Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 15 (1955)

Artikel: Design and construction of reinforced concrete shell structure of non-uniform thickness supported on roller system

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DOI: https://doi.org/10.5169/seals-14498

Design and Construction of Reinforced Concrete Shell Structure of Non-Uniform Thickness Supported on Roller System

Etude et construction d'un ouvrage en voûte mince en béton armé, d'épaisseur non uniforme, supporté par un système de galets

Berechnung und Konstruktion einer beweglich gelagerten Eisenbetonschale von veränderlicher Dicke

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PART I. DESIGN

Synopsis

The structure described in this report is a multi-purpose building which has the characters of gymnasium, auditorium, concert hall and exhibition gallery and was built at Matsuyama in Japan in 1954. The building plan consists of the main area of about 1,000 m^2, the stand of 1,400 seats and other areas (projection booth and broadcasting room, etc.). The floor plan is a circle of 50 m diameter. The radius of the shell covering this floor is 50 m and the rise is 6.70 m. The height from the ground to the crown of the shell is 14 m. The total weight of the shell roof is 1,500 tons.

The shell is supported by steel rollers on 20 columns. The thickness of the shell is constant 12 cm in the central main part but variable gradually in the circumferential part up to more than 70 cm at the edge.

The author's object is to investigate an analysis based on the variable thickness for more general case. We believe that shell structure should be analysed even if the membrane stress condition is not established. We consider the sub-structure which supports the shell separately from the super shell-structure, so the analysis also should be proceeded individually and the suitable separation and supporting method should be provided not to give any thrust from the shell to the sub-structure. We are very glad to have agreed with the

Fig. 1. View from the hill of Matsuyama Castle

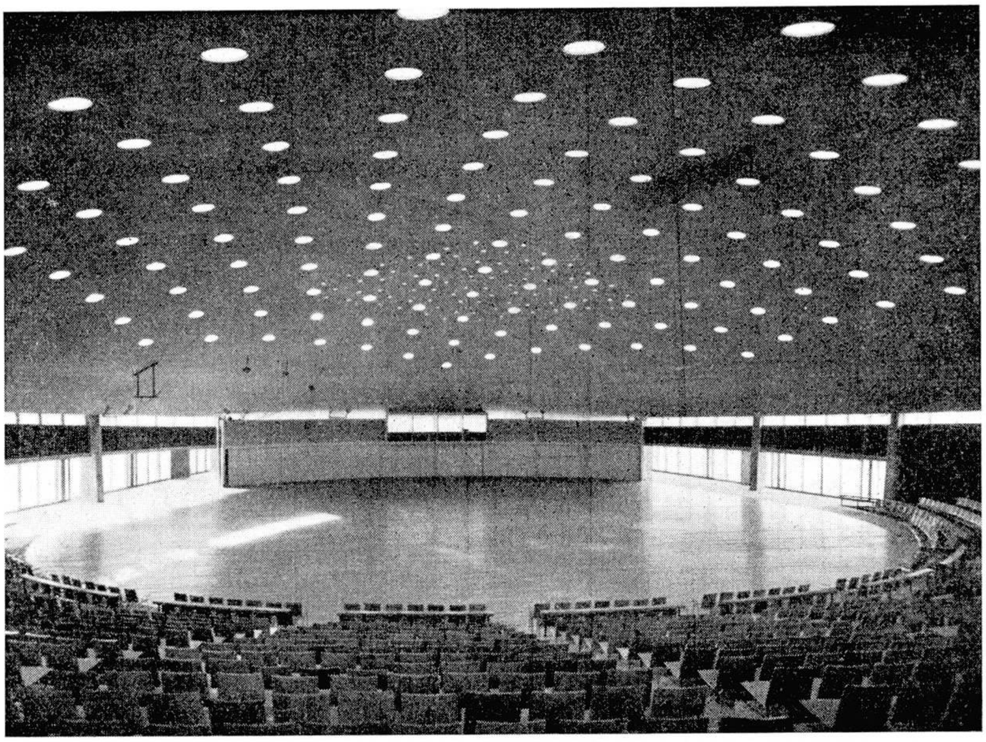

Fig. 2. Inside view (wooden floor, seats and skylights)

architect Kenzo Tange's[1]) idea, who was in charge of the architectural design.

In our investigation the bending stresses are not secondary but to be computed strictly, and the thermal difference and the seismic force also should be taken into account besides the vertical loading. To perform this analysis,

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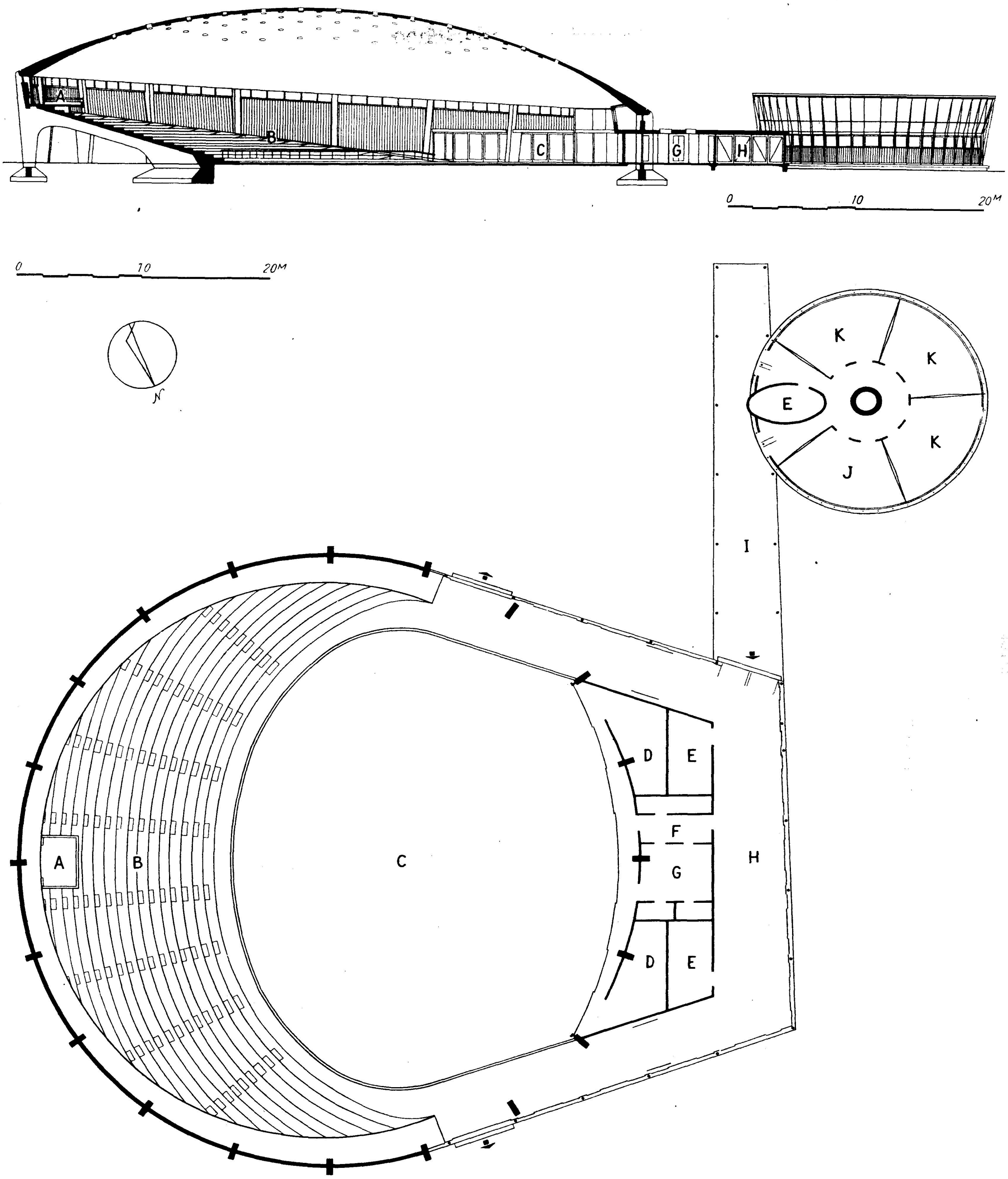


Fig. 3. Plan and section.

A Projection booth. *B* Stands. *C* Multi-purpose floor. *D* Storage. *E* Lavatory. *F* Preparation. *G* Locker room. *H* Lobby. *I* Covered walk. *J* Council room. *K* Office room.

we will discuss in this paper an advantageous generalized analytical method for a flat dome submitted to various loads. The results are so satisfactory that we can apply it to general cases. The thickness of the shell is variable in the circumferential part, so we introduce an approximation for this non-uniform thickness part. We will check that the values of bending stresses are rather small and converge fairly quickly.

Besides the theoretical analysis, we have undertaken experimental investigations using a model of 1/20 scale made from mortar and wire reinforcement.

For the support system of the shell on the columns, rollers were considered to eliminate the cracks which may occur because of displacement after removal of the forms if the shell is fixed to the columns, but we may allow larger bending stresses in the shell. After the forms are removed and the displacement is completely set, we fill up the roller supporting system with dry pack.

Part I consists of the following articles.

1. General analytical method for flat dome.
2. Boundary conditions and applications for the actual design.
3. Analysis of spherical shell of non-uniform thickness.

1. General Analytical Method for Flat Dome

I. Introduction

Today there is no novelty in expressing an analytical method for the spherical shell, which has the simplest form of any surface of revolution, but any analytical expression which has been developed involving not only membrane forces but bending forces is limited to problems symmetrical with respect to the axis of rotation. For the case of symmetrical loading and uniform change of temperature we can state that the results may be satisfactorily applied to an actual design. What is desired now, however, is to develop a method which is also applicable to any unsymmetrical loading case because in practical condition the load may act entirely arbitrarily.

The equation which is derived for the above mentioned purpose is of course a partial differential equation of higher order because one of the variables is φ (meridian direction) and the other is θ (latitude direction). Accordingly, the solution becomes much complicated. Whatever efforts we make, it would be quite difficult to find the accurate solution. That is the reason why we proceed to an approximation in which we simplify the expression neglecting some terms.

In this paper an approximate method which is applicable to only a comparatively flat spherical shell, is discussed. Applying this method to rather high dome, we have found that the value of bending stress is larger than estimated.

We may consider any arbitrary boundary and loading conditions for shells, for instance, own weight, thermal difference (uniform variable radially), snow load at one side and seismic force. We shall derive the solution (membrane stresses and bending stresses) for those prescribed conditions. It may be very advantageous to provide some tables of functions at hand, the solution being given in Bessel functions. Otherwise, our numerical computation becomes very cumbersome. In case of unsymmetrical loading the solution consists of Bessel functions of higher order and the values of these functions have never yet been calculated, and so for simplification we will use only the first term of Fourier series for the unsymmetrical load.

In the following discussion the fundamental equation will first be derived from the equilibrium conditions of forces and the compatible relationships between stresses, strains and displacements. Then the equation will be conformed to our structure and the stresses will be calculated. The first fundamental equation is applicable only to the spherical shell of uniform thickness, but actually we have a significantly thicker circumferential part, so we have to consider the effect on the stress calculation due to this variation of the thickness. We proceed to find the bending stress depending on the variation of the intensity of own weight in the circumferential part of the shell with roller support condition. Moreover, we shall deduce the stresses due to the thermal difference and the symmetrical and unsymmetrical loading and obtain the more advantageous results for more general cases. We go further to investigate the effect due to openings in the shell top but we check that the circumferential condition is not affected because the openings are located fairly far from the edge.

II. Derivation of the fundamental equation

We begin with the consideration of the equilibrium condition of a shell having the form of a surface of revolution. We take x axis tangentially to the latitude, y axis tangentially to the meridian and z axis normal to the surface radially. The signs of coordinates and stresses are indicated in fig. 4.

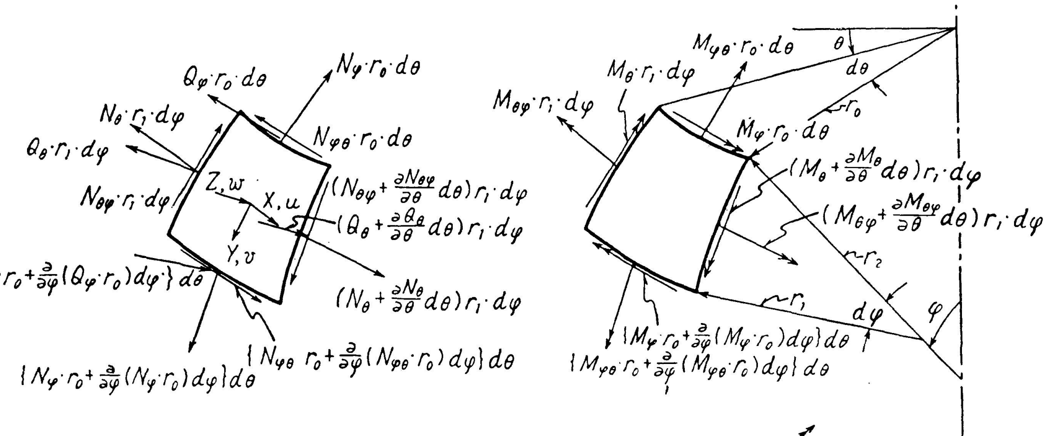


Fig. 4.

The equilibrium of an element which is cut from the shell by two infinitesimal adjacent meridian and latitude planes gives three equations of summation of projections of forces and three equations of moments with respect to those axis. Hence we obtain the following system of six equations which represent the equilibrium conditions.

$$\begin{aligned}
&\frac{\partial}{\partial \varphi}(r_0 N_{\varphi\theta}) + r_1 \frac{\partial N_\theta}{\partial \theta} + r_1 N_{\theta\varphi}\cos\varphi - r_1 Q_\theta \sin\varphi + r_0 r_1 X = 0 \\
&\frac{\partial}{\partial \varphi}(r_0 N_\varphi) + r_1 \frac{\partial N_{\theta\varphi}}{\partial \theta} - r_1 N_\theta \cos\varphi - r_0 Q_\varphi + r r_1 Y = 0 \\
&r_1 N_\theta \sin\varphi + r_0 N_\varphi + r_1 \frac{\partial Q_\theta}{\partial \theta} + \frac{\partial}{\partial \varphi}(r_0 Q_\varphi) + r_0 r_1 Z = 0 \\
&\frac{\partial}{\partial \varphi}(r_0 M_\varphi) - r_1 \frac{\partial M_{\theta\varphi}}{\partial \theta} - r_1 M_\theta \cos\varphi - r_0 r_1 Q_\varphi = 0 \\
&r_1 \frac{\partial M_\theta}{\partial \theta} + \frac{\partial}{\partial \varphi}(r_0 M_{\varphi\theta}) - M_{\theta\varphi} r_1 \cos\varphi - r_0 r_1 Q_\theta = 0 \\
&\frac{M_{\theta\varphi}}{r_2} + \frac{M_{\varphi\theta}}{r_1} = N_{\varphi\theta} - N_{\theta\varphi}
\end{aligned} \tag{1.1}$$

In these six equations there are ten unknowns; two normal forces (N_φ and N_θ), two shear forces ($N_{\varphi\theta}$ and $N_{\theta\varphi}$), two bending moments (M_φ and M_θ), two torsional moments ($M_{\varphi\theta}$ and $M_{\theta\varphi}$) and two transverse (shear) forces (Q_φ and Q_θ). We use the summary designation direct (or membrane) forces for these normal and shear forces, and bending forces for these bending and torsional moments and transverse forces. We take a meridian plane as the basic plane of rotation and a latitude plane perpendicular to that meridian plane and the corresponding radii of curvature of these planes are denoted by r_1 and r_2, respectively. X, Y and Z are the components of the intensity of the load.

Since we limit the problem only to spherical shell, $r_1 = r_2$ and $M_{\varphi\theta} = -M_{\theta\varphi}$ because $N_{\varphi\theta} = N_{\theta\varphi}$. Next, we shall introduce the most important condition in which the shell is a flat dome. Hence,

$$\begin{aligned}
&r_1 = r_2 = a & &r_0 = r_2 \sin\varphi \\
&\sin\varphi \approx \varphi, & &\cos\varphi \approx 1 \\
&N_{\theta\varphi} = N_{\varphi\theta}, & &M_{\theta\varphi} = -M_{\varphi\theta}
\end{aligned} \tag{1.2}$$

using eqs. (1.2), eqs. (1.1) are simplified as follows:

$$\begin{aligned}
&\frac{\partial N_{\theta\varphi}}{\partial \theta} + \frac{2}{\varphi} N_{\theta\varphi} + \frac{1}{\varphi}\frac{\partial N_\theta}{\partial \theta} - Q_\theta + aX = 0 \\
&\frac{\partial N_\varphi}{\partial \varphi} + \frac{1}{\varphi} N_\varphi + \frac{1}{\varphi}\frac{\partial N_{\theta\varphi}}{\partial \theta} - \frac{1}{\varphi} N_\theta - Q_\varphi + aY = 0 \\
&N_\theta + N_\varphi + \frac{1}{\varphi}\frac{\partial Q_\theta}{\partial \theta} + \frac{\partial Q_\varphi}{\partial \varphi} + \frac{1}{\varphi} Q_\varphi + aZ = 0
\end{aligned} \tag{1.3}$$

$$\frac{\partial M_\varphi}{\partial \varphi}+\frac{1}{\varphi}M_\varphi-\frac{1}{\varphi}\frac{\partial M_{\theta\varphi}}{\partial \theta}-\frac{1}{\varphi}M_\theta-a\,Q_\varphi=0$$
$$\frac{1}{\varphi}\frac{\partial M_\theta}{\partial \theta}+\frac{\partial M_{\theta\varphi}}{\partial \varphi}-\frac{2}{\varphi}M_{\theta\varphi}-a\,Q_\varphi=0 \tag{1.3}$$

In our problem the thickness of the shell is very small as compared with the radius of curvature. So, in eqs. (1.3) we neglect the terms of transverse forces because the effective values are small. Thus we take Q_φ and Q_θ equal to zero in the first two equations. Finally eqs. (1.3) becomes

$$\varphi\frac{\partial N_{\theta\varphi}}{\partial \varphi}+2\,N_{\theta\varphi}+\frac{\partial N_\theta}{\partial \theta}+a\,X\,\varphi=0 \qquad Q_\theta\to 0$$
$$\varphi\frac{\partial N_\varphi}{\partial \varphi}+\frac{\partial N_{\theta\varphi}}{\partial \theta}+(N_\varphi-N_\theta)+a\,Y\,\varphi=0 \qquad Q_\varphi\to 0$$
$$(N_\theta+N_\varphi)+\frac{1}{a}\left\{\left(\frac{\partial^2}{\partial\varphi^2}+\frac{2}{\varphi}\frac{\partial}{\partial\varphi}\right)M_\varphi-\left(\frac{1}{\varphi}\frac{\partial}{\partial\varphi}-\frac{1}{\varphi^2}\frac{\partial^2}{\partial\theta^2}\right)M_\theta-\right.$$
$$\left.-2\left(\frac{1}{\varphi}\frac{\partial^2}{\partial\varphi\,\partial\theta}+\frac{1}{\varphi^2}\frac{\partial}{\partial\theta}\right)M_{\theta\varphi}\right\}+a\,Z=0 \tag{1.4}$$

III. Load

It is very advantageous to define the load as a potential function of φ and θ for more general description. The potential function is given by the product of unit load and distance. We denote the loading conditions as below using such a function. For uniformly distributed vertical load (i.e. own weight).

$$\Omega^{(V)}=p\,a\cos\varphi$$
$$X^{(V)}=-\frac{1}{r_0}\frac{\partial\Omega^{(V)}}{\partial\theta}=0$$
$$Y^{(V)}=-\frac{1}{r_1}\frac{\partial\Omega^{(V)}}{\partial\varphi}=p\sin\varphi\approx p\,\varphi \tag{1.5a}$$
$$Z^{(V)}=-\frac{1}{r_1}\frac{\partial\Omega^{(V)}}{\partial\varphi}\cot\varphi=p\cos\varphi\approx p$$

For uniformly distributed horizontal load (i.e. seismic force).

$$\Omega^{(H)}=q\,r_0\cos\theta=q\,a\sin\varphi\cos\theta$$
$$X^{(H)}=-\frac{1}{r_0}\frac{\partial\Omega^{(H)}}{\partial\theta}=q\sin\theta$$
$$Y^{(H)}=-\frac{1}{r_1}\frac{\partial\Omega^{(H)}}{\partial\varphi}=-q\cos\varphi\cos\theta\approx-q\cos\theta \tag{1.5b}$$
$$Z^{(H)}=-\frac{1}{r_1}\frac{\partial\Omega^{(H)}}{\partial\varphi}(-\tan\varphi)=q\sin\varphi\cos\theta\approx q\,\varphi\cos\theta$$

IV. Stress function

In two-dimensional analysis it is well known that the stress function can be used and it satisfies the equations of equilibrium. In case of plane stress distribution the potential function also is introduced into those equations. From the first two eqs. of (1.4) the direct forces become

$$\begin{aligned} N_\varphi &= \frac{1}{\varphi}\frac{\partial \Phi}{\partial \varphi} + \frac{1}{\varphi^2}\frac{\partial^2 \Phi}{\partial \varphi} + \Omega \\ N_\theta &= \frac{\partial^2 \Phi}{\partial \theta^2} + \Omega \\ N_{\theta\varphi} &= \frac{1}{\varphi^2}\frac{\partial \Phi}{\partial \theta} - \frac{1}{\varphi}\frac{\partial^2 \Phi}{\partial \varphi\, \partial \theta} \end{aligned} \tag{1.6}$$

Substituting eqs. (1.6) in the last eq. of eq. (1.4), we obtain

$$\begin{aligned} \Delta\Phi + \frac{1}{a}\Bigg\{ \left(\frac{\partial^2}{\partial \varphi^2} + \frac{2}{\varphi}\frac{\partial}{\partial \varphi}\right) M\varphi - \left(\frac{1}{\varphi}\frac{\partial}{\partial \varphi} - \frac{1}{\varphi^2}\frac{\partial^2}{\partial \theta^2}\right) M_\theta - \\ -2\left(\frac{1}{\varphi}\frac{\partial^2}{\partial \varphi\, \partial \theta} + \frac{1}{\varphi^2}\frac{\partial}{\partial \theta}\right) M_{\theta\varphi}\Bigg\} + (2\Omega + aZ) = 0 \end{aligned} \tag{1.7}$$

where $$\Delta = \frac{\partial^2}{\partial \varphi^2} + \frac{1}{\varphi}\frac{\partial}{\partial \varphi} + \frac{1}{\varphi^2}\frac{\partial^2}{\partial \theta^2}$$

This is the final equation which involves the stress function developed from the previously mentioned equation of equilibrium.

V. Relationships between stresses, strains and displacements

In the design of shells, we want to find not only membrane stresses but also bending stresses, introducting the relations between stresses, strains and displacements. Our problem is statically indeterminate and we can not find the stresses by the equilibrium conditions only, as it is same as in the two dimensional problem of elasticity.

We begin with the introduction of two assumptions for the purpose of simplification of the equations. The first one is that the deformation is so small that we can neglect the effect due to the deflection and the second one is that the deformation due to the transverse forces Q_φ and Q_θ is negligible. The latter assumption is equivalent to stating that during bending of a shell the linear elements normal to the middle surface remain straight and become normal to the deformed middle surface. This is called Bernoulli's law.

In considering the changes of the curvature, besides the above assumptions we may neglect the displacements in the tangential directions. In other words those displacements are independent of the bending stresses, so we finally get the following relationships:

$$
\begin{aligned}
\chi_\varphi &= \frac{1}{a^2}\frac{\partial^2 w}{\partial \varphi^2} \\
\chi_\theta &= \frac{1}{a^2}\left\{\frac{1}{\varphi}\frac{\partial w}{\partial \varphi}+\frac{1}{\varphi^2}\frac{\partial^2 w}{\partial \theta^2}\right\} \\
\chi_{\theta\varphi} &= \frac{1}{a^2}\left\{\frac{1}{\varphi}\frac{\partial^2 w}{\partial \varphi\,\partial \theta}-\frac{1}{\varphi^2}\frac{\partial w}{\partial \varphi}\right\}
\end{aligned}
\tag{1.8}
$$

Then the relations between the bending stresses and the changes of curvatures are as follows:

$$
\begin{aligned}
M_\varphi &= -D(\chi_\varphi+\nu\chi_\theta) \\
M_\theta &= -D(\chi_\theta+\nu\chi_\varphi) \qquad D=\frac{E t^3}{12(1-\nu^2)} \\
M_{\theta\varphi} &= D(1-\nu)\,\chi_{\theta\varphi}
\end{aligned}
\tag{1.9}
$$

D is called the flexural rigidity.

The relations between stresses, strains and displacements are a little complicated. We assume that the deformation is so small that we may handle this case the same as previously. The effect of strain in the middle surface on curvature is neglected. Finally, the relations between the strains and the displacements are as following:

$$
\begin{aligned}
\epsilon_\varphi &= \frac{1}{a}\left\{\frac{\partial v}{\partial \varphi}-w\right\} \\
\epsilon_\theta &= \frac{1}{a}\left\{\frac{1}{\varphi}\frac{\partial u}{\partial \theta}+\frac{v}{\varphi}-w\right\} \\
\gamma_{\theta\varphi} &= \frac{1}{a}\left\{\frac{\partial u}{\partial \varphi}-\frac{u}{\varphi}+\frac{1}{\varphi}\frac{\partial v}{\partial \theta}\right\}
\end{aligned}
\tag{1.10}
$$

where, u, v and w denote the displacements in the x, y and z directions, respectively.

From the above equations we can derive the next relation in which these strains are connected together by remaining w.

$$
\left(\frac{1}{\varphi}\frac{\partial^2}{\partial \theta^2}-\frac{1}{\varphi}\frac{\partial}{\partial \varphi}\right)\epsilon_\varphi+\frac{1}{\varphi}\frac{\partial^2}{\partial \varphi^2}(\varphi\,\epsilon_\theta)-\frac{1}{\varphi^2}\frac{\partial^2}{\partial \varphi\,\partial \theta}(\varphi\,\gamma_{\theta\varphi})+\frac{1}{a}\Delta w=0 \tag{1.11}
$$

This is called the condition of compatibility of the strains. Consequently, the relations between the strains and the direct forces including the thermal difference are as follows:

$$
\begin{aligned}
\epsilon_\varphi &= \frac{1}{Et}\{N_\varphi-\nu N_\theta\}+\alpha T \\
\epsilon_\theta &= \frac{1}{Et}\{N_\theta-\nu N_\varphi\}+\alpha T \\
\gamma_{\theta\varphi} &= \frac{2(1+\nu)}{Et}N_{\theta\varphi}
\end{aligned}
\tag{1.12}
$$

where, α is the coefficient of expansion of reinforced concrete and T is the function representing the difference of temperature.

VI. Differential equation and solution

The equation which is now established finally is still a general expression and involves the initial stresses.

Substituting eq. (1.8) into eq. (1.9) and eq. (1.9) into eq. (1.7), we find

$$\varDelta\Phi-\varDelta\varDelta w\frac{D}{a^3}+(2\Omega+aZ)=0 \tag{1.13}$$

Substituting eq. (1.6) into eq. (1.12) and eq. (1.12) into eq. (1.11), we get

$$\varDelta\varDelta\Phi+\frac{Et}{a}\varDelta w+(1-\nu)\varDelta\Omega+Et\varDelta(\alpha T)=0$$

$$\text{where}\quad \varDelta\varDelta\Phi=\left(\frac{\partial^2}{\partial\varphi^2}+\frac{1}{\varphi}\frac{\partial}{\partial\varphi}+\frac{1}{\varphi^2}\frac{\partial^2}{\partial\theta^2}\right)\left(\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{1}{\varphi}\frac{\partial\Phi}{\partial\varphi}+\frac{1}{\varphi^2}\frac{\partial^2\Phi}{\partial\theta^2}\right) \tag{1.14}$$

For the case of the unsymmetrical problem, the next solution is obtained.

$$\varDelta\Phi=-\left\{\frac{Et}{a}w+(1-\nu)\Omega+Et(\alpha T)\right\}+C_1\varphi+\frac{C_2}{\varphi} \tag{1.15}$$

From eqs. (1.13) and (1.14)

$$\varDelta\varDelta w+\beta^4 w-\frac{a^3}{D}\left\{(1+\nu)\Omega+aZ+C_1\varphi+\frac{C_2}{\varphi}\right\}+\beta^4 a(\alpha T)=0$$

$$\text{where}\qquad \beta^4=12(1-\nu^2)\frac{a^2}{t^2} \tag{1.16}$$

To get the solution of eq. (1.16), we introduce the complementary function of w, which is denoted by

$$\varDelta\varDelta w_c-\beta^4 w_c=0 \ \text{ or } \ \varDelta w_c\pm i\beta^2 w_c=0$$

a) In case $w_c=w_c(\varphi)$ i.e. the displacement w is the function of φ alone, eq. (1.16) is equivalent to the following:

$$\frac{d^2 w_c}{d\varphi^2}+\frac{1}{\varphi}\frac{dw_c}{d\varphi}\pm i\beta^2 w_c=0 \tag{1.17}$$

If we take $\xi=\beta\varphi$, the solution of eq. (1.17) is a modified Bessel function of zero order. Hence,

$$w_c=A\,\mathrm{ber}\,\xi+B\,\mathrm{bei}\,\xi+C\,\mathrm{ker}\,\xi+D\,\mathrm{kei}\,\xi \tag{1.18}$$

b) $w_c=\zeta\cos\theta,\ \zeta=\zeta(\varphi)$.

Now, instead of eq. (1.17), we find

$$\frac{d^2\zeta}{d\xi^2}+\frac{2}{\xi}\frac{d\zeta}{d\xi}+\left(\pm i-\frac{1}{\xi^2}\right)\zeta=0 \tag{1.19}$$

If we take the positive sign of i, ξ is replaced by x i.e.

$$\xi=i^{3/2}x=i^{-1/2}x.$$

Hence,
$$\frac{d^2\zeta}{dx^2}+\frac{1}{x}\frac{d\zeta}{dx}+\left(1-\frac{1}{x^2}\right)\zeta=0$$

and then
$$\zeta=J_1(x)+Y_1(x)=J_1(i^{-3/2}\xi)+K_1(i^{1/2}\xi)$$

next, taking the negative sign of i and by differentiation of the zero order we have the Bessel function of the first order and finally we obtain the following:

$$\zeta=C_1\frac{d}{d\xi}\{J_0(i^{-3/2}\xi)\}+C_2\frac{d}{d\xi}\{K_0(i^{1/2}\xi)\}+ \\ +C_3\frac{d}{d\xi}\{J_0(i^{3/2}\xi)\}+C_4\frac{d}{d\xi}\{K_0(i^{-1/2}\xi)\} \tag{1.20}$$

Consequently, having the particular solution, w can be represented in the following form:

$$w=\zeta\cos\theta+w_p=\left[A\frac{d}{d\xi}(\mathrm{ber}\,\xi)+B\frac{d}{d\xi}(\mathrm{bei}\,\xi)+C\frac{d}{d\xi}(\mathrm{ker}\,\xi)+D\frac{d}{d\xi}(\mathrm{kei}\,\xi)\right]\cdot \\ \cdot\cos\theta+\left\{(1+\nu)\Omega+aZ+C_1\varphi+\frac{C_2}{\varphi}\right\}\frac{a}{Et}-a\alpha T \tag{1.21}$$

w is now determined from the above, and we can obtain the stress function Φ as follows:

$$\Delta\Phi+\frac{Et}{a}w=0 \tag{1.22}$$

If we set
$$\frac{Et}{a}w_c=\pm i\Phi_p$$

we obtain
$$\Delta\Phi_p\pm i\Phi_p=0 \tag{1.23}$$

Φ_p is the particular solution of the stress function Φ. Eq. (1.23) is equivalent to eq. (1.19) and then we have

$$\Phi_p=\left[C_1\frac{d}{d\xi}(\mathrm{ber}\,\xi)+C_2\frac{d}{d\xi}(\mathrm{bei}\,\xi)+C_3\frac{d}{d\xi}(\mathrm{ker}\,\xi)+C_4\frac{d}{d\xi}(\mathrm{kei}\,\xi)\right]\cos\theta$$

Taking $C_1=\dfrac{Et}{a\beta^2}B,\quad C_2=-\dfrac{Et}{a\beta^2}A,\ \cdots\cdots$ etc., Φ_p is combined with w_c.

Finally it may be concluded that the following formulas are the desired solutions in which the superscript (s) indicates the solution for the symmetrical case and the superscript (i) indicates the one for the unsymmetrical case including the seismic force and the thermal difference.

$$\begin{aligned} w^{(s)}&=A\,\mathrm{ber}\,\xi+B\,\mathrm{bei}\,\xi+C\,\mathrm{ker}\,\xi+D\,\mathrm{kei}\,\xi+ \\ &\quad+(2+\nu)\frac{pa^2}{Et}-a\alpha T_0+K_1\log\varphi+K_2 \\ \Phi^{(s)}&=\{B\,\mathrm{ber}\,\xi-A\,\mathrm{bei}\,\xi+D\,\mathrm{ker}\,\xi-C\,\mathrm{kei}\,\xi\}\frac{Et}{a\beta^2}- \\ &\quad-\frac{3}{4}pa\varphi^2+K_3\log\varphi+K_4 \end{aligned} \tag{1.24}$$

where $T=T_0=\text{const.}$

$$w^{(i)} = \Big[\bar{A}\,\mathrm{ber}'\xi + \bar{B}\,\mathrm{bei}'\xi + \bar{C}\,\mathrm{ker}'\xi + \bar{D}\,\mathrm{kei}'\xi +$$

$$+ (2+\nu)\frac{q\,a^2\varphi}{E\,t} - a\,\alpha\,T_0\left(\frac{\varphi}{\varphi_0}\right) + \bar{K}_1\varphi + \frac{\bar{K}_2}{\varphi}\Big]\cos\theta$$

$$\Phi^{(i)} = \Big[\{\bar{B}\,\mathrm{ber}'\xi - \bar{A}\,\mathrm{bei}'\xi + \bar{D}\,\mathrm{ker}'\xi - \bar{C}\,\mathrm{kei}'\xi\}\frac{E\,t}{a\,\beta^2} - \tag{1.25}$$

$$-\frac{3}{8}q\,a\,\varphi^3 + \bar{K}_3\varphi + \bar{K}_4\Big]\cos\theta$$

where $T = T_0\left(\dfrac{\varphi}{\varphi_0}\right)\cos\theta,\quad \mathrm{ber}'\xi = \dfrac{d}{d\xi}\mathrm{ber}\,\xi$, etc.

In the above equations $A, B, \cdots\cdots, \bar{A}, \bar{B}, \cdots\cdots, K_1, K_2, \cdots\cdots, \bar{K}_1, \bar{K}_2, \cdots\cdots$ are unknown constants which may be determined by the boundary conditions.

2. Boundary Conditions and Application to the Actual Design

I. Introduction

The structure which is discussed here is an inclined spherical shell of 50 m radius covering a circular plan of 25 m radius, and although it may be not quite right to say that this is a fully flat dome the character of this analysis has a fairly extensive application. To obtain further accuracy, we shall note the comparison between this analysis and other methods in the next section.

The circumferential part is quite thick and has much reinforcement, so the shell is fairly rigid all around. Here is our justification for thinking that the circumferential edge acts partly as a tension ring besides the shell action. It is quite difficult of course to prove this reality by analytical methods in conformity with the above hypothesis. Therefore we assume that the shell is simply supported.

In this paragraph are denoted the following notations and dimensions:

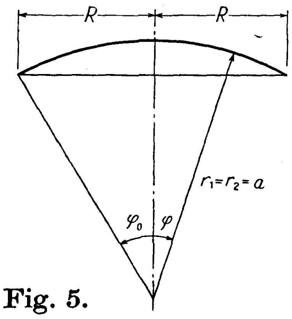


Fig. 5.

$p = 0.5\ \mathrm{t/m^2}$:	Own weight.
$q = 0.1\ \mathrm{t/m^2}$:	Seismic force.
$W = 1{,}500\ \mathrm{t}$:	Total weight of the shell.
$a = 50\ \mathrm{m}$:	Radius of shell roof.
$R = 25\ \mathrm{m}$:	Radius of circle plan.
$\varphi_0 = 30° = 0.5236$:	Angle subtended by the edges of shell.
$t = 0.12\ \mathrm{m}$:	Thickness.

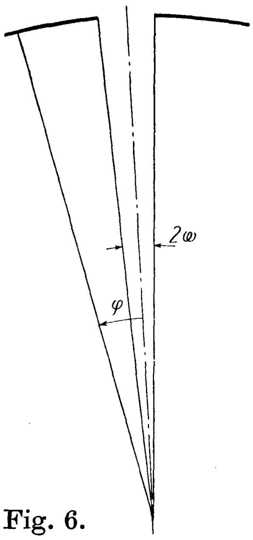


Fig. 6.

β	$= 37.606$:	Coefficient.
E	$= 2.1 \cdot 10^6\,\mathrm{t/m^2}$:	Young's modulus.
ν	$= 0.2$:	Poisson's ratio.
α	$= 1.2 \cdot 10^{-5}$:	Coefficient of thermal expansion of concrete.
T	$= 20°$:	Maximum amount of change for uniform difference of temperature.
T	$= 10°$:	Maximum amount of change for unsymmetrical difference of temperature.
ω	$= 0.006$:	2ω is the angle subtended by the edges of the opening.

II. For the case under the action of own weight, simply supported at the edge

The general solution of the deflection is as follows, having only the terms which converge to zero at $\zeta = 0$.

$$w = \{C_1 \operatorname{ber} \xi + C_2 \operatorname{bei} \xi\} + p\,(2+\nu)\,\frac{a^2}{E\,t} \tag{2.1}$$

By the differentiation of the above equation, we obtain

$$\begin{aligned}
M_\varphi &= -D\,\frac{\beta^2}{a^2}\left\{C_1\left[\operatorname{bei}\xi - (1-\nu)\,\frac{1}{\xi}\,\operatorname{ber}'\xi\right] - C_2\left[\operatorname{ber}\xi + (1-\nu)\,\frac{1}{\xi}\,\operatorname{bei}'\xi\right]\right\}\\
M_\theta &= -D\,\frac{\beta^2}{a^2}\left\{C_1\left[(1-\nu)\,\frac{1}{\xi}\,\operatorname{ber}'\xi + \nu \operatorname{bei}\xi\right] + C_2\left[(1-\nu)\,\frac{1}{\xi}\,\operatorname{bei}'\xi - \nu\operatorname{ber}\xi\right]\right\}\\
Q_\varphi &= -\frac{D}{a^3}\left\{\frac{\partial^3}{\partial\varphi^3} + \frac{1}{\varphi}\,\frac{\partial^2}{\partial\varphi^2} - \frac{1}{\varphi^2}\,\frac{\partial}{\partial\varphi}\right\} w\\
&= -D\,\frac{\beta^3}{a^3}\left\{C_1\left(\operatorname{bei}'\xi - \frac{1}{\xi^2}\,\operatorname{ber}'\xi\right) - C_2\left(\operatorname{ber}'\xi - \frac{1}{\xi^2}\operatorname{bei}'\xi\right)\right\}
\end{aligned} \tag{2.2}$$

As the boundary conditions, we have

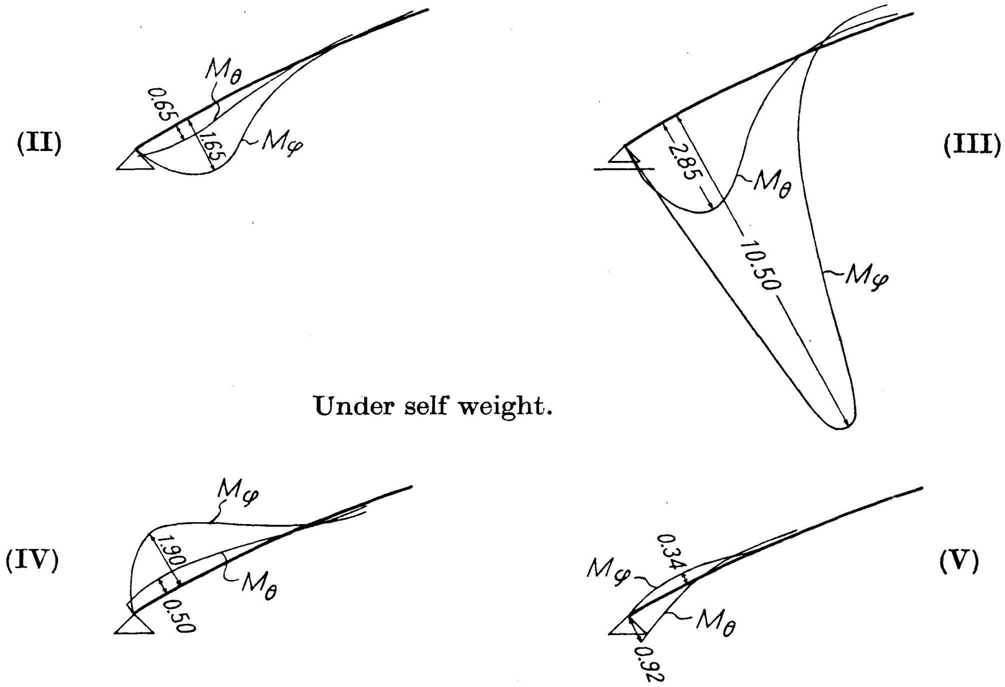
$$\begin{aligned}
&1. \qquad M_\varphi\,|_{\varphi=\varphi_0} = 0\\
&2. \qquad w\;|_{\varphi=\varphi_0} = 0
\end{aligned} \tag{2.3}$$

Hence,

$$C_1 = -\frac{p\,a^2}{E\,t}\,(2+\nu)\,\frac{1}{\operatorname{ber}(\varphi_0\beta) + \dfrac{\operatorname{bei}(\varphi_0\beta) - (1-\nu)\,\dfrac{1}{\varphi_0\beta}\,\operatorname{ber}'(\varphi_0\beta)}{\operatorname{ber}(\varphi_0\beta) + (1-\nu)\,\dfrac{1}{\varphi_0\beta}\,\operatorname{bei}'(\varphi_0\beta)}\,\operatorname{bei}(\varphi_0\beta)} \tag{2.4}$$

$$C_2 = -\frac{p\,a^2}{E\,t}(2+\nu)\frac{1}{\operatorname{bei}(\varphi_0\beta) + \dfrac{\operatorname{ber}(\varphi_0\beta)+(1-\nu)\dfrac{1}{\varphi_0\beta}\operatorname{bei}'(\varphi_0\beta)}{\operatorname{bei}(\varphi_0\beta)-(1-\nu)\dfrac{1}{\varphi_0\beta}\operatorname{ber}'(\varphi_0\beta)}\operatorname{ber}(\varphi_0\beta)} \tag{2.4}$$

The results are indicated in fig. 7.



For symmetrical case. For unsymmetrical case.

Under temperature difference.

Fig. 7. Bending stresses $(t\cdot m/m)$.

III. For the case under the action of own weight with roller support at the edge

As the boundary conditions, we have

$$\begin{aligned} &1. \qquad M_\varphi\,|_{\varphi=\varphi_0} = 0 \\ &2. \qquad Q_\varphi\,|_{\varphi=\varphi_0} = -\frac{W}{2R\pi}\cos\varphi_0 \end{aligned} \tag{2.5}$$

Using eqs. (2.2) and (2.5) and neglecting the terms of small quantities, we obtain

$$C_1 = \frac{W}{2R\pi}\,\frac{a\beta}{Et}\cdot \frac{\operatorname{ber}(\varphi_0\beta)+(1-\nu)\dfrac{1}{\varphi_0\beta}\operatorname{bei}'(\varphi_0\beta)}{\operatorname{ber}(\varphi_0\beta)\operatorname{bei}'(\varphi_0\beta)-\operatorname{ber}'(\varphi_0\beta)\operatorname{bei}(\varphi_0\beta)+(1-\nu)\dfrac{1}{\varphi_0\beta}\{\operatorname{ber}'^2(\varphi_0\beta)+\operatorname{bei}'^2(\varphi_0\beta)\}} \tag{2.6}$$

$$C_2 = \frac{W}{2R\pi}\frac{a\beta}{Et} \cdot \tag{2.6}$$

$$\cdot \frac{\text{bei}(\varphi_0\beta) - (1-\nu)\frac{1}{\varphi_0\beta}\text{ber}'(\varphi_0\beta)}{\text{ber}(\varphi_0\beta)\,\text{bei}'(\varphi_0\beta) - \text{ber}'(\varphi_0\beta)\,\text{bei}(\varphi_0\beta) + (1-\nu)\frac{1}{\varphi_0\beta}\{\text{ber}'^2(\varphi_0\beta) + \text{bei}'^2(\varphi_0\beta)\}}$$

The results are indicated in fig. 7.

IV. For the case under the action of uniform difference of temperature with simply supported edge

The general equation of deflection is as follows in place of eqs. (1.24)

$$w = \{C_1\,\text{ber}\,\xi + C_2\,\text{bei}\,\xi\} - a\cdot\alpha\cdot T_0 \tag{2.7}$$

The boundary conditions are

$$\begin{aligned} &1. \quad M_\varphi\big|_{\varphi=\varphi_0} = 0 \\ &2. \quad w\,\big|_{\varphi=\varphi_0} = 0 \end{aligned} \tag{2.8}$$

Hence, the constants of integration become

$$\begin{aligned} C_1 &= a\cdot\alpha\cdot T_0 \frac{1}{\text{ber}(\varphi_0\beta) + \dfrac{\text{bei}(\varphi_0\beta) - (1-\nu)\frac{1}{\varphi_0\beta}\text{ber}'(\varphi_0\beta)}{\text{ber}(\varphi_0\beta) + (1-\nu)\frac{1}{\varphi_0\beta}\text{bei}'(\varphi_0\beta)}\,\text{bei}(\varphi_0\beta)} \\ C_2 &= a\cdot\alpha\cdot T_0 \frac{1}{\text{bei}(\varphi_0\beta) + \dfrac{\text{ber}(\varphi_0\beta) + (1-\nu)\frac{1}{\varphi_0\beta}\text{bei}'(\varphi_0\beta)}{\text{bei}(\varphi_0\beta) - (1-\nu)\frac{1}{\varphi_0\beta}\text{ber}'(\varphi_0\beta)}\,\text{ber}(\varphi_0\beta)} \end{aligned} \tag{2.9}$$

The results are shown in fig. 7.

V. For the case under the action of unsymmetrical difference of temperature and seismic force with simply supported edge

The equation of deflection is as follows, taking the terms which diverge at $\varphi = 0$ in eqs. (1.25).

$$w = \left\{[C_1'\,\text{ber}'\,\xi + C_2'\,\text{bei}\,\xi] + \left[q\,(2+\nu)\frac{a^2}{Et} - \alpha\cdot T_0\cdot a\frac{1}{\varphi_0}\right]\varphi\right\}\cos\theta \tag{2.10}$$

The moments are obtained as below by neglecting the terms involving $\frac{1}{\xi^2}$.

$$\begin{aligned} M_\varphi &= -D\frac{\beta^2}{a}\left\{C_1'\left[\text{bei}'\,\xi - \nu\,\text{ber}'\,\xi - \frac{1-\nu}{\xi}\,\text{bei}\,\xi\right] - \right. \\ &\left. - C_2'\left[\text{ber}'\,\xi + \nu\,\text{bei}'\,\xi - \frac{1-\nu}{\xi}\,\text{ber}\,\xi\right]\right\}\cos\theta \end{aligned} \tag{2.11}$$

$$M_\theta = -D\frac{\beta^2}{a^2}\left\{C_1'\left[-\mathrm{ber}'\xi+\nu\,\mathrm{bei}'\xi+\frac{1-\nu}{\xi}\,\mathrm{bei}\,\xi\right]-\right.$$
$$\left.-C_2'\left[\mathrm{bei}'\xi+\nu\,\mathrm{ber}'\xi+\frac{1-\nu}{\xi}\,\mathrm{ber}\,\xi\right]\right\}\cos\theta \qquad (2.11)$$
$$M_{\theta\varphi} = -D(1-\nu)\frac{\beta^2}{a^2}\left[C_1'\frac{1}{\xi}\,\mathrm{bei}\,\xi - C_2'\frac{1}{\xi}\,\mathrm{ber}\,\xi\right]\sin\theta$$

As the boundary conditions, we have

$$\begin{array}{ll} 1. & w\,|_{\varphi=\varphi_0} = 0 \\ 2. & M_\varphi\,|_{\varphi=\varphi_0} = 0 \end{array} \qquad (2.12)$$

Comparing with the particular solution in eq. (2.10).

$$\alpha\cdot T_0\frac{a}{\varphi_0} = 1.1459\cdot 10^{-2}\ \text{metre}$$
$$q\,(2+\nu)\frac{a^2}{E\,t} = 2.1825\cdot 10^{-3}\ \text{metre}$$

It is seen from the above results that the value due to the thermal change $\pm 10°$ C is much larger than the one due to the seismic force based on seismic coefficient 0.2. Therefore, we neglect the effect of the seismic force, and obtain the constants of integration as follows:

$$C_1' = \alpha\cdot T_0\,a\,\frac{1}{\mathrm{ber}'(\varphi_0\beta)+\dfrac{\mathrm{bei}'(\varphi_0\beta)-\nu\,\mathrm{ber}'(\varphi_0\beta)-\dfrac{1-\nu}{\varphi_0\beta}\,\mathrm{bei}(\varphi_0\beta)}{\mathrm{ber}'(\varphi_0\beta)+\nu\,\mathrm{bei}'(\varphi_0\beta)-\dfrac{1-\nu}{\varphi_0\beta}\,\mathrm{ber}(\varphi_0\beta)}\,\mathrm{bei}'(\varphi_0\beta)}$$
$$C_2' = \alpha\cdot T_0\,a\,\frac{1}{\mathrm{bei}'(\varphi_0\beta)+\dfrac{\mathrm{ber}'(\varphi_0\beta)+\nu\,\mathrm{bei}'(\varphi_0\beta)-\dfrac{1-\nu}{\varphi_0\beta}\,\mathrm{ber}(\varphi_0\beta)}{\mathrm{bei}'(\varphi_0\beta)-\nu\,\mathrm{ber}'(\varphi_0\beta)-\dfrac{1-\nu}{\varphi_0\beta}\,\mathrm{bei}(\varphi_0\beta)}\,\mathrm{ber}'(\varphi_0\beta)} \qquad (2.13)$$

The results are shown in fig. 7.

VI. The effect of an opening

In this shell structure there are 133 circular skylight openings of 60 cm diameter and the sizes and the locations are determined so that the stress disturbance may be ignored.

At the circular edge of the hole, the boundary condition is that the normal stress perpendicular to the edge is zero, and the stress component in the radial direction is $p\,a\,\omega/2$.

The equation of deflection is as follows, taking only the terms which converge at $\varphi=\varphi_0$ in eqs. (1.24)

$$w = C_3 \ker \xi + C_4 \operatorname{kei} \xi + p(2+\nu)\frac{a^2}{Et} \tag{2.14}$$

The moments and the shear force become as follows:

$$\begin{aligned}
M_\varphi &= -D\frac{\beta^2}{a^2}\left\{C_3\left[\operatorname{kei}\xi - (1-\nu)\frac{1}{\xi}\ker'\xi\right] - C_4\left[\ker\xi + (1-\nu)\frac{1}{\xi}\operatorname{kei}'\xi\right]\right\} \\
M_\theta &= -D\frac{\beta^2}{a^2}\left\{C_3\left[(1-\nu)\frac{1}{\xi}\ker'\xi + \nu\operatorname{kei}\xi\right] + C_4\left[(1-\nu)\frac{1}{\xi}\operatorname{kei}'\xi - \nu\ker\xi\right]\right\} \\
Q_\varphi &= -D\frac{\beta^3}{a^3}\left\{C_3\left[\operatorname{kei}'\xi - \frac{1}{\xi^2}\ker'\xi\right] - C_4\left[\ker'\xi - \frac{1}{\xi^2}\operatorname{kei}'\xi\right]\right\}
\end{aligned} \tag{2.15}$$

The boundary conditions are

1. $$M_\varphi\big|_{\varphi=\omega} = 0$$
2. $$Q_\varphi\big|_{\varphi=\omega} = \frac{p\cdot a}{2}\,\omega \tag{2.16}$$

We obtain the constants of integration as follows by neglecting the small terms.

$$C_3 = -\frac{p\,a^2\,\omega\cdot\beta}{2\,Et}\cdot \frac{\ker(\omega\cdot\beta) + (1-\nu)\frac{1}{\omega\cdot\beta}\operatorname{kei}'(\omega\cdot\beta)}{\ker(\omega\cdot\beta)\operatorname{kei}'(\omega\cdot\beta) - \ker'(\omega\cdot\beta)\operatorname{kei}(\omega\cdot\beta) + (1-\nu)\frac{1}{\omega\cdot\beta}\{\ker'^2(\omega\cdot\beta) + \operatorname{kei}'^2(\omega\cdot\beta)\}}$$

$$C_4 = -\frac{p\,a^2\,\omega\cdot\beta}{2\,Et}\cdot \frac{\operatorname{kei}(\omega\cdot\beta) - (1-\nu)\frac{1}{\omega\cdot\beta}\ker(\omega\cdot\beta)}{\ker(\omega\cdot\beta)\operatorname{kei}'(\omega\cdot\beta) - \ker'(\omega\cdot\beta)\operatorname{kei}(\omega\cdot\beta) + (1-\nu)\frac{1}{\omega\cdot\beta}\{\ker'^2(\omega\cdot\beta) + \operatorname{kei}'^2(\omega\cdot\beta)\}} \tag{2.17}$$

We obtain the stress function from eqs. (1.24) and the direct stresses by differentiation as follows:

$$\begin{aligned}
\Phi &= \frac{Et}{a\beta^2}\{C_4\ker(\xi) - C_3\operatorname{kei}(\xi)\} - \frac{3}{4}p\,a\,\varphi^2 + K_1\log\varphi \\
N_\varphi &= \frac{Et}{a}\left[C_4\frac{1}{\xi}\operatorname{kei}'(\xi) + C_3\frac{1}{\xi}\ker'(\xi)\right] - \frac{1}{2}p\,a + \frac{K_1}{\varphi^2} \\
N_\theta &= \frac{Et}{a}\left\{C_4\left[\operatorname{kei}(\xi) - \frac{1}{\xi}\ker'(\xi)\right] - C_3\left[\ker(\xi) + \frac{1}{\xi}\operatorname{kei}'(\xi)\right]\right\} - \frac{1}{2}pa - \frac{K_1}{\varphi^2}
\end{aligned} \tag{2.18}$$

The result shows that the values of bending moments are quite small and at the edge of the hole the compressive direct stress becomes three times that in the case of no hole and this fact coincides with the condition of a rivet hole in the two-dimensional problem.

3. Analysis of Spherical Shell of Non-uniform Thickness

I. Introduction

In the conventional design method of the spherical shell, it is common practice to deal with the circumferential beam as an individual tension ring. In case the thickness of the shell is not sufficient for the bending stresses which are calculated, we have to increase the thickness. We quite often disregard the variation of the thickness of the shell when we compute the internal stresses and especially where the variation of the thickness is very small and the solution is obtained by the traditional method we may neglect any effect due to the variation. But in our shell structure the difference of the thickness is quite large and the effect due to that difference should be considered in the computation of the internal stresses.

Since we have considered the edge part acting as tension ring, we may regard the tensile force in the ring as a statically indeterminate reaction and we treat the whole structure as a statically indeterminate structure. Of course there are many questions in such a concept about the estimation of the deformation of the tensile member of reinforced concrete from the shell proper. But it may be a closer solution than the previously mentioned method if we analyze the stresses taking account of the tension ring which consists of some part of the shell although its boundary is not so distinct.

From the above consideration, the analytical method for a spherical shell of non-uniform thickness will be discussed in the following.

It is, however, inevitable to be limited to the approximate method based on problems symmetrical with respect to the axis of revolution.

II. Differential equation for the bending theory of the spherical shell of non-uniform thickness

The equations of equilibrium and compatibility for the spherical shell of non-uniform thickness based on the symmetrical loading with respect to the axis of revolution are expressed in the following system by taking account of the transverse shear force and the angle of rotation V of a tangent to a meridian.

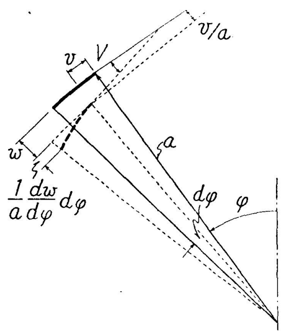


Fig. 8.

$$\begin{aligned}&\frac{d^2 Q_\varphi}{d\varphi^2}+\left\{\cot\varphi-\frac{1}{t}\frac{dt}{d\varphi}\right\}\frac{dQ_\varphi}{d\varphi}-\left\{\cot^2\varphi-\nu\left(1+\cot\varphi\frac{1}{t}\frac{dt}{d\varphi}\right)\right\}Q_\varphi=E\,t\,V\\&\frac{d^2 V}{d\varphi^2}+\left\{\cot\varphi+\frac{3}{t}\frac{dt}{d\varphi}\right\}\frac{dV}{d\varphi}-\left\{\cot^2\varphi+\nu\left(1-\cot\varphi\frac{3}{t}\frac{dt}{d\varphi}\right)\right\}V=-\frac{a^3}{D}Q_\varphi\end{aligned}\tag{3.1}$$

where $$V=\frac{1}{a}\left(v+\frac{dw}{d\varphi}\right),\qquad D=\frac{E\,t^3}{12\,(1-\nu^2)}$$

v: displacement in the direction tangential to the meridian.
w: displacement in the direction normal to the middle surface.

We can get the stresses and displacements in the following expression.

$$\begin{aligned}N_\varphi&=-\cot\varphi\,Q_\varphi\\N_\theta&=-\frac{dQ_\varphi}{d\varphi}\\M_\varphi&=-\frac{D}{a}\left\{\frac{dV}{d\varphi}+\nu\,V\cot\varphi\right\}\\M_\theta&=-\frac{D}{a}\left\{V\cot\varphi+\nu\frac{dV}{d\varphi}\right\}\\v&=\frac{a}{E\,t}(1+\nu)\,Q_\varphi\\w&=\frac{a}{E\,t}\left\{\frac{dQ_\varphi}{d\varphi}+\cot\varphi\,Q_\varphi\right\}\end{aligned}\tag{3.2}$$

Now, we assume that the variation of cross-sectional thickness can be expressed as follows by the exponential function.

$$t=t_0\,e^{2\kappa\varphi}\tag{3.3}$$

where t_0 and κ are the constants which are determined from the initial and final values of the variable thickness.

Substituting eq. (3.3) in eqs. (3.1) and eliminating Q_φ, we have the following:

$$\begin{aligned}&\frac{d^4 V}{d\varphi^4}+\psi_1\kappa\frac{d^3 V}{d\varphi^3}+\psi_2\kappa^2\frac{d^2 V}{d\varphi^2}+\psi_3\kappa^3\frac{dV}{d\varphi}+(\psi_4\kappa^4+\lambda^4 e^{-4\kappa\varphi})\,V=0\\&\psi_1=16+2\cot\varphi/\kappa\\&\psi_2=84+2\,(11+4\nu)\cot\varphi/\kappa-(2+\cot^2\varphi)/\kappa^2\sin^2\varphi\\&\psi_3=144+60\,(1+\nu)\cot\varphi/\kappa+2\,(4+3\nu)\cot^2\varphi/\kappa^2-\\&\qquad-\cot^3\varphi/\kappa^3-\{2\,(6-\kappa+6\nu)-5\cot^2\varphi+12\cot^2\varphi\}/\kappa^3\sin^2\varphi\\&\psi_4=144\,\nu\cot\varphi/\kappa-12\,(2-3\nu-\nu^3)\cot^2\varphi/\kappa^2-\\&\qquad-2\,(3+4\nu)\cot^3\varphi/\kappa^3+\cot^4\varphi/\kappa^4-2\,\{30\,\kappa^2\nu-\\&\qquad-\kappa\,(10+3\nu)\cot\varphi+\cot^2\varphi+1/\sin^2\varphi\}/\kappa^4\sin^2\varphi\end{aligned}\tag{3.4}$$

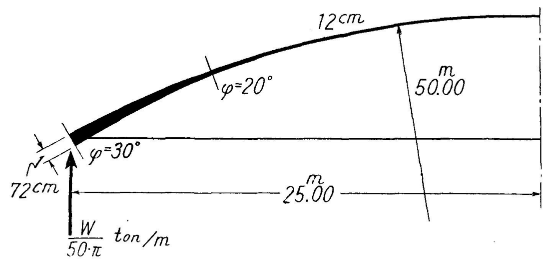


Fig. 9.

Now, we compute the values of ψ_1, ψ_2, ψ_3 and ψ_4 for our shell of which dimensions are shown in fig. 9 and these values are shown in the following table:

Table

	ψ_1	ψ_2	ψ_3	ψ_4	$\lambda^4 e^{-4\kappa\varphi}$	Remark
$\varphi=20°$	17.071	92.046	181.555	9.0465	2,000,000	$\cot\varphi=2.7435$
$\varphi=30°$	16.675	89.956	169.401	6.7646	55,557	$\cot\varphi=1.7321$
Value adopted	17.000	88.375	171.0625	29.00390675	$\lambda^4 e^{-4\kappa\varphi}$	Eq. (3.5)
Remark						$\kappa=5.1330$ $\nu=0.2$

The table shows that the values of ψ_1, ψ_2 and ψ_3 are not sensitive to the difference of $\cot\varphi$ and ψ_4 is small as compared with $\lambda^4 e^{-4\kappa\varphi}$. We can determine the values of constants ψ_1, ψ_2 and ψ_3 and then we select the suitable value of ψ_4 which is in the functional relation of those constants and with their values, the differential equation (3.13) can be solved. In this way, finally we get the following expression:

$$\frac{d^4 V}{d\varphi^4}+17\kappa\frac{d^3 V}{d\varphi^3}+88.375\kappa^2\frac{d^2 V}{d\varphi^2}+171.0625\kappa^3\frac{dV}{d\varphi}+$$
$$+(29.00390675\kappa^4+\lambda^4 e^{-4\kappa\varphi})V=0 \qquad (3.5)$$

III. Membrane stresses of the spherical shell of non-uniform thickness

Since the load term is not introduced in the differential equation of the above bending theory, we consider the load as the particular solution. To combine the effect of the load with the bending stresses, we calculate the membrane stresses due to the load and superpose on the bending stresses.

For the membrane stresses the equations of equilibrium are expressed as follows:

$$\begin{aligned}
&\frac{\partial}{\partial \varphi}(r_0 N_\varphi)+r_1 \frac{\partial N_{\theta\varphi}}{\partial \theta}-r_1 N_\theta \cos\varphi+r_0 r_1 Y = 0\\
&\frac{\partial}{\partial \varphi}(r_0 N_{\theta\varphi})+r_1 \frac{\partial N_\theta}{\partial \theta}+r_1 N_{\theta\varphi} \cos\varphi+r_0 r_1 X = 0\\
&\frac{N_\varphi}{r_1}+\frac{N_\theta}{r_2} = -Z
\end{aligned} \tag{3.6}$$

If we limit our problem to the spherical shell and employ eq. (3.3) for the variation of the thickness, we have the following relations:

$$\begin{aligned}
&r_1 = r_2 = a \qquad\qquad r_0 = a\sin\varphi\\
&p = \gamma t = \gamma t_0 e^{2\kappa\varphi}
\end{aligned} \tag{3.7}$$

where γ is specific gravity.

We consider only the vertical load i.e. the own weight. Then eqs. (3.6) become

$$\begin{aligned}
&\frac{dN_\varphi}{d\varphi}+(N_\varphi-N_\theta)\cot\varphi+\gamma t_0 a e^{2\kappa\varphi}\sin\varphi = 0\\
&N_\varphi+N_\theta = -\gamma t_0 a e^{2\kappa\varphi}\cos\varphi
\end{aligned} \tag{3.8}$$

By eliminating N_θ,

$$\frac{dN_\varphi}{d\varphi}+2N_\varphi\cot\varphi+\gamma t_0 a \frac{e^{2\kappa\varphi}}{\sin\varphi} = 0 \tag{3.9}$$

Hence, we get

$$N_\varphi = -\frac{\gamma t_0 a}{\sin^2\varphi}\{\int e^{2\kappa\varphi}\sin\varphi\, d\varphi + C\} \tag{3.10}$$

Evaluating the integral and using eqs. (3.7), we find

$$\begin{aligned}
&N_\varphi = -\frac{\gamma t_0 a}{(4\kappa^2+1)\sin^2\varphi}\{e^{2\kappa\varphi}(2\kappa\sin\varphi-\cos\varphi)+C'\}\\
&N_\theta = -(N_\varphi+\gamma t_0 a e^{2\kappa\varphi}\cos\varphi)
\end{aligned} \tag{3.11}$$

By substituting the specific gravity including the final weight in eqs. (3.11), we can have the final values of the membrane stresses.

IV. Solution of the differential equation for the bending theory

Eqs. (3.5) may be expressed in the following form:

$$\frac{d^4V}{dx^4}-\frac{11}{x}\frac{d^3V}{dx^3}+\frac{44.375}{x^2}\frac{d^2V}{dx^2}-\frac{64.6875}{x^3}\frac{dV}{dx}+\left(\frac{29.0039675}{x^4}+\rho^4\right)V = 0 \tag{3.12}$$

where $\qquad x = e^{-\kappa\varphi}, \quad \rho = \frac{\lambda}{\kappa}$

Now, we will use the following notation,

$$L(---) = \left\{\frac{d^2}{dy^2} - \frac{5.5}{y}\frac{d}{dy} + \frac{1.5625}{y^2}\right\}(---)$$

where $y = \rho x$, then eqs. (3.12) becomes

$$LL(V) + V = 0$$

or

$$L(V) \pm i V = 0$$

Finally, eqs. (3.12) are represented as follows:

$$\frac{d^2 V}{dy^2} - \frac{5.5}{y}\frac{dV}{dy} + \left\{\pm i - \frac{1.5625}{y^2}\right\} V = 0 \tag{3.13}$$

Now, we introduce the following Bessel's differential equation and the solution:

$$\frac{d^2 y}{dx^2} - \frac{(1-2\alpha)}{x}\frac{dy}{dx} + \left(\lambda^2 \gamma^2 x^{2\gamma-2} + \frac{\alpha^2 - n^2\gamma^2}{x^2}\right) y = 0$$
$$y = x^{\alpha} Z_n(\lambda x^{\gamma}) \tag{3.14}$$

Referring to the above formula, we get the following solution of eqs. (3.13)

$$V = y^{3.25}\{J_3(i^{\pm 3/2} y) + K_3(i^{\pm 1/2} y)\} \tag{3.15}$$

Substituting V in the second equation of eqs. (3.1) for solving Q_φ, we can determine the stresses and displacements from eqs. (3.2). But $J_3(i^{\pm 3/2} y)$ is not the comformable term for the bending stresses in the circumferential part of the spherical shell, and therefore we adopt the term $K_3(i^{\pm 1/2} y)$ only.

We use the following notations which are derived by differentiating $K_3(i^{\pm 1/2} y)$ with respect to y and involve the imaginary number i in the unknown constant A.

$$K^{(0)} = K_3(i^{\pm 1/2} y) = A\,\mathrm{ker}_3\, y + B\,\mathrm{kei}_3\, y$$

$$K^{(1)} = \frac{dK^{(0)}}{dy} = -A\left\{\frac{3}{y}\mathrm{ker}_3\, y + \frac{1}{\sqrt{2}}(\mathrm{ker}_2\, y + \mathrm{kei}_2\, y)\right\} - $$
$$-B\left\{\frac{3}{y}\mathrm{kei}_3\, y + \frac{1}{\sqrt{2}}(\mathrm{kei}_2\, y - \mathrm{ker}_2\, y)\right\}$$

$$K^{(2)} = \frac{d^2 K^{(0)}}{dy^2} = A\left\{\frac{12}{y^2}\mathrm{ker}_3\, y - \mathrm{kei}_3\, y + \frac{1}{\sqrt{2}}\frac{1}{y}(\mathrm{ker}_2\, y + \mathrm{kei}_2\, y)\right\} +$$
$$+B\left\{\frac{12}{y^2}\mathrm{kei}_3\, y + \mathrm{ker}_3\, y - \frac{1}{\sqrt{2}}\frac{1}{y}(\mathrm{ker}_2\, y - \mathrm{kei}_2\, y)\right\} \tag{3.16}$$

$$K^{(3)} = \frac{d^3 K^{(0)}}{dy^3} = -A\left\{\frac{60}{y^3}\mathrm{ker}_3\, y - \frac{4}{y}\mathrm{kei}_3\, y + \frac{11}{\sqrt{2}}\frac{1}{y^2}(\mathrm{ker}_2\, y + \mathrm{kei}_2\, y) +\right.$$
$$\left. + \frac{1}{\sqrt{2}}(\mathrm{ker}_2\, y - \mathrm{kei}_2\, y)\right\} - B\left\{\frac{60}{y^3}\mathrm{kei}_3\, y + \frac{4}{y}\mathrm{ker}_3\, y - \right.$$
$$\left. - \frac{11}{\sqrt{2}}\frac{1}{y^2}\mathrm{ker}_2\, y - \mathrm{kei}_2\, y) + \frac{1}{\sqrt{2}}(\mathrm{ker}_2\, y + \mathrm{kei}_2\, y)\right\}$$

where, $\ker_3 y$, $\mathrm{kei}_3 y$, $\ker_2 y$ and $\mathrm{kei}_2 y$ indicate 3rd and 2nd order of the modified Bessel function. And the factor $E t_0 \lambda^2/\kappa^3$ is involved in the unknown constant A and B. Using eqs. (3.2), the stresses appear as follows:

$$\begin{aligned}
Q_\varphi &= \frac{1}{\kappa}\left\{8.9375\, y^{-2.75} K^{(0)} - 1.5\, y^{-1.75} K^{(1)} - y^{-0.75} K^{(2)}\right\} \\
M_\varphi &= \frac{a}{\kappa^2}\left\{(3.25\, y^{-2.75} K^{(0)} + y^{-1.75} K^{(1)}) - \frac{\nu}{\kappa}\, \kappa^{-2.75} K^{(0)} \cot\varphi\right\} \\
M_\theta &= \frac{a}{\kappa^2}\left\{\nu\,(3.25\, y^{-2.75} K^{(0)} + y^{-1.75} K^{(1)}) - \frac{1}{\kappa}\, y^{-2.75} K^{(0)} \cot\varphi\right\} \qquad (3.17) \\
N_\varphi &= -\frac{1}{K}\left\{8.9375\, y^{-2.75} K^{(0)} - 1.5\, y^{-1.75} K^{(1)} - y^{-0.75} K^{(2)}\right\} \cot\varphi \\
N_\theta &= -\left\{24.5781\, y^{-2.75} K^{(0)} - 11.5625\, y^{-1.75} K^{(1)} + 0.75\, y^{-0.75} K^{(2)} + y^{0.25} K^{(3)}\right\}
\end{aligned}$$

While the displacements are similarly given as follows:

$$\begin{aligned}
v &= \frac{a\,(1+\nu)}{\kappa E t_0 \rho^2}\left\{8.9375\, y^{-0.75} K^{(0)} - 1.5\, y^{0.25} K^{(1)} - y^{1.25} K^{(2)}\right\} \\
w &= \frac{a}{\kappa E t_0 \rho^2}\left\{\kappa\,[24.5781\, y^{-0.75} K^{(0)} - 11.5625\, y^{0.25} K^{(1)} + 0.75\, y^{1.25} K^{(2)} + \right. \qquad (3.18) \\
&\quad \left. + y^{2.25} K^{(3)}] + \cot\varphi\,[8.9375\, y^{-0.75} K^{(0)} - 1.5\, y^{0.25} K^{(1)} - y^{1.25} K^{(2)}]\right\}
\end{aligned}$$

V. Results calculated and check

So far, the author has shown estimates of the bending stresses of his shell under the action of own weight by various methods and with various assumptions in order to compare the results with each other. The boundary conditions, however, in all calculations are identical, in that the shell is supported by the roller system all around the periphery and they may be expressed by the following:

$$\begin{aligned}
M_\varphi/_{\varphi=\varphi_0} &= 0 \\
Q_\varphi/_{\varphi=\varphi_0} &= -\frac{w}{\pi R}\cos\varphi_0
\end{aligned} \qquad (3.19)$$

The calculated results are shown in fig. 10 in which the full lines (A) indicate the effects that are analyzed by the method based on the non-uniform thickness. The resulting bending moments M_φ and M_θ are shown.

The chain lines (B) represent the stresses which are calculated by the general approximate method mentioned in the previous section based on a uniform thickness of 12 cm and the dotted lines (C) indicate the stresses by the same method based on a uniform thickness of 42 cm which is the mean value of the 12 cm and the 72 cm maximum. The broken lines (D) indicate

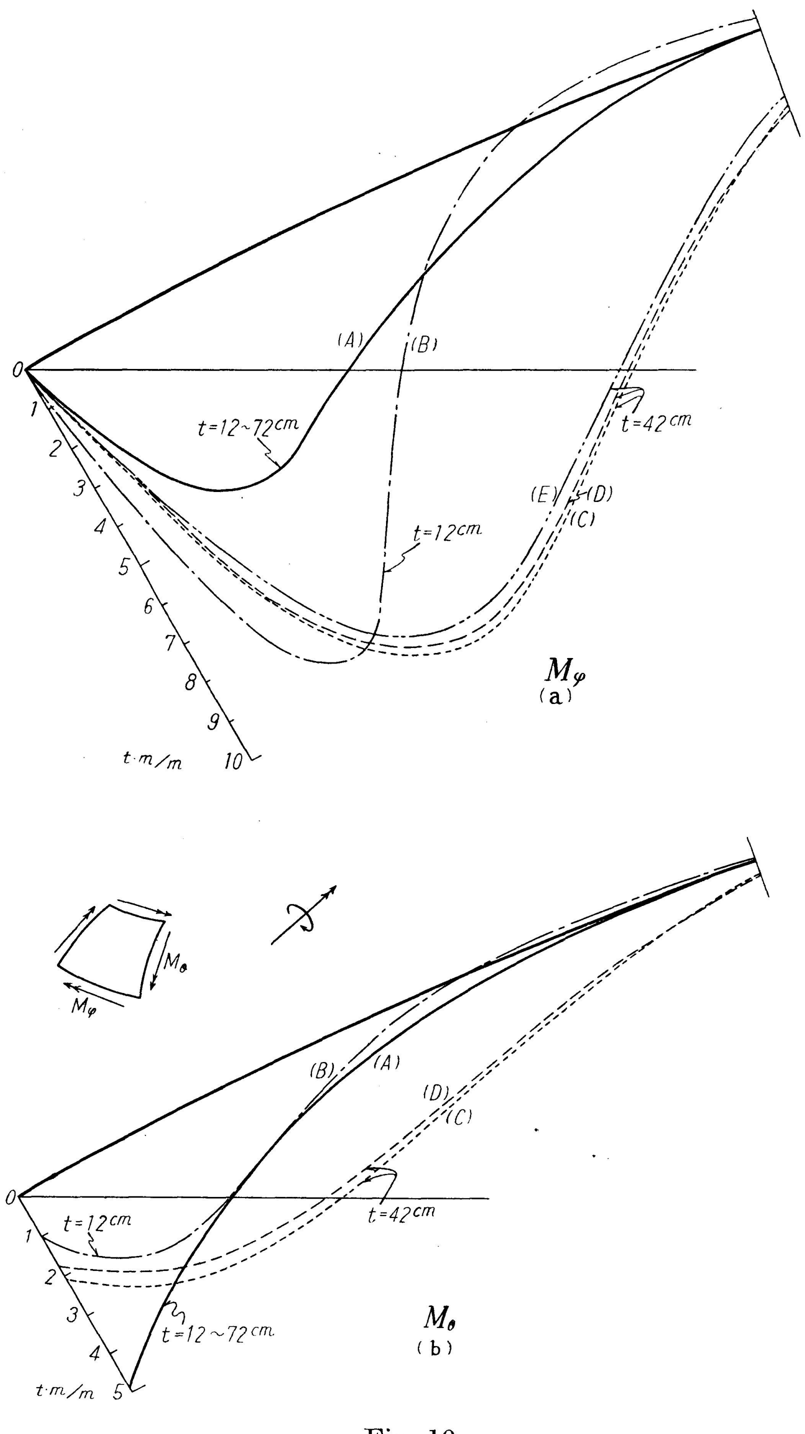


Fig. 10.

the solution of the second approximation method by M. HETENYI[1]) depending on the average thickness 42 cm. It is seen that this method is sufficiently accurate compared with the exact solution.

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[1]) M. HETENYI: Spherical Shells subjected to axial symmetrical Bending. Abhandlungen. 1937/38.

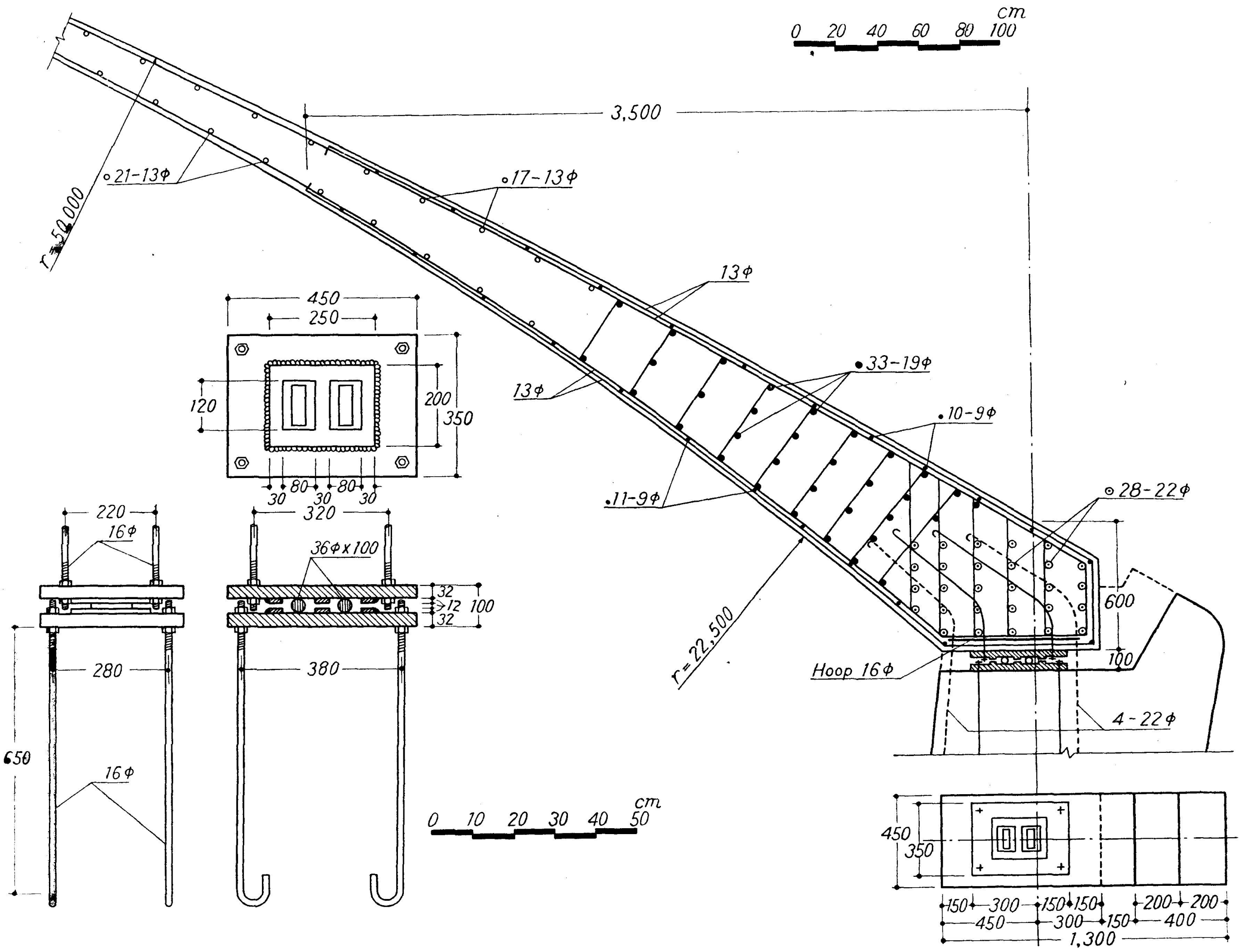

Fig. 11. Detail of the circumferential part and the roller system.

Moreover, the other double dot-dash lines (E) show the results by Y. YOKOWO and O. MATSUOKA's[2]) method.

From these results shown in fig. 10, we see that the values of the bending stresses obtained by these approximate methods are quite close to each other for a uniform thickness of 42 cm, although each method has its own certain character. For practical application it can be concluded that whichever of those methods we use, we can expect satisfactory values of the stresses in the case of a spherical shell of uniform thickness.

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[2]) Y. YOKOWO and O. MATSUOKA: Proceeding of the 4th J.N.C. for Appl. Mech. 1954.

The author also invites readers to refer to the following paper in which the same subject is treated. P. M. NAGHDI and C. NEVIN DE SILVA: On the Deformation of Elastic Shells of Revolution. Quarterly of Appl. Math. Vol. XII, January 1955.

Finally, considering the effect of the variable thickness it can be concluded that if we take the thickness uniform in the preliminary design and increase the thickness towards the circumference in practice, as regards M_φ the consequent error is on the safe side but as regards M_θ it is critical in the part close to the edge.

It should be noted that in estimating M_θ we cannot use the simple expression $M_\theta = \nu M_\varphi$ at the edge zone, as this depends on the assumption of uniform thickness.

PART II. CONSTRUCTION

1. Form-Work

Owing to the necessity of making full-size details to achieve the accurate dimensions of the forms, 400 tsubo (1,320 m) of plank was boarded on the framing of struts and short piles every 3 shaku (1 m). Each plank was jointed together with grooves. During the construction of full size details over a fairly long time, neither warping nos distortion appeared in spite of working in the open-air, so the form work was satisfactorily executed.

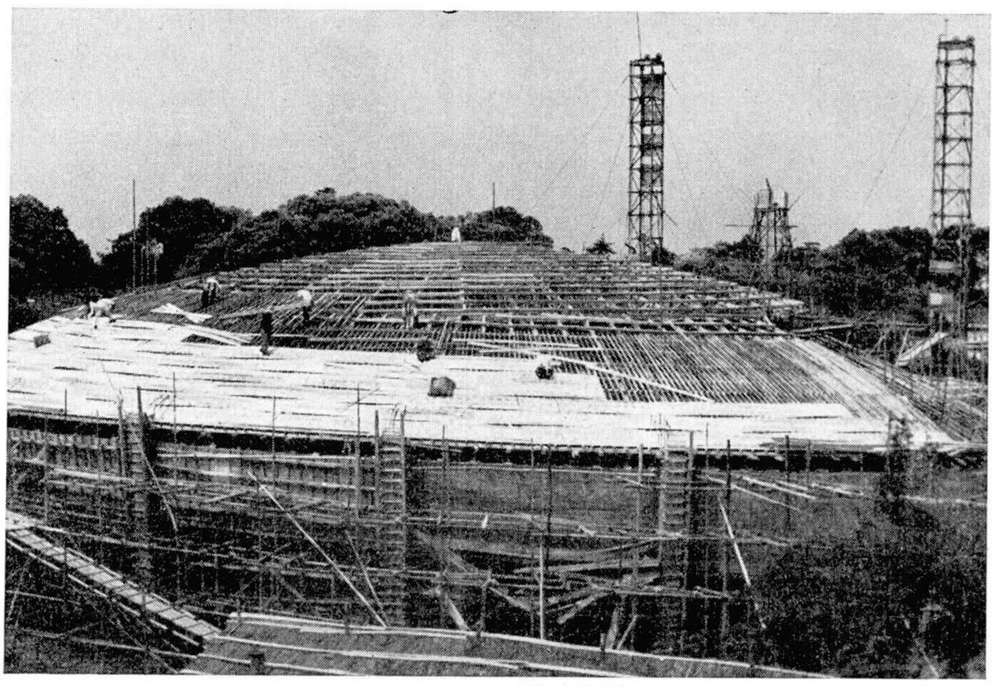


Fig. 12. Form work.

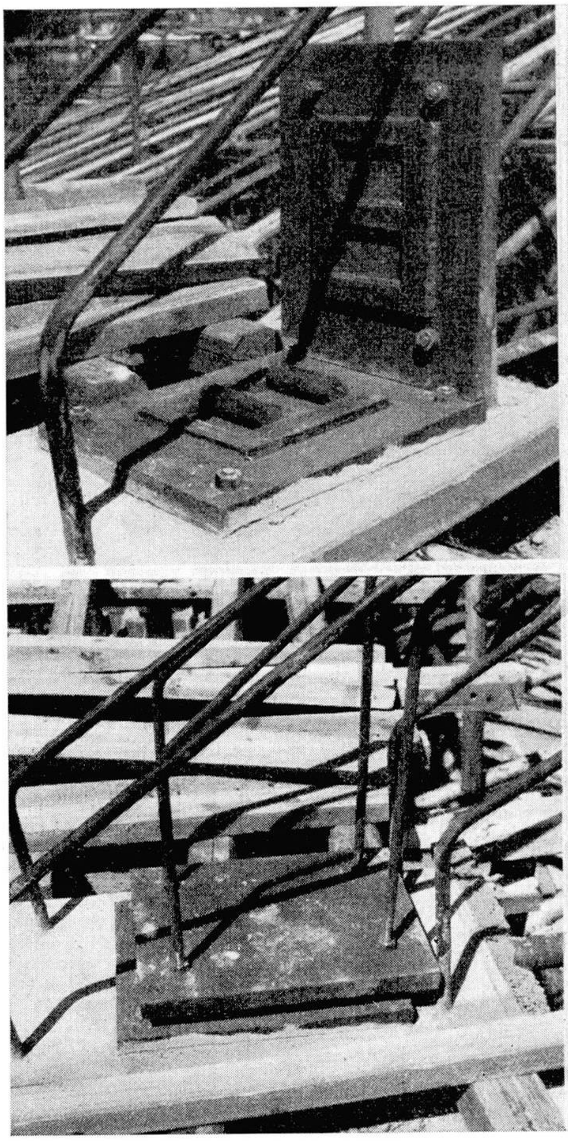


Fig. 13. Detail of the roller on a piller.

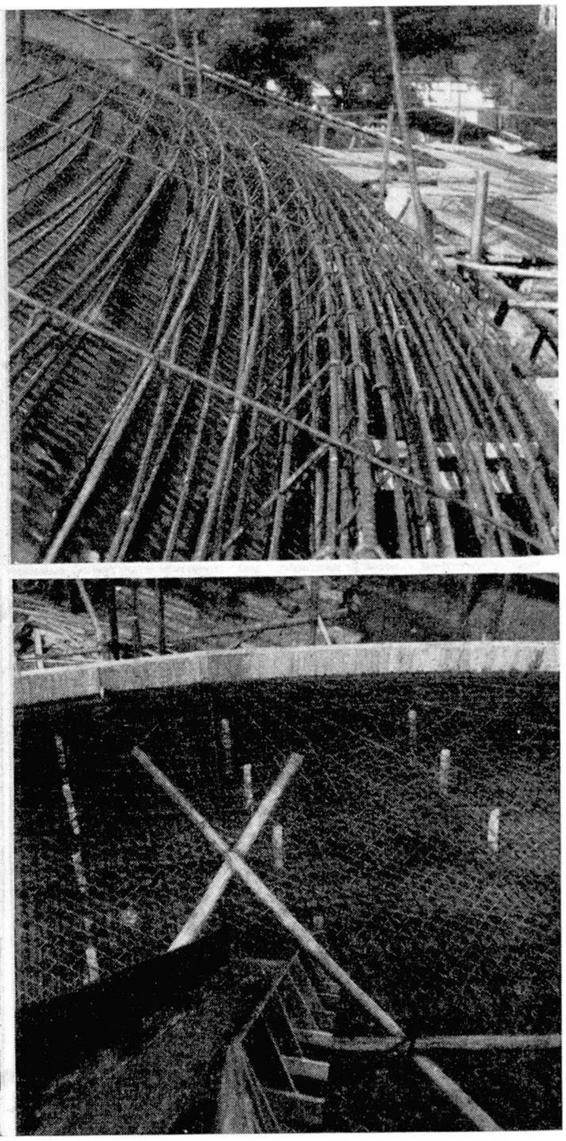


Fig. 14. Detail of the placed bars.

To prevent the supports from sagging so as to keep the shape of the shell accurately, the floor slab of the main hall was poured after the foundation was completed. Then, on this stable floor the centers of the above structures were marked and the supports of the forms were installed exactly and so the sag of the form-work could be kept as small as possible.

In the central part the supports were located every 1.8 m radially from the apex (different from the center of the plan) at which 2 pieces of scaffold batten were jointed. In the circumferential part along the edge the spacing of the supports was closed up as the loading increased.

As horizontal bracings, scaffold battens were crossed in two directions every 5 or 6 shaku (1.5 or 2 m) and many other battens were used as diagonal bracings to strengthen the supporting system. Landings were installed at heights of 6.1 m and 9.7 m according to the ceiling height for finishing. Above the supporting systems shorter posts were set up and many struts and bracings were used for strengthening. The beams which support the forms were installed on this system using shorter diagonal bracings. The struts perpendicular to the above beams were located every 3 shaku (1 m) and the joists were laid on every 1 or 1.5 shaku (30 or 45 cm) radially from the apex like the frame of an umbrella. And then, 6 bu (2 cm) thick plank was nailed and bent according to the curvature of the shell and jointed together with grooves. Thus the entire form work was completed. Moreover, from the consideration of prevention of warping due to eccentric loading in pouring concrete, the hooked bars were anchored at the necessary places in the ground and the wire was tightened up with turnbuckles to prevent the forms from distortion. This idea was very good for keeping the form work stable and strong.

2. Reinforcment

Along the circumferential edge the reinforcment was very close with large main bars and at the apex the thickness is only 12 cm and therefore pouring concrete had to be executed very carefully. To save the splice length of bars all joints were gas-welded. For 22 mm bars 465 places were welded. Before actual placing, the welded joint in the bar was tested in tension, the strength was checked and the result was very good. Only the intersections were tied with wire and the shifting of the reinforcment could be kept as small as possible in pouring. According to the thickness of concrete special spacer blocks were made for fixing the position of both top and bottom bars at same time and so the required clearances and thickness were maintained.

3. Concrete Work

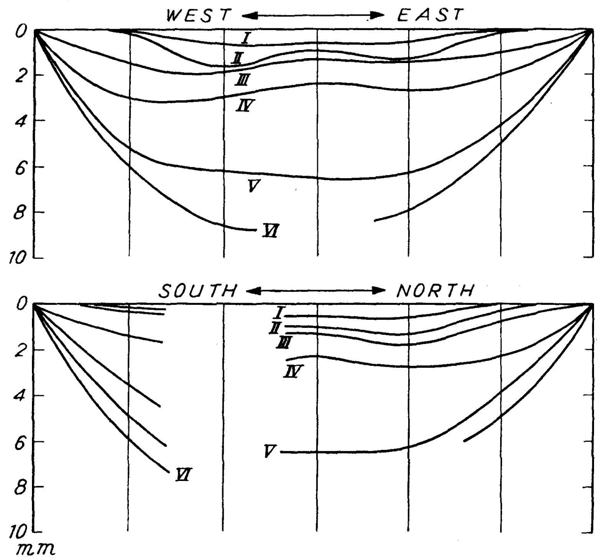
First of all, we ordered the making of special coarse cement which had still the required strength for this shell construction to eliminate as far as possible the cracks due to shrinkage. Actually since the last war Japanese cement is very fine to obtain higher strength and so we feared that shrinkage and cracking might occur if such market cement were used. Also, we selected the aggregate carefully and decided the mix-ratio and to obtain uniform concrete we had 2 minutes of mixing after putting the materials into the mixer, using a sand glass. Moreover, above the top bars of the shell part we put $\#8 \times \#8 / 3.5$ inch $\times$ 3.5 inch wire mesh over all surface and then poured concrete. Around the circumferential edge of the shell the slope was fairly steep but the wire mesh prevented the concrete from sliding in pouring and we completed the concrete work without any top forms. As the result of the above procedures, during 2 months before water proofing we did not find any noticeable cracks with the naked eye.

The concrete volume of the shell part of the main hall is 556 m^3 and even if we exclude the eaves part of the circumferential edge which was to be poured afterwards, we have 82.6 cubic tsubo (486 m^3). We had to pour continually without any construction joint to the line along which the thickness was changed to constant. The circumferential part has 63.7 cubic tsubo (372.8 m^3) and the central part of 12 cm th. has 18.9 cubic tsubo (113.6 m^3). As mentioned above, we had 2 minutes of mixing so we had enough time to go on pouring. We provided 2 mixers of 21 cubic shaku (feet) capacity and we could operate one mixer even at lunch time and poured concrete ceaselessly.

The execution time just fell in the rainy season and we worried about the spell of rain at the time of estimating. Actually we had much rain in that year and it was difficult to get 2 continuous fine days, but we could not wait for the fine weather to keep the completion date. We started to pour at 6:30 a.m. on July 19th with the postponement of only one day. We continued to pour about 23 hrs. to get the proposed construction joint along the line of inflexion until we ceased the pouring because of the rainfall. The construction joint was scarfed and cured with mat cover about 26 hrs. till it stopped raining and then the surface of the joint was roughened and we poured the central part of 18.9 cubic tsubo (113.6 m^3). Concrete was put in compactly by vibrator for the edge beam and by hand for the other places. Finally the shell was covered and cured with wet mat by sprinkling of water every day for 4 weeks until the forms were removed. Incidently we had much rain, so it suited our requirements.

4. Removal of Forms

The removal of forms was started from the horizontal and diagonal bracings. Next the shell area was divided by 4 concentric circles radially from the apex and we loosened the wedges at the bottom of the supports at every concentric portion from the center, measuring the sags and then the supports were moved slowly from the central portion. After the beams were removed the joists and the form boards were still self supported under the shell concrete like a membrane, so actually it happened that the central part of about 15 meter diameter fell down as a whole losing such membrane action when we were just pulling one portion of the circumferential part as it was coming detached. The results of measuring the strain of the shell concrete were quite satisfactory, and we could not find any cracks in the circumferential part.



I At the removal of the first concentric circle part.
II At the removal of the second concentric circle part.
III At the removal of the third concentric circle part.
IV At the complete removal of the forms.
V After 7 hours since IV.
VI After 16 hours since IV.

Fig. 15. Deflections at the removal of the forms.

Table 1. Quantities of materials of form work

description	quantity	remark
plywood	70 sheets	for scale
scaffold board	400 tsubo	(1 tsubo = 3.3 m^2)
scaffold batten	1,300 pieces	
camben	4,600 pieces	
post batten	4,000 pieces	3.5—4.0 ken (1 ken = 6 feet)
batter	250 koku	(1 koku = 1 ft. × 1 ft. × 10 ft.)
strut	160 koku	
joist	170 koku	
form board	130 koku	
wire	680 kg	
rail	1,500 kg	
clamp	10,000 pieces	
total	1,445 koku (lumber only)	2,493 koku per unit tsubo area of bldg.

Table 2. Quantities of steel reinforcement

diameter	deformed	plain	total
22 ∅	13.100		13.100
19 ∅	10.960		10.960
16 ∅		0.070	0.070
13 ∅	4.746	41.700	45.916
9 ∅		1.010	1.010
6 ∅		1.560	1.560
total	28.806	44.280	73.086
per 1 cubic tsubo of concrete	0.345	0.536	0.881
per 1 tsubo area of bldg.	0.050	0.076	0.126

1 cubic tsubo = 6.01 m^3

Table 3. Labour hrs. of concreting for shell

	regular daytime	overtime	over night	total
concrete man	1,124 hr.	909.5 hr.	648 hr.	2,681.5 hr.

PART III. CONCLUSIONS

This Matsuyama Convention Hall which has such a large span, that has never been built before in Japan, has been completed with the expense of Yen 60,000,000 ($ 140,000) and 9 months time. In the designing stage, the designers' intention was to investigate how to establish such a large multipurpose space with 1,400 seats within the limited budget.

From the structural view-point, considerable efforts were made to analyse the stresses under the conditions in which the membrane theory cannot be applied. In the case in which the internal forces virtually consist of the extensional forces in the plane only, like the case of the auditorium of M.I.T.[2]) (it can be considered as a complete membrane condition if the shell is supported on more than 3 points rigidly), the structural design is all based on the determination of the rise and thickness and is completed with the design of the support system. In our case, however, where the thrust of the shell cannot be transmitted to the supporting columns, i. e. the shell is completely separated from the substructure, we have to dispose its own horizontal reactive forces in the shell super-structure itself and accordingly we have to consider the bending stresses in addition to the direct stresses. Consequently, we had the details as shown in fig. 11 and 14, and we had to place 80% of the total amount of steel which was used in this structure.

After the project plan was established, we made a strength test on the $^1/_{20}$ scale model in our laboratory to confirm the theoretical analysis and prove the safety of the design. The model was made of mortar and wire. We put the load on this model using sand up to 4 tons which is 1.4 times the design load and the maximum capacity of the loading apparatus. But we could not obtain the expected deflection nor any cracks. We could, however, prove sufficient safety for the construction based on this design.

In the field at first we measured the deflection of the forms due to pouring concrete and we got 1 cm uniform sag of the cast portion but we could not observe any partial sag or uplift. This shows that the form work had no defects. The above uniform sag came from the inevitable small gaps between the timbers. Next, we measured the sag at each removal time of the forms, but this measurment was made during construction, so we could not expect satisfactory results. In the following we show the results. At the apex of the shell we had about 3 mm sag immediately after the removal of the supports, and more than 8 mm later 15 hrs. and still increasing but we could not continue the measurement and we might have had more than 1 cm when the deflection

[2]) Engineering News Record. May 27, 1954.

set. The computed value based on 2.1×10^5 kg/cm^2 for Young's modulus of concrete is 1.13 cm. It should be noted that the deflection is settling with time after removal of the supports and this is a problem of reinforced concrete structure which remains to be clarified.

Summary

The paper relates to the structural design and construction of a reinforced concrete thin spherical shell which has 50 m radius and covers a circular plan of 50 m diameter. It is a pure spherical shell and the large thrust from the shell is not resisted by the supporting sub-structure. Such a structural system has actually been built in Japan.

The approximate analytical methods for a flat dome and for the case of non-uniform thickness are discussed, the solutions being based on Bessel functions for both cases. These solutions can be applied to practical design. With this design method the construction work which involves many technical problems can be completed.

Résumé

L'auteur traite de l'étude et de la construction d'une voûte sphérique mince en béton armé dont le rayon est de 50 m et qui couvre au sol un cercle de 50 m de diamètre. Il s'agit d'une voûte purement sphérique, dont l'infrastructure n'est pas prévue pour faire face à une forte poussée mise en jeu par la voûte elle-même. Il a effectivement réalisé un ouvrage de ce genre au Japon.

L'auteur discute les méthodes analytiques permettant le calcul approché de cette voûte mince, tant dans le cas de l'épaisseur uniforme que de l'épaisseur non uniforme. Dans les deux cas, il introduit des solutions basées sur les fonctions de Bessel. Le résultat ainsi obtenu peut être appliqué au calcul pratique. Grâce à ces méthodes, il a été possible de mener a bien la construction de cette voûte, bien qu'elle implique de nombreux problèmes techniques.

Zusammenfassung

Der Aufsatz behandelt Berechnung und konstruktive Ausführung einer dünnwandigen, flachen Kugelschale. Diese weist einen Krümmungsradius von 50 m auf und überspannt eine Kreisfläche von 50 m Durchmesser. Die Tragkonstruktion der Schale ist nicht so ausgebildet, daß sie den Horizontalschub aufnehmen kann. Ein solches Bauwerk wurde kürzlich in Japan errichtet.

Die Näherungsmethode der Berechnung für flache Gewölbe und auch für den Fall veränderlicher Schalendicke beruht auf der Einführung einer Lösung mit Besselschen Funktionen. Diese Lösungen können für die praktische Berechnung verwendet werden. Die Methode wurde beim erwähnten Bauwerk angewendet, das verschiedene technische Probleme stellte.