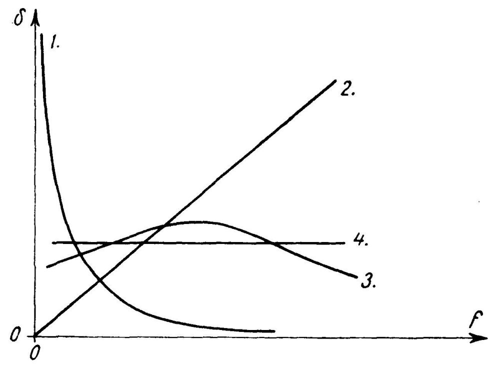


Fig. 6. The logarithmic decrement versus the frequency in the case of 1. external (air) damping, 2. internal (viscous) damping according to SEZAWA [2], 3. internal damping according to [4], 4. internal damping according to the "constant Q theory" [5].

Thus the tests seem to indicate that the internal damping is of the type that is shown by curve 3 in fig. 6. A more definite statement is hardly to be made from this small and special test material.

Choosing between the two theories that are mathematically the most convenient — the theory of the viscous damping and the "constant Q theory" — it is found that the former theory can only be used for vibrations with few high modes whereas the "constant Q theory" seems to constitute a fair approximation for vibrations including modes of high orders.

B. Frequency measurement

The differential equation that is generally used for solution of the problem of vibrating beams and which does not consider the influence of rotatory inertia, shear forces or damping is [6]

$$E\,I\,\frac{\partial^4 y}{\partial x^4}+A\,\rho\,\frac{\partial^2 y}{\partial t^2}=0 \tag{7}$$

The symbols are the same as in equation (3). In the case of a beam with hinged ends the frequency of the n-th mode of vibration will be (l = the span of the beam)

$$f_n^I=\frac{n^2\,\pi}{2\,L^2}\sqrt{\frac{E\,I}{A\,\rho}} \tag{8}$$

If, in addition, the rotatory inertia of the beam elements is taken into account, the differential equation is [7]

$$EI\frac{\partial^4 y}{\partial x^4} - I\rho\frac{\partial^4 y}{\partial x^2\,\partial t^2} + A\rho\frac{\partial^2 y}{\partial t^2} = 0 \tag{9}$$

and the frequency of the n-th mode will be

$$f_n^{\mathrm{II}} = \frac{n^2\pi}{2l^2}\sqrt{\frac{EI}{A\rho}}\cdot\frac{1}{\sqrt{1+\frac{I}{A}\frac{n^2\pi^2}{l^2}}} \tag{10}$$

If the beam is subjected to damping forces, so that the vibrations are damped exponentially, the frequency will be (considering the influence of rotatory inertia as well as external and internal damping)

$$f_n^{\mathrm{III}} = \frac{n^2\pi}{2l^2}\sqrt{\frac{EI}{A\rho}}\frac{\sqrt{1-\left(\frac{\delta_n}{2\pi}\right)^2}}{\sqrt{1+\frac{I}{A}\cdot\frac{n^2\pi^2}{l^2}}} \tag{11}$$

where δ_n is the total logarithmic decrement of the n-th mode of vibration in the case of both external and internal damping.

If consideration is taken to both rotatory inertia and shear forces but not to damping the differential equation is [8]

$$EI\frac{\partial^4 y}{\partial x^4} - I\rho\left(1+\frac{2(1+\nu)}{\Lambda}\right)\frac{\partial^4 y}{\partial x^2\,\partial t^2} + A\rho I\rho\frac{1}{AAG}\frac{\partial^4 y}{\partial t^4} + A\rho\frac{\partial^2 y}{\partial t^2} = 0 \tag{12}$$

where ν = Poisson's ratio

Λ = a constant relating the shearing forces with the angle of shear (for a rectangular section $\approx 0{,}9$)

G = modulus of rigidity $= \frac{E}{2(1+\nu)}$

In the case of a beam with hinged ends the frequency of the n-th mode of vibration will be [9]

$$f_n^{\mathrm{IV}} = \frac{1}{2\pi}\sqrt{\frac{s_n}{u} - \sqrt{\left(\frac{s_n}{u}\right)^2 - \frac{t_n}{u}}} \tag{13}$$

where

$$s_n = \frac{1}{2}\left\{1+\frac{I}{A}\frac{n^2\pi^2}{l^2}\left[1+\frac{2(1+\nu)}{\Lambda}\right]\right\}$$

$$t_n = \frac{EI}{A\rho}\cdot\frac{n^4\pi^4}{l^4}$$

$$u = \frac{I\rho}{AE}\cdot\frac{2(1+\nu)}{\Lambda}$$

In table 3 the four frequencies theoretically calculated in the above-mentioned way are compared to the average values of the frequencies measured experimentally according to table 2. In the computations E is taken equal to $2{,}1 \cdot 10^6$ kg/cm^2 and $\gamma = 7{,}85$ gm/cm^3, which are the actual values for the test beams. E has been measured both statically and dynamically. The frequencies f^{III} have been computed from eq. (11) with values of the logarithmic decrement taken from table 2.

A comparison between the frequencies f^{II} and f^{III}, where in the former case no consideration is taken to internal and external damping, shows that the damping has very little influence upon the frequencies. Hence it is probably of no importance for the computation of frequencies, that in the theory according to TIMOSHENKO [8] (the frequencies f^{IV}) the damping is neglected.

From the table it is apparent that the agreement between f^{IV} and the experimental values is very close, especially when considering that the sweep-time circuit of the oscilloscope does not guarantee a greater accuracy than $\pm 5\%$. Yet the calibrations that have been made indicate that the accuracy of the determination of the times has been about $\pm 2\%$ in these cases.

Table 3. Comparison between theoretical and experimental values of frequencies

m	Theoretical values				Experimental values	$\frac{f^{IV}}{f^{exp}}$
	f^{I}	f^{II}	f^{III}	f^{IV}	f^{exp}_{mean}	
1	116,2	116,1	116,1	115,7	116,0	1,00
3	1046	1036	1036	1010	979	0,97
5	2905	2833	2831	2653	2679	1,01
7	5694	5423	5418	4861	4962	1,02
9	9412	8692	8689	7442	7484	1,01
11	14060	12509	12506	10268	10568	1,03

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Summary

This paper presents a method by which the magnitude of damping of vibrating constructions (e.g. beams) can be determined. The method can also be used for experimental determination of frequencies of high modes of vibration.

The experimental method used for this purpose consists in segregating the various component frequencies of vibration of the construction (e.g. a beam) in such a way that the damping corresponding to each component frequency can be measured separately. This segregation was achieved, first, by an appropriate adjustment of the load, i.e. by ensuring that the load shall be as closely affine as possible to the characteristic function of that component frequency which is required to be amplified, and second, by a suitable location of two wire resistance strain gauges, coupled in series. The points at which these gauges should be placed were determined by means of a mathematicographical procedure. Owing to the simultaneous use of the two methods outlined in the above, it was possible to determine the damping and the frequencies of the odd harmonics ($m = 1$, 3, 5, 7, 9 and 11) of a steel beam with hinged ends. A comparison of the theoretically calculated and the observed frequencies is made in table 3, where f^{I} is the frequency computed from the Bernoulli-Euler theory, f^{II} is the frequency calculated from Lord Rayleigh's theory without regard to damping, f^{III} is the frequency calculated from the same theory so as to take damping into account, and f^{IV} is the frequency computed from the Timoshenko theory, taken into account both rotatary inertia and shearing forces. The latter theory is found to be in very close agreement with the experimental results.

Résumé

L'auteur expose un procédé pour la détermination de la grandeur de l'amortissement sur les ouvrages soumis à des vibrations. Ce procédé permet d'ailleurs de déterminer aussi expérimentalement les fréquences des harmoniques supérieurs.

Il consiste à séparer les différentes fréquences partielles de la vibration (d'une poutre, par exemple), de telle sorte qu'il soit possible de déterminer individuellement l'amortissement qui correspond à chaque vibration partielle. Cette séparation est réalisée tout d'abord par une disposition appropriée de la charge, c'est-à-dire par la mise en jeu d'une charge aussi affine que possible vis-à-vis de la fonction caractéristique des composantes de fréquence à amplifier, puis par la disposition appropriée de deux jauges à fil résistant montées en série. La position de ces points de mesure peut être déterminée par des moyens mathématiques et graphiques. L'emploi conjoint de ces deux méthodes de séparation permet de déterminer l'amortissement et les fréquences des vibrations d'ordre impair ($m = 1, 3, 5, 7, 9$ et 11) d'une poutre en acier avec supports articulés aux extrémités. Le tableau 3 fournit une comparaison entre les fréquences déterminées théoriquement et les fréquences mesurées. Les calculs ont été faits dans les conditions suivantes: fréquence f^{I} d'après Bernoulli-Euler, fréquence f^{II} d'après Lord Raleigh sans tenir compte de l'amortissement, fréquence f^{III} d'après Lord Raleigh en tenant compte de l'amortissement et fréquence f^{IV} d'après Timoshenko en tenant compte de la résistance à la distorsion et des efforts tranchants. Cette dernière théorie donne une très bonne concordance avec les résultats des mesures.

Zusammenfassung

Die Arbeit zeigt ein Verfahren zur Bestimmung der Größe der Dämpfung an schwingenden Konstruktionen. Sie erlaubt zugleich die versuchsmäßige Bestimmung der Oberschwingungsfrequenzen.

Diese Versuchsmethode besteht in der Trennung der verschiedenen Teilfrequenzen der Schwingung (z. B. eines Trägers) derart, daß die zu jeder Teilschwingung gehörende Dämpfung einzeln gemessen werden kann. Diese Auftrennung wurde erstens durch eine passende Lastanordnung, d. h. durch eine möglichst affine Belastung zur charakteristischen Funktion derjenigen Frequenzkomponente, die verstärkt werden soll, und zweitens durch eine geeignete Anordnung zweier in Serie geschalteter Widerstandsmeßstreifen erreicht. Die Lage dieser Meßpunkte läßt sich auf mathematisch-graphischem Weg ermitteln. Mit der gemeinsamen Anwendung dieser beiden Trennungsmethoden war es möglich, Dämpfung und Frequenzen der ungeraden Schwingungen ($m = 1, 3, 5, 7, 9$ und 11) eines Stahlbalkens mit gelenkigen Endlagen zu bestimmen. Ein Vergleich der theoretisch ermittelten und der gemessenen Frequenzen ist in Tafel 3 dargestellt, wobei die Frequenz f^{I} nach Bernoulli-Euler, f^{II} nach Lord Raleigh ohne Berücksichtigung der Dämpfung, f^{III} nach Lord Raleigh unter Zuziehung der Dämpfung und f^{IV} nach Timoshenko mit Berücksichtigung von Verdrehungssteifigkeit und Schubkräften berechnet wurde. Diese letztere Theorie zeigt eine sehr gute Übereinstimmung mit den Meßergebnissen.