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Thin Walled Box Beams Under Pure Bending

Les poutres en caisson à parois minces sollicitées à la flexion pure

Dünnwandige Kastenträger bei reiner Biegung

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I.

The study of built-up structures embraces actually an extremely wide field. Numerous contributions to this field are made by solutions of related part-problems, which not always further our better understanding of the whole. The designer's general approach to the actual problems consequently is still an oversimplified one in sharp contrast to the actually very complex behavior of some details.

The term built-up structure is applied to a very great variety of structures including anything from box girders to airplane fuselages. All such structures actually could be broken up for stress analysis in a number of details. Even a box girder can be considered as an assembly of its webs and flanges. Whenever we attempt to analyse the structure as a whole we are referring to it by the term built-up member. Thus, correctly, some term like this one should designate the approach used in the analysis rather than the constructional properties of the member. However, it would be difficult to be right since type of analysis and type of construction are so tightly interrelated. If we took a box girder as an example, the ratio of plate thicknesses to the overall cross sectional dimensions might be such that the stress analysis would not require any special considerations and the classical flexure formula could be successfully applied. Another box girder might be built with thin plates, in which case the stress analysis would require a more complex procedure and might never leave us with the same assurance of accuracy that the first simple case did. This is because we had to utilize a number of assumptions in describing the behavior of details. The uncertainties in the assumptions will be added to the general uncertainty of any analysis.

We singled out as the object of the present study the behavior of a rectangular, thin walled box section under bending with no shearing forces. The rectangular thin walled box is the basic type of a great variety of structures. The bending without shearing force, or pure bending, is a fundamental case. Since this study is the first part of a planned sequence, the reason for this selection of object is obvious.

The purpose of this study is in the first place the search for an improved overall approach. The soundness of any such approach should be continuously compared with experimental facts. The more elaborate theoretical and experimental analyses of the details involved should go on separately. New results should gradually be incorporated and the general approach may thus be expanded and revised. The process should always try to supply the designer with the tools, which are based on simple and generally well-known principles, which agree reasonably well with the results of basic experience and which can easily be adapted to practical computations.

II.

Since the generally known flexure formulæ are the most adaptable tools of the engineer, an attempt is presented in which these formulæ are expanded in order to approach better the actual behavior of thin walled box sections in pure bending. The new adaptation is in the turn compared with experimental results. A further improvement and expansion for bending and shear should be the next step in this line of investigation.

The compression plating of boxes with thin walls usually buckles in a very early state of loading, which is far less than the maximum load carrying capacity of the structure. The importance of this strictly local deformation should not be minimized, although such deformation is a normal follower of the functioning of the plating. This local buckling of the compressed plating might by itself constitute a structural damage and determine the useful range of loading of the whole structure. However, we should search for a simple method to predict the overall load carrying capacity of the structure.

Early investigation of a similar problem, that of thin walled tubes under pure bending by Brazier [1] and Chwalla [2, 3] took into consideration the flattening of the cross section and the resulting reduction in the moment of inertia. The same line of thought was used by Timoshenko [4] in connection with box sections. Experiments show that the decrease of the moment of inertia is not as significant as it was assumed [1, 2] and that the arbitrarily assumed deformed shape [4] can not be found on models.

Eggwertz [5] and others made a significant contribution in giving practical ways to determine the buckling stress of the compressed plating of box sections. The predicted buckling stresses seem to agree with experimentally determined values, but the question of the overall maximum load carrying capacity is left open.

Very important results were published by CHAPMAN, FALCONER [6, 7] and others, both from the standpoint of the local buckling problem of the compressed plating and the treatment of overall load carrying capacity. Their approach to the overall load carrying capacity is through the assumption of an equivalent section and an arbitrary stress distribution.

Our approach to the problem is similar but somewhat more elaborate. We make the main assumption that the post buckling behavior of the compressed plating is such that it keeps on supporting the buckling stress while the load on the structure is increased. This actually creates an analogy between the effect of local buckling and of local yielding, by which we understand a process, under which the deformation of a part of the structure may progress without any increase of the corresponding stress.

III.

Analysing the pure bending of a member of an arbitrary cross section, parts of which might locally yield (fig. 1 a), we make the following assumptions.

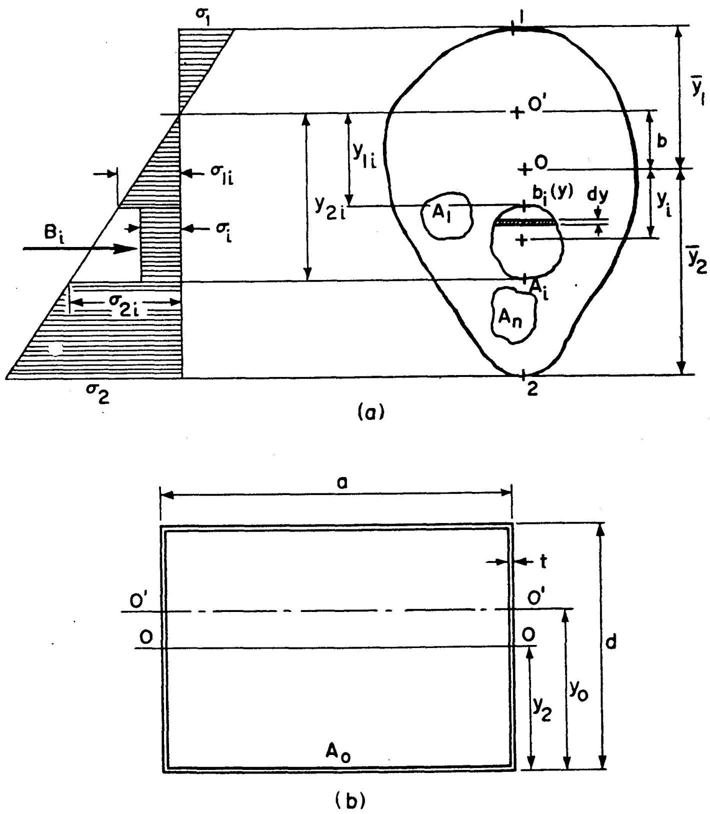

Fig. 1. a) Sketch of a general cross section A with part areas A_i, which yield at a stress σ_i.
Fig. 1. b) Sketch of a box section.

1. The area A is homogeneous, with the exception of the A_i areas.
2. The areas A_i are homogeneous within their boundaries, each area has a specific σ_i yield stress.

Investigating the case when each A_i area reaches its specific σ_i stress, the extreme fiber stress σ_2 being greater than σ_i max., we find that the location of zero stress, the neutral axis, moves from 0 to 0′. The distance $0-0'$ is b.

In order to establish a relation between the M moment on the section and the σ_2 stress, we have to find this distance b.

In the place of the stress distribution shown and described above, we introduce the following system of equivalent stresses:

a) A linearly distributed stress with zero value at 0′, σ_1 at fiber 1 and σ_2 at fiber 2, and intermediate values of $\sigma = \frac{y}{y^2}\sigma_2$.
b) At the centroids of each A_i stresses opposite in sign to the general stress σ and with values varying between $\sigma_{1i}-\sigma_i$ and $\sigma_{2i}-\sigma_i$ in which expression σ_{1i} and σ_{2i} are the values at the fibers of A_i nearest to 1 and to 2, σ_i being the specific yield stress of A_i.

By this we apply the following forces at any one A_i

$$B_i = \int_{A_i} (\sigma - \sigma_i)\, d A$$

The equilibrium of the horizontal forces in the above case and in the above terms equals the sum of the B_i forces:

$$\Sigma B_i = \Sigma \int_{A_i} (\sigma - \sigma_i)\, d A$$

Introducing $$\sigma = \frac{y}{y_i}\sigma_2, \quad b_i(y)\, dy = dA$$

we obtain $$\Sigma B_i = \Sigma \frac{\sigma_2}{y_2}\left(\int_{y_{1i}}^{y_{2i}} y\, b_i(y)\, dy - \frac{\sigma_i}{\sigma_2} y_2 \int_{y_{1i}}^{y_{2i}} b_i(y)\, dy\right)$$

if we further introduce the parameter

$$\alpha_i = \frac{\sigma_i}{\sigma_2}$$

and express the first integral, which is the first moment of the A_i area about the axis through 0′, by $y_i A_i$ understanding that y_i is the distance of the centroid of A_i from the 0′ axis, we obtain

$$\Sigma B_i = \sigma_2 \Sigma A_i \left(\frac{y_i}{y_2} - \alpha_i\right) \tag{1}$$

the total horizontal force due to the σ stresses

$$F = \int_A \sigma \, d A = \frac{\sigma_2}{y_2} A b \tag{2}$$

where we again find the first moment of the total A area about $0'$, which equals $b A$.

From the equilibrium condition

$$F - \Sigma B_i = 0, \quad F = \Sigma B_i$$

from (1) and (2)

$$A b = \Sigma A_i (y_i - \alpha_i y_2) \tag{3}$$

according the notations on fig. 1.

$$y_i = \bar{y}_i + b, \quad y_2 = \bar{y}_2 + b$$

which introduced in (3) give

$$A b = \Sigma A_i (\bar{y}_i + b - \alpha_i [\bar{y}_2 + b])$$

and finally

$$b = \frac{\Sigma A_i (\bar{y}_i - \alpha_i \bar{y}_2)}{A - \Sigma A_i (1 - \alpha_i)} \tag{4}$$

Thus b, and so the location of the new $0'$ axis can be determined by the original dimensions and the parameter α_i.

In the following derivations we need now the distance c from $0'$ of the resultant of all B_i forces.

These can be found from the following elementary consideration:

$$c \Sigma B_i = \Sigma \int_{A_i} (\sigma - \sigma_i) \, y \, d A$$

using

$$\sigma = \frac{\sigma_2}{y_2} y$$

and rewriting the expression

$$c \Sigma B_i = \Sigma \left(\frac{\sigma_2}{y_2} \int_{A_i} y^2 \, d A - \sigma_i \int_{A_i} y \, d A \right)$$

since

$$\int_{A_i} y^2 \, d A = I_i$$

is the second moment of A_i about $0'$, and

$$\int_{A_i} y \, d A = A_i y_i$$

is the first moment of A_i about $0'$, and $\frac{\sigma_i}{\sigma_2} = \alpha_i$, we have

$$c\,\Sigma B_i = \Sigma \frac{\sigma_2}{y_2}\,(I_i - \alpha_i A_i y_i y_2) \tag{5}$$

using $I_i = \overline{I}_i + \overline{y}_i^2 A_i$ where $\overline{I}_i$ means the second moment of A_i about its own gravity axis and ΣB_i from Equ. (1)

$$c = \frac{\Sigma A_i\,(k_i^2 + y_i^2 - \alpha_i y_i y_2)}{\Sigma A_i\,(y_i - \alpha_i y_2)} \tag{6}$$

where
$$k_i^2 = \frac{\overline{I}_i}{A_i}$$

The stress-moment relation can now be derived, using the equilibrium of moments on the section, if M is the bending moment, we write

$$M = \frac{\sigma_2}{y_2}\int_A y^2\,dA - c\,\Sigma B_i$$

since
$$\int_A y^2\,dA = I_A$$

is the second moment of the whole area A about $0'$ and this can be written as

$$\overline{I} + A\,b^2 = I_A$$

and using $c\,\Sigma B_i$ as expressed in Aq. (5) the moment equation becomes

$$M = \sigma_2 \frac{A}{y_2}\left([k_A^2 + b^2] - \Sigma \frac{A_i}{A}\,[k_i^2 + y_i^2 - \alpha_i y_i y_2]\right) \tag{7}$$

which is analogue to the fundamental flexure formula, in which the place of the section modulus is taken by

$$\frac{A}{y_2}\left([k_A^2 + b^2] - \Sigma \frac{A_i}{A}\,[k_i^2 + y_i^2 - \alpha_i y_i y_2]\right), \quad k_A^2 = \frac{\overline{I}_A}{A}$$

in which expression A, A_i, k_a^2, k_i^2 refer to the cross section and its original dimensions, b y_i and y_2 can be determined, since

$$y_i = \overline{y}_i + b, \quad y_2 = \overline{y}_2 + b$$

and b can be found from eq. (4) in which original coordinates $\overline{y}_i$ and y_2 are used. All important is the parameter

$$\alpha_i = \frac{\sigma_i}{\sigma_2}$$

which on the other hand for any σ_2 can be determined, knowing the characteristic σ_i of any A_i part area.

For but one yielding area A_0 instead of i number of A_i areas, the basic formulæ become the following:

$$b = \frac{A_0(\bar{y}_0 - \alpha \bar{y}_2)}{A - A_0(1-\alpha)} \tag{8}$$

$$M = \sigma_2 \frac{A}{y_2} \left([k^2 + b^2] - \frac{A_0}{A} [k_0{}^2 + y_0{}^2 - \alpha y_0 y_2] \right) \tag{9}$$

In a practical problem if we accepted a certain solution furnishing the buckling (or yielding, as we called it in the previous text) stress σ_i in some detail, for arbitrarily chosen values of σ_2 the parameters α_i can be calculated and eq. (7) will supply us with points of a moment-stress relation.

Using the above derived generalization of the flexure formula, we want now to compare the results furnished by it with experimental results. Since the experiments were carried out on a simple box section, we had to adapt eq. (9) to that case.

In doing this, we assumed that the cross section of the box, being thin walled, can be represented by the one shown on fig. 1b.

a) t is so small compared to the other dimensions, that it can be neglected in several cases, thus

$$\bar{y}_0 = \bar{y}_2 = \frac{d}{2}, \quad c = y_2, \quad I_0 = 0$$

b) We assume, that the entire lower flange yields (buckles)

$$A_0 = a\,t$$

Thus we have the eqs. (8) and (9) yielding the following:

$$b = \frac{A_0 d (1-\alpha)}{2(A - A_0(1-\alpha))} \tag{10}$$

From $$M = \sigma_2 \frac{A}{y_2} \left(k_A^2 + b^2 - \frac{A_0}{A} [\kappa_0{}^2 + y_0{}^2 - \alpha y_0 y_2] \right)$$

the latter allows further simplifications, if we consider eq. (5) from which

$$c A_0 (y_0 - \alpha y_2) = A_0 (k_0{}^2 + y_0{}^2 - \alpha y_0 y_2)$$

and eq. (3) from which (for $i = 1$, subscript 0)

$$A_0 (y_0 - \alpha y_2) = A\,b$$

the expression on right hand side equals

$$A\,b\,c$$

and for the box section, fig. 2 $c = y_2$ we obtain

$$M = \sigma_2 \left(\frac{A\,k_A^2}{y_2} + \frac{A\,b^2}{y_2} - \frac{A\,b\,y_2}{y_2} \right)$$

and introducing $\bar{y}_2 + b = y_2$

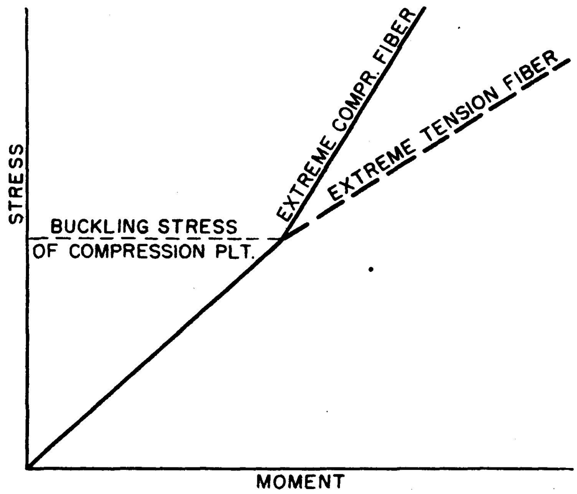

Fig. 2. The basic stress-moment relation for a thin walled box section.

$$M = \sigma_2 \left(\frac{\overline{I}_A + A\,b^2 - A\,b\,(\overline{y}_2 + b)}{\overline{y}_2 + b} \right)$$

since in the numerator $A\,b^2 - A\,b\,\overline{y}_2 - A\,b^2$

cancel the $A\,b^2$ terms: $M = \sigma_2 \dfrac{\overline{I}_A - A\,b\,\overline{y}_2}{\overline{y}_2 + b}$

Substituting now $\dfrac{d}{2} = \overline{y}_2$

and calling $\dfrac{2\,\overline{I}_A}{d} = \overline{Z}_A$

the section modulus of the box, referring to its gravity center and extreme fiber

$$M = \sigma_2 \frac{\overline{Z}_A - A\,b}{1 + \frac{2\,b}{d}}$$

substituting eq. (10) for b and rearranging

$$M = \sigma_2 \left(\overline{Z}_A + \left[\frac{A_0\,d}{2} - \frac{\overline{Z}_A\,A_0}{A} \right] - \alpha \left[\frac{A_0\,d}{2} - \frac{\overline{Z}_A\,A_0}{A} \right] \right) \qquad (11)$$

which further transformed becomes:

$$M = \sigma_2\,\overline{Z}_A - \left(\frac{\overline{Z}_A\,A_0}{A} + \frac{A_0\,d}{2} \right) \sigma_2 + \left(\frac{\overline{Z}_A\,A_0}{A} - \frac{A_0\,d}{2} \right) \sigma_0 \qquad (12)$$

$\overline{Z}_A$, A, A_0 and σ_0 being constants, the new stress-moment relation is a linear one. It should also be remarked, that for all values $\sigma_2 < \sigma_0$

$$M = \sigma_2\,\overline{Z}_A$$

the convention flexure formula.

Consequently, the stress-moment relation will be described for a thin walled rectangular box under pure bending by two straight lines, as shown on fig. 2.

This is the stress-moment relation, which we compared with experimentally established values.

IV.

Following some exploratory tests on plastic models, two aluminium boxes were made with cemented connections and two with riveted connections, the dimensions and details of which are shown on fig. 3. The boxes were tested in pure bending, applied through cantilever loads as shown in fig. 4 and the photographs on fig. 5. The deflections were measured by dial gages and the

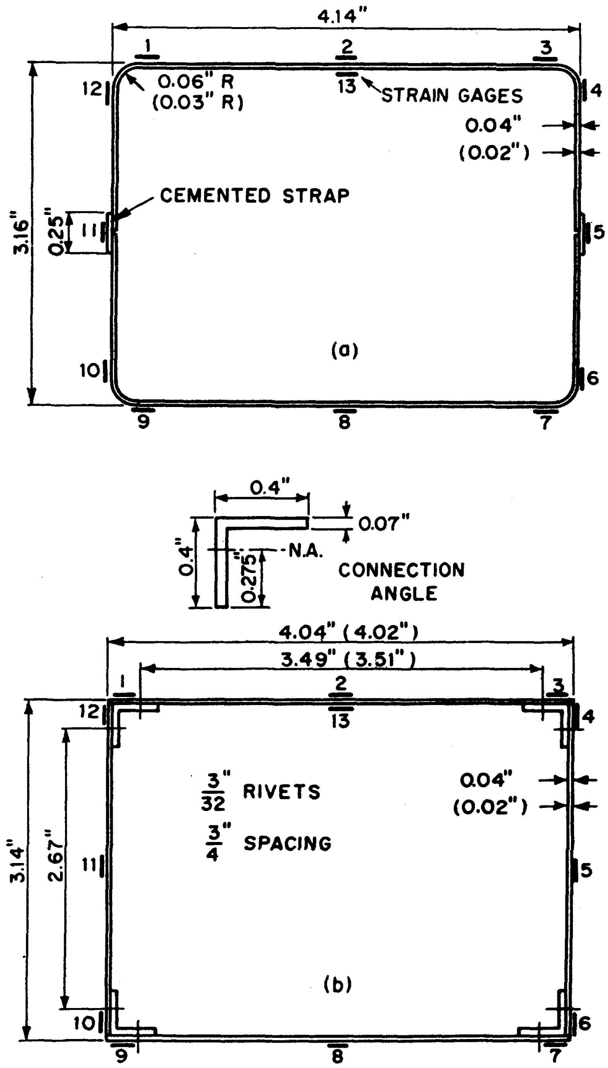

Fig. 3. Top: Experimental box beam section, cemented connection. Bottom: Experimental box section in riveted construction.

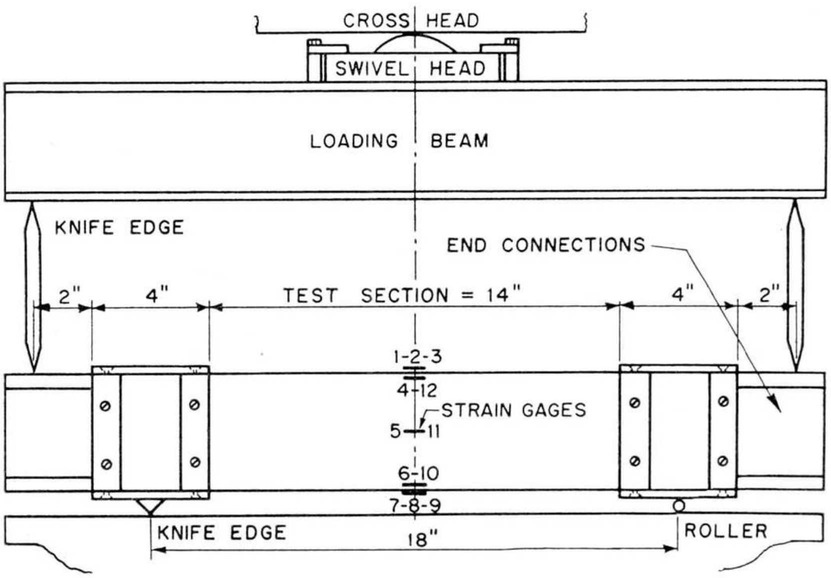

Fig. 4. Experimental setup used in testing the box beams.

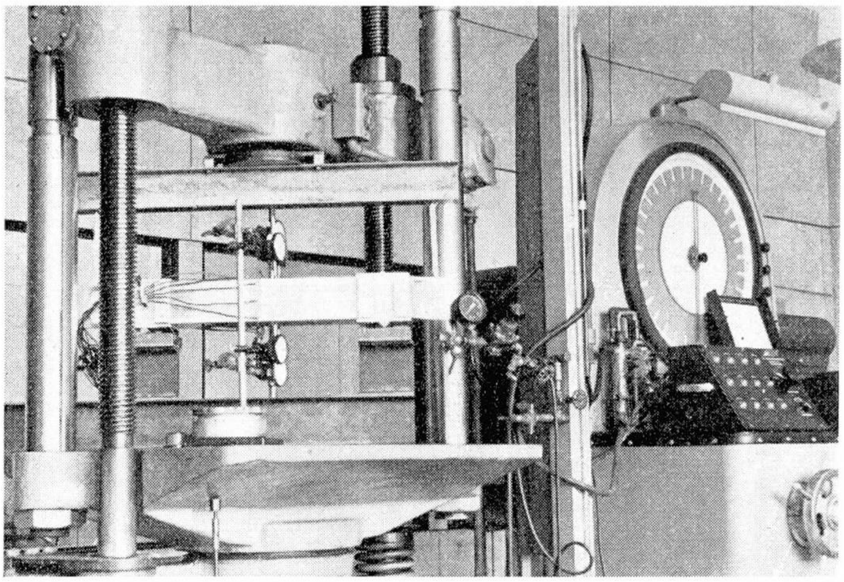

Fig. 5. Photograph of the experimental setup.

strains on the center cross section by SR-4 gages, as indicated on the figures. The boxes with the cemented connections could not be loaded to fracture due to the earlier breakdown of the joints. However, they could be loaded well past the load at which the compressed flange buckled. The riveted boxes could and were loaded, until secondary localized buckling of the compressed plate within rivets occurred almost simultaneously with the buckling of the corner angles and final breakdown of the whole structure. At this point the buckling of the compressed plate was far advanced.

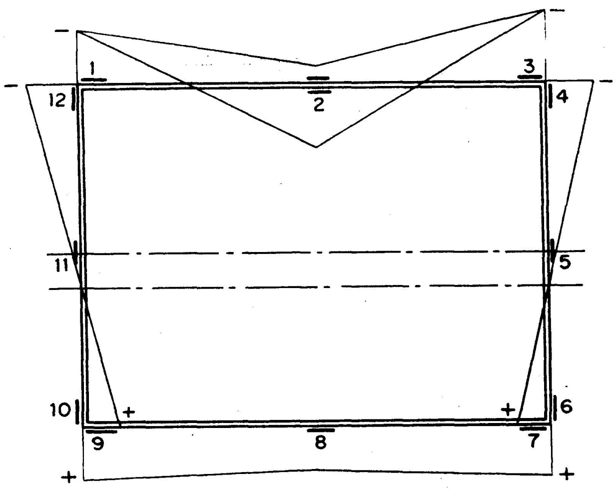

Fig. 6. Typical distribution of stress, as found by SR-4 gage readings. Top: compression.

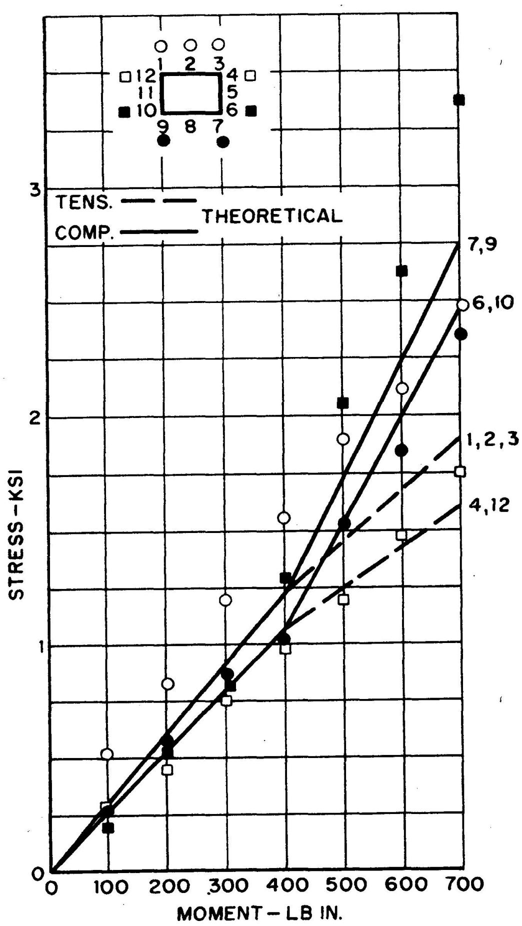

Fig. 7. Actual and theoretical stress values. Cemented box beam, 0.02 in. plate thickness. Compression gages: 7, 8, 9.

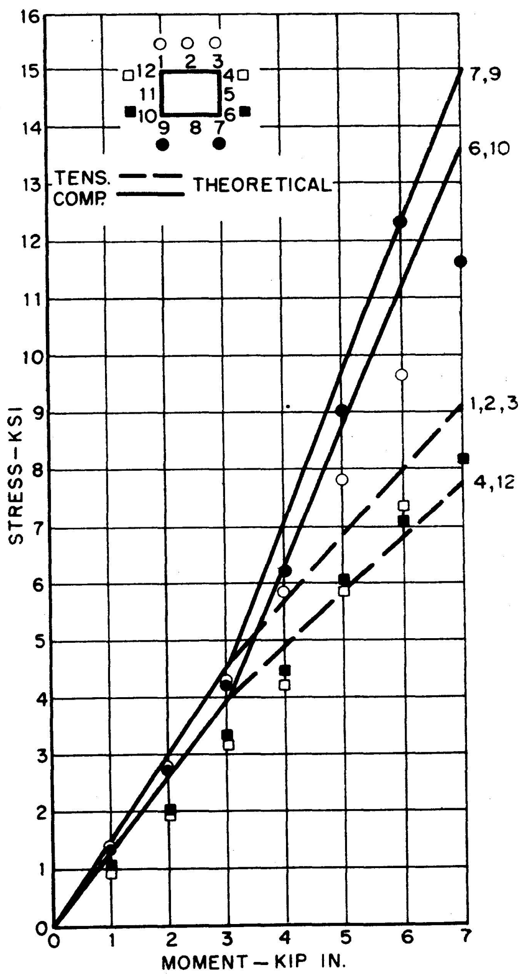

Fig. 8. Actual and theoretical stress values. Cemented box beam, 0.04 in. plate thickness. Compression gages: 7, 8, 9.

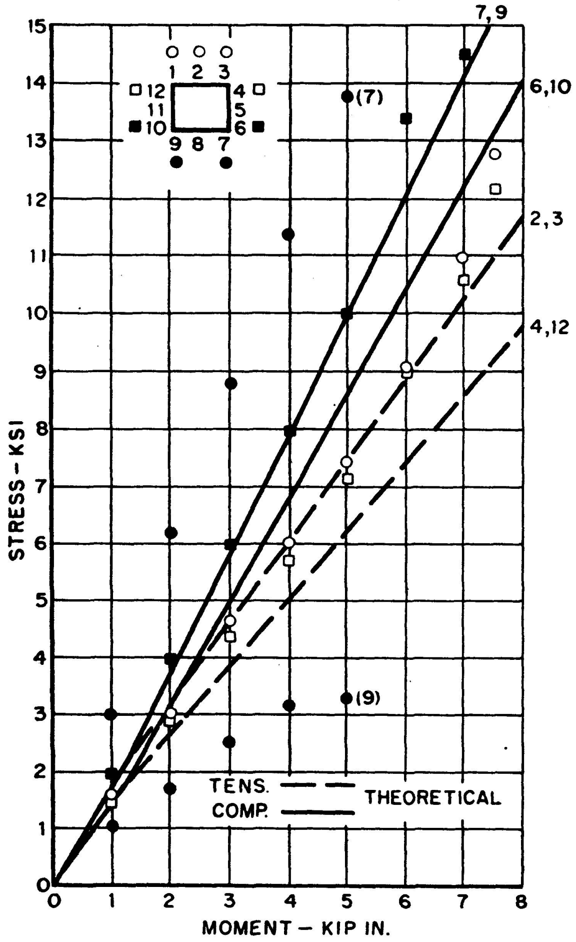

Fig. 9. Actual and theoretical stress values. Riveted box beam, 0.02 in. plate thickness. Compression gages: 7, 8, 9.

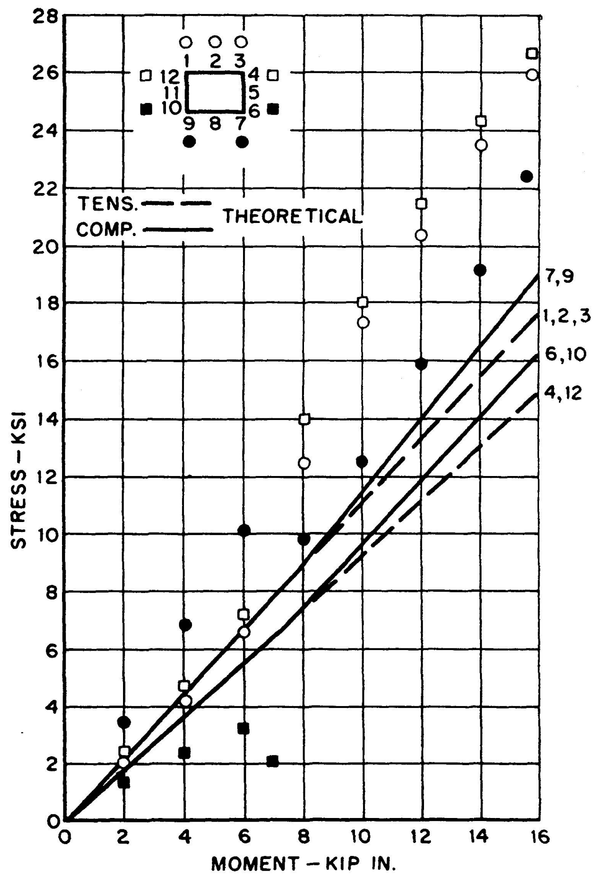

Fig. 10. Actual and theoretical stress values. Riveted box beam, 0.04 in. plate thickness. Compression gages: 7, 8, 9.

The general distribution of the stresses is shown schematically on fig. 6. The distribution of the web stresses was assumed to be linear, which was also proved to be justifiable by other tests [6, 7].

The actual stresses as they increased with the moment are compared on figs. 7—10 to the theoretical relationship.

In order to establish the theoretical lines for the particular case the following assumptions were used:

a) The buckling stress of the compressed plate is given by EGGWERTZ's results [5].

b) No inelastic action occurred.

c) The buckling cross sectional area A_0 in the case of the cemented boxes was the area of the compressed plate between the webs. In the case of the riveted boxes the buckling area is the area of the compressed plating between the center lines of the rivets.

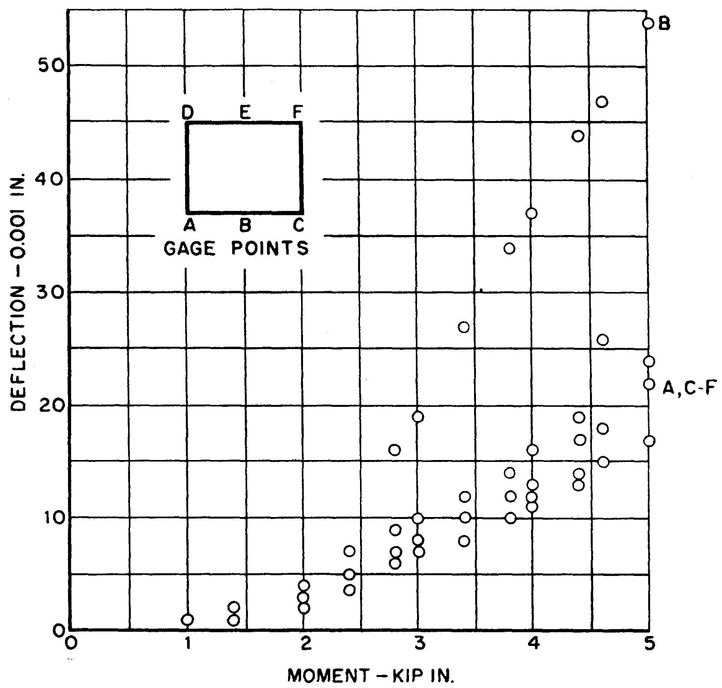

Fig. 11. Deflections at the midspan of the riveted box beam, 0.02 in. plate thickness. Compression on A, B, C.

It is fully recognized that these assumptions are arbitrary and they were chosen before even all the experiments were run. A slight modification of the assumptions could actually result in a much better agreement between computed and measured values. This, however, was not attempted. We think a very general agreement of the main trends of stress moment relations is enough to show, that the way to improvement and expansion of the process is open.

The experimental results show indeed, that the relation is really very closely linear, if there is a deviation, it indeed can be broken into two straight lines, the intersection of which actually falls in the vicinity of a probable buckling stress in the compressed plate. The actual buckling of the compressed plate was always observed in that vicinity. Thus computations based on eq. (9) approach more closely the actual situation than either the plain flexure formula taking no account of the local buckling or the plain flexure formula by dropping the whole compressed plate area from the useful cross section.

In thin walled boxes probably the maximum load carrying capacity of the structure can not be utilized anyhow, since considerations of local deformations might be the determining factor.

Plots on fig. 11 and 12 show the measured moment-deflection relations of the riveted models. The same basic fact can be pointed out, that the relation follows the general pattern of a two straight line relation. The only exception is, of course, the local deflection of the gage point located on the locally buckling compressed plate.

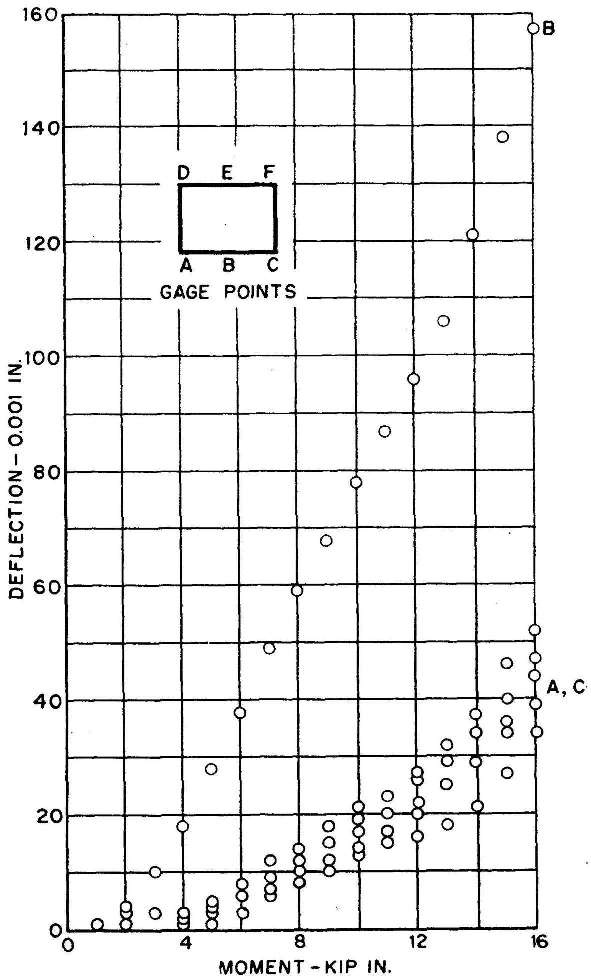

Fig. 12. Deflections at the midspan of the riveted box beam, 0.04 in. plate thickness. Compression on *A*, *B*, *C*.

V.

The adapted flexure formula seems to agree satisfactorily with experimental results and could actually be used to describe the stress-moment relation in thin walled boxes under pure bending. The analysis of this type of structure must always be based on a set of well-chosen assumptions. The formula appears to be a good frame in which the results of an improved knowledge of the buckling and postbuckling behavior of the compressed plates can fit later, thus leading to a more accurate and simple approximative description of the general behavior of a box girder.

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Summary

A general formula is presented for the stress-bending moment relation in pure bending of sections, parts of which yield or buckle.

The formula is applied in the case of a thin walled box beam. The values so obtained are compared with experimental results and a satisfactory agreement of theory and experience is shown.

Résumé

Les auteurs indiquent une formule générale pour le rapport entre la tension et le moment fléchissant, dans le cas du travail en flexion pure, pour des sections dont certaines parties se trouvent soumises au voile ou à l'écoulement.

Cette formule est appliquée au cas d'une poutre en caisson à parois minces. La comparaison entre les valeurs calculées et les valeurs déterminées par des essais met en évidence une concordance satisfaisante entre la théorie et l'expérimentation.

Zusammenfassung

Eine allgemeine Formel für das Verhältnis von Spannung zu Biegungsmoment bei reiner Biegung wird für Querschnitte gezeigt, von welchen sich Teile im Fließ- oder Beulzustand befinden.

Diese Formel wird auf den Fall eines dünnwandigen Kastenträgers angewendet. Im Vergleich der rechnerisch erhaltenen Werte mit den durch Versuche ermittelten, ergibt sich eine befriedigende Übereinstimmung zwischen Theorie und Versuch.

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